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# A TD-CIS study of high-harmonic generation of uracil cation fragments

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## Abstract

High-harmonic generation (HHG) spectroscopy was investigated from laser ablation plasmas of uracil targets in the experimental work of Hutchinson *et al.*, [*Phys. Chem. Chem. Phys. Comparison of high-order harmonic generation in uracil and thymine ablation plumes*, 2013, **15**, 12308] and of Lopez-Quintas *et al.*, [*Appl. Surf. Sci., Characterization of laser-induced plasmas of nucleobases: Uracil and thymine*, 2014, **302**, 299]. Inspired by these works we computed HHG spectra of uracil cation fragments by means of real-time time-dependent configuration interaction with single excitations (RT-TD-CIS) and continuum-optimal Gaussian basis sets. We calculated the rotationally-averaged HHG spectra of different cation fragments analysing the relation between HHG, geometry of the fragments (linear/nonlinear) and electron dynamics (bound and continuum energy states). Moreover, to simulate the complexity of a sample containing fragments upon ablation, we calculated weighted-rotationally-averaged HHG spectra where the contribution of each fragment is included through weights assigned ac-

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cordingly to the experimental mass spectra.

*Keywords:* nonlinear optics; quantum chemistry; ultrafast dynamics

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## 1. Introduction

Attosecond atomic and molecular processes are nowadays intensively investigated thanks to the dramatic advances of ultrafast laser technology [1, 2, 3, 4, 5, 6, 7, 8]. Generation of attosecond pulses are indeed applied to study ultrafast electron dynamics in atomic and molecular systems with unprecedented time and spatial resolution [5, 9, 10, 11, 12].

Attosecond laser resolution is achieved by nonlinear optical processes, such as high-harmonic generation (HHG) [9, 10, 11, 12]. The HHG spectra are composed of the emitted harmonics of the infrared pulse frequency, from which one extracts information on the electronic structure and dynamical of the target [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. HHG spectra are used for molecular orbital tomography [14, 29, 30], to study interference among different electron channels characterising ionisation or recombination [31, 19, 18, 26], the role of quantum coherence [32], the ultrafast charge migration [33], and vibrational signatures emerging in the strong-field dynamics [34].

Application of HHG spectroscopy to organic and biological molecules presented some technical issues. [35] In fact, these molecules in ambient conditions are not easily volatile, and thus creating an high-density gas-phase sample is challenging. Innovative experimental works [36, 37] measured HHG spectra of uracil and thymine using laser ablation technique capable to produce plasma plumes from solid samples of the two nucleobases. The recorded signal originated from a complex relation between single-fragment features, phase matching and interaction among fragments [36, 37].

Theory helps the interpretation of the HHG measurements in complex molecular systems and time-resolved ab initio methods demonstrated to be successfully in this context. [24, 25, 38, 26, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52].

Recently, some of us studied the rotational-averaged HHG spectra of uracil and thymine [51] within real-time time-dependent configuration interaction with single excitations (RT-TD-CIS) and continuum-optimal Gaussian basis sets. We developed this theoretical strategy for HHG in the recent years. [42, 43, 44, 45, 46, 51, 52] This approach represents a reasonable compromise between accuracy and computational cost, which permits an affordable treatment of many-electron systems interacting with strong fields.

Moreover, this approach allows one to control and systematically improve Gaussian basis sets.

Using the same theoretical approach, in this work our goal is computing and analysing the HHG spectra of the uracil cation fragments, in order to unravel the contribution of the single fragment in the plumes. [36, 37] Since Hutchinson *et al.* [36] and Lopez-Quintas *et al.* [37] do not give a fully molecular interpretation of the mass spectra, we used the fragmentation study of Arani *et al.* [53] for getting the molecular moieties for the HHG simulations. We calculated the rotationally-averaged HHG spectra of different cation fragments analysing the correlation between the shape of an HHG spectrum and the geometry of the fragments (linear/nonlinear) together with the electron dynamics (bound and continuum energy states). Moreover, to simulate the complexity of a sample containing a certain number of fragments after ablation, we calculated a weighted-rotationally-averaged HHG where the contribution of each fragments is included. The weights have been assigned accordingly to the measured peak heights in the experimental mass spectrum in Refs [36, 37].

The article is organised as follows: in Section 2 the theoretical framework for computing the HHG spectra is reviewed, computational details of our simulations are given in Section 3, and the results are presented and discussed in Section 4, while in Section 5, we summarize the main achievements of the present work and give perspectives for future developments.

