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# Mathematicians and experienced teachers: crossing the boundary

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*Academic mathematics (AM) has a central role in the preparation and professional development of secondary mathematics (SM) teachers, yet in recent years there is growing evidence that realizing the affordances of AM for SM teaching is not straightforward. This study is part of a long-term research program named M-Cubed, that seeks to investigate the processes involved in utilizing AM for informing SM teaching. In M-Cubed, mathematicians and experienced SM teachers watch videotaped SM lessons and discuss teaching alternatives. This study explores how, in this setting, mathematicians and teachers make sense of authentic teaching moves and decisions, how they invite each other to adopt new perspectives, and how these invitations are met. Findings provide insight into the social boundary between mathematicians and teachers, and into how opportunities for learning through crossing this boundary may be realized. We conclude by discussing possible implications for research and practice.*

*Keywords: knowledge for teaching, advanced mathematics, teacher education, cross-community encounters, boundary-crossing.*

## INTRODUCTION

Teachers' engagement with academic mathematics (AM) and interaction with mathematicians are key components in the secondary mathematics (SM) teacher preparation programs of mathematics teachers in many countries (Tatto et al., 2010). The literature suggests various potential benefits of teacher-mathematician encounters for SM teachers. For example, it has been suggested that such encounters may support the development of teachers' Horizon Content Knowledge (Ball & Bass, 2009; Wasserman, 2018); develop teachers' understandings *of* and *about* the discipline of mathematics (CBMS, 2012; Even, 2011); and inform instructional decision-making (Cooper & Pinto, 2017; Wasserman & McGuffey, 2021). Nevertheless, empirical studies have indicated that in practice, AM studies may only have a limited impact on SM teaching, thereby suggesting that it is far from trivial to translate knowledge of AM into knowledge for teaching SM (Biza et al., 2022; Zazkis & Leikin, 2010).

Relatively little is known about the processes involved in translating AM knowledge into knowledge for teaching SM, in part because such processes are often tacit and highly personal, in the sense that they are inspired by experiences of individual teachers in their particular AM studies (Zazkis, 2020). Research suggests that these translation processes build not only on deep understanding of AM but also on knowledge for teaching SM, as well as on knowledge about connecting AM and SM teaching (Dreher et al., 2018; Pinto & Cooper, 2022; Wasserman et al., 2018; Wasserman & McGuffey, 2021). In addition, Cooper and Pinto (2017) suggest that such a translation may manifest in back-and-forth moves between different perspectives on mathematics, its

teaching and its learning, mutually enriching each other. All this suggests that realizing the potential affordances of AM for SM teaching is far from trivial, and that teacher education should look beyond development of teachers' AM knowledge in order to support and encourage teachers' in utilizing AM in practice, specifically in the context of teacher-mathematician interaction (Biza et al., 2022; Wasserman et al., 2018).

This study is part of a long-term research program, named M-Cubed, that investigates processes involved in utilizing AM for SM teaching (Pinto & Cooper, 2022). In M-Cubed (*Mathematicians, Mathematics teachers, Mathematics teaching*), small groups of mathematicians and experienced SM teachers view videotaped SM lessons and jointly inquire into mathematical issues and pedagogical dilemmas that they recognize therein. This research setting can be seen as a laboratory for generating and studying implications of ideas and perspectives from AM, for the consideration of authentic instructional situations. Within M-Cubed, the present case study investigates processes underlying realized and unrealized opportunities for utilization of the mathematicians' perspectives and knowledge for informing SM teaching. We conceptualize and study these processes in terms of *boundary crossing*, as we discuss in the next section.

