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High Reynolds Channel Flows: Upstream interaction of various wall deformations

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Summary. The flow at high Reynolds number in the entrance of distorted channel is considered. We analyse the anticipated fluid responds to a downstream wall distortion, and we find that the non linear upstream length $\Delta = O(R_e^{1/7})$, using either a new asymptotic approach called Successive Complementary Expansions Method (SCEM) with generalized asymptotic expansions and a modal analysis of the perturbed flow. Comparisons with Navier-Stokes solutions show that the mathematical model is well founded.

1 Introduction

This paper considers the upstream interaction of flows in a two-dimensional channel at high Reynolds number with wall deformations. An asymptotic model using the successive complementary expansion method with generalized asymptotic expansions, called GIBL for Global Interactive Boundary Layer [1, 3], is used. The aim is to analyse the non linear asymptotic length Δ of the upstream influence of an *accident* at $x = x_0$ at the walls. As Smith [2] we found that $\Delta = O(R_e^{1/7})$, where R_e is the Reynolds number. The only hypothesis on the wall *accident* is that it is significant enough to perturb the Poiseuille flow, so that the Poiseuille flow is no more a good approximation in the boundary layer.

Then by assuming an exponential variation in x of the perturbed flow, in order to obtain the Poiseuille flow as $x \rightarrow -\infty$ (i.e. far upstream the wall deformations), we perform an eigenvalue analysis. We thus found that the first mode is related to non-symmetric wall deformations. Two kind of wall deformations are considered (local and global distortions) and comparisons between GIBL, Navier-Stokes solutions and eigenmodes show that the model is well founded.

2 Geometrical configuration

Two kind of geometrical configuration have been considered for the *accident*: (i) a local wall perturbation as in figure 1, or (ii) a global wall curvature as in figure 2.

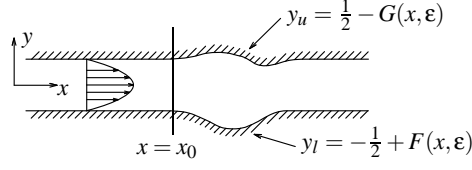


Fig. 1. (i) case: Local wall perturbation; location of the accident at $x = x_0$.

In the test case (i), the walls are deformed in a domain $x_0 \leq x \leq x_0 + L$ such as:

$$F = \frac{h_l}{2} \left(1 + \cos \frac{2\pi x}{L} \right) ; G = -\frac{h_u}{2} \left(1 + \cos \frac{2\pi x}{L} \right) . \quad (1)$$

where h_u and h_l are small parameters.

In the test case (ii), we use a generalized system of coordinates, where X and Y are distances along and perpendicular to the line $H = 0$. We call it the median line if the upper (or inner) and lower (or external) walls are respectively given by $Y = \pm \frac{1}{2}$.

For a point M with general coordinates X and Y , we can write $\overrightarrow{OM} = \overrightarrow{OM_0} + Y\mathbf{n}$, where \mathbf{n} is the unit normal vector. Then, $d\overrightarrow{M} = dX(1 + KY)\boldsymbol{\tau} + dY\mathbf{n}$, where $\boldsymbol{\tau}$ is the unit vector tangent at M_0 to the median line in such a way that $(\boldsymbol{\tau}, \mathbf{n})$ is direct; $K(X)$ is the algebraic curvature of this line. Thus, $K < 0$ in the case of figure 2. The curvature K and its variation in X are small. We thus describe the channel variable curvature for $X > 0$ by $K = \delta k(X)$, where δ is a small positive parameter. Let U and V denote the velocity components parallel and perpendicular to the line $H = 0$, then, as $\mathbf{V} = U\boldsymbol{\tau} + V\mathbf{n}$, the full equations of motion written in generalized coordinates are given in [4]. These equations must be solved with boundary conditions: $U = V = 0$ for $Y = \pm \frac{1}{2}$.

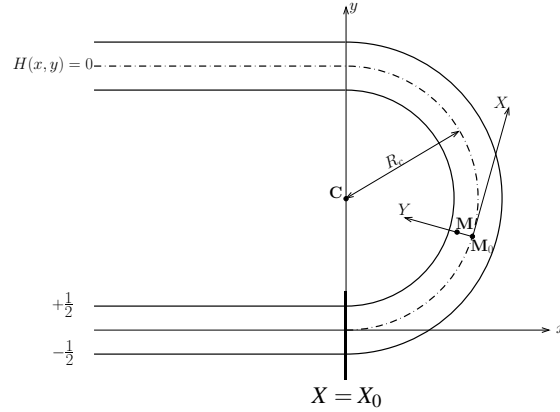


Fig. 2. (ii) case: Global wall curvature; location of the accident at $X = X_0$.

