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Informal learning situations in the context of mathematics studies - development of an analysis framework

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Abstract: In mathematics-related university study programmes, self-study has a special importance. This gives rise to individual and self-directed learning situations that trigger phases of informal learning within the formal learning contexts of mathematics studies. For a differentiated description of such learning situations, an analysis framework is presented that enables an analysis of the subphases of individual learning actions in a gradual spectrum between formal and informal. Using an exemplary learning situation, the framework presented allows a detailed view of independent learning in mathematics studies and is intended to identify starting points for the promotion of individual, self-directed learning processes.

Keywords: Teachers' and students' practices at university level, Transition to, across and from university mathematics, digital and other resources in university mathematics, informal learning, analysis framework.

INTRODUCTION

In the formal context of university mathematics, self-study takes on a special significance, whereby various parts of student learning actions take place in individual and self-designed learning situations, both within and outside of curricular and didactic frameworks (Liebendörfer, 2018). Such learning situations are often referred to as *informal learning*. According to Jadin et al. (2008) this term describes individual, self-initiated and self-regulated acquisition of knowledge, which is generally distinguished from *formal learning*. The latter designates institutionalised and structured learning that takes place within the framework of educational institutions and leads to certification.ⁱⁱ

In recent decades, the benefits of informal learning contexts for sustainable knowledge acquisition have been repeatedly demonstrated empirically (e.g. BMBF, 2001; Cedefop, 2003; Chrishol et al., 2005). However, it remains unclear how specific learning situations (such as the understanding of a certain mathematical concept) are formed within different learning contexts (such as lectures, tutorials, study groups) (Jadin et al., 2008, p. 170). Moreover, recent publications express that the dichotomous view on formal vs. informal learning, as described above, is “less informative than the differentiated analysis of the various dimensions in which learning activities vary”. (Callanan et al., 2011, p. 648, author's transl.). In this sense, concepts have been proposed which describe the tension between formal and informal as gradual and do not refer macroscopically to entire learning contexts, but rather to individual learning

situations and their specific learning actions (e.g. Arnold, 2016; Callanan et al., 2011; Jadin et al., 2008).

Aiming towards analysing such individual, mathematics-related learning situations and the inherent informal learning, it's necessary to classify independent learning actions of mathematics students in the field of tension between formal and informal. With the help of a differentiation between learning contexts (macro-level) and individual learning situations embedded in them (micro-level), an analysis framework will be presented in the following. It looks at learning situations and their sub-phases and enables a classification of those within a gradual spectrum between formal and informal. Such an analysis can reveal new, individual starting points for support measures. This approach ties in with recent publications that highlight educational successes as well as positive social and personal developments through the promotion of informal learning in formal contexts (Peeters et al., 2014).

Derived from the primary research concern three subordinate questions arise: How can learning contexts and learning situations be distinguished from each other? In which sub-phases do learning situations proceed (especially in mathematics studies)? Which gradual spectrum describes the field of tension between formal and informal learning in a useful way?

Based on this line of thought, in the following chapters a gradual spectrum between informal and formal learning in the context of expansive learning processes will be discussed before a subdivision of learning situations into analysable sub-phases is proposed. Afterwards the application of the proposed framework will be exemplary presented and possible implications for support measures in mathematics-related studies will be discussed.

INFORMAL LEARNING

Proceeding from the first description by Dewey in 1899 (cf. Archambault, 1966), over the last century numerous definitions of and perspectives on formal and informal learning were stated. The development of the terms and their interrelationship has already been described in several review articles (e.g. Harring et al., 2018; Rohs, 2016), although there is still no general and comprehensive definition (Jadin et al., 2008). However, Jadin et al. (2008) summarise the demarcation as follows:

“Formal learning is institutional, highly structured, takes place within the framework of educational institutions and is concluded with a certificate. [...] Informal learning can [...] take place in institutions but is characterised by low structuring and does not lead to a certificate of completion. The initiative and control of learning is not dependent on an institution, but lies in the hands of the learners themselves.” (Jadin et al., 2008, pp. 170-171, author's transl.)