## 2. Theory

### 2.1. Real-Time Time-Dependent Configuration-Interaction Singles (RT-TD-CIS)

In RT-TD-CIS, the time-dependent electronic wave function is written as

$$|\Psi(t)\rangle = \sum_{k \geq 0} c_k(t) |\Psi_k\rangle, \quad (1)$$

where  $|\Psi_k\rangle$  for  $k=0$  is the Hartree-Fock ground state  $|\Psi_0\rangle$ . For  $k > 0$  we use the field-free (time-independent) CIS excited states, i.e.  $|\Psi_k\rangle = \sum_{ia} d_{i,k}^a |\Phi_i^a\rangle$ , constructed on singly-excited Slater determinants  $|\Phi_i^a\rangle$ , with an electron moving from the occupied molecular orbital  $i$  to the virtual one  $a$ , while  $d_{i,k}^a$  are the linear coefficients. [54] The  $c_k(t)$  represents the expansion coefficients that are time dependent and thus responsible for the time dependence in RT-TD-CIS. [41, 55]

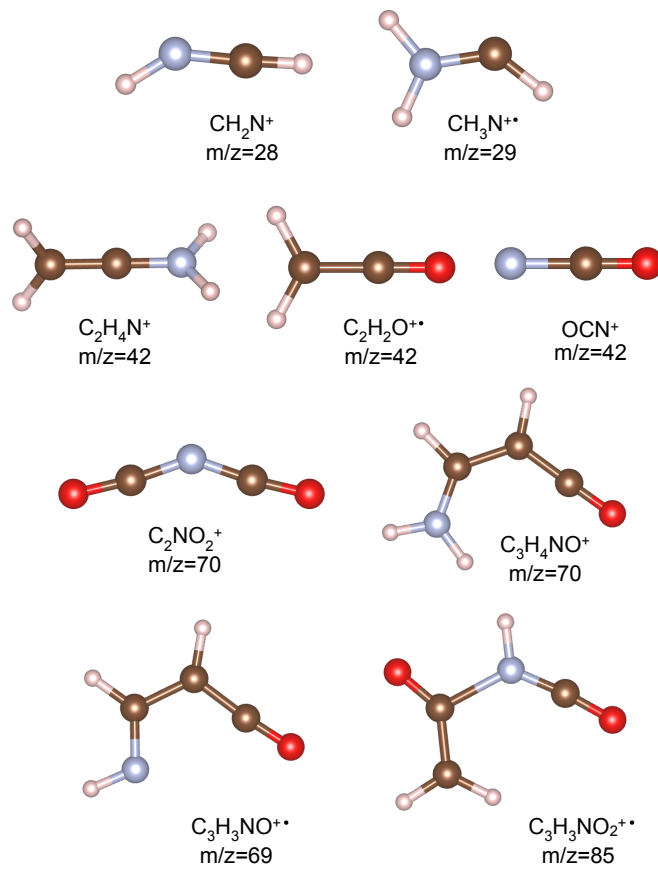


Figure 1: Fragments of the uracil cation and mass/charge ( $m/z$ ) ratio. O atoms are red, C atoms are brown, N atoms are grey and H atoms are white.

The time-dependent Schrödinger equation (TDSE) for the coefficients  $c_k(t)$  is (atomic units) [42, 43, 44, 45, 46, 51]

$$i \frac{\partial c_k(t)}{\partial t} = \sum_s (E_k^{\text{CIS}} \delta_{ks} - \boldsymbol{\mu}_{ks} \cdot \mathbf{E}(t)) c_s(t), \quad (2)$$

where  $E_k^{\text{CIS}}$  are the HF ( $k = 0$ ) and CIS excitation energies,  $\boldsymbol{\mu}_{ks} = \langle \Psi_k | \hat{\boldsymbol{\mu}} | \Psi_s \rangle$  are the matrix elements of the dipole moment operator, and  $\boldsymbol{\mu}_{ks} \cdot \mathbf{E}(t)$  refers to the time-dependent molecule-field interaction. The solution to this equation can be obtained making use of an operator-splitting technique [40, 44]. The time-dependent dipole moment is computed as

$$\boldsymbol{\mu}(t) = \sum_{sk} c_s(t) c_k^*(t) \boldsymbol{\mu}_{ks}, \quad (3)$$

and the HHG spectrum is then obtained as the power spectrum  $P(\omega)$

$$P(\omega) = \left| \frac{1}{t_f - t_i} \int_{t_i}^{t_f} W(t) \boldsymbol{\mu}(t) \cdot \hat{\mathbf{n}} e^{-i\omega t} dt \right|^2, \quad (4)$$

where  $\hat{\mathbf{n}}$  is the unit vector in the polarization direction,  $W(t)$  is the Hann window function and  $t_i$  and  $t_f$  are, respectively, the initial and the final times for the Fourier transform.

TDSE propagation in presence of an intense external field must account for ionisation, which is modelled in the present work by resorting to the heuristic lifetime model [38].

### 2.1.1. Rotationally-averaged HHG spectra

In order to simulate the optical response of a randomly oriented molecular systems, we carried out laser-driven electron dynamics with many different polarisation directions of the laser. We used the unit vector  $\hat{\mathbf{n}}_s = (\hat{n}_{s,x}, \hat{n}_{s,y}, \hat{n}_{s,z})$  to identify the pulse polarisation direction, where  $\hat{n}_{s,x}$ ,  $\hat{n}_{s,y}$  and  $\hat{n}_{s,z}$  are random numbers in  $[-1:1]$  with the constraint  $(\hat{n}_{s,x}^2 + \hat{n}_{s,y}^2 + \hat{n}_{s,z}^2) \leq 1$ .