## **THEORETICAL BACKGROUND**

The design and conduct of M-Cubed are informed by the literature on boundary-crossing, which characterizes dialogical learning processes in cross-community interactions (Akkerman & Bakker, 2011). They define *boundaries* as a social and cultural discontinuity in action or in communication. Researchers have suggested that teacher-mathematician interactions may be framed as boundary encounters (Goos & Bannison, 2018; Pinto & Cooper, 2018, 2022). Pinto and Cooper (2018, 2022) have stressed that discontinuity in these interactions is manifested not merely in disagreements between the two parties but rather in tacit, incommensurable perspectives, which are difficult to recognize and make explicit, and that such discontinuity can hinder communication and collaboration between mathematician and teachers. Specifically, in terms of the *practical rationality* of teaching (Herbst & Chazan, 2020), mathematicians tend to have a strong obligation to the discipline, i.e., to mathematical precision and rigor, even when faced with pedagogical dilemmas, while teachers tend to have a strong obligation to students and their wellbeing and to the functioning of a class as a social entity. These different obligations can lead teachers and mathematicians to endorse conflicting courses of action. Pinto and Cooper (2018, 2022) have shown that such tensions can challenge both parties to engage in boundary crossing (Akkerman & Bakker, 2011), namely, to make explicit and possibly reconsider their positions, thus making public and visible their processes of exploring, elaborating and refining potential utilizations of AM for informing SM teaching.

The literature indicates various factors that may encourage and support boundary-crossing (Akkerman & Bakker, 2011). For example, productive communication across discursive boundaries can rely on *boundary objects* (Star, 2010), that is, objects that are, on the one hand, robust enough to maintain a common identity across communities working with them, yet, on the other hand, flexible enough for the parties to interact

with them differently. In M-Cubed, the videotaped SM lesson episodes function as boundary objects. The episodes are selected by the researchers from the archive of the VIDEO-LM Project (Karsenty & Arcavi, 2017). Selection is based on various criteria that help in identifying potential opportunities for boundary crossing. The research question that guided this case study was: What are the opportunities for boundary crossing in teacher-mathematician interactions, and how may they be realized?

## **METHODOLOGY**

Data for this study consist of 7.5 hours of videotaped discussions in three M-Cubed sessions (2.5 hours each) held in 2020, with the participation of five mathematicians (herein marked as M1-M5), five SM teachers (marked T1-T5), and two mathematics educators, the first and second authors of this paper. All of the participants expressed interest in, and openness to, the project's premise of learning from and with one another about the affordances of AM for teaching SM. All the mathematicians and teachers have more than a decade of teaching experience. Three of the five mathematicians have taught mathematics courses for teachers. All five teachers have experience as students in AM courses. The first session of the three documented for this case study was held in-person, and the other two were conducted remotely due to the COVID-19 pandemic. Within data reduction, we analyzed plenary discussions that took place before and after participants watched the videotaped SM lessons, as these discussions were particularly rich in cross-interactions between teachers and mathematicians. These discussions were fully transcribed and selectively translated to English by the authors.

The data analysis had three focal points: the emergence of a boundary in the M-Cubed discussions; opportunities for boundary crossing; and the realization of these opportunities. The first stage of analysis focused on identifying potential boundaries. We examined how different participants interpreted the goals of the joint inquires and how they approached them: the dilemmas they addressed, claims and observations they made, and justifications they provided. We then considered how different speaking turns relate to one another, not only in the sense of whether the participants agree or not, but also in the sense of being comparable, in terms of the questions that are being explored, and the implied grounds for endorsing or rejecting answers. The second stage of analysis focused on identifying opportunities for boundary-crossing. Here we looked for explicit or implied invitations from a participant to participants from the other community to make explicit and explain tacit aspects of their perspectives, or to adopt a new perspective. Finally, in the third stage of analysis, we explored whether and how invitations for boundary crossing were accepted, guided by different processes of dialogical learning identified in the literature, such as reflection and hybridization (Akkerman & Bakker, 2011; Pinto & Cooper, 2018, 2022).

## **FINDINGS**

Our analysis highlighted various manifestations of boundary and boundary-crossing. In this paper we focus on one particular instance from the M-Cubed sessions that illustrates both realized and unrealized opportunities for boundary crossing. We first

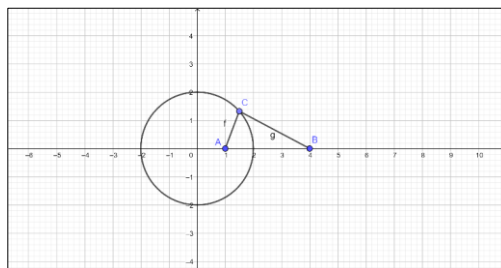
provide an overview of the videotaped lesson episode that the participants discussed, and of mathematical ideas that were explored or implied in this discussion. Then, we discuss the boundary that emerged between the teachers and the mathematicians, three opportunities for boundary crossing that were unrealized, and one opportunity that was.

