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Activity Theory as a Base for Course Design in Pre-Service Teacher Education: Design Principles and Their Application in Two Examples

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Observations in practice show that pre-service teachers do not always experience and acquire mathematics subject knowledge and mathematics education knowledge in such a way that they are ready to effectively use this knowledge in their future careers when designing and implementing lessons. We develop an activity-theoretical framework that contributes to describing and explaining underlying discontinuity obstacles, and, as a developmental contribution, we use the framework to formulate and implement design principles for courses in pre-service teacher education aimed at counteracting discontinuity phenomena in the area of argumentation and proving.

Keywords: Teachers' and students' practices at university level; Transition to, across and from university mathematics; Teaching and learning of logic, reasoning and proof; Activity theory.

INTRODUCTION

Discontinuities between school mathematics and university mathematics have been recognized as a problem at least since Felix Klein's time (Klein, 1908). The first transition (secondary-tertiary) has been the focus of considerable attention in recent years (see Gueudet, 2008, for an analysis of phenomena and causes). A number of projects try to mitigate this “first discontinuity” by helping students to establish connections between the two “worlds”, for instance by engaging them in interface tasks (Bauer & Partheil, 2009) that address the specific differences. The second critical transition, which occurs when pre-service teachers (PST) go back to school after graduation, has received less attention so far. The issue is whether students have acquired mathematical content knowledge and mathematics education knowledge in such a way that they can effectively use this knowledge in their future careers to design and implement lessons. Experience from practice, as well as from capstone courses as described by Winsløw & Grønbaek (2014), shows that one should not expect that this occurs automatically. In extreme cases, novice teachers may regard academic knowledge as unworkable in the “real world” of the classroom (Cavanagh and Prescott, 2007). They are then susceptible to the “familiarity pitfall” (Feiman-Nemser and Buchmann, 1985, p. 56), i.e., they might identify teaching with classroom practices that they experienced as pupils themselves – just as already Felix Klein observed.

The transition from university to school in particular means a transition from one mathematical practice to another, where PSTs experience discontinuous changes in various respects. Our hypothesis is that part of the problem of the second discontinuity is de facto based on a poorly mediated, possibly distorted experience of *differences between core activities* within different mathematical practices at school and at

university. In the present paper we focus on the core mathematical activities of proving and argumentation. The following two desiderata motivate our work. (1) Provide a theoretical framework that would allow to grasp (that is, observe, describe, and explain) the practical impressions of discontinuity effects regarding the second transition as possibly interconnected general phenomena. (2) Find starting points for developing suitable formats for PST training courses in order to effectively address the respective issues of teacher education.

First, as a theoretical contribution, we develop an activity-theoretical framework that helps to analyze differences and commonalities between mathematical practices at school and university, and contributes to describing and explaining discontinuity obstacles. Second, as a developmental contribution, we use the framework to formulate design principles for courses in PST education and we will present two course designs where we have implemented these principles. Our concrete implementations show in particular that the three principles can be employed in a longitudinal setting (i.e., in a sequence of consecutive modules) as well as in a single module.

ACTIVITY THEORY AS A GENERAL CONCEPTUAL FRAMEWORK ON CORE MATHEMATICAL ACTIVITIES AND DISCONTINUITY PHENOMENA

In our approach, we build upon the work of Leontjew (1982) as developed further by Lompscher and Giest (see Bruder and Schmitt, 2016; Giest, 2008). Based on foundational work in the 1980s, a spectrum of activity-theoretical approaches emerged in the mathematics education literature, e.g. Engeström (2001), Jaworski et al. (2017), Cerulli et al. (2005). The approaches differ in particular in terms of the fields of activity they address. In our work, we consider argumentation and proving as a field of activity in school and university mathematics to study respective discontinuities between them.

From the point of view of activity theory, human activities always have a “dual character”: Namely, on the one hand as activities in the context of a communal, collaborative practice, and on the other hand as individual action.

Mathematical argumentation and proving as activities in collaborative practices

In our application and adaptation, we first look at mathematical argumentation and proof as activities in the context of collaborative practices. We distinguish three structural components: Motive, object and ways and means of action. The interplay of these three components in terms of activity theory can be described – in a very compact form – as follows. The motive drives activities that are directed towards an object. The object thus becomes the object *of* the activity. The concrete goals that can be pursued in an activity, as a realization of the motive, depend on available and suitable ways and means of action.

