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# **Questioning the quantum world?**

## **A priori analysis of an SRP at the interface between mathematics and quantum mechanics**

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*This work is at the confluence of two groups of studies: on the one hand, research on didactic phenomena (and their consequences for teaching and learning) when mathematics is taught in an engineering or a physics class; on the other hand, research on conditions and constraints allowing to carry out the evolution from the prevalent didactic paradigm of visiting works to the novel paradigm of questioning the world, which are both brought out by the Anthropological Theory of the Didactic (ATD). In this dual context, we present in this paper the preliminary and a priori analyses of a Study and Research Path (SRP) set at the interface between mathematics and introductory quantum mechanics, in third year of bachelor's degree, at the University of Montpellier.*

*Keywords: Teachers' and students' practices at university level, Novel approaches to teaching, Teaching and learning of mathematics in other fields, Curricular and institutional issues concerning the teaching of mathematics at university level, anthropological theory of the didactic.*

### **INTRODUCTION**

In recent years, much work has been carried out on the teaching and learning of mathematics for engineers or on the role of physical modelling in mathematics curricula (see for instance Hausberger, Bosch & Chellougi, 2020, TWG2). However, the relative roles of mathematics and advanced physics are quite different: rather than seeing mathematics merely as being used, both fields are then more on an equal footing (Lombard & Hausberger, to appear). In this work, we focus on the relationship between algebraic structures and theoretical physics, based on the example of Hilbert spaces in quantum mechanics. The study is carried out in the framework of the Anthropological Theory of the Didactic (ATD), which “aim (...) is the elucidation of human societies' relation to “the didactic,” that is to say, to all the possible factors of learning. By adopting an anthropological point of view, it purports to embrace the didactic wherever it may show itself around us, paying special attention to the institutional constructions of knowledge and the conditions established to disseminate it.” (Chevallard & Bosch, 2020a, p. 53).

Within the research program of the ATD falls the identification of “study paradigms”, and the “contribution (...) to the paradigm of questioning the world” (Chevallard & Bosch, 2020a, p. 59; see also the next section). The study and development of Study and Research Paths (SRP) plays a key role in this endeavour (Bosch, Barquero, Florensa & Ruiz-Munzon, 2020). In this context, we have undertaken to set up an SRP at the interface between mathematics and introductory quantum mechanics at the

University of Montpellier. More specifically, this SRP is set in the context of quantum computing, and starts with the following generating question  $Q_0$ : *In what respect are quantum computers indeed quantum?* It is designed for students that are in third year of bachelor's degree (sixth semester). It consists of nine two- to three-hour sessions, distributed into three lab sessions and six classroom sessions. In this paper we present the results of the first two phases of the implementation of an SRP: the preliminary and a priori analyses.

Our work is thus in line with the past INDRUM conference “instructional proposals for university mathematics to move towards a change of paradigm, such as problem-posing activities, interdisciplinary projects or study and research paths” (Hausberger et al., 2020, p. 167). In this context, it aims at addressing the following issues (Hausberger et al., p. 167-168):

How to find a “good” generating question for an SRP? Can the design and implementation of SRPs help us to rethink the contents of the course? [...] How to look at university mathematics curricula from an interdisciplinary approach? How can the perspective of mathematical modelling contribute to it?

## **THEORETICAL FRAMEWORK AND METHODOLOGY**

### **SRPs as part of ATD**

In the paradigm of visiting works, also called paradigm of visiting monuments, works under study at school or university (theorems, formulae, methods) are “approached as (...) monument(s) stand(ing) on (their) own” (Chevallard 2015, p. 3). Notably, the *raison d'être* of these pieces of knowledge is never specified. The paradigm of questioning the world, as its name indicates, focuses rather on questions. In this paradigm, one sees pieces of knowledge as answers to be considered only when judged relevant to solve given problems, hence highlighting their *raison d'être*.

Study and Research Paths (Winsløw, Matheron & Mercier, 2013) constitute a contribution to the advent of this “oncoming counter paradigm” (Chevallard, 2015) in teaching institutions. “An SRP is an inquiry process which starts from an open-ended question and leads to a combination of investigation activities - to explore the question, and study processes - to obtain new information that will help in the research” (Bartolomé, Florensa, Bosch & Gascón, 2018).

