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# Model Order Selection for Collision Multiplicity Estimation

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**Abstract**—The collision multiplicity (CM) is the number of users involved in a collision. The CM estimation is an essential step in multi-packet reception (MPR) techniques and in collision resolution (CR) methods. We propose two techniques to estimate collision multiplicities in the context of IEEE 802.11 networks. These two techniques have been initially designed in the context of source separation. The first estimation technique is based on eigenvalue statistics. The second technique is based on the exponentially embedded family (EEF). These two techniques outperform current estimation techniques in terms of underestimation rate (UNDER). The reason for this is twofold. First, current techniques are based on a uniform distribution of signal samples whereas the proposed methods rely on a Gaussian distribution. Second, current techniques use a small number of observations whereas the proposed methods use a number of observations much greater than the number of signals to be separated. This is in accordance with typical source separation techniques.

## I. INTRODUCTION

We investigate the way collisions are resolved in IEEE 802.11-based wireless local area networks (WLANs), more precisely when multiple users simultaneously transmit to an access point (AP) (see Fig. 1). Recent advances in MPR [1], [2] and in CR [3], [4] allow now the recovery of data packets from collision signals. These techniques usually start with the estimation of the CM  $K$ , i.e., the number of nodes involved in the collision. CM estimation techniques (CMETs) are all based on model order selection (MOS) methods [5]. They all rely on eigenvalue statistics [3], [4], [6].

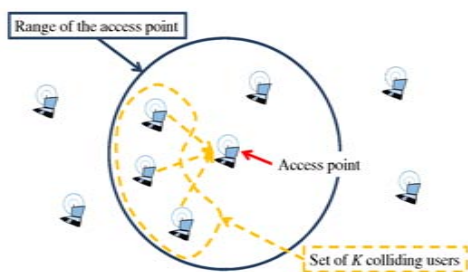


Fig. 1. Collision Scenario with  $K = 3$  colliding users.

The use of current CMETs in the context of IEEE 802.11 networks raises two main issues. The first issue deals with the distribution of signal samples. Current CMETs assume that signal samples are uniformly distributed over a finite alphabet, i.e., signal samples are modulation symbols [1], [2], [6]. However, the IEEE 802.11 standard implements orthogonal frequency division multiplexing (OFDM) transmission so that signal samples are Gaussian distributed. Thus, current CMETs may not perform well in IEEE 802.11 networks. Note that the MPR protocols in [1], [2] have been specially designed for IEEE 802.11 networks but they implement a blind separation of the colliding users that relies on a uniform distribution of signal samples [6]. The second issue deals with the number of observations ("snapshots")  $T$  that is needed to perform the CM estimation. In current CMETs,  $T$  is not much greater than the number of colliding users  $K$ . In [3] (resp. [4]),  $T$  is not greater than  $K + 1$  (resp.  $K + 2$ ). In [1], [2],  $T$  is limited by the number of receive antennas at the AP, i.e.,  $T \leq 4$  in typical implementations of IEEE 802.11 WLANs [7]. These settings are not in accordance with the settings that are used in typical source separation algorithms where  $T \gg K$  [8]–[10]. The issue here is to decide whether current CMETs can still operate with a small number of snapshots in IEEE 802.11 networks, or not.

The purpose of this paper is twofold: (a) to show that current CMETs do not perform well in the context of IEEE 802.11 networks, and (b) to propose new CMETs for this context. The work presented in this paper leverages recent advances in both information criterion design and random matrix theory (RMT) to tackle these issues. We propose two new criteria for CM estimation: the EEF criterion and the Tracy-Widom inference test (TWIT).

The rest of the paper is organized as follows. The system model is presented in Section II and the proposed CMETs are described in Section III. Simulation results are presented in Section IV and a conclusion is drawn in the last section.

