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# Promoting Inquiry in Mathematics: Professional Development of University Lecturers

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*Abstract. The paper reflects on the Inquiry Based Mathematics Education approach and the professional development of university mathematics lecturers. The main focus is on the design of tasks as a key aspect in lecturer training. The professionalisation activity we have established is rooted in a framework developed by the European project PLATINUM. We describe its basic ideas, the methodological approach and a didactical tool and focus on challenges and issues that arise in the design and implementation of mathematical tasks for an inquiry-oriented teaching. For empirical analyses, we use some data from a professional activity implemented at the Complutense University of Madrid in Spain and carried out jointly by a team from Madrid and Hanover.*

*Keywords: inquiry approach, professional development, university education*

## INTRODUCTION

The present study is located at the intersection of two issues: the methodological revitalisation of mathematics teaching towards inquiry-oriented approaches in the classroom, and the reflection about professional development of university teachers in a collaborative way. It is framed within the European project PLATINUM developed by researchers from seven European countries (Gómez-Chacón, et. al, 2021). One of the goals of the project was to develop and pilot a platform for the professional development of mathematics lecturers on a regular basis in the format of a “hands-on” workshop. The need for such a training platform reflects the current situation in university mathematics teaching, where the lecturer has to find a balance between preparing knowledge and reflecting on pedagogic methods, as well as bringing complementary areas of expertise together in subject-didactic considerations. On the one hand, mathematics lecturers often have limited or no access to information about contemporary pedagogical and didactic methods<sup>1</sup>, which in turn might contribute to a lack of motivation to use them. On the other hand, lecturers, whose expert field is not mathematics education, cannot be expected to fully immerse themselves in mathematics education research<sup>2</sup> to become experts in contemporary mathematics pedagogy and didactics. We are aware of these boundaries and therefore focus on a developmental process that different tools can foster in terms of knowledge about different teaching methods and how to apply them. This can then also have a positive impact on student performance.

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<sup>1</sup> While didactics is a discipline that is essentially concerned with the science of teaching and instruction for any given field of study, pedagogy is focused more specifically on the strategies, methods and various techniques associated with teaching and instruction.

<sup>2</sup> This is due to a number of institutional restrictions, e.g. time, interest, possibility, institutional position, and expectations.

PLATINUM and the workshop we conducted are grounded in, among other things, the idea of creating Communities of Inquiry (CoI) (Jaworski, 2020). Collaborating in CoIs effectively supports lecturers in IBME and fosters their professional development in teaching mathematics, such that teaching of mathematics on the university level supports the aim to achieve students' conceptual learning of mathematics. The theoretical model of IBME in higher education by the PLATINUM project (Gómez-Chacón, et. al., 2021) introduces three levels which all approach teaching and learning through principles of development and interaction.

The first level describes the inquiry in mathematics that is carried out by students and the lecturer in the classroom. Here student – lecturer interactions and student - material (e.g. tasks) interactions are essential. In the second level, lecturers reflect on the processes in the first level. Teaching material, e.g. learning tasks, are designed and adapted collaboratively based on the experiences in the classroom (i.e. first level). The lecturers discuss teaching and learning and give and receive feedback on the design of the learning tasks and their implementation. Together with lecturers, more experienced members and invited experts can promote and support professional development. In the third level, didacticians and educational researchers reflect together with the lecturers on the developmental process that takes place in the first and the second level, which also supports the developmental research. The boundaries between the first and second and second and third levels are crucial nodes in the development process. They are the communication and critical reflection and feedback nodes connecting lecturers and students and lecturers, researchers, and peers respectively. It is at this third level that the tools and reflections presented in this contribution are discussed, offering overviews of theoretical backgrounds for teaching. From this level, we carried out a previous collaborative work of deepening and discussing theoretical and practical aspects of IBME. We intend to support the reflective and evidence-informed teaching attitude of the workshop participants and promote different categories of reflections on the teaching and learning process.

Following this model and taking the institutional restrictions of university mathematics lecturers into account, we focus here on a small-scale approach of task (re)design. The question is how or in which directions tasks could be developed or modified, essentially what makes a task an IBME task. In the international workshops of the PLATINUM project, we dealt with this question in different ways and developed various instruments. For the present paper we choose one of the tools developed and used to characterise and further develop inquiry-based tasks previously at local level in Leibniz University of Hannover (see section LUH workshop in Gómez-Chacón, et.al., (2021)). Two central characteristics of this tool are, on the one hand, its simple and direct usability by lecturers and, on the other hand, it enables to the interpretation of the effects of the tool against an elaborated subject didactic background (here ATD was chosen).