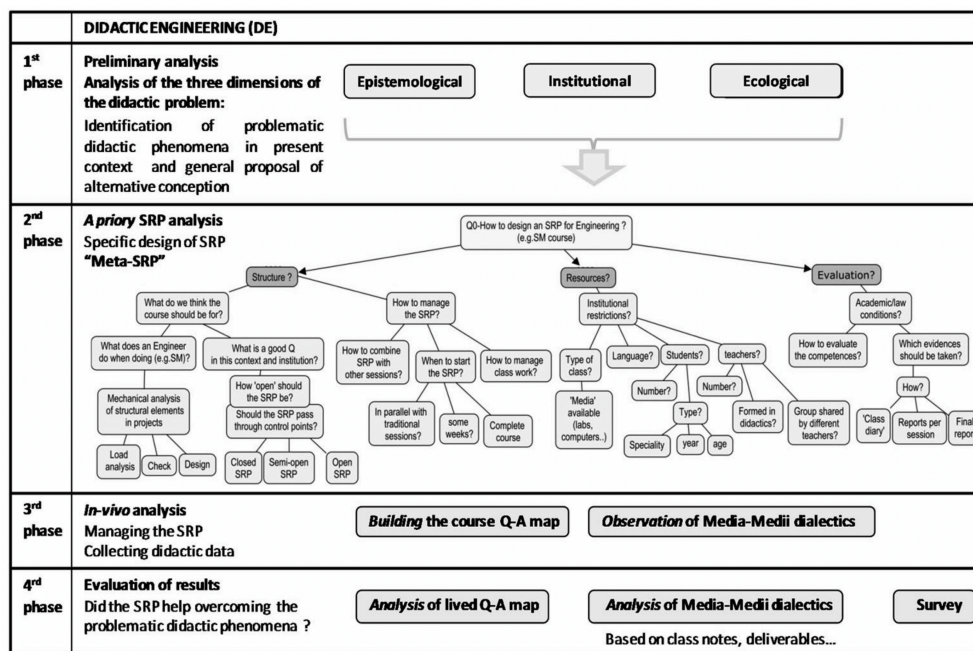
Throughout an SRP, students are looking for answers to intermediary questions in lectures, in (text)books, on the internet, etc. All these “social system pretending to inform (...) some group of people about the natural or social world” are media in the sense of ATD, and they can become components of the inquiry “milieu”, under the “dialectic of media and milieus” (Chevallard, 2006, p. 9). During the study of questions, the notion of systems and models (Barquero, Bosch & Gascón, 2019, p. 9) is also to play an important role, especially in the context of interface between disciplines we consider in this paper. The dialectics of media and milieu and systems and models should be important theoretical tools to monitor our experimentation.

## Research questions

By means of this theoretical framework, we will now consider the following research issues: what are the conditions and constraints imposed on the implementation of an SRP at this level of studies, at the interface of mathematics and physics? What lies behind the choice of its generating question? How to avoid visiting works while maintaining research and teaching objectives?

## Methodology

In order to prepare this module, we proceeded according to the Didactic Engineering methodology as it was applied to SRPs by Bartolomé et al. (2018): see fig. 1.



**Fig. 1: The Didactic Engineering methodology applied to the implementation of an SRP (Bartolomé et al., 2018, p. 5)**

First, we performed a preliminary analysis consisting in three parts:

- the study of epistemological aspects underlying the project, based on the study of primary and secondary sources in history of quantum physics and mathematics
- the institutional conditions of the SRP, based on interviews with professors at the University of Montpellier
- the ecological context of the SRP, based on the same interviews as well as analyses of relevant course material

Then, we carried out the a priori analysis which “includes the specific design of the SRP, including the selection of a generating question starting the study process, taking into consideration the conditions and constraints identified in the first phase.” (Bartolomé et al., 2018, p. 6).

## **PRELIMINARY ANALYSIS**

### **Epistemological aspects**

From a study of historical epistemology (in the sense of « Histories of epistemic things » (Feest and Sturm, 2011, p. 288)) dedicated to the interplay between quantum theory and functional analysis during their respective early developments (1900-1930), we could extract several aspects that seem particularly relevant for the design of this SRP.

First, mathematics and physics exerted a mutual influence throughout the development of quantum mechanics. In particular, from 1925-1927, mathematicians could draw results from questionings arising from physics (Lacki, 2011). This culminated in von Neumann's introduction of Hilbert spaces as an abstract structure encompassing the variety of theories of quantum mechanics known at that time. So, a physical context may be fruitful to introduce higher level mathematics via the dialectics of questions and answers.