## II. SYSTEM MODEL

We consider the scenario in which  $K$  users simultaneously transmit to an AP. Each user is equipped with a single antenna. The AP receives  $T$  snapshots. These snapshots are obtained through two possibly complementary ways. The AP can be equipped with more than one antenna [1], [2] and several retransmissions from the colliding users can be triggered by a feedback frame from the AP [3], [4]. The scenario is similar

to a source separation problem when  $K$  signals are impinging on a  $T$  sensor array. We do not consider here the case of multiple transmit antennas at each user. We assume that the  $K$  users are coarsely synchronized in time. Each user transmits OFDM signals that are Gaussian distributed and that have a white power spectral density. More precisely, each snapshot  $\mathbf{y}(n)$  is a  $T \times 1$  vector

$$\mathbf{y}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{w}(n), \quad n = 0, 1, \dots, N$$

where  $N$  denotes the number of samples per snapshot,  $\mathbf{s}(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_s)$  is a  $K \times 1$  complex Gaussian vector of OFDM samples with covariance matrix  $\mathbf{R}_s$  and  $\mathbf{w}(n) \sim \mathcal{N}(\mathbf{0}, \Sigma)$  is a  $T \times 1$  complex Gaussian noise vector with noise covariance matrix  $\Sigma$ . In the white noise case, the covariance  $\Sigma$  is  $\sigma^2 \mathbb{I}_T$  where  $\mathbb{I}_T$  is the  $T \times T$  identity matrix. The channel matrix  $\mathbf{A}$  is a  $T \times K$  unknown matrix with circularly symmetric Gaussian elements with power unity (Rayleigh fading). A block-fading wireless channel is considered here so the coefficients in  $\mathbf{A}$  have constant values during an OFDM block of  $N$  samples, and then change randomly from one block to another. When the noise covariance matrix  $\Sigma$  is known *a priori* and is nonsingular, the snapshots  $\mathbf{y}(n)$  can be "whitened" by the following transformation

$$\mathbf{y}^\dagger(n) = \Sigma^{-1/2} \mathbf{y}(n)$$

where  $\Sigma^{-1/2}$  is the Hermitian nonnegative definite square root of  $\Sigma$ . This transformation simply reduces to a normalization step in the case of a white Gaussian noise.

The signal and noise vectors being independent, the covariance matrix  $\mathbf{R}_y$  of the snapshots is given by

$$\mathbf{R}_y = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \Sigma$$

with  $^H$  denoting the complex conjugate. We assume that the matrix  $\mathbf{A}$  is full rank and that the signal covariance matrix  $\mathbf{R}_s$  is nonsingular. Hence the rank of  $\mathbf{A}\mathbf{R}_s\mathbf{A}^H$  is  $\min(K, T)$ , i.e.,  $\mathbf{A}\mathbf{R}_s\mathbf{A}^H$  has exactly  $T$  non-zero eigenvalues when  $T \leq K$  and  $K$  non-zero eigenvalues when  $T > K$ . One would expect that  $T$  is strictly higher than  $K$  in such a way that  $\min(K, T) = K$  but, in [3] and [4], the algorithms start with  $T = 1$  and  $T$  is then incremented. Hence,  $\min(K, T) = T$  for the first iterations. When the whitening transformation is applied, the covariance matrix  $\mathbf{R}_y^\dagger$  of the whitened snapshots is defined as

$$\mathbf{R}_y^\dagger = \Sigma^{-1/2} \mathbf{R}_y \Sigma^{1/2} = \Sigma^{-1/2} \mathbf{A}\mathbf{R}_s\mathbf{A}^H \Sigma^{1/2} + \mathbb{I}_T$$

Let  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_T$  and  $\lambda_1^\dagger \geq \lambda_2^\dagger \geq \dots \geq \lambda_T^\dagger$  denote the eigenvalues of  $\mathbf{R}_y$  and  $\mathbf{R}_y^\dagger$  respectively. We have that

$$\lambda_i > \sigma^2 \text{ and } \lambda_i^\dagger > 1 \quad \text{for } 1 \leq i \leq \min(K, T) \quad (1)$$

$$\lambda_i = \sigma^2 \text{ and } \lambda_i^\dagger = 1 \quad \text{for } \min(K, T) < i \leq T \quad (2)$$