3 Fully established flow in a curved channel

For a channel of constant curvature δ , the fully established flow U_0 is solution of

$$(1 + \delta Y) \frac{d^2 U_0}{dY^2} + \delta \frac{dU_0}{dY} - \frac{\delta^2}{1 + K_0 Y} U_0 = -GR_e \quad (2)$$

where $G = -\frac{\partial P}{\partial X}$ is constant, and with $U_0 = 0$ for $Y = \pm \frac{1}{2}$. Notice that for $\delta = 0$ we

retrieve the equation for the Poiseuille flow: $\frac{d^2 U_0}{dY^2} = -2$.

The exact solution is given by:

$$U_0(Y) = \frac{1}{64} GR_e \frac{f(\delta, Y)}{(\delta^2 (1 + \delta Y))} \quad (3)$$

where

$$\begin{aligned} f(\delta, Y) = & [\delta^3(1 - 4Y^2) + 8\delta^2 Y(2Y - 1) + 4\delta(-4Y^2 + 8Y - 3) + 16(1 - 2Y)] \ln\left(\frac{2 - \delta}{2\delta}\right) + \\ & [-\delta^3(1 - 4Y^2) + 8\delta^2 Y(2Y + 1) + 4\delta(4Y^2 + 8Y + 3) + 16(1 + 2Y)] \ln\left(\frac{2 + \delta}{2\delta}\right) - \\ & 32(2\delta Y + \delta^2 Y^2 + 1) \ln\left(\frac{1 + \delta Y}{\delta}\right) \end{aligned}$$

As shown in figure 3, the corresponding exact solution $U_0(y)$ bends towards the internal wall of the bend. Notice that, for a small constant curvature δ and for a flow

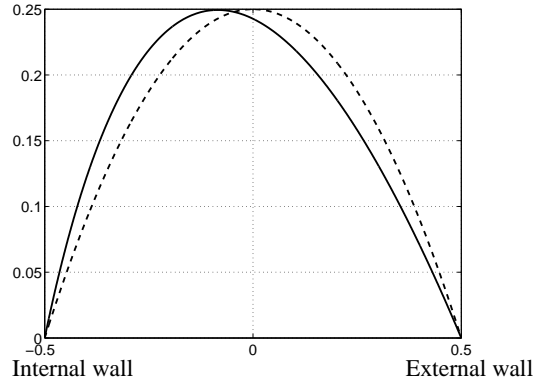


Fig. 3. Velocity profile $U_0(Y)$; Poiseuille flow (dashed line); profile for $\delta = 1$ (straight line).

rate of $1/6$, an approximate solution $O(\delta)$ is $U_0 = \left(\frac{1}{4} - Y^2\right) \left(1 - \frac{2\delta}{3} Y\right)$, which implies a skin friction of $C_f \frac{Re}{2} = 1 \mp \frac{\delta}{3}$.

4 Global Interactive Boundary Layer (GIBL) model

According to the SCEM, a Uniformly Valid Approximation (UVA) for the velocity and pressure fields (U, V, P) is obtained by complementing the core approximation $(U_1 = u_0 + \delta u_1, V_1 = \delta v_1, P_1 = p_0 + \delta p_1)$ such as:

$$\begin{aligned} U &= u_0(Y) + \delta[u_1(X, Y, \delta) + U_{BL}(X, \eta, \delta)] \\ V &= \delta[v_1(X, Y, \delta) + \varepsilon V_{BL}(X, \eta, \delta)] \\ P &= p_0(X) + \delta[p_1(X, Y, \delta, \varepsilon) + \Delta(\varepsilon)P_{BL}(X, \eta, \delta, \varepsilon)] \end{aligned} \quad (4)$$

where $\lim_{\eta \rightarrow \infty} U_{BL} = 0$, $\lim_{\eta \rightarrow \infty} V_{BL} = 0$ and $\lim_{\eta \rightarrow \infty} P_{BL} = 0$ (see [4] for more details).