Then, models and formulations were abundant at the interface with successive attempts at unifications and simplifications, which the structuralist stance in mathematics finally helps to achieve (von Neumann, 1955, p. 28). We should thus expect the dialectics of objects and structures (Hausberger, 2017) will play a role.

Finally, interviews we performed with professors of physics and mathematical physics at the University of Montpellier lead us to consider such phenomena still occur in their day-to-day activity. More precisely, though there seems to exist a vivid practice at the interface between mathematics and physics when considering “scholarly knowledge”, this does not seem to be the case anymore regarding “taught knowledge”, showing a lack in the “didactic transposition” at the interface (Chevallard & Bosch, 2020b). This is one of the issues this SRP attempts to address.

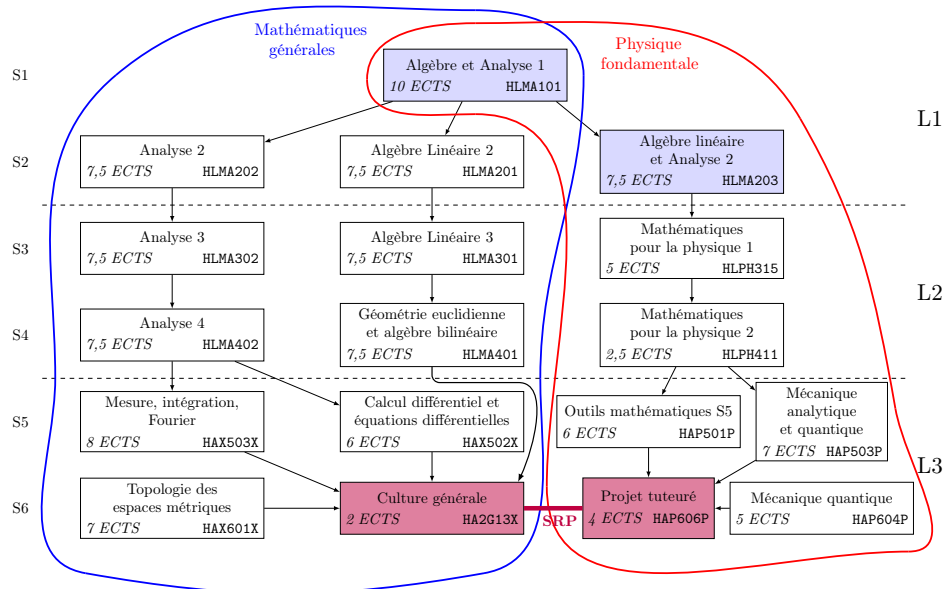
Quantum computing appeared to be a subject that could put at play the aforementioned epistemological aspects. In addition, as the setting of an SRP should “be regarded – by the students, by their teachers (...) – as crucial to a better understanding and mastery of their lived world” (Chevallard, 2006, p. 7-8), quantum computing seemed to fit all the more. Lastly, specific institutional conditions in Montpellier would facilitate the implementation of an SRP on this topic.

### **Institutional conditions**

Indeed, in 2019, the technology company IBM initiated a partnership with the University of Montpellier, in which several members of both the mathematics and physics departments are involved. This is why, in the first place, quantum computing emerged as a potentially workable setting for the upcoming SRP. However, it was yet to be found how to include such a project into the sequence of teaching units (TU) taught in this university. In Montpellier, physics and mathematics curricula are quite detached (see fig. 2). The bachelor (*Licence*) lasts three years (L1-L3), and it is divided into six semesters (S1-S6). Some TU taken by physics students are

nevertheless taught by mathematicians: we filled them in blue. Lastly, our experimentation takes place during the sixth semester (S6), as a mixed TU (see below). It is shown in purple in the figure.

Several constraints weighted on the institutional implementation of the SRP, as it was to develop at the interface between mathematics and quantum mechanics. Firstly, physics students take their first quantum mechanics class at the fifth semester (S5, *Mécanique analytique et quantique*). Then, during the sixth semester, on the one hand, mathematics students have to take a “common knowledge” class (*Culture générale*), whereas, on the other hand, physicists take a TU devoted to doing an experimental research project supervised by a professor (*Projet tuteuré*). So, we could set up the SRP as a mixed teaching unit registered with both math and physics departments. It is the only such TU at this level of studies at the University of Montpellier. This way we could project nine two- to three-hour time-slots.