When  $\mathbf{R}_y$  and  $\Sigma$  are known, the CM estimation can be easily performed either from the multiplicity of the  $\lambda_i$  equalling  $\sigma^2$  or from the multiplicity of the  $\lambda_i^\dagger$  equalling one. When  $\mathbf{R}_y$  and  $\Sigma$  are unknown and have to be estimated, another approach

must be used. We defined the sample covariance matrix (SCM) of  $\mathbf{y}(n)$ , denoted  $\hat{\mathbf{R}}_y$ , by

$$\hat{\mathbf{R}}_y = \frac{1}{N} \sum_{n=1}^N \mathbf{y}(n) \mathbf{y}^H(n)$$

The SCM of  $\mathbf{R}_y^\dagger$ , denoted  $\hat{\mathbf{R}}_y^\dagger$ , is defined by

$$\hat{\mathbf{R}}_y^\dagger = \hat{\Sigma}^{-1/2} \hat{\mathbf{R}}_y \hat{\Sigma}^{1/2}$$

where  $\hat{\Sigma}$  denotes the noise SCM

$$\hat{\Sigma} = \frac{1}{N} \sum_{n=1}^N \mathbf{w}(n) \mathbf{w}^H(n)$$

The noise SCM can be computed because we have access to noise-only samples<sup>1</sup>. This assumption is reasonable since empty time slots can provide the  $N$  samples that are needed to compute the SCM  $\hat{\Sigma}$ .

### III. COLLISION MULTIPLICITY ESTIMATION TECHNIQUES

There are two approaches to estimate  $K$ . The first approach relies on the eigenvalue statistics. The second one is based on information criteria. The two approaches are presented next.

#### A. CME based on eigenvalue statistics

According to (1) and (2), counting the number of eigenvalues that are significantly higher than one or  $\sigma^2$  could serve as a CMET. This approach is rather impractical since it relies on subjective adjustments of thresholds [8]. The proposed TWIT criterion uses the properties that have been highlighted recently in [10], [11] and in the references therein:

- There is a detectability threshold below which it is not possible to identify signal from noise. This threshold  $\gamma$  depends on system parameters:  $\gamma = (1 + \sqrt{T/N})\sigma^2$  [11], [12].
- In the signal-free case, the distribution of the largest noise eigenvalue of the SCM  $\hat{\mathbf{R}}_y^\dagger$  is approximated by a complex Tracy-Widom distribution.
- In the non signal-free case, the distribution of the signal eigenvalues that are below  $\gamma$  is also approximated by a Tracy-Widom distribution and the signal eigenvalues that are above  $\gamma$  are distributed according to a Gaussian law [10]. Note that all approximations are performed after appropriate centering and scaling of the eigenvalues. Noise eigenvalues are still distributed according to a Tracy-Widom distribution.

The noise samples are Gaussian distributed, so we have that  $N\hat{\Sigma} \sim \mathbb{CW}_T(N, \Sigma)$  where  $\mathbb{CW}_T(N, \Sigma)$  denotes the  $T$ -variate complex Wishart law with  $N$  degrees of freedom, and parameterized by the covariance matrix  $\Sigma$ . Similarly, we have that  $N\hat{\mathbf{R}}_y \sim \mathbb{CW}_T(N, \mathbf{R}_y)$ . We investigate the distribution of the largest eigenvalue  $\lambda_1^\dagger$  that satisfies

$$(N\hat{\mathbf{R}}_y)v_1 = \hat{\lambda}_1^\dagger(N\hat{\Sigma})v_1 \quad (3)$$

<sup>1</sup>We assume that the noise variance  $\sigma^2$  can be estimated by different other means at the AP when  $\sigma^2$  is the only parameter that is needed.

TABLE I  
TWIT ALGORITHM

---

Set $\hat{K}_{\text{TWIT}} = 0$ and Test=false
While (Test=false and $\hat{K}_{\text{TWIT}} < T$ ) do
$\hat{K}_{\text{TWIT}} = \hat{K}_{\text{TWIT}} + 1$
Compute $\mu = \mu(T - \hat{K}_{\text{TWIT}}, N, N)$
Compute $\sigma = \sigma(T - \hat{K}_{\text{TWIT}}, N, N)$
Test={ $\sigma^{-1}[\log(\hat{\lambda}_{\hat{K}_{\text{TWIT}}+1}^\dagger) - \mu] < \tau_\alpha$ }
If (Test=false) then do $\hat{K}_{\text{TWIT}} = \hat{K}_{\text{TWIT}} + 1$ else break End if
End while

