

How to choose relevant mathematical content to address Klein's second transition? The case of the inner product

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INTRODUCTION AND RESEARCH QUESTION

Research dedicated to issues in relation with the transition between secondary and university education have been developed for long (Gueudet 2008). More recently, specific work has been carried out on what is referred to in the literature as “Klein's double discontinuity” (Winsløw & Grønbaek, 2013), and more specifically with “Klein's second transition” (Gueudet et al., 2016), that is the transition between academic mathematics and secondary mathematics for (future) teachers.

The report of TWG3 during INDRUM2020 highlighted Klein's second transition as a promising avenue of research in many regards. Among other things, the issue to find relevant topics with strong epistemological foundation that could allow teacher students to deeply understand crucial links between university mathematics and secondary mathematics in a professional perspective was particularly emphasized (Biehler & Durand-Guerrier, 2020). The research presented aims at contributing to the following related main issue: how to choose relevant advanced mathematical content to address or study Klein's second transition and to create and design capstone courses?

MAIN ASPECTS

The purpose of the proposed poster is to present a methodological and inductive preliminary work in order to highlight possible main challenges, tools or levers or hypotheses with respect to the above main issue. To do this, we focus on the notion of inner product in the Euclidean plane, as it is taught in secondary education in France. In this regard, the underlying theoretical framework is minimal and based upon didactic transposition between scholarly knowledge and knowledge to teach, and between knowledge to teach and taught knowledge.

By considering the case of the inner product, we first illustrate in which regard the relevance of a notion for future teachers with respect to Klein's second transition could be measured. For this purpose, several examples are developed in the poster that highlight different aspects – either related to university curriculum, secondary curriculum or to the relations between them (both directions) – that can help teacher students to mobilize a “higher standpoint on mathematics”:

- the choice of a definition, and the articulation between the definition and proof processes, these aspects being particularly rich in the case of inner product;

- the use of specific tools that shed light on the resolution of a problem: the example of the distance between a point and a line will be developed;
- the choice and use of an appropriate representation register, and the underlying flexibility to mediate between them;
- the importance to search for and solve inconsistencies and “vicious circles” in the curricula.

HYPOTHESES AND CHALLENGES

The above considerations give us the opportunity to formulate hypotheses with respect to teachers or future teachers that are consistent with respect to Klein's second transition. One of those is the fact that academic knowledge must allow teachers to have tools to control and analyse the choices related to didactic transposition on the one hand, and "everyday teaching" on the other hand. Also, in our view, academic knowledge is knowledge that is useful for the profession of mathematics teacher. It is the responsibility of mathematics education researchers to find relevant content for Klein's second transition, to create links and finally to design situations or capstones courses with respect to these dimensions.

In our view, this preliminary study highlights a new issue that need to be addressed in future didactic research: the “coherence” of taught knowledge in classrooms from the point of view of didactic transposition. In view of the various aspects presented above, it seems to us that this is a relevant criterion for designing capstone courses. This study could also be a first step to constitute a reference for researchers to model the possible mathematical activities for the considered notions within several theoretical frameworks.

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