Thus, we obtain Uniformly Valid Approximation (UVA) equations:

$$\begin{aligned} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\partial P_1}{\partial X} + \frac{1}{Re} \frac{\partial}{\partial Y} \left[(1 + KY) \frac{\partial U}{\partial Y} \right] \end{aligned}$$

with the following boundary conditions, $U = V = 0$, for $Y = \pm \frac{1}{2}$. The core equations being:

$$\begin{aligned} u_0 \frac{\partial V_1}{\partial X} - K u_0^2 &= -\frac{\partial P_1}{\partial Y} \\ -u_0 \frac{\partial V_1}{\partial Y} + V_1 \frac{du_0}{dY} &= -\frac{\partial(P_1 - p_0)}{\partial X} \end{aligned}$$

A simplified model for the pressure gives $\frac{\partial P_1}{\partial X} = \frac{dp_0}{dX} + \delta(A''' + k') \int_{\eta_c}^{\eta} u_0^2(\eta') d\eta' + \delta B'(X)$. At the medline, i.e. for $\eta = \eta_c$, since the UVA V should match the core approximation V_1 , we impose the coupling condition $V = V_1 = -A'(X)u_0$.

For more details about GIBL, see the companion paper [5].

5 Upstream interaction

5.1 Upstream length

In a straight channel, upstream of the wall *accident*, for $x < 0$, the GIBL and core equations become:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial P_1}{\partial x} + \frac{1}{Re} \frac{\partial^2 U}{\partial y^2} \quad (5)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (6)$$

$$-U_0 \frac{\partial V_1}{\partial y} + V_1 \frac{dU_0}{dy} = -\frac{\partial(P_1 - P_0)}{\partial x} \quad (7)$$

$$U_0 \frac{\partial V_1}{\partial x} = -\frac{\partial(P_1 - P_0)}{\partial y} \quad (8)$$

We now consider perturbations of the following form: $U = U_0 + \varepsilon u$, $V = \varepsilon v$ and $P_1 = P_0 + \lambda p_1$.

If the critical unknown streamwise length scale is Δ , then, with $\bar{x} = \frac{x}{\Delta}$ and thus $\bar{V} = \Delta V$, we obtain from (5,6,7,8) the following perturbation equations:

$$\frac{\partial u}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial y} = 0 \quad (9)$$

$$U_0 \frac{\partial u}{\partial \bar{x}} + \bar{v} \frac{dU_0}{dy} + \varepsilon \left(u \frac{\partial u}{\partial \bar{x}} + \bar{v} \frac{\partial u}{\partial y} \right) = -\frac{\lambda}{\varepsilon} \frac{\partial p_1}{\partial \bar{x}} + \frac{\Delta}{Re} \frac{\partial^2 u}{\partial y^2} \quad (10)$$

$$-U_0 \frac{\partial \bar{v}_1}{\partial y} + \bar{v}_1 \frac{dU_0}{dy} = -\frac{\lambda}{\varepsilon} \frac{\partial p_1}{\partial \bar{x}} \quad (11)$$

$$U_0 \frac{\partial \bar{v}_1}{\partial \bar{x}} = -\frac{\lambda \Delta^2}{\varepsilon} \frac{\partial p_1}{\partial y} \quad (12)$$

If ε is the boundary layer thickness, the first significant perturbation is such as $U_0 = O(\varepsilon)$, $\bar{v} = O(\varepsilon)$ in the boundary layers, which implies from (10) that ε , $\frac{\lambda}{\varepsilon}$ and $\frac{\Delta}{\varepsilon^2 Re}$ are of same order.

An upstream interaction takes place if we have a generation of a significant transverse pressure gradient in the core flow, which implies from (12) that $\frac{\lambda \Delta^2}{\varepsilon} = O(1)$. Thus, we easily obtain (as did Smith [2] by regular asymptotic expansions) the following crucial orders:

$$\Delta = O(Re^{1/7}), \quad \varepsilon = O(Re^{-2/7}) \quad \text{and} \quad \lambda = O(Re^{-4/7}). \quad (13)$$

5.2 Eigenmode analysis

For $x < 0$, the linearized UVA system of equations may be written as :

$$\begin{cases} U_0 \frac{\partial u}{\partial x} + U_0' v = -\frac{\partial p_1}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ U_0 \frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} \end{cases} \quad (14)$$