---

where  $v_1$  is the eigenvector corresponding to the eigenvalue  $\hat{\lambda}_1^\dagger$ . Let  $\lambda_1^{[\text{DW}]}$  be the largest eigenvalue that satisfies

$$(N\hat{\mathbf{R}}_{\mathbf{y}})v_1^{[\text{DW}]} = \lambda_1^{[\text{DW}]}(N\hat{\Sigma} + N\hat{\mathbf{R}}_{\mathbf{y}})v_1^{[\text{DW}]} \quad (4)$$

where  $v_1^{[\text{DW}]}$  is the eigenvector corresponding to the eigenvalue  $\lambda_1^{[\text{DW}]}$ . From Theorem 2 in [13], when  $N \rightarrow \infty$  as  $T \rightarrow \infty$  with  $N > T$ , we have that

$$\text{P}\left[\frac{W(\lambda_1^{[\text{DW}]}) - \mu(T, N, N)}{\sigma(T, N, N)} \leq x\right] \rightarrow TW_{\mathbb{C}}(x) \quad (5)$$

where  $TW_{\mathbb{C}}(x)$  is the Tracy-Widom distribution function for complex data and  $W(\lambda) = \log[\lambda/(1-\lambda)]$ . The mean  $\mu(n, m, N)$  and the standard deviation  $\sigma(n, m, N)$  are defined in Table II. From (4), we have that

$$(N\hat{\Sigma})^{-1}(N\hat{\mathbf{R}}_{\mathbf{y}})v_1^{[\text{DW}]} = \left(\frac{\lambda_1^{[\text{DW}]}}{1 - \lambda_1^{[\text{DW}]}}\right)v_1^{[\text{DW}]} \quad (6)$$

So, from (3) and (6), and using (5), we have that

$$\text{P}\left\{\frac{\log(\hat{\lambda}_1^\dagger) - \mu(T, N, N)}{\sigma(T, N, N)} \leq x\right\} \rightarrow TW_{\mathbb{C}}(x)$$

when  $N \rightarrow \infty$  as  $T \rightarrow \infty$ . The algorithm presented in Table I exploits these properties. Basically the estimate  $\hat{K}_{\text{TWIT}}$  is initialized to zero and incremented by one for each iteration as long as the eigenvalue  $\hat{\lambda}_{\hat{K}_{\text{TWIT}}+1}^\dagger$  is not considered as being an eigenvalue of the noise subspace, i.e., as being Tracy-Widom distributed [10]. The threshold  $\tau_\alpha$  is defined as  $TW_{\mathbb{C}}^{-1}(1-\alpha)$  where  $TW_{\mathbb{C}}(x)$  denotes the Tracy-Widom distribution for complex-valued data and  $\alpha$  is some significance level. Note that this criterion is designed for arbitrary (or colored) noise.

### B. CME based on information criteria

Information criteria, such as the minimum description length (MDL) or the Akaike's information criterion (AIC)<sup>2</sup>, have been originally designed in order to avoid subjective threshold settings in MOS techniques [8]. The MDL has been widely used over the past two decades and is still used in current CMETs. However, this criterion has been shown to be inconsistent when the noise variance of the channel tends to zero [14]. This means that, even in the high signal to noise ratio (SNR) regime, the MDL still tends to overestimate the model

<sup>2</sup>We shall not refer to the AIC hereafter since the criterion has been proven to be inconsistent in the  $N \rightarrow \infty$  sense [8].