Depending on the discipline, the descriptions of the terms are more open- or closed-ended and the used terminologies vary. These differences can usually be explained by the diverse contexts of application within the disciplines and refer primarily to the

inclusion or exclusion of the following characteristics: Intentionality, structure and structuredness, institutional dependency, learning for certification, self-direction as well as randomness of learning (cf. e.g. Livingstone, 1999; Molzberger & Overwien, 2004; Münchhausen & Seidel, 2016; Rohs, 2016; for informal learning in the context of mathematics learning see e.g. Pattison et al., 2016). Nevertheless, the definitions have a conceptual dichotomy between formal and informal in common, which can be traced back to the focus on learning contexts. These are understood as the overarching, macroscopic frameworks and environments of learning, e.g. university studies themselves, lectures, tutorials, student interactions or exercises (see below, Jadin et al., 2008).

LEARNING CONTEXTS AND SITUATIONS IN MATHEMATICS STUDIES

Studying mathematics, whether as a stand-alone course or as a (sub)module of another, is fundamentally different to the forms of teaching and learning in school. In addition to participation in curricular courses (e.g. lectures, seminars, exercise sheets, examinations), university students are required to learn in “substantial self-study” (Liebendörfer, 2018, p. 342). Its intensity and scope differs considerably from preparation and follow-up work in school (e.g. homework), since it includes not only the training of familiar procedures, but rather independent problem-solving and special precision through formalism and abstraction (Hochmuth et al., 2021; Liebendörfer, 2018). In addition, the high complexity and barriers of comprehension make independent learning university mathematics indispensable (Liebendörfer, 2018; Pritchard, 2015).

However, the self-study of mathematics often does not proceed in an unproblematic and straight forward manner. Due to the large amounts of complex and formalised subject content and methods, learning meanders between extrinsic and intrinsic motivation (Bauer et al., 2020). From the perspective of a subject-scientific theory of learning, such learning actions can be described as expansive and defensive. Defensive learning is “primarily externally controlled and [...] fact-bound” (Marvakis & Schraube, 2016, p. 212) and focussed on the achievement of an extrinsic goal (e.g. successful examination performance). In expansive learning the learning problem and thus the learning object are in the foreground of the learning action (Holzkamp, 1993). At the same time, expansive learning is at least indirectly influenced by interaction between teachers and learners. Marvakis and Schraube (2016) refer to this as the fluidity of learning:

“The learning process of individual subjects is always a social process and situated in relation to others, unfolding as a constant back-and-forth between learning and teaching in and between persons. This fluidity of learning and teaching forms a basic element of expansive learning and the nucleus of a productive and lively learning practice.” (Marvakis & Schraube, 2016, author's transl.)

If such an understanding of learning – from the perspective of the individual – is assumed, it is not sufficient to consider only its learning contexts for the detailed analysis of mathematics learning. Rather, individual learning situations, which as emergent processes contain the concrete moments of learning within those contexts, would have to be analysed in more detail: Solving a single problem, understanding a practice task or an unknown concept, getting stuck in a proof... Furthermore, the analysis would have to be done from the perspective of the individual (cf. Göller, 2020; Jadin et al., 2008).

Inspired by models of inquiry-based and self-regulated learning in the study of mathematics (Göller, 2020; Roth & Weigand, 2014; Wildt, 2009; Zimmerman, 2000) six inherent sub-phases of a concrete learning situation can be described: *learning occasion*, *goal*, *(subject-related) content*, *methodology*, *feedback* and *reflection*. Based on Holzkamp's "learning problem" (1993) the *learning occasion* describes the trigger, the problem, the call to action of a learning situation, e.g. concrete contents of a course, an exercise or a statement by a teacher or a peer. It leads to a *learning objective*, which names the desired gain of knowledge or the final state of the learning situation. The learning action, which can be expansive or defensive, can be divided into a content-related and a methodological component. The *subject content* refers to all subject-mathematical terms and procedures that are needed during the learning action to achieve the objective. The strategies used for this are summarised within the *methodology*. This includes conscious and unconscious, independent and group-based as well as assimilated, accumulated and acquired methods. *Feedback* is any kind of response to the four previous sub-phases by teachers, peers, media, experience or oneself (Hattie and Timperley, 2007, p. 81; cf. Pepin, 2014). Finally, *reflection* is the personal metacognitive perception, discussion and, if necessary, future adaptation of the learning process and/or its individual sub-phases.

