

# Investigation of metarules in lecturing for enabling students' participation

Thomais Karavi<sup>1</sup>, Dimitri Lipper<sup>1</sup>, and Angeliki Mali<sup>2</sup>

<sup>1</sup>University of Groningen, Faculty of Science and Engineering, The Netherlands, t.karavi@rug.nl, <sup>2</sup>University of Crete, Greece

*Keywords: Teachers' and students' practices at university level, teaching and learning of analysis and calculus, commognition, university lecture, metarule*

Participation in the university mathematics community usually starts with the students attending mathematical lectures. One of the lecturers' main goal in teaching is to facilitate students' participation in this community (Sfard, 2008). The actions of the lecturer in the teaching that may assist students' participation are still underexplored in the university mathematics literature (Melhuish et al., 2022). Thus, in this study, we propose the commognitive framework to investigate the discursive actions that may assist students' participation in the university mathematical discourse as it gives a fine-grained analysis for a micro-level investigation.

For the investigation of the discursive actions, awareness of the metarules in the teaching is necessary. A mathematical discourse has its own sets of metarules, which are narratives that define patterns in the activity of the participants. These metarules result in routines, which are patterns of discursive actions. Consequently, our research question is "which are the discursive actions and the underlying metarules of the lecturer for supporting students' participation from the lecturer's perspective?".

## METHODOLOGY

For this study, we investigated the teaching in online lectures of an introductory real analysis course. The lecturer in this case study is a mathematician with six years of teaching experience in this course. For the analysis, we coded the seven lectures using inductive and deductive thematic analysis with theoretical codes from Karavi et al. (2022). Through constant comparisons of the quotes under the same code, preliminary themes of discursive actions emerged. Then, while further investigating the discursive actions in relation to their appearance while proving, we identified implicit metarules. Operationalisation of the discursive actions and metarules occurred through constant comparisons.

## RESULTS

We present briefly the results using excerpts from the episode of teaching of the characterization of compact sets:  $K \subseteq \mathbb{R}$  compact  $\Leftrightarrow K$  closed and bounded. In our interpretation, the lecturer supported students' participation in the proving processes through the performance of the following discursive actions: *making decisions on how to start the proving process* (e.g., "Let's assume that  $K$  is a compact set and then show the  $K$  is both closed and bounded. Well, let's do a proof by contradiction. Let's assume

that a set  $K$  is not bounded”), *sharing the key idea of the proof* responding to the question “And now what?” that came after the statement of the theorem (e.g., “So, if you look very carefully what I'm doing in this proof, I'm almost using the same sort of proof that I use to show that the set of real numbers is not compact. Yeah, I am almost using the same trick here”), and *bringing the means for the emergence of the proof* (e.g., “Okay, so now my assumption is  $K$  is closed and bounded. And I'm going to reason from right to left. So, how do I show that  $K$  is compact? Well, the only thing I can do is to verify the definition”). These discursive actions are governed by the implicit metarule *while proving, an idea of how to start is needed*. The metarule is related to students’ de-ritualization and an independent, product-oriented engagement with proving processes. The lecturer’s discursive actions shifted the attention to the product and gave an idea to the students why specific actions took place.

## DISCUSSION

Identifying the discursive actions and the metarules could give in future studies valuable insights into the ways of possible facilitation of the newcomers’ participation in the mathematical community. Commognition can support micro-level investigation in the observational data from the lectures, aiming to explore lecturers’ practices towards students’ learning. As Pinto (2019) highlighted the differences among the examined lecturers were on a meta-level and related to their different views and experiences with teaching. Following Pinto (2019), the metarule affected the discursive actions that appeared in the lectures, shaping the lectures. In our case, the identification of the metarule facilitated the observation of de-ritualization instances behind the proving processes that may assist students’ explorative participation. Awareness about the metarules and lecturers’ views on them could provide understanding into the university mathematics lecturing than considering it as a monologue.

## REFERENCES

- Karavi, T., Mali, A., & Avraamidou, L. (2022). Commognition as an approach to studying proof teaching in university mathematics lectures. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(7). <https://doi.org/10.29333/ejmste/12173>
- Melhuish, K., Fukawa-Connelly, T., Dawkins, P. C., Woods, C., & Weber, K. (2022). Collegiate mathematics teaching in proof-based courses: What we now know and what we have yet to learn. *The Journal of Mathematical Behavior*, 67, 100986.
- Pinto, A. (2019). Variability in the formal and informal content instructors convey in lectures. *The Journal of Mathematical Behavior*, 54, 100680. <https://doi.org/10.1016/j.jmathb.2018.11.001>
- Sfard, A. (2008). *Thinking as communicating: Human development, development of discourses, and mathematizing*. Cambridge University Press.