TABLE II  
TWIT PARAMETERS  $\mu(n, m, N)$  AND  $\sigma(n, m, N)$

---

$\mu(n, m, N)$	$= \left(\frac{u_{\tilde{N}}}{\tau_{\tilde{N}}} + \frac{u_{\tilde{N}-1}}{\tau_{\tilde{N}-1}}\right)\left(\frac{1}{\tau_{\tilde{N}}} + \frac{1}{\tau_{\tilde{N}-1}}\right)^{-1}$
$\sigma(n, m, N)$	$= 2\left(\frac{1}{\tau_{\tilde{N}}} + \frac{1}{\tau_{\tilde{N}-1}}\right)^{-1}$
$\tilde{N}$	$= \min(n, m)$
$\sin^2(\gamma_{\tilde{N}}/2)$	$= (\tilde{N} + 1/2)(2\tilde{N} + N - n +  m - n  + 1)^{-1}$
$\sin^2(\phi_{\tilde{N}}/2)$	$= (\tilde{N} +  m - n  + 1/2)$
	$\times (2\tilde{N} + N - n +  m - n  + 1)^{-1}$
$\tau_{\tilde{N}}^3$	$= 16(2\tilde{N} + N - n +  m - n  + 1)^{-2}$
	$\times \sin^{-2}(\phi_{\tilde{N}} + \gamma_{\tilde{N}}) \sin^{-1}(\phi_{\tilde{N}}) \sin^{-1}(\gamma_{\tilde{N}})$
$u_{\tilde{N}}$	$= 2 \log[\tan(\frac{\phi_{\tilde{N}} + \gamma_{\tilde{N}}}{2})]$

---

order. In this context, the EEF criterion has been proposed in [9]. This new criterion has been proven to overcome the shortcomings of the MDL criterion, so it offers new opportunities for CM estimation. We first describe the MDL criterion and then present the EEF criterion.

1) *MDL criterion:* The MDL criterion is defined as

$$\hat{K}_{\text{MDL}} = \underset{k=1, \dots, T}{\text{argmin}} \{\text{MDL}(k)\}$$

where

$$\text{MDL}(k) = -N(T - k) \log\left[\frac{g(k)}{a(k)}\right] + \frac{1}{2}k(2T - k) \log(N)$$

where  $g(k) = \prod_{i=k+1}^T \hat{\lambda}_i^{\frac{1}{T-k}}$  and  $a(k) = \frac{1}{T-k} \sum_{i=k+1}^T \hat{\lambda}_i$  where the  $\hat{\lambda}_i$  denote the eigenvalues of  $\hat{\mathbf{R}}_{\mathbf{y}}$  with  $1 \leq i \leq T$ . This estimator is consistent in the  $N \rightarrow \infty$  sense.

2) *EEF criterion:* The EEF criterion is defined as

$$\hat{K}_{\text{EEF}} = \underset{k=1, \dots, T}{\text{argmax}} \text{EEF}(k)$$

where

$$\begin{aligned} \text{EEF}(k) &= \{L_{G_k}(\mathbf{y}) - n_k [\log(\frac{L_{G_k}(\mathbf{y})}{n_k}) + 1]\} \\ &\times u[\log(\frac{L_{G_k}(\mathbf{y})}{n_k}) - 1] \end{aligned}$$

where  $u(\cdot)$  is the step function defined as  $u(t) = 1$  for  $t \geq 0$  and  $u(t) = 0$  for  $t < 0$ . The number of free adjustable parameters in the  $k^{\text{th}}$  model, denoted  $n_k$ , is defined as

$$n_k = k(2T - k) + 1$$

The likelihood ratio  $L_{G_k}(\mathbf{y})$  is given by

$$\begin{aligned} L_{G_k}(\mathbf{y}) &= -2N \left\{ \log\left(\prod_{i=1}^k \hat{\lambda}_i\right) - T \log\left[\frac{1}{T} \text{tr}(\hat{\mathbf{R}}_{\mathbf{y}})\right] \right\} \\ &+ (T - k) \log\left(\frac{1}{T - k} \sum_{i=k+1}^T \hat{\lambda}_i\right) \end{aligned}$$

Note that the computational load of the EEF is similar to that of the MDL.

#### IV. SIMULATION RESULTS

CMETs are compared in the context of Rayleigh fading channels. User stations are transmitting OFDM signals that are built according to the IEEE 802.11 standard [7]. The signals are composed of 1024 sub-carriers and use BPSK modulation, the guard interval is 1/4 of the total symbol period. There are 48 OFDM symbols per OFDM block, so the total number of samples per snapshot is  $N = 61440$ . The performance of CMETs have been evaluated over ten thousand Monte Carlo trials. The simulation results are expressed in terms of UNDER and OVER. The UNDER (resp. OVER) is the probability of having  $\hat{K}$  strictly lower (resp. higher) than  $K$ .

