

Recognizing matrix equations as eigenequations or not: Examples of student reasoning in quantum mechanics

Megan Wawro¹ and John Thompson²

¹Virginia Tech, Mathematics Department, United States, mwawro@vt.edu;

²University of Maine, Department of Physics and Astronomy, United States

Keywords: Teaching and learning of linear and abstract algebra, teaching and learning of mathematics in other fields, eigentheory, quantum mechanics.

Reasoning about mathematics is central in many of the scientific disciplines. Students often utilize mathematical concepts and procedures, mathematize physical constructs, and interpret mathematical entities in terms of physics (Uhden et al., 2012). For example, quantum mechanics problems often involve reasoning about linear algebra content such as matrix-vector operations, change of basis, eigentheory, projection, orthonormality, and inner products (e.g., Schermerhorn et al., 2019; Serbin & Wawro, 2022). Our broad research project investigates students' understanding, symbolization, and interpretation of eigentheory in quantum mechanics (US NSF #1452889). This poster will focus on the following research questions: in what ways do students recognize if quantum mechanical matrix equations are eigenequations, and how does this relate to their reasoning for eigentheory in mathematics and quantum contexts?

The data consist of video, transcript, and written work from individual, semi-structured interviews (Bernard, 1988) with ten volunteers from a senior-level quantum mechanics course at a medium-sized public research university in the United States. One interview question probed students' reasoning about three equations E1-E3 (Figure 1). E2 is a quantum mechanics eigenequation for a spin- $\frac{1}{2}$ system [1], and E3 is an equation in which the operation "flips" the spin state; E3 is *not* an eigenequation.

"I have a few equations prepared. For each one, I want you to explain what the equation means to you."

[E1] $A\vec{x} = \lambda\vec{x}$, where A is a 2×2 matrix, \vec{x} is a 2×1 vector, and λ is a scalar

[E2] $\hat{S}_x|+\rangle_x = \frac{\hbar}{2}|+\rangle_x$

"You mentioned both related to eigentheory. Please compare and contrast how you personally conceptualize eigentheory in the two situations."

[E3] $\hat{S}_z|+\rangle_x = \frac{\hbar}{2}|-\rangle_x$

Figure 1. The interview question used to gather the data analysed in this poster.

In this work, we adopt a theoretical stance consistent with the Knowledge in Pieces framework (diSessa, 1993). This assumes that students' intuitively held knowledge pieces are productive in some context and that knowledge change involves evolutionary refinement and reorganization of ideas. We conducted our analysis by iteratively examining the data for nuance in student imagery and noting relevant discursive cues (Gee, 2005) that we then organized into themes.

The results presented in the poster will focus mostly on student reasoning about E3. One aspect will delineate results related to student reasoning about if E3 was a valid

equation. Of the four students who this engaged in this way, two of them used written calculations to eventually convince themselves of the equation's validity, and two of them believed it to be an untrue equation. The second aspect will delineate results related to student reasoning about if E3 was an eigenequation, with eight of them eventually determining that E3 was not an eigenequation. All eight in some way discussed the two kets in the equation not matching, either by reasoning about co-existing distinct vectors (a static view of the equation) or reasoning about not getting same vector back (a dynamic view of the equation); these are synergistic with results about E1 and E2. For example, one student stated, "So not so much an eigenvalue equation because we don't have the same vector on either side." In their examination of E3's structure, they seemed to leverage a static view of the equation as they looked for the same vector on both sides of the equal sign. This is consistent with Sherin's work on symbolic forms (2001), which interprets students' understanding of equations in terms of pairing symbol templates with conceptual justifications for the structure of the equation. The remaining two students displayed reasoning that indicated they knew E3 contained some aspects that related to eigentheory but were not sure if it was an eigenequation. For example, one student explained E2 in terms of measuring spin but voiced uncertainty about interpreting E3 with respect to measurement. The poster will include a broad synthesis of results across all three equations, highlighting instances of synergistic and potentially incompatible interpretations of the three equations, will offer pedagogical implications related to linear algebra, and will discuss avenues for future research such as the use of symbolic forms in mathematics education research.

NOTES

1. Spin is a measure of a particle's intrinsic angular momentum. Possible spin states are represented by normalized kets $|\psi\rangle$ which behave mathematically like vectors. The eigenstates $|+\rangle_x$ and $|-\rangle_x$ for the spin- $\frac{1}{2}$ operator \hat{S}_x correspond to the two possible spin measurements of $\pm \frac{\hbar}{2}$ along the x -axis, encapsulated in the eigenequations $\hat{S}_x|\pm\rangle_x = \pm \frac{\hbar}{2}|\pm\rangle_x$.

REFERENCES

- Bernard, H. R. (1988). *Research methods in cultural anthropology*. Sage Publications.
- diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition and Instruction*, 10(2–3), 105–225.
- Gee, J. P. (2005). *An introduction to discourse analysis: Theory and method*.: Routledge.
- Schermerhorn, B. P., Passante, G., Sadaghiani, H., & Pollock, S. J. (2019). Exploring student preferences when calculating expectation values using a computational features framework. *Physical Review Physics Education Research*, 15(2), 020144.
- Serbin, K. S., & Wawro, M. (in press). The inextricability of students' mathematical and physical reasoning in quantum mechanics problems. Accepted with revision, *International Journal of Research in Undergraduate Mathematics Education*.
- Uhden O., Karam, R. Pietrocola, M., & Pospiech, G. (2012). Modelling mathematical reasoning in physics education. *Science and Education*, 21(4), 485–506.