By replacing v_1 by v in the transverse core momentum equation, and by assuming the following form for u , v and p_1 :

$$u(x, y) = \hat{u}(y) e^{\theta x}, \quad v(x, y) = \hat{v}(y) e^{\theta x}, \quad p_1(x, y) = \hat{p}_1(y) e^{\theta x} \quad (15)$$

we obtain for the perturbations:

$$\theta \begin{pmatrix} U_0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & U_0 & 0 \end{pmatrix} \hat{q} = \begin{pmatrix} \frac{D^2}{Re} & -U'_0 & 0 \\ 0 & -D^1 & 0 \\ 0 & 0 & -D^1 \end{pmatrix} \hat{q} \quad (16)$$

where $\hat{q} = \begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{p}_1 \end{pmatrix}$, $D^1 = \frac{\partial}{\partial y}$ and $D^2 = \frac{\partial^2}{\partial y^2}$.

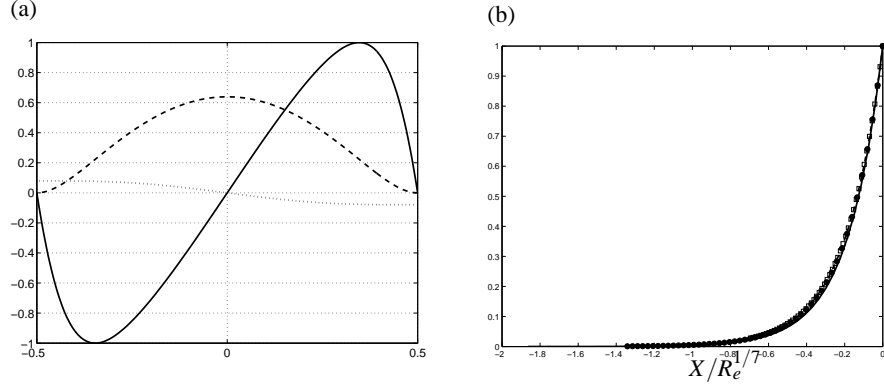


Fig. 4. (a) Profiles of the first mode eigenfunctions \hat{u} (straight line), \hat{v} (dashed line) and \hat{p}_1 (dotted line) for $Re = 1000$; (b) Upstream influence of the first mode for $Re = 10^3$ (straight line), 10^4 (black circle), 10^5 (dashed line), 10^6 (white square).

We just have now to find the eigenvalues and eigenfunctions of the matrix $B^{-1}A$, where :

$$A = \begin{pmatrix} \frac{D^2}{Re} & -U'_0 & 0 \\ 0 & -D^1 & 0 \\ 0 & 0 & -D^1 \end{pmatrix} \text{ and } B = \begin{pmatrix} U_0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & U_0 & 0 \end{pmatrix} \quad (17)$$

For $Re = 1000$, the first positive eigenvalue found is $\theta_1 \simeq 2.0441$. The figure 4(a) represents the eigenfunctions of this mode. As shown in figure 4(b), by computing this first positive eigenvalue for different Reynolds number ranging from 10^3 to 10^6 , we obtain that the corresponding upstream influence $\Delta = O(Re^{1/7})$ as in the analysis of the section 5.1.

6 Results

Both the order analysis of section 5.1 and the eigenmode analysis of section 5.2 show that $\Delta = O(Re^{1/7})$. We now compute the flow field using the GIBL model described in section 4 for different accident types at $x = 0$.

First, we have considered a straight channel connected at $x = 0$ to a curved channel of constant curvature. The figures 5 (a) and (b) represent the median curved length evolution of $V(X, \eta_c)$ for, respectively, a fixed $\delta = 0.2$ at different Reynolds numbers, and a fixed $R_e = 1000$ at different wall curvature. These two results confirm that $\Delta = O(R_e^{1/7})$.