Although the learning contexts of mathematics studies can be usefully described by a dichotomous demarcation of formal and informal learning, this is not possible with learning situations and their inherent subphases as described above: They move between defensive and expansive phases of learning, on the one hand through individual motivations and actions, on the other open to fluid interaction with learners and teachers inside and outside courses and sometimes have more, sometimes less, sometimes no reference to certification. Therefore, the analysis of learning situations requires less rigid, dichotomous criteria, but more "gradual characteristics" (Jadin et al., 2008, p. 171) in a field of tension between formal and informal. Besides Decius et al. (2021), Jadin et al. (2008) and Callanan et al. (2011) also describes Arnold (2016) such a scale.

DIFFERENTIATED ANALYSIS OF LEARNING SITUATIONS

In reference to Holzkamp (1993) Arnold describes informal learning as "the self-organised, often accidental biographical learning in which the person intentionally

strives for the transformative search for new and more functional solutions” (Arnold, 2016, p. 483) and differentiates it into three intertwined degrees of informalisation:

Implicit learning describes unconscious and inherent learning. It takes place in everyday situations as well as interactions between people. In this sense, it is the least self-determined, rather accidental and unavoidable learning. *Reactive learning* describes a conscious learning process that is experienced by the individual in response to a new problem or challenge. Finally, there is metacognitive *reflective learning* “in the aftermath or in preparation of experiences and actions” (Arnold, 2016, p. 486), which is aimed at optimising one's own behaviour. It represents the transition between informal and formal learning and can be observed in both (Arnold, 2016).

Together with the sub-phases described above and supplemented by the perspective on formal learning of Jadin et al. (2008), these degrees of informalisation create a possible basis for the differentiated analysis of individual learning situations in mathematics studies. Independent of the assignment of the superordinate learning context, concrete learning situations can thus be described in their subphases between *formal*, *reflexive informal*, *reactive informal* and *implicit informal*.

APPLICATION TO A MATHEMATICAL LEARNING SITUATION

In the following, the proposed analysis framework will be applied to a real learning situation in the context of university mathematics studies. Since empirical studies based on this framework will not be conducted until the winter semester 2022/2023, a sufficiently described situation from a study conducted by Heinrich and Hattermann (2021) will be used here as an example to enable an authentic and realistic application.

In the learning context observed by Heinrich and Hattermann, two fellow students deal with descriptive statistics by means of instructional texts and tasks in the digital learning environment Moodle as part of an assessed study and examination performance. Firstly, the arithmetic and harmonic means were introduced through definitions and explanations. The learners now work on application tasks, of which they only enter the solution into the digital learning environment and receive a binary evaluation (“correct”/“incorrect”). They have several attempts to solve the tasks.

In the learning situation relevant to this paper, one of the learners (L1) asks the fellow learner (L2) for support (Heinrich & Hattermann, 2021, pp. 182-183, author's transl.):

- L1: But why do I need the harmonic mean and the arithmetic mean? I don't understand that.
- L2: There [pointing to the screen] he always drives the same time and there [pointing to the other example] he always drives the same distance but takes different lengths of time.

Based on this information L1 looks at the definitions and explanations in the learning environment and tries to establish a connection between the means and the clues

received. L1 starts a new attempt to solve, the result of which is marked as “correct” by the digital learning environment. L1 makes notes.

Based on Jadin et al. (2008) the learning context can be described as formal, since the learners work on a task given by the teacher in connection with a curricular course, which is relevant for completing the course itself (part of the course work and content-related preparation for examination work), but consequently also relevant for the success of studying mathematics on university level.

Nevertheless, characteristics of informal learning are also recognisable: By implementing the tasks in a digital learning environment, the duration, pace, and location of learning as well as the process and methodology of knowledge acquisition are determined by the learners or the learning group. Self-organised learning or self-study is thus initiated. Within this framework, the two learners L1 and L2 decide to learn and work on the given contents and tasks at a common place and at the same time.

Based on this, the learning situation that arose between L1 and L2 will be classified in the following within the field of tension between formal-informal based on an interpretation of the speech acts using the proposed analysis framework.

The learning situation begins with L1's speech act. It can be assumed that it was preceded by a discussion about the task. In this, L2 seems to have assigned the arithmetic or harmonic mean to the two sub-elements of the task. This represents an incomprehensible step for L1, which can be classified as a learning occasion. Since, in summary, this is an affinitive, reactive and self-directed cognition on the part of L1, the occasion can be classified as reactively informal.