In this contribution, we focus on our experience and data from the development of an International Workshop *Inquiry-based education in mathematics and professional*

*development for university teaching* that took place at Complutense University of Madrid (UCM) in 2021, which focused on the implementation of models and materials in Inquiry based mathematics education (IBME) developed in the PLATINUM project. In this workshop we introduced, among other things, the tool to support lecturers and teacher students in developing and characterising IBME tasks (Gómez-Chacón, et.al., 2021). The research question that we try to answer in this contribution is: What leeways and scope for actions could we observe with this tool? What difficulties occurred?

The structure of this contribution is as follows: In the next section we describe the professionalisation workshop jointly conducted at the UCM. In what follows, we then focus on analysing data from this workshop and answering our research questions. To this end, we first address underlying theoretical and methodological aspects. We then present results of our analyses. Finally, we formulate some conclusions.

## **IBM PLATINUM INTERNATIONAL WORKSHOP**

In the following we describe the institutional context and goals, the participants, and the pedagogy of the workshop.

### **Institutional context and goals**

The concept of the PLATINUM workshop held in Madrid was developed with two boundary conditions in mind. First, we had to consider the methods and materials created locally by the three PLATINUM partners (Germany, Norway and Spain), that were developed specially to take the mathematical content into account. Second, the context of Professional development of mathematics teachers and lecturers at the institutional level at UCM. Here, the academic training of novice lecturers was a priority. IBME aspects worked out in the PLATINUM project had to be addressed within existing course structures for PhD students or assistant lecturers. From this point of view, there are principles of effectiveness in professional development that are promoted both in the content and in the training processes used. The following principles are focussed on the workshop: a) To have as a fundamental objective the participants' learning; b) To be based on the mathematical knowledge that the participants must teach and c) To be connected to the teaching practices of the participants, serving as a support for them. So, the objectives of the workshop were:

- 1) Practical initiation of university lecturers into an inquiry-based approach to teaching and learning of mathematics.
- 2) Development of methodological skills to (re)design inquiry-based tasks.
- 3) Knowledge of resources: examples of inquiry-based tasks and -projects in university mathematics teaching.

In the following, we will focus on 2), specifically taking the mathematical knowledge to be taught into account. The specific approach of the workshop was the focus on modifying already existing tasks to become more inquiry oriented. With this focus we

also take into account institutional restrictions that lecturers have to face when they would have to develop new material from scratch.

## **Participants**

Fifty participants from 8 different universities or mathematical research centres took part at the workshop. The profiles of the attendees were: new university lecturers of mathematics, mathematics research assistants, mathematical student-teachers of the master's degree in mathematics education<sup>3</sup>. In this contribution, we focus on this last group (21 participants from UCM).

## **Pedagogy of the workshop**

The workshop was organised in the following steps:

*Step 1. Task 1, given before the start of the workshop.* The participants were given the following task: For the development of the workshop bring one or two math activities that you find interesting or that may be problematic to work with the Inquiry Based-Learning approach in university mathematics teaching. For those of you who are teaching, you can take it from the courses you teach. For those of you who are teacher students, you can take them from manuals and focus on the transition between high school and university or for the first academic year at the university.

*Step 2. Information given in the workshop about characteristics of inquiry-based tasks.* Working groups were formed and each group chose a task for the group work. In the workshop, presentations about IBME aspects of tasks were given and a didactical tool to help characterising IBME aspects in tasks was provided (cf. next section and Table 1). The task given to the groups was: Which aspects of IBME does your task already fulfil? Which aspects (and why) does/should a modified task fulfil? Prepare a presentation of your group work result.

*Step 3. Sharing.* Each group presented its modified task in the whole class group.

*Step 4. Going back and reflecting.* Each group returns to the design of the proposed task and analyses the following aspects in depth: 1) Formulate learning goals for the original task and for the modified tasks; 2) Formulate an expectation horizon for solving the modified task; 3) Fill in the table and explain your choices. What did you find useful for IBME teaching, why did you not use the table, or why would you change it.