Figures 2 and 3 show the comparison for five CMETs: the method designed in [3], the method designed in [4], the MDL criterion, the EEF criterion, and the TWIT. The first three methods belong to the current CMETs. The results have been obtained for  $1 \leq K \leq 10$ , a fixed number of snapshots ( $T = 15$ ), and an SNR of 10 dB. The method designed in [3] relies on a threshold test on the lowest eigenvalue of the SCM  $\hat{\mathbf{R}}_y$ ,  $\lambda_T$ . According to (1) and (2),  $\lambda_T$  can be considered as belonging to the noise subspace when  $\lambda_T < \tau$  where  $\tau$  is a threshold that depends on the noise variance  $\sigma^2$ . Hence, the number of snapshots  $T$  is initialized to one and incremented by one for each iteration as long as  $\lambda_T$  is above the threshold  $\tau$ . So, as soon as  $\lambda_T < \tau$ , we have that  $\hat{K} = T - 1$ . In [4] the threshold test is based on the two lowest eigenvalues:  $\lambda_T + \lambda_{T-1} < 2\tau$ . Once the threshold test is passed, an MDL criterion is applied in order to estimate  $K$ . Note that these first two methods do not use  $T = 15$  snapshots since the estimation is triggered by the threshold test. The threshold is set to  $\tau = \sigma^2 + 0.005$  as in [4]. Finally, the significance level  $\alpha$  for the TWIT is set to 0.01 [10]. The EEF criterion and the TWIT both outperform the first three methods, i.e., the methods that belong to current CMETs, in terms of UNDER. The EEF criterion and the TWIT exhibit almost the same results in terms of UNDER. Note that the first three methods exhibit good performance in terms of OVER but this does not compensate for the poor UNDER performance<sup>3</sup>. The performance of the two proposed CMETs, in terms of UNDER, is poor when  $K$  gets closer to  $T$ . At this point, three conclusions can be drawn. First, the proposed techniques outperform current CMETs. Second, current CMETs do not perform well in a IEEE 802.11 context. This is a major result since several MPR protocols rely on these CMETs in order to separate the colliding users. Third,  $T$  should be significantly larger than  $K$  in order to achieve performance levels that could allow a practical implementation in IEEE 802.11-based WLANs. This also confirms that the setting of  $T$  is consistent with the settings that are used in other domains. Figures 4 and 5 show the UNDER and the OVER of both the EEF criterion and the TWIT, for a fixed number of colliding users ( $K = 4$ ), different numbers of snapshots  $4 \leq T \leq 20$ , and SNRs from 5 dB to 20 dB. Here also, these two techniques give similar results in terms of UNDER, except for an SNR

<sup>3</sup>The sum of the UNDER and OVER is dominated by the UNDER indicator.

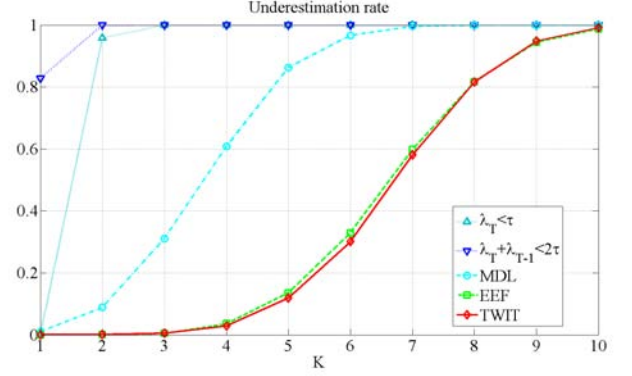


Fig. 2. Underestimation rates of five CME techniques: the method designed in [3] ( $\lambda_T < \tau$ ), the method designed in [4] ( $\lambda_T + \lambda_{T-1} < 2\tau$ ), the MDL criterion, the EEF criterion and the TWIT (SNR = 10 dB).