The rotationally-averaged time-dependent dipole for a molecular fragment labelled with F is computed as

$$\bar{\boldsymbol{\mu}}_F(t) = \frac{1}{S} \sum_s^S \boldsymbol{\mu}_F(t) \cdot \hat{\mathbf{n}}_s, \quad (5)$$

where  $S$  is the number of pulse polarisations for a random electric-field polar-

isation  $\hat{\mathbf{n}}_s$  and  $\boldsymbol{\mu}_F(t)$  is the corresponding time-dependent dipole. Therefore, the averaged HHG spectrum for the fragment F is

$$\bar{P}_F(\omega) = \left| \frac{1}{t_f - t_i} \int_{t_i}^{t_f} W(t) \bar{\boldsymbol{\mu}}_F(t) e^{-i\omega t} dt \right|^2. \quad (6)$$

Moreover, we also considered that the experimental sample of the uracil cation upon fragmentation is constituted of various fragments, each of them expected in the sample with a different probability. Therefore, we defined a weighted-averaged time-dependent dipole moment for the sample as

$$\bar{\boldsymbol{\mu}}_F(t) = \sum_F \omega_F \bar{\boldsymbol{\mu}}_F(t), \quad (7)$$

where the weights  $\omega_F$  are extracted from the experimental mass spectrum of Hutchison *et al.* [36] and of Lopez-Quintas *et al.* [37].

The HHG spectrum of the weighted-averaged rotationally-averaged time-dependent dipole moment is finally computed as

$$\bar{P}_{\bar{F}}(\omega) = \left| \frac{1}{t_f - t_i} \int_{t_i}^{t_f} W(t) \bar{\boldsymbol{\mu}}_{\bar{F}}(t) e^{-i\omega t} dt \right|^2. \quad (8)$$

### 2.1.2. Bound- and continuum-state analysis

Analysing the HHG spectrum in terms of bound- ( $B$ ) and continuum-  
85 state ( $C$ ) contributions allows one to individuate possible interference effects in the time evolution of the electronic wavefunction [40].

The bound-state contribution to the time-dependent dipole moment of the fragment  $F$  is

$$\boldsymbol{\mu}_{F,B}(t) = \boldsymbol{\mu}_{F,GG}(t) + \sum_{i \in B} \boldsymbol{\mu}_{F,Gi}(t) + \sum_{ij \in B} \boldsymbol{\mu}_{F,ij}(t), \quad (9)$$

while, considering only the continuum states, the time-dependent dipole  
90 moment of  $F$  is

$$\boldsymbol{\mu}_{F,C}(t) = \boldsymbol{\mu}_{F,GG}(t) + \sum_{i \in C} \boldsymbol{\mu}_{F,Gi}(t) + \sum_{ij \in C} \boldsymbol{\mu}_{F,ij}(t). \quad (10)$$

In the Equations 9 and 10  $\boldsymbol{\mu}_{F,GG}(t) = |c_0(t)|^2 \boldsymbol{\mu}_{F,00}$  is the ground-ground contribution,  $\boldsymbol{\mu}_{F,Gi}(t) = c_0^*(t) c_i(t) \boldsymbol{\mu}_{F,Gi}$  is the ground-excited contribution with the index  $i$  running over the bound ( $B$ ) or continuum ( $C$ ) states, and  $\boldsymbol{\mu}_{F,ij}(t) = c_i^*(t) c_j(t) \boldsymbol{\mu}_{F,ij}$  is the excited-excited contribution with both the

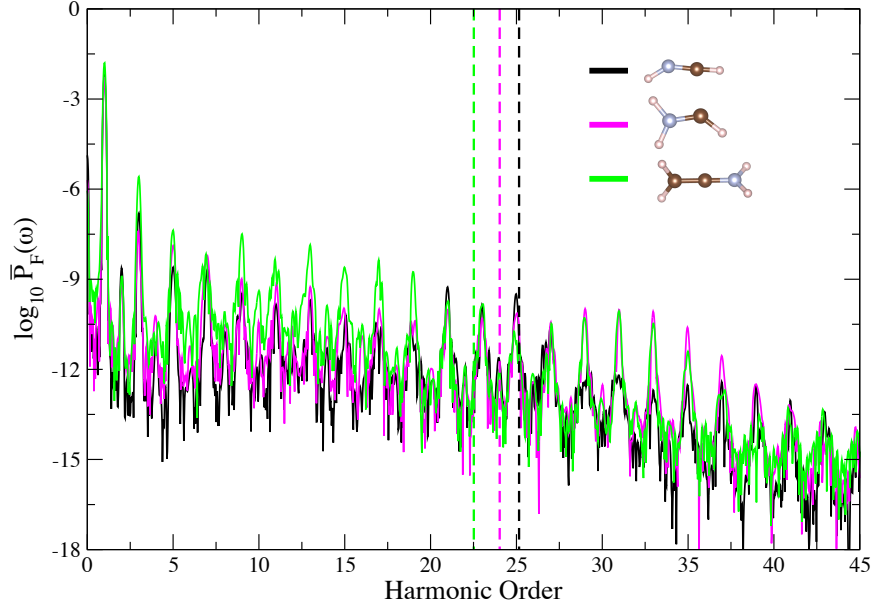


Figure 2: HHG rotationally-averaged spectra of  $\text{CH}_2\text{N}^+$ ,  $\text{CH}_3\text{N}^{+\bullet}$  and  $\text{C}_2\text{H}_4\text{N}^+$  fragments.