**Fig. 2: Overview of some TU from the “General Mathematics” and “Fundamental Physics” programs that are taken by students participating in the experimentation**

With this organisation came further conditions and constraints, mainly from the physics department. For instance, evaluation should include a peer-reviewing process among students taking this TU. This was actually a favorable condition. Indeed, defining the recipients of the answer to be given to the generating question is a crucial step. So, we extended the physics instructions to all students: to write a report and make a presentation their third-year colleagues could read and understand.

## Ecological context

Physics students has already had a quantum mechanics course during the first semester, whereas students from mathematics never did (at least at the University of Montpellier). The content of this quantum mechanics course corresponded to parts of

the first two chapters of *Quantum Mechanics, vol. 1* by Cohen-Tannoudji, Diu & Laloe (1991) which is often used in introductory quantum mechanics courses in France. We could analyse its content in a study which provides additional inputs into the preliminary analysis (Lombard et Hausberger, to appear).

For instance, a tension exists between the abstractness of the Hilbert space formalism and the necessity for students to develop operative skills in order to study actual physical systems or models. In particular, the *raison d'être* of some elements of the formalism is never specified (why an infinite number of dimensions? why Hilbert spaces and not Banach spaces or even pre-Hilbertian spaces, which are studied in second year by math students?). Of course, this is consistent with the fact this course provides an example of the paradigm of visiting works. By shifting towards the paradigm of questioning the world, this is another aspect this SRP wishes to address.

This tension particularly transpires when it comes to the passage from finite- to infinite-dimensional Hilbert spaces, as was actually acknowledged by a professor we interviewed. As a matter of fact, the course we studied began with the infinite-dimensional case, even though the students never encountered them, especially to solve eigenvalue problems as is customary in quantum mechanics. In this SRP we chose to go the other way, starting with what students may already be familiar with and going from finite- to infinite-dimensional models (for such a treatment in the common paradigm of study, see for instance Le Bellac (2013)).

## A PRIORI ANALYSIS

### Resources and assessment

In light of the previous considerations we undertook the a priori analysis of an SRP at the interface between mathematics and quantum mechanics, with quantum computing as its background. It would include both classroom sessions (six) and computing sessions (three), the latter being taught by an IBM representative already working with the University of Montpellier (denoted by  $P_{qc}$ ). Besides, to enforce the conveyance of mathematics and physics content, we chose to let one professor from each field ( $P_\varphi$  and  $P_\mu$  for physics and mathematics, respectively) teach once during the SRP. Finally, in order to comply with the constraints imposed on this TU by each department, we decided students would work in groups of three, by curriculum. So, the set of students may be denoted by  $X = \{g^3_{\mu 1}, g^3_{\mu 2}, g^3_{\varphi 1}, g^3_{\varphi 2}\}$  (where  $g^3_\mu$  and  $g^3_\varphi$  are math and physics trios, respectively) and the team of teachers by  $Y = \{P_{qc}, P_\varphi, P_\mu\}$ . So, we end up considering the didactic system  $S(X, Y, Q_0)$  (Chevallard 2019, p. 72), where  $Q_0$  is our generating question (see next section).

Before addressing the choice of the generating question, we can complete the answers to the following questions in our meta-SRP: “Q2) Human and physical resources” and “Q3) Student assessment” (Bartolomé et al., 2018, p. 6-7, and fig. 1 here), which otherwise almost fully ensue from our preliminary analyses. First, we planned on collecting question-answer maps per group in order to monitor the evolution of students’ questioning, actually letting students draft them themselves.



Then, complying with the institutional constraints coming from physics, we opted for a final answer in the form of a written report and an oral presentation. This implied, in particular, to let a significant part of the investigation on the “other stage” (Chevallard, 1998, p. 17). This is a questionable choice, as our research would gain in monitoring as closely as possible students’ questioning. On the other hand, it enforces the customisation of the final answer  $A^\heartsuit$  produced by students, making it indeed close to their hearts (Chevallard, 2019, p. 100).

### **Generating question**

We may now answer the “Q1) The SRP structure” block of the meta-SRP. The most critical point here is our objectives are always two-fold, as this SRP is at the same time a course and a research experiment (see also Markulin, Bosch, Florensa & Montañola, 2022, p. 3). For instance, regarding Hilbert spaces, we wish to put two phenomena at play: on the one hand, in relation to the paradigm shift we investigate, we wish to reinstall the *raison d’être* of their use in quantum mechanics; on the other hand, in the context of a PhD devoted to the didactics of algebraic structures, we wish to see them play a unifying and simplifying role. Actually, our epistemological analyses lead to the conclusion we could meet both these targets at once, provided the use of the structures comes as an answer to the generating question or to a question in the process.