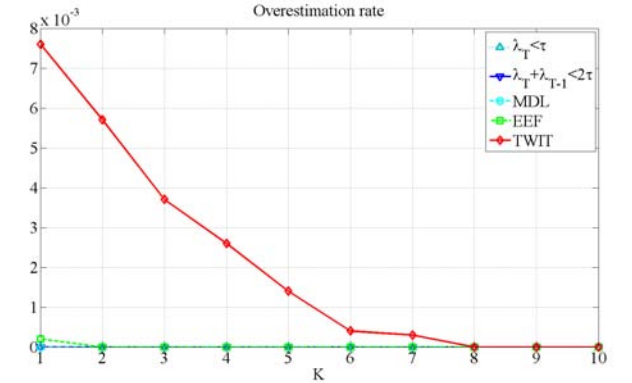


Fig. 3. Overestimation rates of five CME techniques: the method designed in [3] ( $\lambda_T < \tau$ ), the method designed in [4] ( $\lambda_T + \lambda_{T-1} < 2\tau$ ), the MDL criterion, the EEF criterion and the TWIT (SNR = 10 dB).

of 5 dB. Note that, even for a high SNR (20 dB), at least  $T = 6$  snapshots are needed in order to get an UNDER lower than 10%. The results are better in terms of OVER since this indicator is always lower than 2% for any value of  $T$ . These results further preclude the use of current CMETs in the context of IEEE 802.11 networks since these techniques rely on either small values of  $T$  [1], [2] or values of  $T$  close to  $K$  [3], [4].

Figure 6 shows the minimum number of snapshots  $T$  that is needed in order to achieve a 10% UNDER and a 10% OVER for both the EEF criterion and the TWIT, for different values of  $K$  and different values of SNR. Once again, the most significant result is that the number of snapshots that is needed to achieve a relevant performance level must be much higher than the number of colliding users.

#### V. CONCLUSION

In this paper, two new CMETs have been proposed. They rely on the EEF criterion and the TWIT. These two techniques have been shown to outperform typical CMETs based on the MDL criterion. Moreover, simulation results have shown that a large number of snapshots  $T$  is needed in order to allow a good estimation of  $K$  in terms of UNDER and OVER. Furthermore,



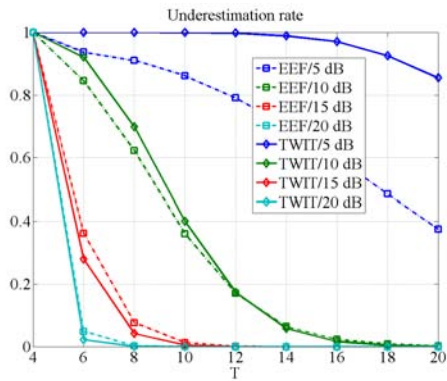


Fig. 4. Underestimation rates for the EEF criterion and the TWIT with  $K = 4$ ,  $K \leq T \leq 20$ , and  $\text{SNR}(\text{dB}) \in \{5, 10, 15, 20\}$ .

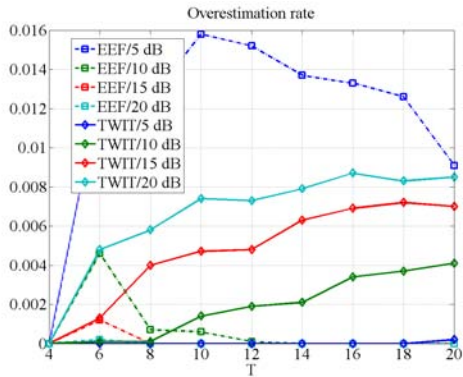


Fig. 5. Overestimation rates for the EEF criterion and the TWIT with  $K = 4$ ,  $K \leq T \leq 20$ , and  $\text{SNR}(\text{dB}) \in \{5, 10, 15, 20\}$ .

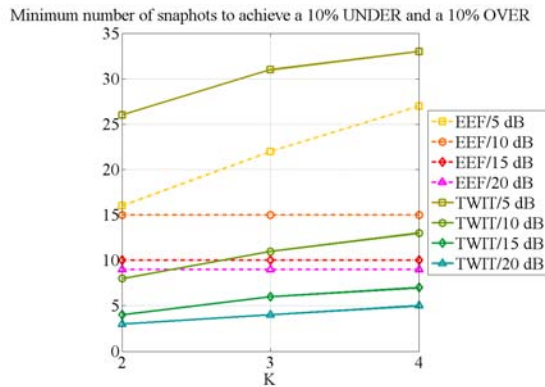


Fig. 6. Minimum number of snapshots to achieve a 10% UNDER and a 10% OVER for  $K \in \{2, 3, 4\}$  and  $\text{SNR}(\text{dB}) \in \{5, 10, 15, 20\}$ .

the number of snapshots must be significantly higher than the number of colliding users  $K$  ( $T \gg K$ ). These settings are similar to the settings that are used in MOS techniques for signal array processing.