## **THEORETICAL AND METHODOLOGICAL ASPECTS UNDERLYING THE DIDACTICAL TOOL AND THE ANALYSES**

Tasks are an important part of learning environments. Lecturers should develop their design skills on IBME tasks and to modify existing tasks in the direction of promoting more IBME. To support professional development of lecturers in this

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<sup>3</sup> There are two types of participants, students who are going to teach at secondary level, but also PhD students in mathematics who are doing it for qualification. In Spain there is a figure of university teacher, called "associate" and they are teachers who are going to teach at both levels, secondary and university

respect, one challenge is to reflect about what it means for a task to be an inquiry-based task in mathematics. For this we used a tool developed at the Leibniz University of Hannover to help identifying characteristics of inquiry-based tasks that focusses also on the mathematical content. The following dimensions are considered: a) Openness of the tasks (process open, open-ended, content-open), b) Enabling specific inquiry strategies (mathematical heuristics and developing solutions strategies), c) Enabling discourses on techniques, d) Enabling inner-mathematical knowledge linking, e) Enabling interdisciplinary knowledge linking<sup>4</sup>. The dimensions are compiled in form of a table to facilitate the use by lecturers (Table 1).

Content dimension \ Task format	Inquiry - strategies are to be learned.	A discourse on techniques should be stimulated.	Innermathematical knowledge linking is to take place.	Interdisciplinary knowledge linking is to take place.
Closed				
Process-open				
Open-ended				
Content-open				

Table 1. Table of dimensions for inquiry-based tasks

An important aspect of using this tool is that it is not meant to classify tasks to be IBME tasks or not, so it is not meant for easy assessment. We see IBME-tasks as an open range of possible IBME orientations, where the dimensions above play major roles for (re-)designing tasks that allow to foster (eventually more) inquiry activities by students. The table and its use in the workshop represent a tool to help lecturers to collaboratively reflect about existing tasks, to redesign tasks with respect to the dimensions and to use it as a focus for discussion about tasks with colleagues.

The development of this tool incorporated basic concepts of the *Anthropological Theory of the Didactic (ATD)* (Chevallard, 1999). In the following we will shortly introduce the concept of praxeology from the ATD that will serve as an analysis tool later. In ATD a praxeology is a basic tool to model knowledge in so called “4T-models (T,τ,θ,Θ)”. They consist of a practical block (i.e. the praxis, T and τ) and a theoretical block (i.e. the logos, θ and Θ). Bosch and Gascón (2014) concretise:

A praxeology is thus an entity formed by four components, usually called the “four Ts”: a type of tasks, a set of techniques, a technological discourse, and a theory. As activities and knowledge can be described considering different delimitations or granularities, a distinction is made between a “point praxeology” (containing a single type of task), a “local praxeology” (containing a set of types of tasks organized around a common

<sup>4</sup> Due to limitations of space we cannot go into details here. Please refer to (Gómez-Chacón, Hochmuth, et.al., 2021).

technological discourse) and a “regional praxeology” (which contains all point and local praxeologies sharing a common theory). (p. 69)

We will mainly use the concepts of point praxeology and local praxeology to analyse how the participants used the table and what kind of modifications they made to their tasks. The overall idea is to analyse the works of the participants and then draw conclusions for the impact and success. For the analysis of the group work and the interpretation of modifications in view of the table and its dimensions we reformulate them and their aspects in praxeological terms.

A next methodological step consists in embedding the tasks in the respective curricular context, outlining praxeological landscapes and locating the initial tasks and the modified tasks in this map. Due to limitations of space, we will not go into the methodological details here, but we will illustrate the last aspect with an analysis vignette considering examples from the group results. We would like to notice that the participants did not know the ATD, nor was it our aim for them to learn about this theory. The restriction of the table to terms that seem technically evident, i.e. that can be used “superficially”, makes sense in view of the participants and the goals of the workshop. We introduced the dimensions and gave short explanations in the workshop but didn’t stress a precise definition of each dimension. They are formulated quite openly and such that there is room for interpretation by the participants of the workshops. This openness is deliberate because we did not want to focus on teaching the underlying ATD background of the dimensions but on promoting discussions among the participants. The meaning of the dimensions is allowed to be adapted to the context relevant for the participants. How the participants interpreted the meanings of the dimensions is also of interest.

## **RESULTS**

In the following we present the results, regarding the dimensions of IMBE represented in the table and the usefulness of the table as a didactical reflection tool.

### **Inquiry-based tasks and developed dimensions**

There were five groups, each (re)designing a task. Three groups focused on Solving Systems of Equations, one on the Derivative of Functions and one on the Concept of Vector Space. The transformation of the tasks essentially consisted of:

- a transition from specific more or less straight forward calculations in view of tasks which address several independent punctual praxeologies (focussing mostly on techniques), to extended tasks addressing local praxeologies, i.e. tasks mobilising also technology aspects of the content (describing, validating, questioning, exploring etc.) and fostering a common technological discourse.
- a problematisation from fixed procedural knowledge to a deeper understanding of definitions and concepts; the tasks were extended to focus on inner-mathematical knowledge linking that aims at connecting different mathematical concepts and

overcoming the compartmentalisation of mathematical knowledge (Kondratieva & Winsløw, 2018).