Along these three structural components, we can contrast two mathematical practices that are relevant for PSTs, university mathematical practice and school mathematical

practice, and identify characteristic differences that are often experienced by students as dominant, for example:

- On the motive level, the deductive derivability of statements within globally ordered mathematical theories plays an important role in university mathematics, whereas school mathematics is more concerned with the verification of statements, which is conducted, if at all, with reference to locally ordered systems of statements.
- On the level of objects, we find a variety of explicit objects of proving in university mathematics, and in contrast often only implicit, hidden objects of argumentation and proving in school mathematical practice.
- On the level of modes and means of action, a difference between the two practices that is experienced as dominant can be identified with regard to the different roles of heuristic or generic arguments in the two practices.

In line with our hypothesis stated in the introduction, PSTs who experience such differences as dominant might not be able to integrate the meaning that argumentation and proving have as core activities in these different mathematical practices into their individual activity in a coherent way. Hence, they might also struggle to stage these activities properly in class, thus going through a “second discontinuity” in a problematic way.

Mathematical argumentation and proving as individual action

In order to capture this issue fully from the point of view of activity theory, we have, in addition to the collaborative community perspective, to consider the activities of mathematical argumentation and proving also with respect to individual action. Under this perspective, the triad of constitutive elements is similar to activity in collaborative practice, with a level of motives, a level of objects, and a level of ways and means of action.

We conceptualize the individual concretization of these three levels and their interplay in individual activity as follows (see Fig. 1): The objects that the individual has made their own and the available repertoire of ways and means of action form individual preconditions for action. These are at the same time preconditions for the individual to be able to consciously set concrete goals of action, and, accordingly, to be able to act on the object directed towards these goals. Such consciously set goals for action are influenced by superordinate motives for activity, which the individual, however, is not usually aware of at the stage of their concrete action. Rather, the goals mirror individual constructions of meaning in relation to the activity.

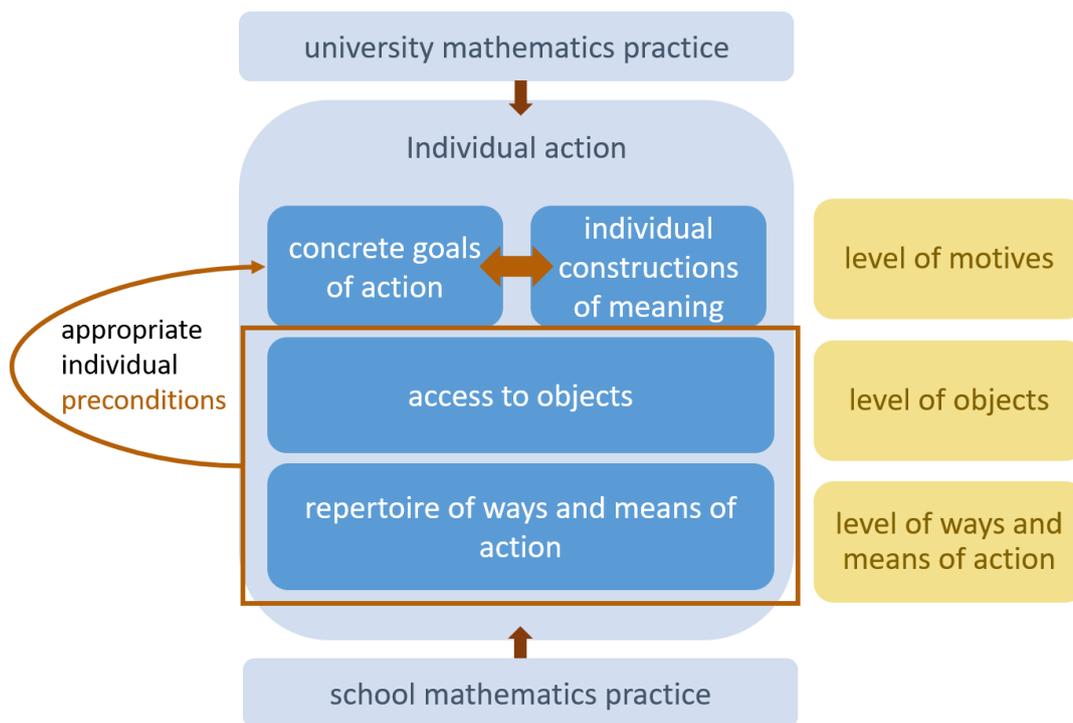


Fig. 1. Constitutive elements of individual action

First results of explorative empirical investigations of PSTs' lesson planings and stagings, which we carried out on the basis of this activity theoretical framework (Bauer & Müller-Hill, 2022), indicate that there is indeed a need for action in PST educational practice: We exhibited phenomena and patterns in PSTs' stagings that indicate that they apparently lack effective background motives and corresponding goals and constructions of meaning, appropriate ways of action and access to suitable objects, in order to stage argumentation and proving activities in a meaningful way in the mathematics lessons they plan and conduct.