95 indexes  $i$  and  $j$  running over the the bound ( $B$ ) or continuum ( $C$ ) states. The average of only bound and only continuum time-dependent dipole moment of the fragment is respectively given by

$$\bar{\mu}_{F,B}(t) = \frac{1}{S} \sum_s^S \boldsymbol{\mu}_{F,B}(t) \cdot \hat{\mathbf{n}}_s \quad (11)$$

and

$$\bar{\mu}_{F,C}(t) = \frac{1}{S} \sum_s^S \boldsymbol{\mu}_{F,C}(t) \cdot \hat{\mathbf{n}}_s. \quad (12)$$

### 3. Computational details

100 The geometry of the uracil cation fragments were taken from the work of Arani *et al.* [53] in which the fragmentation process of the uracil is investigated *via* Density Functional Theory (DFT). The only exception is the  $\text{OCN}^+$  fragment which was optimised in its triplet state at HF/cc-pVDZ



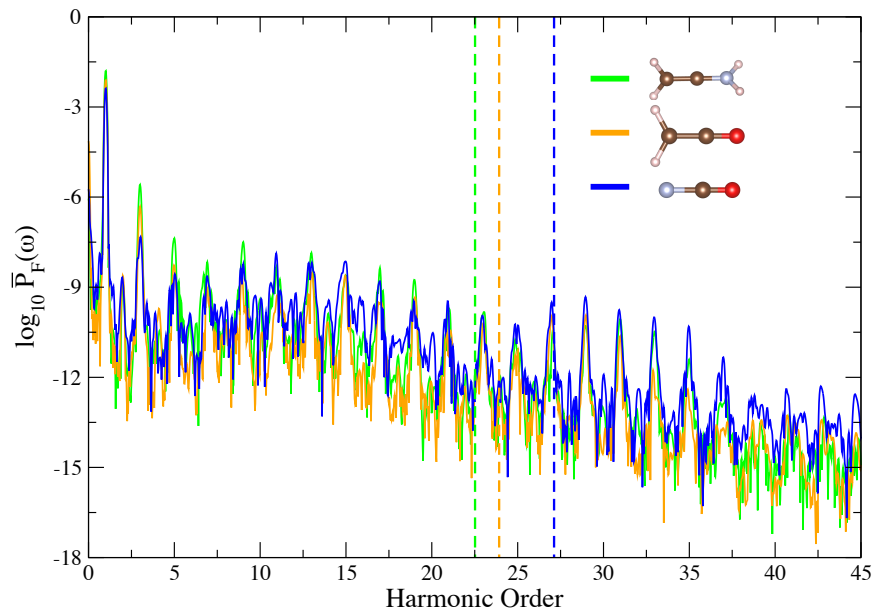


Figure 3: HHG rotationally-averaged spectra of  $\text{C}_2\text{H}_4\text{N}^+$ ,  $\text{C}_2\text{H}_2\text{O}^+$  and  $\text{OCN}^+$  fragments.

level, using Q-Chem [56]. In fact, using the already optimised DFT geometry  
 105 from Ref. [53] in combination with optimal-continuum Gaussian basis sets  
 used in the present work led to unphysical convergence of the HF SCF  
 procedure for the CIS calculation.

RT-TD-CIS calculations were performed using the dipole matrix ele-  
 ments and energies from field-free calculations by means of the Q-Chem  
 110 software package [56], and employed in the homemade code Light [39, 40,  
 41, 42, 43, 44, 46, 50, 51] that propagates the wavepacket under the influence  
 of a time-dependent field. All results correspond to electronic dynamics at  
 fixed nuclear geometries.

To represent the laser pulse, we have considered an electric field  $\mathbf{E}(t)$   
 polarised along the unit vector  $\hat{\mathbf{n}}$

$$\mathbf{E}(t) = E_0 \hat{\mathbf{n}} \sin(\omega_0 t + \phi) f(t), \quad (13)$$

where  $E_0$  is the maximum field strength,  $\omega_0$  is the carrier frequency,  $\phi$  is the

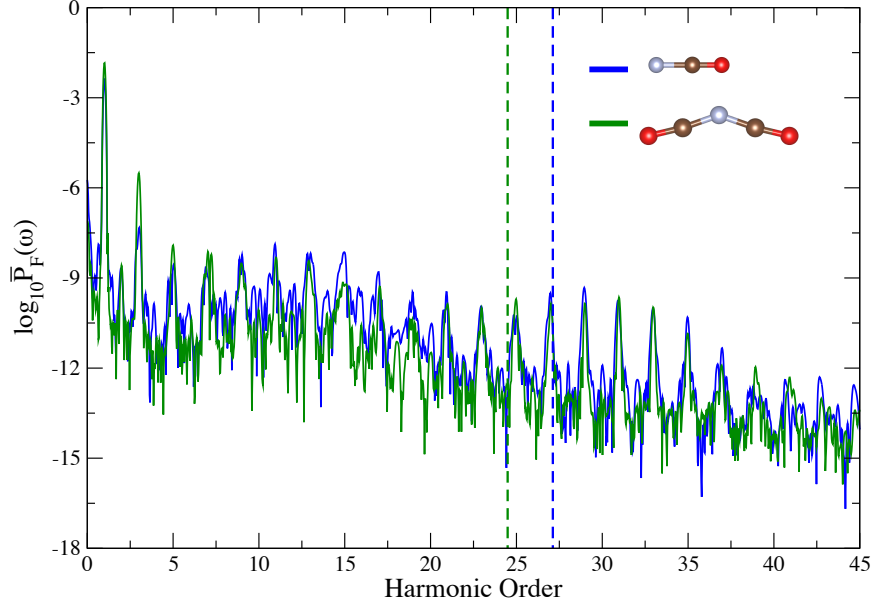


Figure 4: HHG rotationally-averaged spectra of  $\text{OCN}^+$  and  $\text{C}_2\text{NO}_2^+$  fragments.