As was already mentioned, the passage from the finite- to the infinite-dimensional settings is a crucial step in quantum mechanics, and on in which the Hilbert space structure plays an important role. The latter structure is indeed the good framework where the practice acquired in low dimension can the most easily be transposed to infinite dimension. Though it is common in introductory quantum mechanics to cover several situations where both finite and infinite-dimensional frameworks play a role, even though the peculiarities of the infinite-dimensional case are often hidden — and with it the necessity of a more general, hence abstract, framework (see Lombard & Hausberger, to appear). This is particularly so when it comes to quantisation (eigenvalue problems), as show for instance most treatments of the infinite quantum well, the simplest realistic model of a quantum computer. That is, quantisation in this case can only be accounted for on the base of truly quantum mathematics.

Actually, the context of quantum computing puts at play numerous models to describe the machine, from two-level systems to anharmonic oscillators. Each time, the question of quantisation is crucial, as quantum bits are quantised states spanning two levels (usually marked  $|0\rangle$  and  $|1\rangle$ ). So, working on models of quantum computers surely brings about many epistemological aspects our analyses have highlighted so far. Consequently, we put forward the following generating question  $Q_0$ : *In what respect are quantum computers indeed quantum?* As it stands, the questioning is however quite open, so we chose to add three questions, as is for instance done in Barquero (2009, p. 198):

$Q_{0a}$ : *What are quantum bits and how can you calculate with them?*



*Q<sub>0b</sub>: What phenomena allow quantum computers to operate?*

*Q<sub>0c</sub>: What characteristics of quantum systems are shared by quantum computers?*

All that being said, a pitfall consists in wishing students go through certain questions for the sake of our research (for instance), thus leading us back to the “monumentalist” paradigm we wished to quit (on this point, see Chevallard, 2006, p. 8). Consequently, given this setting, we can only hope students will opt for a mathematically-leaning answer to our generating question.

## Media and models

The choice of media gives us further latitude though, especially regarding how open the SRP will be. Indeed, in the case students actually enrich their milieu with them, media could channel the questioning towards given works, be they visited or not. So, media both increase the numbers of models of quantum computers under study and decrease the openness of the generating question, in order to balance its scope. During lab sessions, students will manipulate the software Qiskit. During class sessions, book excerpts or videos will enunciate facts about physical or mathematical models of quantum computers (such as the Bloch sphere, abstract two-level systems or quantum wells). In addition, the professors taking part in the experiment should play an important role in the media environment (see fig. 3). Lastly, we plan on producing tailor-made pieces of media, for instance to encourage the process of questioning about the links between the various models so introduced.

Date	Topic/model	Pedagogical and epistemic context
January 20 <sup>th</sup> , 2022	First encounter and opening of the questioning	Video presentation of quantum mechanics and quantum computers
January 27 <sup>th</sup>	First lab session	Taught by $P_{qc}$ : first manipulations with Qiskit
February 3 <sup>rd</sup>	Questions following the first lab session	
February 10 <sup>th</sup>	Double quantum well	Intervention of $P_{\phi}$ : two-level systems in two-dimensional complex vector space
February 17 <sup>th</sup>	Second lab session	Taught by $P_{qc}$
February 24 <sup>th</sup>	Infinite well	Discrete infinite spectrum of eigenfunctions
March 3 <sup>rd</sup>	Winter break	
March 10 <sup>th</sup>	Third lab session	Taught by $P_{qc}$
March 17 <sup>th</sup>	Finite Well	Intervention of $P_{\mu}$ ; spectrum of eigenfunctions, both discrete and continuous
March 24 <sup>th</sup>	Conclusion	Hilbert space as a common framework
April 15 <sup>th</sup>	Peer-reviewing report	
April 18 <sup>th</sup>	Written report	
April 21 <sup>st</sup>	Oral exam	

**Fig. 3: Practicable sequence of sessions and prefiguration of media and models at play throughout the SRP**

## CONCLUSION

In this article, we have given a concrete instance of the implementation of both the preliminary and a priori analyses of an SRP set up at the interface between mathematics and quantum mechanics. We have shown by way of example that careful studies of the institutional and epistemological contexts are in order. Lastly, we have described the rationale underlying our choice of the generating question, which is a crucial step in the design of an SRP.