ACTIVITY THEORY AS A DESIGN BASE: DESIGN PRINCIPLES AND THEIR APPLICATION IN TWO COURSE DESIGNS

We understand the empirical observations mentioned above as a variant of double discontinuity, namely as an impact of transition obstacles that emerge between three areas of action and experience: the mathematics lessons experienced at school, university mathematics and didactics, and PSTs' own lesson stagings at school.

Accordingly, our main idea in using activity theory as a design base is that the presented activity-theoretical considerations on constitutive elements of core mathematical activities, together with empirical results such as those reported above, form a basis for the development of design principles for PST courses. The application of such principles is intended to help to turn discontinuity experiences productively, and hence to mitigate second transition issues such as, e.g., the observed phenomena described in Bauer & Müller-Hill (2022).

Design Principles

We suggest the following three design principles, each of which is formulated with reference to one of the constitutive elements of activity. A combined application of these principles is intended to counter the specific discontinuity issues that are revealed by the activity-theoretical explanations for the empirical phenomena mentioned in the previous section.

Principle 1: Engage PSTs in getting to know, applying, and assessing a variety of appropriate objects and ways/means of action regarding core mathematical activities, both in the PSTs' role as mathematically active individuals and as members of the collaborative mathematical practices of school and university.

Principle 2: Engage PSTs in explicating, reflecting and relating motives and goals of core mathematical activities, with respect to individual activity and with respect to activity within the collaborative mathematical practices of school and university.

Principle 3: Engage PSTs in explicating and reflecting content-related and didactical decisions within their own mathematical working processes as well as within their planning and implementation of mathematics lesson stagings, with the aim to support the development of appropriate and coherent individual constructions of meaning for core mathematical activities.

We will now show how these three design principles can be brought to fruition in concrete course designs.

ProPraxis – A Longitudinal Implementation of the Three Design Principles

The design principles are in this case implemented in a series of consecutive modules (see Fig. 2). In these modules, proving, learning to prove, and teaching proof is addressed both on the object level and on the meta level.

The subject-matter modules in the first three semesters (Linear Algebra and Analysis) serve as the starting point. They are related to Principle 1, as they engage students in getting to know a variety of appropriate objects as well as ways and means of action – both as mathematically active individuals (e.g. when solving homework problems) and as members of a collaborative practice of university mathematics (e.g. when they are exposed to the norms of the discipline through lectures and books).

The subsequent mathematics education module *ProfiWerk* (the title is a German shorthand for “*Professionalization Workshop*”) then implements in its first part the Principles 1 and 2, as it engages students in applying and assessing a variety of appropriate objects and ways/means of action, as well as in explicating and reflecting motives and goals of core mathematical activities. For instance, when the Toulmin model (Toulmin, 1958) is used to analyze proofs from university mathematics courses as well as proof products by pupils, students assess appropriate objects and establish connections between school and university practices. Or, when they practice and reflect on generic proofs, they connect proving as experienced in subject-matter modules with

proving as an activity to be staged in the classroom. (See Bauer, Müller-Hill & Weber, 2021a, for details on Part 1 of ProfiWerk.)