phase and  $f(t)$  is the envelope function chosen as

$$f(t) = \begin{cases} \cos^2(\frac{\pi}{2\sigma}(\sigma - t)) & \text{if } |t - \sigma| \leq \sigma, \\ 0 & \text{else.} \end{cases} \quad (14)$$

where  $\sigma$  is the width of the field envelope.

115 We used the same pulse wavelength of the experiment of Hutchison *et al.* [36], i.e.  $\lambda_0 = 780$  nm ( $\omega_0 = 1.59$  eV). We used pulse intensities of  $I = 3 \times 10^{13}$  W/cm<sup>2</sup> and  $I = 10^{14}$  W/cm<sup>2</sup>. All the spectra reported here have been calculated with  $I = 10^{14}$  W/cm<sup>2</sup> while the spectra calculated with  $I = 3 \times 10^{13}$  are reported in the Supplementary Information (SI). The duration of the pulse was 23 optical cycles (oc), where  $1 \text{ oc} = 2\pi/\omega_0$ . The time step was 1.21 fs (0.05 a.u.). Value of ionisation energy  $I_p$  is taken as minus HOMO energy, following Koopmans theorem. [57] The three-step model (3SM) cutoff used in this work is computed as  $1.32I_p + 3.17U_p$ , where  $U_p$  is the electron ponderomotive energy. [43, 45, 40, 58, 59, 60]

125 Concerning the choice of the basis sets, we used the computational strategy developed in the recent years by some of us, which demonstrated to be successful to describe HHG for atomic [43, 46, 32] and molecular species

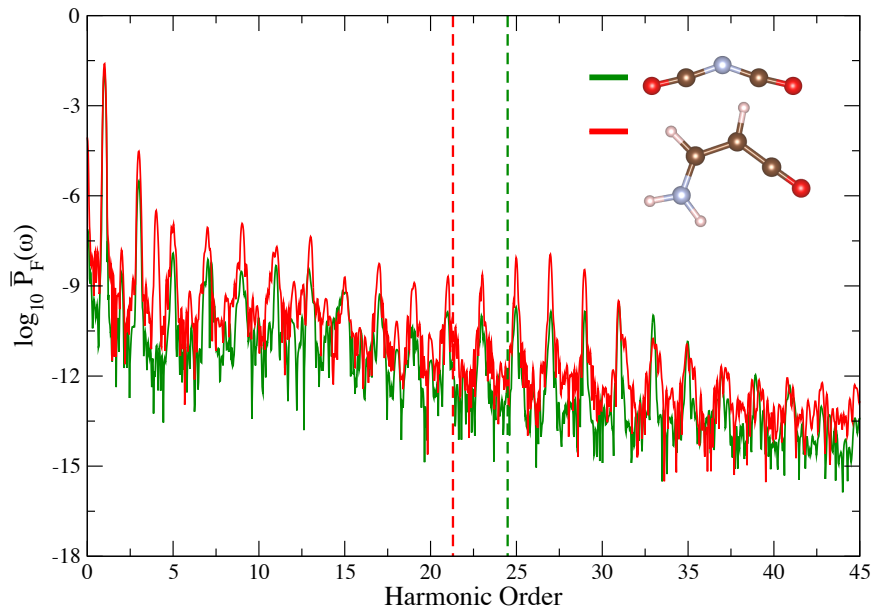


Figure 5: Rotationally-averaged HHG spectra of  $C_2NO_2^+$  and  $C_3H_4NO^+$  fragments.

[51, 45, 41]. We combined Gaussian continuum functions (K) [61] with the heuristic lifetime model. [38, 43, 46, 32, 45] This model prevents unphysical reflections of the wave function in the laser-driven dynamics. The heuristic lifetime model was originally proposed by Klinkusch *et al.* [38] and it consists of adding to the energies calculated from the field-free Hamiltonian a complex term  $\Gamma_k = \sum_{ia} |r_{i,k}^a|^2 \sqrt{2\epsilon_a/d}$  ( $\epsilon_a > 0$ ) which represents the inverse lifetime. The parameter  $d$  is empirical representing the characteristic escape length that the electron is allowed to travel during the lifetime  $1/\Gamma_k$ . According to Refs. [45, 51], the value of  $d$  is given by  $\frac{E_0}{\omega_0^2}$ . A 5aug-cc-pVDZ-3K Gaussian basis set has been used for all the fragments, with the exception of  $OCN^+$ , for which the basis set has been reduced to 5aug-cc-pVDZ to overcome convergence issues of the triplet ground state. K functions have been generated using  $\zeta=1$  in Eq. 15 of Ref. [61]. Density of states and HHG spectra obtained with  $\zeta = 2$  are practically superimposed to the  $\zeta = 1$  ones.