### A student's unexpected question on Apollonian circles

The M-Cubed session discussed herein revolved around a short exchange between a SM teacher, whom we will call Ms. L., and three SM students in a 12<sup>th</sup>-grade advanced-track class. The exchange took place in an Analytic Geometry lesson, after the teacher presented and solved the following problem:

*What is the locus of points whose distance from (4,0) is twice their distance from (1,0)?*

The answer is an Apollonian circle of radius 2 centered at the origin (see Figure 1).



**Figure 1: The locus of points whose distance from (4,0) is twice their distance from (1,0)**

After Ms. L. concluded the solution, a student, whom we will call Ophir, asked a question, which triggered the following short exchange:

- Ophir      Can I define circle in this manner?
- Ms. L.      You're asking if it will always come out to be a circle?
- Ophir      Of course, it will.
- Ms. L.      You all heard his question? Can he define a circle according to this property?... If the points are not (4,0) and (1,0)?
- Daniel      It depends on the relationship between the two points.
- Ms. L.      What if they are general points? (k,0) and (p,0)?
- Gefen      Why zero?
- Ophir      I can take those two points and do this (gestures a translation), and the circle will simply move together with them.
- Ms. L.      Ok. This will be part of what I will assign, as general proofs. Your questions are excellent, I just want to know if it will always be a circle, and if it will always be [centered at the origin]. I see you're starting to guess. And now a new question.

Before moving to the M-Cubed discussion, we offer a few observations. It is important to note that in Ophir's question – "Can I define a circle in this manner?" – the term *in this manner* is not clear. It appears that Ophir suggested generalizing the solution in some manner, but exactly how is open for interpretation. For example, Ophir may have

meant that two points in the plane, other than  $(4,0)$  and  $(1,0)$ , may be considered. Ms. L. seemed to have interpreted Ophir's question as referring to *any* two points on the x-axis, and Gefen and Ophir generalized even further to *any* two points on the plane with the same y value. It is also not clear what Ophir meant by *define*. Ophir may have referred, as Ms. L.'s response seems to suggest, to the existence of a sufficient condition ("it will always come out to be a circle"), but he may have implied a necessary condition, or a definition, i.e., a sufficient *and* necessary condition. Thus, there are many different ways to derive a general mathematical statement from Ophir's question. Here are a few options:

- For any two given points  $A$  and  $B$  on the plane, the locus of points whose distance from point  $A$  is twice their distance from point  $B$  is a circle.
- For any positive  $k$ , the locus of points whose distance from  $(4,0)$  is  $k$  times their distance from  $(1,0)$  is a circle.
- For any positive  $k$  and for any circle  $C$  with radius  $k$  on the plane, there exist points  $A$  and  $B$  on the plane such that  $C$  is the locus of the points whose distance from point  $A$  is  $k$  times their distance from point  $B$ .
- A circle is the locus of the points whose distance from some point  $A$  is  $k$  times their distance from some point  $B$ .

Notably, some statements are true, and some are not. In the discussion between Ms. L. and her students, various statements were implied, but none were made fully explicit.

### **Unrealized opportunities for boundary crossing**

After watching this exchange between Ms. L and her students, the participants engaged in exploring alternative responses to Ophir's question. The conversation involved 57 talk turns among two mathematicians and four teachers. At first, even though all the discussants engaged with the same task, two distinct conversations took place. In one conversation, the mathematicians made mathematical observations while suggesting interpretations to Ophir's questions. In the second conversation, the teachers made observations about the classroom environment, while considering merits and risks of alternative teacher responses. These conversations were distinct in the sense that even though the mathematicians and teachers took turns and appeared to be responding to one another, they did not address questions and observations raised by discussants from the other community, as the following talk turns exemplify:

- 90 M1 By the way, the student at the end who realized that it is possible to obtain all circles in this manner, it would be very interesting to unpack his thinking, what he fixed and what he changes in his head.
- 101<sup>1</sup> M2 At first, [Ms. L.] did not even understand what [Ophir] was asking. She said: "You ask if it will always turn out that way". That is not what he asked at all. He asked if it could be taken as a definition of a circle.