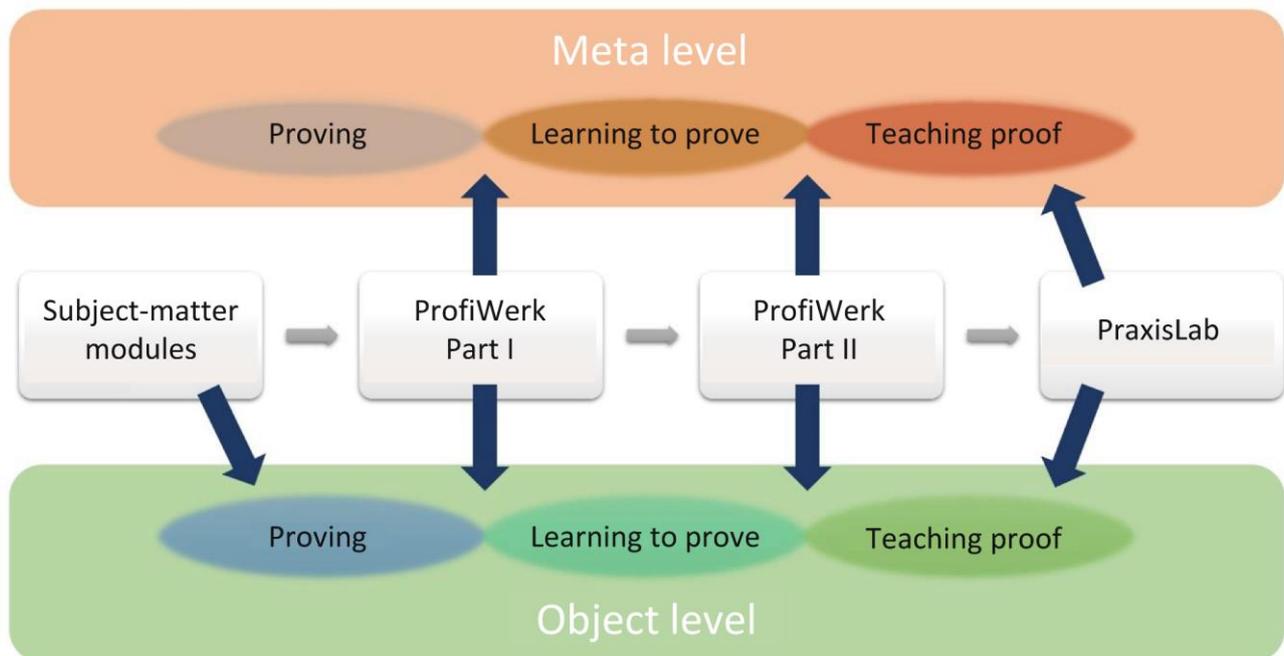


Fig. 2: Components and sequential structure of the longitudinal implementation

The second part of *ProfiWerk* focuses on problem solving as a core mathematical activity. Students solve problems and then analyze their problem solving process in terms of the heuristics strategies they used and in terms of the problem solving phases that occurred. They thus analyze and reflect on motives, objects, content-related decisions, goals, and ways and means of action in their working processes. So, in addition to Principles 1 and 2, Principle 3 is implemented here in a focused way. It is a decisive feature of the course that students work on problem tasks both at university level and at school level. Through explicit assignments they compare the respective problem solving processes, thus comparing and connecting the two practices. (See Bauer, Müller-Hill & Weber, 2021b, for details on Part 2 of ProfiWerk.)

The final module of the sequence, *PraxisLab*, consists of a field experience and an accompanying seminar. Here students get the opportunity to observe lessons and to try out their own implementations, which they plan based on motives, goals, suitable objects, and ways and means of action developed in ProfiWerk. As this entails explicating and reflecting content-related and didactical decisions within their own working, planning and staging processes, Principle 3 is fully implemented in this module.

ABEB – An Implementation of the Three Design Principles in a Single Module

The seminar design of ABEB (ABEB is a German shorthand for “Argumentieren, Begründen, Erklären und Beweisen im Mathematikunterricht”, in English: “Argumentation, reasoning, explanation and proof in mathematics lessons”) is one

possible way to apply the design principles within a single PST seminar to initiate a productive turn of discontinuity issues.

The seminar agenda falls into four parts (see Fig. 3): The first and the last part are conducted mainly asynchronously (the closing session is the only exception), and include, as an application of Principle 2, individual reflexive writing tasks as core elements: The guided explication and reflection of motives, goals, and subjective core ideas regarding argumentation and proof in mathematics class as well as in university mathematics are central topics of the initial writing assignments. At the end of the seminar, the PST participants have to do an overall reflection that asks about changes in the views that they had written down at the beginning of the seminar.

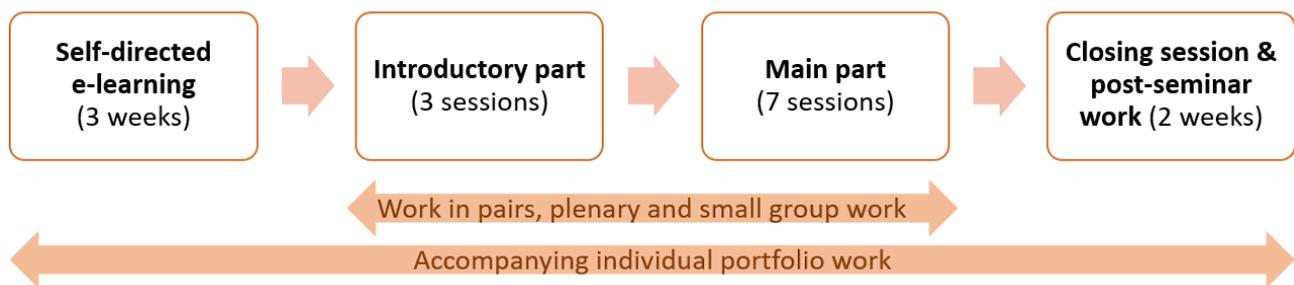


Fig. 3: Components and structure of the single-module implementation

These asynchronous parts also serve as the start and end points of accompanying individual portfolio work, which is one important form of learning during the whole seminar.