The number  $S$  of pulse propagation directions was chosen in the 150-350 range, according to the studied fragment. For each fragment, we implemented a convergence study of the HHG spectrum by increasing the number of random orientations. The convergence is achieved when the ratio between odd and even harmonics is high enough to distinguish the physi-

cal signal, and when the shape of the spectrum (intensity and number of harmonics, cutoff region) keeps unchanged with a larger number of random orientations.

#### 150 4. Results

To compare and analyse the HHG spectra of the different fragments, we followed a *rationale* which correlates the degree of linearity of the fragment with its mass/charge ratio ( $m/z$ ). This choice follows the idea that starting from physical models, such as Lein model, [62] one can interpret the  
155 HHG spectral features from the interference effects between electrons and atoms during the recombination. This is certainly true also in more complex molecules and can be used as criterion of analysis and interpretation of the results. Moreover, we show here only rotationally-averaged HHG spectra in order to compare as close as possible with experimental findings; how-  
160 ever, investigating the fragment response to a pulse polarised along its main pseudo-axis could be a valuable tool to understand HHG fine features.

In Figure 2 the HHG rotationally-averaged spectra of the fragments  $\text{CH}_2\text{N}^+$  ( $m/z=28$ ),  $\text{CH}_3\text{N}^{+\bullet}$  ( $m/z=29$ ) and  $\text{C}_2\text{H}_4\text{N}^+$  ( $m/z=42$ ) are shown. The energy cutoff from the 3SM is also shown as a qualitative reference.  
165 These are the fragments with the smallest mass/charge ratio and, neglecting the H atoms, are diatomics or linear molecules. It is interesting to observe that fragments  $\text{CH}_2\text{N}^+$  and  $\text{CH}_3\text{N}^{+\bullet}$  show the same order of magnitude of the peak intensities - before the three-step cutoff - and significantly lower than those of fragment  $\text{C}_2\text{H}_4\text{N}^+$ ; this difference may be caused by the  
170 presence of one more carbon atom in  $\text{C}_2\text{H}_4\text{N}^+$ . Instead, beyond the three-step cutoff, at higher harmonics, the behaviour of  $\text{CH}_2\text{N}^+$  and  $\text{CH}_3\text{N}^{+\bullet}$  and changes considerably: while peak intensities for  $\text{CH}_2\text{N}^+$  decrease dramatically,  $\text{CH}_3\text{N}^{+\bullet}$  shows a peak intensity profile similar to  $\text{C}_2\text{H}_4\text{N}^+$ . As specified above,  $\text{CH}_2\text{N}^+$  and  $\text{CH}_3\text{N}^{+\bullet}$  differ only for one hydrogen atom,  
175 except for which they have the same geometric structure; following the two-interference model [62] they should have the minimum almost at the same position as for the  $\text{CH}_2\text{N}^+$  the CN bond length is 1.24 Å and for the  $\text{CH}_3\text{N}^{+\bullet}$  the CN bond length is 1.26 Å. Indeed both fragments share a minimum at the 19th harmonic. The presence of an other hydrogen in fragment  $\text{CH}_3\text{N}^{+\bullet}$   
180 creates an asymmetry, which is reflected in the spatial distribution of electronic orbitals and thus in affecting ionisation and recombination processes, thus leading to a different higher-energy portion of the spectrum. This is translated, at the spectrum level, into a further minimum after the three-step cutoff (25th harmonic), shared with fragment  $\text{C}_2\text{H}_4\text{N}^+$ , and into the

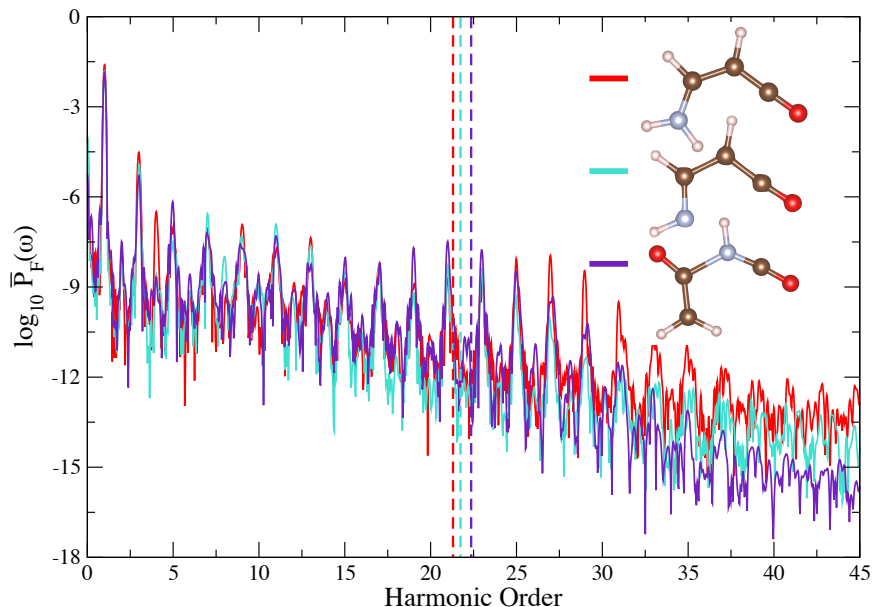


Figure 6: Rotationally-averaged HHG spectra of  $C_3H_4NO^+$ ,  $C_3H_3NO^{+\bullet}$  and  $C_3H_3NO_2^{+\bullet}$  fragments.