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<sup>1</sup> Some turns were omitted to maintain coherency, as evident in the turn numbering.



- 104 T4 It's a dilemma I have in nearly every lesson, especially with high-ability students. There will always be one or two [students] who will drag me there. It comes with a price. My first inclination is to play along, but then I lose all the rest [of the students].
- 105 M1 And if Ophir would have been told "let's find the parameterization, just be careful of straight lines". Is it too dangerous?
- 107 T1 Why not put it to everyone? "This is what he asked, what do you think?"
- 111 T3 [...] They could benefit from it.

We consider these distinct conversations as an indication of a boundary. The teachers and the mathematicians appeared to be interpreting the goals of inquiry differently and to be utilizing distinct types of knowledge, consequently leading the two communities to develop insights separately instead of learning from and with one another.

Our analysis identified three unrealized opportunities for boundary crossing, where implicit invitations made by discussants to the whole group, to examine a specific issue from a particular perspective, were not picked up by members from the other community. For example, in turn 90, M1's comment "it would be very interesting to unpack his thinking" can be seen as an invitation for an exploration of different mathematical statements that could underlie Ophir's question, yet in the turns that followed, the teachers did not react to this invitation. In turn 105, M1 asks a question that is within the teachers' area of expertise ("Is it too dangerous?"). In doing so, M1 invites teachers to examine a concrete teaching reaction, albeit from a perspective focused on how the mathematical discourse could be developed in the classroom. Here again, M1's invitation is left unanswered. Conversely, in turn 107, T5 invites all participants to consider a possible teacher response ("Why not put it to everyone?") without specifying what mathematical idea could be addressed. This question initiated a discussion among the teachers about how different students may respond (turns 108-127), without any participation on the part of mathematicians.

### **A realized opportunity for boundary crossing**

Although M1's first two invitations for an exploration of what mathematical statement could underlie Ophir's question were left unanswered, he continued raising this issue. In turn 128, M1 circles back to Ophir's intentions:

- 128 M1 It's not clear what [Ophir] meant. That is, I would really like to know what quantifiers he had in mind. Maybe in high school it's difficult to expect it [...], but in university I would have said: okay, tell me "there exists such and such", phrase it as a clear logical statement. What do you mean?

Here, M1 interweaves the two different conversations. He places a focus on the mathematics but also elaborates a concrete teaching response and questions its appropriateness for SM, while addressing his own teaching approach as a university lecturer. Presumably, this interweaving may have been the trigger that led the teachers to respond to M1's third invitation, leading to a boundary-crossing event where M1

and the teachers built on each other's statements in a joint exploration. This exploration started with one teacher somewhat contesting M1's comment ("Isn't it clear what he meant? To us as teachers?"), which led M1 to further elaborate his perspective:

- 131 M1 Is it clear to you? I mean, is it that for any circle there exist two points such that the ratio [of distances] is one to two? Or that for any circle there exists [a ratio] such that for these two points [this is the ratio of distances]? What is [Ophir] quantifying on?
- 132 T1 The way I understood it is: can you define a circle in this manner.
- 134 M1 What is "in this manner"?
- 135 T1 With two points...
- 139 T3 In the same form.
- 140 T4 The distance from A is twice the distance from B.
- 141 M1 Perhaps not twice the distance? Perhaps [the ratio] is a parameter?
- 142 T3 Not necessarily twice.

We observe that in this short exchange, the teachers started considering various interpretations to Ophir's question, addressing different 'parametrizations' and 'quantifiers' as M1 suggested, such as the relative position of the two points (turn 139) or the ratio between the distances (turns 140-142). Thus, the teachers crossed the boundary, no longer focusing only on pedagogical risks and merits, but actively investigating Ophir's question from a mathematical perspective. Whereas this conversation was cut short, and the teachers and the mathematicians did not follow through and explicated the possible interpretations of Ophir's question, nor have they derived pedagogical implications, there are still indications that this brief boundary crossing was meaningful for the teachers, for example by helping them to make explicit why they appreciated Ophir's question. On several occasions, teachers remarked that Ophir's question is 'beautiful' but without elaborating why. Following this exchange, one teacher remarked:

- 147 T5 Still, it is a beautiful question because [Ophir] knows one definition [for a circle] and the question was whether the same shape can be defined in another way.