The middle part of the seminar agenda consists of two live parts, where work in pairs, plenary work and small group work are the crucial forms of learning. In the introductory part, participant PSTs get to know and practice techniques for proof comprehension and proof construction, in accordance with Principle 1. Participants are also engaged in dialogical reflection on explicated motives and goals for argumentation and proving, continuing the application of Principle 2 in the asynchronous parts. In the main part of the seminar, in each session one PST tandem applies and assesses the techniques learned in the introductory part on a given object, which could be a proof or a theorem on advanced school level. The tandem prepares an interactive presentation on their results for their peer students. This exemplifies Principles 1 and also 3, because the PSTs need to explicate and reflect content-related and didactical decisions to prepare their presentations.

A second component of each main part session consists of new input and collaborative activities on mathematical argumentation (e.g., the Toulmin model), mathematical reasoning (e.g., the interplay of abduction, deduction, and induction as in Müller-Hill, 2019, as well as the phases of argumentation and proving according to Boero, 1999), and mathematical explanation (e.g., invariance criteria for explanatory patterns, *ibid.*). This main part can be seen as a combined realization of all three design principles: PSTs are engaged in conducting and analyzing their own argumentation and proving processes, in analyzing pupils' work products and learning environments, and in

developing approaches for possible implementation of argumentation and proof in lesson stagings.

RESULTS

The theoretical framework on core mathematical activities and the design principles, which we developed on its basis, provided strong guidance in finding course designs that address the empirically detected discontinuity issues regarding motives, objects, and way/means of action in their complexity already in early design stages. Evaluations of the ProPraxis module sequence, which has been running continuously since 2016, have been used as guidelines for adjustments and incremental improvements, but they did not point out the need for a fundamental reorientation. The ABEB seminar, first implemented in 2021, has entered a second iteration in 2022 with only slight modifications.

In our longitudinal concept (ProPraxis), the three design principles are implemented in a cumulative ascending manner: 1, 1+2, 1+2+(3), 1+2+3. In the ABEB seminar, we employ Principle 2 as a bracket in the sense of individual pre- and post-seminar engagement with motives, goals and personal core ideas on a meta level: **2**, 1+2, 1+2+3, **2**. This raises both the individual starting level of the participants and the collective starting level of discourse in the group. The intent is to support the subsequent application of all three principles in their quick succession of cumulatively ascending combinations.

In order to understand possible effects of the courses, we examined the reflections that the students wrote as part of their term papers. We briefly discuss two examples. Nadja (in the ProfiWerk seminar) states:

Looking back now, I can say that my understanding developed over the course of the semester: Through various examples from everyday school mathematics, I was able to see that the concepts and explanations were often equivalent to or quite close to those of university mathematics, although at first glance one would not have thought so.

Apparently, Nadja has made new connections on the level of objects – she now relates argumentation in school and university to each other.

Timo states in his overall reflection of the ABEB seminar:

In my initial reflection at the beginning of the seminar I stated that there seems to be a gap between university and school mathematics, which makes it only conditionally possible and useful to incorporate proofs into school lessons. [...] Especially with the help of the methods for understanding and constructing proofs, which were shown and practiced at the beginning of the seminar, are from my point of view well suited to include proving activities into math lessons in different ways at one point or another. [...] Motives like gaining knowledge and comprehensibility were not very important for me, because I was of the opinion that proofs would complicate the lessons in many places and hinder understanding. [...] Through the learned methods and procedures, through which proofs,

but also argumentation, justification and explanation phases can be integrated into the lessons, this point of view has changed.

His writing indicates that Timo has developed new constructions of meaning regarding proof under the impression of experiencing ways and means of action that were previously unknown to him. Furthermore, he has revised and extended his understanding of the range of motives for argumentation and proving in mathematics lessons.

CONCLUSION

We have presented an activity-theoretical conceptual framework on core mathematical activities and have shown how the design principles that we developed within this framework can be brought to fruition in concrete course designs in order to address and productively turn specific discontinuity issues. The interplay of the constitutive elements of the theory is mirrored in their implementation. As a consequence, elements that might otherwise have remained implicit were brought to the surface – they were given substantial weight and were specifically explicated and worked on in explicit student activities. Accordingly, we found evidence of new, coherent constructions of meaning in the students' reflections.

The fact that activity theory could be used so effectively here for the purposes of design convincingly underscores, in our view, the universal nature of the mechanisms it captures.

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