It is important to note that the learning occasion in this particular situation does not arise from the task itself, but only from the non-understanding of the steps of L2. Nevertheless, understanding the application of the theory and subsequently solving the task is the learning goal. This can be seen on the one hand from the concrete work on the task itself, and on the other hand from the exclusive use of the given material. Since the learning objective is curricularly specified and didactically prepared in the context of the course and the digital learning environment, it can be described as formal.

In order to achieve the goal based on the learning occasion, L1 first decides to enquire L2. This seems to be based on the situation analysed above, which also led to the learning occasion. The questioning here is L1's methodology to advance his learning. This is a self-directed reaction to the learning occasion and can be assigned to the reactive-informal.

However, the technical content chosen for comprehension is no longer controlled by L1. It is the speech act or the answer of L2 that determines what the content focus of L1 is directed to, here: The relationship between time, distance and the two means. This dimension is externally controlled – in relation to the prepared task – and can be described as formal with regard to L1.

From L1's reaction to L2's speech act, an indirect conclusion can be drawn regarding the dimensions feedback and reflection: Since L1 does not focus on the solution of the task after L2 has answered, but again looks at the technical contents of the digital learning environment, L2's explanations do not seem to have been sufficient to solve L1's difficulties in understanding. It can be assumed that intrinsic feedback (e.g. "I still haven't understood what he means.") led to the reflection to turn to the subject content again. Both steps can be understood as a follow-up to the first learning action, leading to an attempt at optimisation and are thus reflexive-informal actions.

Which intrinsic process is taking place in L1 can only be extracted indirectly from the example. Based on his action, it is reasonable to assume that L1 enters a secondary learning action and tries to link L2's statement about the dependence of the means on time and distance with the given subject content. Subphases can be derived from this, which on the one hand describe their own learning situation, but at the same time try to achieve the original learning goal.

Consequently, the secondary reactive-informal learning occasion just described arises as well as the secondary reflexive-informal learning goal of wanting to precisely establish this link to achieve the primary learning goal. The learning methodology, the multiple reading of the given learning material and the independent testing of knowledge on the task, represents a reactive-informal action, whereby further processes, which cannot be read from the situation description, may be running internally here. Since L1 refers exclusively to the digitally given materials and L2's statements, the subject content is formal. Also, L1 does not seem to be convinced of his result by his own attempt to solve the problem, but only by the feedback from the learning platform. Therefore, this sub-phase can also be described as formal. Based on L1's action of taking notes after the feedback, a reflexive-informal process seems to take place.

RÈSUMÉ

Based on the high significance of self-study and independent learning actions in mathematics-related study programmes, an analytical approach was presented in this paper that classifies the different subphases of individual learning situations in a gradual spectrum between formal and informal learning. Based on a subject-scientific approach, learning in mathematics studies was seen as an interplay between internal, intrinsic processes, interactions with learners and teachers as well as learning dispositions and materials.

The exemplary application based on a concrete learning situation shows the potential of such a detailed framework. Through its step-by-step breakdown, learning actions and the strategies applied in the process can be worked out and considered in a differentiated and detailed way. This enables a deeper understanding of learning strategies in dealing with subject content, methodologies, obstacles, and coping strategies. Based on this, new or adapted teaching-learning settings as well as indirect

support possibilities for independent learning, e.g. through strategies of research-based learning, can be developed. A possible starting point would be, for example, the primary learning occasion of L1, by looking more closely at the reason for L1's hurdle or comprehension problem. As a result, the learning material could be adapted to either be more detailed or to offer opportunities for independent research.

However, the application also highlights the need for a sufficiently detailed description of the situation and the effort required for the proposed hermeneutic approach. The planned empirical application will thus also be used to optimise and validate the framework as well as to elaborate an analysis methodology.

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ⁱ Website of the Research Training Group LernMINT: <https://lernmint.org> (last visited: July 1st 2022)

ⁱⁱ In this article, the discussion of formal and informal learning does not include non-formal learning processes, primarily because this term can contribute little to the learning situations considered here. For a definition of the term and its classification in the context of (in)formal learning, please refer to Rohs (2016).