185 enhancement of HHG peak intensity at high energy.

It is worth mentioning that, though the rotational average typically makes fine features such as minima in the spectrum weaker, our findings are robust against the number of random orientations.

In Figure 3 the HHG rotationally-averaged spectra of the fragments  
 190  $C_2H_4N^+$ ,  $C_2H_2O^{+\bullet}$  and  $OCN^+$ , which all have  $m/z = 42$ , are reported. The spectrum for the fragment  $C_2H_4N^+$  was already reported in Figure 1. Here, it is considered the reference spectrum with respect to which the HHG spectra of the other fragments should be compared. In this way the differences between the spectra due to the linearity and the  $m/z$  ratio are better  
 195 distinguished.

The general shape of the HHG spectra is very similar for all the fragments. However, the peak intensity of the  $C_2H_2O^{+\bullet}$  is lower than the one of the  $C_2H_4N^+$  and of the  $OCN^+$ . We believe that also in this case an important role is played by the symmetry features of the molecules, which can  
 200 give rise to different probability concerning ionisation and recombination.

The position of the minimum is at H25 for all the fragments. This can be due to the fact that their molecular skeleton is linear, with similar

interatomic distances, behaving as a three-center interference “generator”.  
[62]

205 In Figure 4 the rotationally-averaged HHG spectra of  $\text{OCN}^+$  ( $m/z = 42$ ) and  $\text{C}_2\text{NO}_2^+$  ( $m/z = 70$ ) fragments are shown. The ratio  $m/z$  is very different between the two fragments which, however, are characterised by a similar HHG spectrum. The C and O atoms in the  $\text{C}_2\text{NO}_2^+$  are placed in symmetric positions with respect to the N atom. Therefore, the geometry  
210 of the  $\text{C}_2\text{NO}_2^+$  is similar to two adjacent  $\text{OCN}^+$  moieties. The  $\text{C}_2\text{NO}_2^+$  is the first nonlinear fragment we analyse here. However, the nonlinearity is very small as the CNC angle is around  $140^\circ$ . Therefore, this fragment continues to preserves similar geometrical and orbital symmetry of the  $\text{OCN}^+$ . However, despite the similarities in the case of the  $\text{C}_2\text{NO}_2^+$  the minimum is shifted  
215 lower to the H19 with respect to the H25 of the  $\text{OCN}^+$

Figure 5 shows the rotationally-averaged spectra of the fragments  $\text{C}_2\text{NO}_2^+$  and  $\text{C}_3\text{H}_4\text{NO}^+$ , which have the same  $m/z = 70$ . The  $\text{C}_3\text{H}_4\text{NO}^+$  is the first fragment in our analysis which geometry is highly nonlinear. However, despite the significant different geometrical arrangement of the atoms, we observe that the position of the minimum does not change with respect the  
220  $\text{C}_2\text{NO}_2^+$ , i.e. H19, while the overall intensity of the spectrum is increased. The presence of a strong even harmonics (H4, 6.36 eV) in the  $\text{C}_3\text{H}_4\text{NO}^+$  spectrum can be attributed to a system excitation (6.14 eV).

Finally, in Figure 6 the rotationally-averaged spectra of the larger and  
225 nonlinear fragments  $\text{C}_3\text{H}_4\text{NO}^+$  ( $m/z = 70$ ),  $\text{C}_3\text{H}_3\text{NO}^{+\bullet}$  ( $m/z = 69$ ) and  $\text{C}_3\text{H}_3\text{NO}_2^{+\bullet}$  ( $m/z = 85$ ) are reported. Comparing the  $\text{C}_3\text{H}_3\text{NO}^{+\bullet}$  with the  $\text{C}_3\text{H}_4\text{NO}^+$  we observe that the position of the minimum does not change, i.e. H19. Instead, a strong reduction of the harmonic intensity at higher energy ( $>\text{H27}$ ) is found. This reduction is even stronger for the  $\text{C}_3\text{H}_3\text{NO}_2^{+\bullet}$  which  
230 moreover has the minimum shifted to the H17. The common geometrical feature of these molecules which could influence the ionisation and recombination process is an increasing reduction of the angle opposite to the oxygen on the right. The  $\text{C}_3\text{H}_4\text{NO}^+$  has NCC angle =  $128^\circ$ , the  $\text{C}_3\text{H}_3\text{NO}^{+\bullet}$  has NCC angle =  $115^\circ$  and finally the  $\text{C}_3\text{H}_3\text{NO}_2^{+\bullet}$  CCN angle= $112^\circ$ .

235 Figures S1-S5 in SI show the same analysis done here when the pulse intensity is equal to  $3 \times 10^{13}$  W/cm<sup>2</sup>. By comparing corresponding figures at the two pulse intensities, one observes that the minimum is preserved in some cases, and is shifted in other ones. These findings do not allow us at the present stage to clearly state the structural or dynamical nature of such  
240 a fine structure.