Here, T5 observes that the beauty of the question is linked with the fact that a student, who already knows one definition for a circle, tries to establish another definition. The word "still" implies that even if the student may not have thought of all the mathematical possibilities that the discussants started to unpack, the question is nevertheless beautiful in exploring alternative, equivalent definitions.

## DISCUSSION

Our aim in this study was to investigate *what* opportunities for boundary crossing may rise in teacher-mathematician interactions that revolve around concrete SM instructional situations, and *how* these opportunities are realized. The analysis we presented, of a joint exploration between mathematicians and experienced SM teachers regarding possible reactions to a student's unexpected question, has revealed both realized and unrealized opportunities for boundary crossing.



The study highlights mathematicians' and teachers' different ways of making sense of instructional situations and inquiring into possible pedagogical dilemmas that these situations may introduce, for example in terms of what they notice and how they interpret what they notice. Our analysis demonstrates how these differences may create a boundary within teacher-mathematician interactions. When exploring possible alternative reactions to Ophir's question, at first the conversation divided. Even though the teachers and the mathematicians engaged in the same task, they interpreted the task differently, approached it from different perspectives, and responded to members from their own community. In particular, the teachers rarely engaged in mathematical discourse, whereas the mathematicians expressed less interest in translating their interpretations and observations into practical pedagogical insights. This dynamic does not encourage cross-interactions and thus limit opportunities for boundary-crossing.

However, our analysis also suggests how this dynamic may change. On several occasions, discussants made explicit invitations for the group to inquire into possible reactions to Ophir's question from a specific perspective. At first, these invitations were confined to one perspective – of the teachers' or of the mathematicians' – and were only accepted by members of the same community. But when an invitation by a mathematician interweaved both perspectives, it seemed to have 'broken' the boundary, leading teachers to adopt, even if only momentarily, the mathematician's perspective on the instructional situation, leaving aside the question of possible merits and risks of different reactions to Ophir's question, and exploring instead how the question can be interpreted mathematically in different ways.

Our study underlines the profound difference between interaction and boundary crossing. Previous studies have emphasized the importance of teachers' interaction with mathematicians and of teachers' exposure to the ideas and perspectives of AM (CBMS, 2012; Wasserman et al., 2018). From this perspective, the M-Cubed session we investigated may seem highly beneficial for teachers, since the mathematicians and teachers appeared to be engaged in cross-interactions and responded to one other in what may seem, at least from a surface examination, as a joint inquiry. However, careful analysis of this session reveals that teacher-mathematician interaction, even when it is focused on SM teaching and even when it includes exchange of ideas and perspectives, may not suffice for a meaningful engagement of the teachers with AM or with mathematicians' perspectives. Such interaction can be visualized as a movement – of the mathematicians and the teachers – on two parallel lines of inquiry, leading to different answers to different questions, and thus not providing a real possibility for the two sides to agree or disagree with one another, and learn from their cross-interaction.

Finally, we make three observations on the boundary-crossing event we presented. First, this event focused on different ways of 'mathematizing' a student's question so to unpack implicit mathematical ideas that could be addressed in a response to the question. However, this inquiry did not materialize into pedagogical insights and concrete responses, and more research is necessary to determine whether this particular inquiry represents a more general form of utilization of AM for SM teaching, and what

could be the affordances of such utilization. A second observation is that this boundary-crossing event involved a shift of the teachers from their own inquiry to that of the mathematicians. In this study, such shifts were much more common than shifts of the mathematicians from their inquiries to those of the teachers. Further research may help to shed light into this observed phenomenon. A third observation relates to the key role of brokering in teacher-mathematician interaction (Akkerman & Bakker, 2011; Pinto & Cooper, 2018, 2022). As we have shown, opportunities for boundary crossing can be subtle and nuanced, thus could be easily missed. Further research of the realized and unrealized opportunities for boundary crossing in teacher-mathematician interactions can help facilitators of such interactions to identify and utilize boundaries that may otherwise remain implicit, and proactively encourage boundary-crossing.

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