### *Example 1: Derivative of a Function*

The original task is a point praxeology with task T to “calculate the derivative of a function”, that addresses the techniques of applying rules for derivatives. Technological aspects are not addressed by this task. Group 4 modified this to a task with several subtasks addressing different approaches and changes of representation, e.g. analytical term to graph or table, graphical differentiation<sup>5</sup>. The subtasks become more and more open. Inquiry strategies are interpreted by this group as the use of mathematical heuristics (see Polya or Schoenfeld for other forms of inquiry strategies). Inner-mathematical knowledge linking is understood as changing of representations and accompanying activities (drawing the graph, calculating the derivative graphically). Each subtask, again, represents a point praxeology. But in the course of all subtasks, also technological aspects are addressed that support the overall forming of a local praxeology. Some examples are the request to compare different approaches used and to explain the properties of the studied function based on the results from previous subtasks. The modified task from Group 4 also shows potential for further development of IBME aspects, that are not explicitly realised in this step of the task development. We see both, the progress of Group 4 and the potentials of our ATD based analysis method as very satisfactory.

Task: Study the derivative of the function  $f(x)=(x^3-7x+6)/(x+5)$  with different approaches and using different tools. a) Calculate the derivative of this function:  $f(x)=(x^3-7x+6)/(x+5)$ ; b) Using the definition of derivative, get the derivative again. Compare the result with a); c) Draw the graph of the function, observe the maxima and minima and calculate its derivative at those points; d) Generate a table with the value of the derivative at each point calculated graphically using slopes. Plot this data on a new graph.; e) What are the properties of the function (deduced from its derivative)? f) Try to factor the polynomials of the function and then try to derive taking into account this factorization.

### *Example 2: Solving Systems of Equations*

This topic was worked on by three groups (Group 1, 2 and 3). It is interesting to see the variety of approaches to the same topic by the three groups.

Taking the (re)designed task from Group 2, the original task was a point praxeology with the task T to “find the solution of a system of equations using a specific method”, that address the technique of application of a concrete method. It was modified and opened up in several subtasks with:

- *respect to solution path:* it is proposed to find the solution of a system of equations previously explaining several methods and to let the students use the method they want to solve the problem

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<sup>5</sup> We present the tasks developed by the Groups in the supplementary appendix.

- *respect to solution/result*: it is formulation of a system of equations which is compatible indefinite and propose each group to find a solution which in several cases would be different.
- *respect to knowledge used*: solving a system of equations geometrically and finding the relation with the real problem (giving meaning of the variables).
- *respect to knowledge applied*: geometrical interpretation: plane intersection (inner-mathematical knowledge linking). Science applications: balancing chemical reactions (interdisciplinary knowledge linking).

The result of the modification is the integration of various point praxeologies into local praxeologies. Interesting here is also the interpretation of the group, that openness of a task also means freedom of the students to choose.

