

# Design principles for intertwining local and nonlocal mathematics - The case of relating registers and representations in abstract algebra

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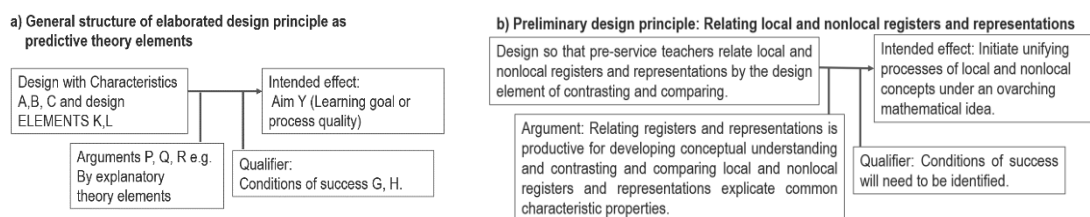
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Pre-service teachers often see no relevance in their mathematical content courses for their mathematics teaching (Ticknor, 2012). In this context, mathematical ideas can be distinguished in the local content that will be taught at school and the nonlocal content that is often addressed in university mathematics (Wasserman, 2018). The nonlocal content is supposed to help restructure and deepen the understanding of the local content from a higher standpoint, but many pre-service teachers are not able to make these connections on their own (Hefendehl-Hebeker, 2013). One manifestation of this restructuring is the unification of seemingly unrelated mathematical objects so that they are grasped more coherently under an overarching mathematical idea (Serbin, 2021).

This project aims at supporting pre-service teachers in relating local and nonlocal mathematical content by the design of profession-specific teaching-learning arrangements for abstract algebra learning. Wasserman, Fukawa-Connelly, Villanueva, Mejia-Ramos and Weber (2017) proposed an instructional model for the overall structure of such intertwined learning trajectories, but further empirical research is needed on a more specific task design level and the design principles.

For task design, *relating registers and representations* has proven as a fruitful principle for developing conceptual understanding of mathematical concepts at school level (Prediger & Wessel, 2013). In addition, also in university mathematics teaching relating registers and representations is an activity central to developing understanding of mathematical content (Moreno-Arotzeno, Pombar-Hospitaler & Barragués, 2021). On a more specific task design level, comparing and contrasting as cognitive activities are often used to identify characteristic properties of a concept and are shown to have positive effects on the learning process (Lipowsky et al., 2019). We see a coordination of relating registers and representations with contrasting and comparing as a potential in regards to intertwining local and nonlocal mathematics (e.g. solving  $x + 5 = 12$  and solving  $d_{120} \circ x = s_1$  in the dihedral group). This leads us to the question: “Which potential and conditions of success can be identified for the design principle *relating registers and representations* in terms of intertwining local and nonlocal algebra?”



**Figure 1: General logical structure (Prediger, 2019, p.6) and project-specific realization**

Methodologically, this project follows the design research approach according to Prediger (2019). The formulation of design principles with a general logical if-then structure (see figure 1a) is a central predictive theory element. The poster will focus on the design principle *relating registers and representations* that arose within the first cycle of the design experiments (see figure 1b). Design experiments with secondary pre-service teachers will be conducted, transcribed and analysed with qualitative content analysis. The poster will present insight into the learning pathways and the resulting implications for the effect of the design principle and its conditions of success.

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