The module we set up is “a subject totally organised as an SRP” (Bosch et al., 2020), which means in particular that no course adopting the paradigm of visiting works supports it. As a consequence, it seems necessary to find a balance between leaving the students’ questioning totally open (which amounts to leave aside tailored planning of learning as well as research goals) and closing it so much that the generating question  $Q_0$  “becomes a mere decorative and opportunistic artefact” (Barquero, 2009, p. 93). So, we have arranged the conditions for a cohabitation of the two main paradigms. Indeed, as put Chevallard (2011, p. 40), the development of the paradigm of questioning the world “does not cancel the paradigm of visiting pieces of knowledge (*le paradigme de la visite des savoirs*), but rather places it otherwise, at both epistemological and didactical levels”. Organising this space needs careful preparation.

## REFERENCES

- Barquero, B. (2009). *Ecología de la Modelización Matemática en la enseñanza universitaria de las Matemáticas*. [Doctoral dissertation, Universitat Autònoma de Barcelona]. <http://hdl.handle.net/10803/3110>
- Barquero, B., Bosch, M., & Gascón, J. (2019). The unit of analysis in the formulation of research problems: The case of mathematical modelling at university level. *Research in Mathematics Education*, 21(3), 314–330.
- Bartolomé, E., Florensa, I., Bosch, M., & Gascón, J. (2018). A ‘study and research path’ enriching the learning of mechanical engineering. *European Journal of Engineering Education*, 44(3), 330-346.
- Bosch, M., Barquero, B., Florensa, I., & Ruiz-Munzon, N. (2020). How to integrate study and research paths into university courses? Teaching formats and ecologies. In T. Hausberger, Bosch, M., & Chellougi, F. (Eds.), *INDRUM2020 Proceedings* (pp. 167-178). University of Carthage and INDRUM.
- Chevallard, Y. (1998). Analyse des pratiques enseignantes et didactique des mathématiques: l’approche anthropologique. *Actes de l’Université d’été de didactique de Clermont-Ferrand* (pp. 91-118). IREM.
- Chevallard, Y. (2006). Steps towards a New Epistemology in Mathematics Education. In M. Bosch (Ed.), *Proceedings of CERME4* (pp. 21–30). Ramon Llull University.

- Chevallard, Y. (2011). La notion d'ingénierie didactique, un concept à refonder. Questionnement et éléments de réponse à partir de la TAD. In C. Margolinas (Ed.), *En amont et en aval des ingénieries didactiques. XVe école d'été de didactique des mathématiques* (pp. 81–108). La Pensée Sauvage.
- Chevallard, Y. (2015). Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counter Paradigm. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer.
- Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, 12, 71–114.
- Chevallard, Y., & Bosch, M. (2020a). Anthropological Theory of the Didactic (ATD). In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 53–61). Springer.
- Chevallard, Y., & Bosch, M. (2020b). Didactic Transposition in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 53–61). Springer.
- Cohen-Tannoudji, C., Diu, B., & Laloe, F. (1991). *Quantum Mechanics, Vol. 1*. Wiley.
- Feest, U., & Sturm, T. (2011). What (Good) is Historical Epistemology? Editors' Introduction. *Erkenntnis*, 75(3), 285–302.
- Hausberger, T. (2017). La dialectique objets-structures comme cadre de référence pour une étude didactique du structuralisme algébrique. *Éducation et didactique*, 11(2), 131–151.
- Hausberger, T., Bosch, M., & Chellougi, F. (Eds.). (2020). *INDRUM2020 Proceedings*. University of Carthage and INDRUM.
- Lacki, J. (2011). From Matrices to Hilbert Spaces: The Interplay of Physics and Mathematics in the Rise of Quantum Mechanics. In K.-H. Schlote & M. Schneider (Eds.), *Mathematics meets physics, A contribution to their interaction in the 19th and the first half of the 20th century* (pp. 297–350). Verlag Harri Deutsch.
- Le Bellac, M. (2013). *Physique quantique, Tome I : Fondements*. EDP Sciences/CNRS Éditions.
- Markulin, K., Bosch, M., Florensa, I., & Montañola, C. (2022). The evolution of a study and research path in Statistics. *ÉpiDEMES*, 1, 1-17.
- Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics*, 83(2), 267–284.
- Von Neumann, J. (1955). *Mathematical foundations of quantum mechanics*. Princeton University Press.