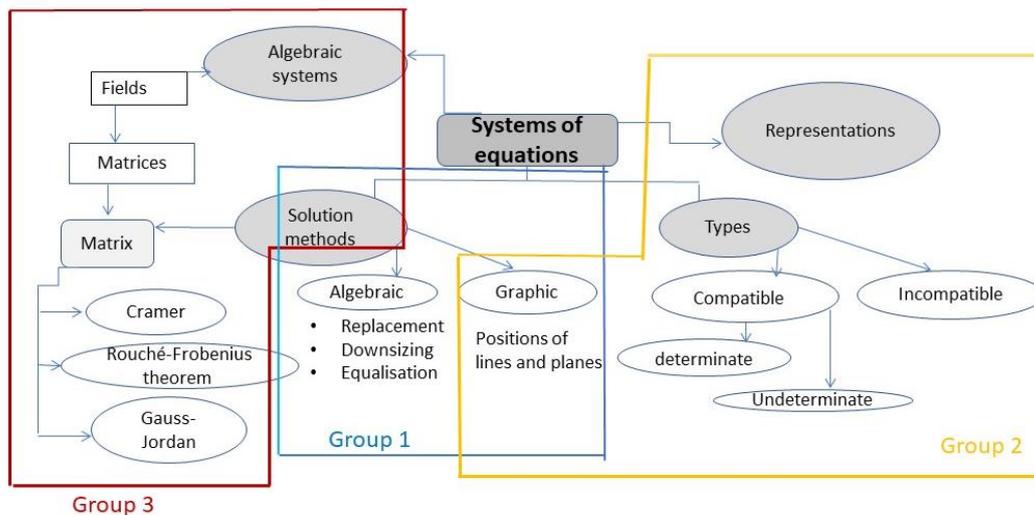


Figure 1. Concept map of the topic

Comparing all contributions of each of the three groups, we find that there is an evolution of the task and the related mathematical domain. The analysis of the conceptual structure of the topic gives rise to the concept map<sup>6</sup> shown in Figure 1, in which four fundamental concepts are organised: representations, types of systems, solution methods and algebraic systems. Some relationships are established between the concepts shown and the semantic structure of the problems. As an example of the relationships established between the concepts, we take the solution methods. These methods are divided into matrix, algebraic and graphical<sup>7</sup>. For the concepts associated with the graphical method, the tasks modified emphasise that concepts, algebraic representations of equations, types of systems of linear equations and the semantic structure of problems are the basis of the study when dealing with the graphical method. For some groups, the design of IBME tasks also entailed the possibility of questioning the curriculum and the predefined organisation of the content. The more or less strict organisation of subtasks seems at first to contradict the possibility of

<sup>6</sup> We also use a concept map instead of a praxeological map for space reasons, to provide a rough overview of the diversity of solutions among three groups. Even though they work on the same topic, they follow quite different paths.

<sup>7</sup> This is associated with the relative position of the lines and planes on the plane or on the space according to the dimensions of the system and the means in the determinate, indeterminate compatible systems or to the incompatible systems definition.

opening up tasks towards more IBME. But for those groups this was a method to keep up the compatibility of task and curriculum.

### **Table-tool valorisation**

The workshop focused on work on inquiry dimensions of mathematical content. Thus, the investigation targeted mathematical knowledge, in particular with the development of technological aspects (from point- to local praxeologies). In view of a particular task, the table used helped to generate questions about its IBME characteristics and further development possibilities. Four of the five groups use the table, even the group that did not use it highlighted its benefits. In their own words: *“We did not use the table as such, but rather talked about the different contents that were in it, since in our case, the opening of activities only occurs in the last section... It is true that there can be several techniques, especially in the simplification of the matrix in section d, which allows us to affirm that in terms of techniques it is an open process”*. The table was not simply applied by the students but provoked intensive group discussions. There were also difficulties that provoked reinterpretations of terms in it, that were interesting for the analysis. The researchers had ATD Theory as a background, so there are many aspects that are implicit and not transparent to the user. However, the analyses show that using the ATD conceptualisations that went into the development of the table also as an analysis tool for the students works, we see a lot more potential in the tasks and possible further developments than the students were able to produce. This is not a deficit of the students. The results are remarkably good for a short workshop with only a short introduction into the table, and without any introduction into the theoretical background. This also promotes the question how much of the theoretical background could be introduced to develop this kind of workshop and instruments further. This is an open balancing act between too much and too little theoretical background. And, so to speak, a basic problem for the further development of professional development workshops.

### **CONCLUSIONS**

In our final section we bring all of the above together, addressing how the development of the workshop and the use of to support lecturer in developing and characterising IBME tasks opened up avenues of professional development. We have indicated the ways in which our work has related to PLATINUM’s three-level model. The observations made can be interpreted against this background. The modification of the initial tasks has led to an increase in the complexity and depth. In the design, the participants tried to state the concepts according to the prescribed curriculum and to establish connections between different registers (geometric, algebraic) and regional praxeologies that are usually less explicit in the lessons. Regarding the three levels mentioned in the introduction the following aspects can be outlined: a) Students: Expansion of the ability to act, explicit addressing of technological/conceptual aspects, etc. b) Lecturers: Requirements for lecturers, pre-structuring etc., necessary or helpful, the exchange on how small-step and guiding

necessary and helpful etc. c) Research: concept of training etc. but also needs for further research with regard to the modified task (beyond anecdotal evidence).

The tool table is one example of promoting connections between layers 3 and 2, i.e. between research and teaching - but not only the table alone. Also, its embedding in the overall workshop. However, these interactions must be balanced in the sense that we as researchers (layer 3) cannot “drag” lecturers into this layer (sometimes researchers are also lecturers, we also reflected on this double position in Ruge & Peters (2021). These results encourage to develop research-based instruments like the table and the workshop for professional development that promote collegial and collaborative reflection and discussion.

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