The impact of these results is twofold. First, some CR techniques such as the network-assisted diversity multiple access (NDMA) [3], [4] cannot be implemented in IEEE 802.11

networks notably because these CR techniques are based on the assumption that  $T$  can be made as small as  $K+1$  or  $K+2$ . Second, the MPR protocols for IEEE 802.11 networks that use the blind user separation in [6] appear to be rather questionable [1], [2]. The blind detection algorithm assumes that  $K$  is known or has been estimated. However, this assumption seems to be impractical since it would imply that the CM estimation has been possible with a single observation of collided request-to-send (RTS) frames. Even if the AP is equipped with four antennas ( $T = 4$ ), our simulation results have shown that the receiver at the AP needs many more snapshots in order to provide a good estimation of  $K$  in terms of UNDER and OVER.

Further investigations are now needed in order to fully characterized the performance of the proposed CMETs in typical operating conditions.

## REFERENCES

- [1] P. X. Zheng, Y. J. Zhang, and S. C. Liew, "Multipacket Reception in Wireless Local Area Networks," in *Proc. IEEE International Conference on Communications (ICC)*, 2006.
- [2] W. L. Huang, K. B. Letaief, and Y. J. Zhang, "Cross-Layer Multi-Packet Reception Based Medium Access Control and Resource Allocation for Space-Time Coded MIMO/OFDM," *IEEE Transactions on Wireless Communications*, vol. 7, no. 9, pp. 3372–3384, 2008.
- [3] R. Zhang, N. D. Sidiropoulos, and M. K. Tsatsanis, "Collision Resolution in Packet Radio Networks Using Rotational Invariance Techniques," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 146–155, 2002.
- [4] B. Özgül and H. Deliç, "Wireless Access with Blind Collision-Multiplicity Detection and Retransmission Diversity for Quasi-Static Channels," *IEEE Transactions on Communications*, vol. 54, no. 5, pp. 858–867, 2006.
- [5] P. Stoica and Y. Selën, "Model-Order Selection : A review of information criterion rules," *IEEE Signal Processing Magazine*, vol. 21, no. 4, pp. 36–47, 2004.
- [6] S. Talwar, M. Viberg, and A. Paulraj, "Blind Separation of Synchronous Co-Channel Digital Signals Using an Antenna Array. Part I. Algorithms," *IEEE Transactions on Signal Processing*, vol. 44, no. 5, pp. 1184–1197, 1996.
- [7] "IEEE 802.11n standard, Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications: Amendment 5: Enhancements for Higher Throughput," IEEE Computer Society, Tech. Rep., 2009.
- [8] M. Wax and T. Kailath, "Detection of Signals by Information Theoretic Criteria," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 2, pp. 387–392, 1985.
- [9] C. Xu and S. Kay, "Source Enumeration via the EEF Criterion," *IEEE Signal Processing Letters*, vol. 15, pp. 569–572, 2008.
- [10] R. R. Nadakuditi and J. W. Silverstein, "Fundamental Limit of Sample Generalized Eigenvalue Based Detection of Signals in Noise Using Relatively Few Signal-Bearing and Noise-Only Samples," *IEEE Transactions on Signal Processing*, vol. 4, no. 3, pp. 468–480, 2010.
- [11] R. R. Nadakuditi and A. Edelman, "Sample Eigenvalue Based Detection of High-Dimensional Signals in White Noise Using Relatively Few Samples," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 2625–2638, 2008.
- [12] P. O. Perry and P. J. Wolfe, "Minimax Rank Estimation for Subspace Tracking," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 3, pp. 504–513, 2010.
- [13] I. M. Johnstone, "Multivariate analysis and Jacobi ensembles: Largest eigenvalue, TracyWidom limits and rates of convergence," vol. 36, no. 6, pp. 2638–2716, 2008.
- [14] Q. Ding and S. Kay, "Inconsistency of the MDL: On the Performance of Model Order Selection Criteria with Increasing Signal-to-Noise Ratio," *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 1959–1969, 2011.