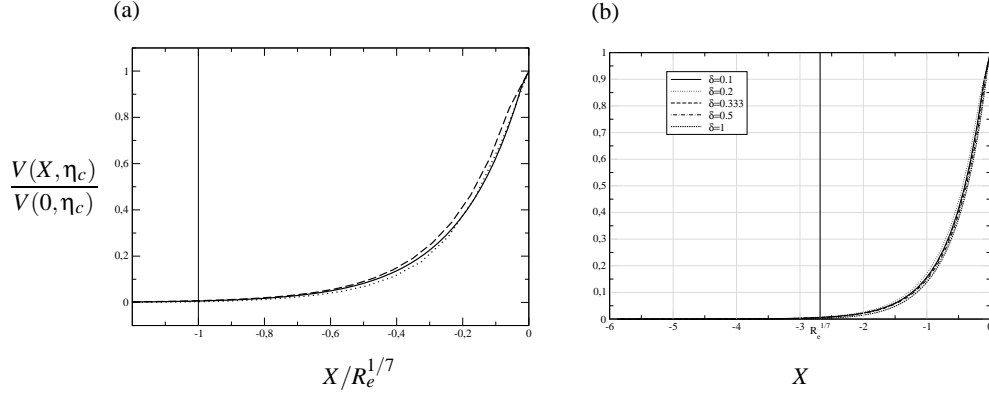


Fig. 5. (ii) case: straight channel connected at $x = 0$ to a curved channel of constant curvature; (a) $\delta = 0.2$, R_e from 100 to 10000; (b) $R_e = 1000$, δ from 0.1 to 1

Then, we have considered an asymmetrically perturbed straight channel at $x = 0$ with $L = 4H$ and $h_u = h_l = 0.3$. The figure 6 represents the streamwise evolution of $V(x, \eta_c)$, where we recover as previously $\Delta = O(R_e^{1/7})$.

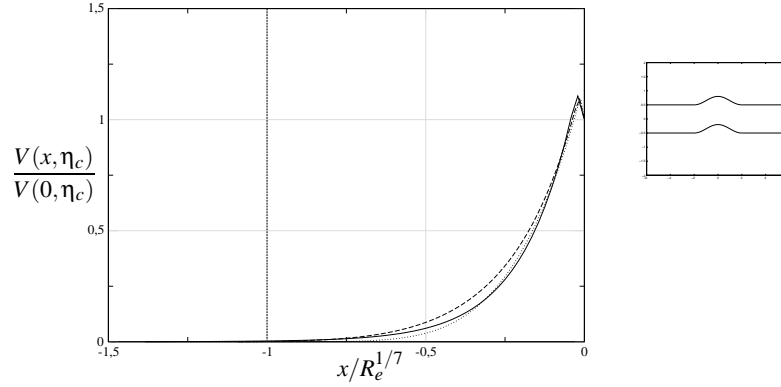


Fig. 6. (i) case: straight channel perturbed at $x = 0$ with $L = 4H$, $h_u = h_l = 0.3$; x -evolution of the adimensionnalized $V(x, \eta_c)$ for different values of R_e (from 100 to 10000).

Finally, we have compared the Navier-Stokes, GIBL and eigenmode analysis results. As shown in figure 7, all the results are very similar.

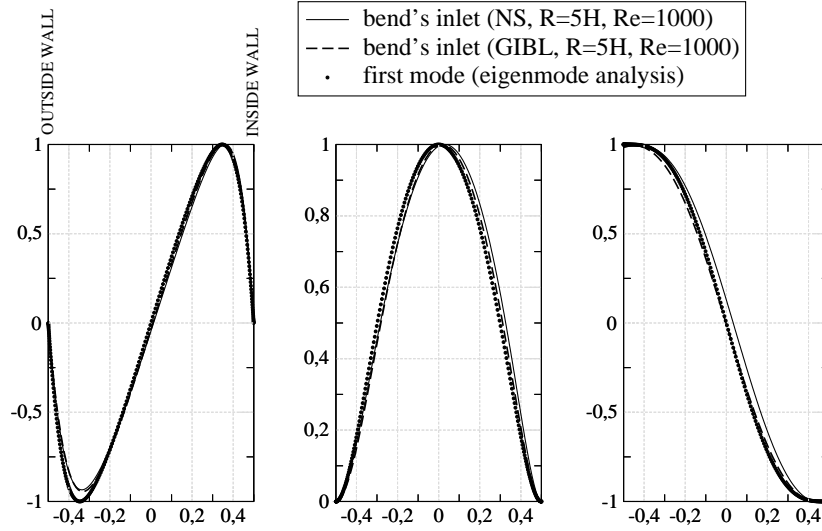


Fig. 7. NS, GIBL, first eigenmode comparison; $R_e = 1000$, $\delta = 0.2$; left: u profile; middle: v profile; right: p_1 profile.

7 Conclusion

The non linear upstream effect on a channel flow submitted to asymmetric disturbance has been studied. By using three different tools, a new asymptotic approach called Successive Complementary Expansions Method (SCEM) with generalized asymptotic expansions, a modal analysis and direct Navier-Stokes computations, we found that the upstream influence length $\Delta = O(R_e^{1/7})$.

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