The interpretation of the HHG spectra of these molecular fragments can be complex because of the interplay of the atomic and many-electron dynam-

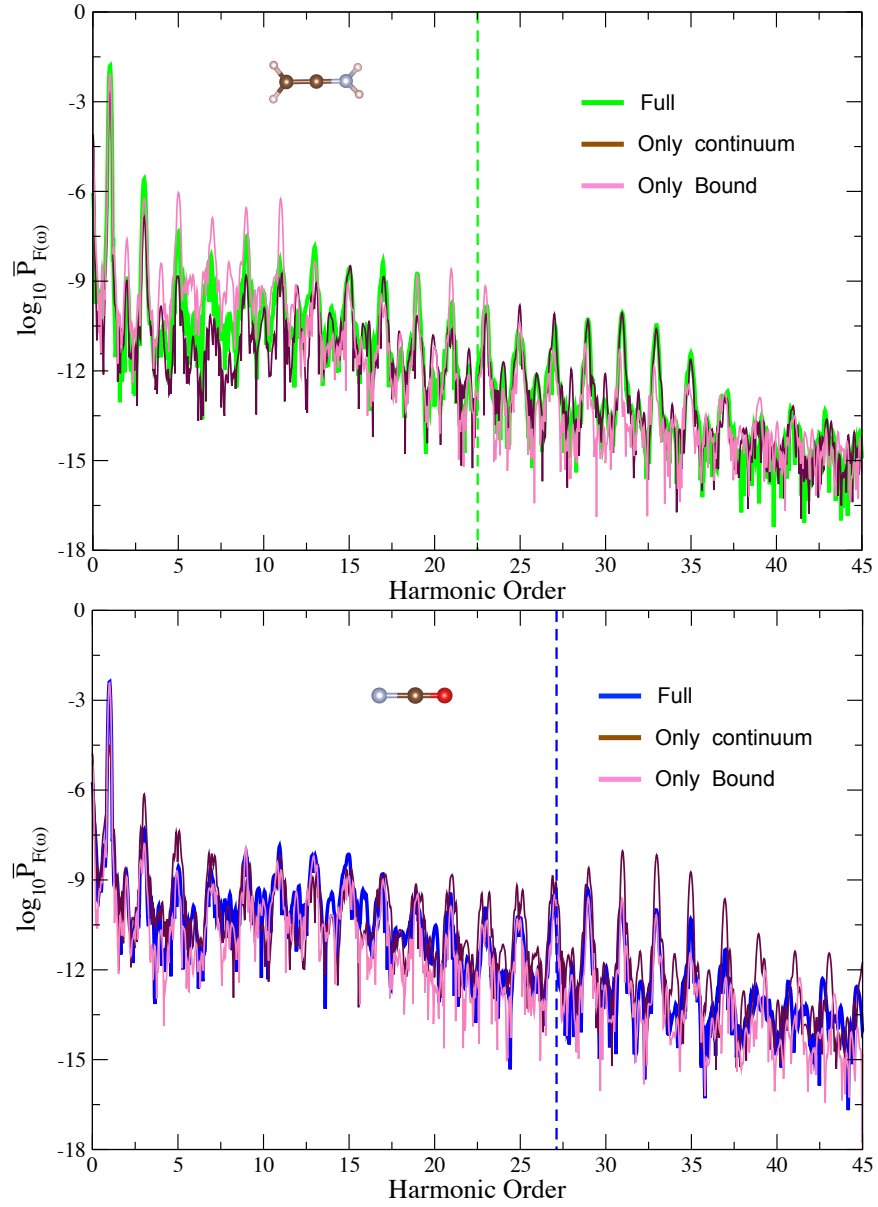


Figure 7: HHG spectra of the  $C_2NH_4$  and  $OCN^+$  fragments compared to the “Only bound” and “Only continuum” HHG spectra of the same fragments.

ics. The geometry plays an important role as well the electron interference between bound and continuum energy states. In Figure 7 we analysed the role of interference between bound and continuum energy states in  $\text{C}_2\text{H}_4\text{N}^+$  (upper panel) and  $\text{OCN}^+$  (lower panel). We considered these particular fragments because they are representative of a singlet and a triplet spin-state among the fragments of the uracil cation.

We calculated the HHG from the time-dependent dipole moment of Equation 11 which includes only bound states (“Only bound”), and from the time-dependent dipole moment of Equation 12 which includes only continuum states (“Only continuum”). For both the fragments, the continuum energy states are particularly important at high energy while the bound energy states at low energy. [40, 51] However, important differences arise when the two fragments are compared. In fact, for  $\text{C}_2\text{H}_4\text{N}^+$  a clear separation of continuum- and bound-state contributions is found: H5-H13 interval in the full spectrum is provided by bound states, which also interfere negatively with the continuum states, since peak intensity for “Only bound” spectrum is higher than that from the “Full” spectrum. In the H27-H35 range, “Full” and “Only continuum” spectra are practically superimposed, indicating a leading contribution in the cutoff region from continuum states. In  $\text{OCN}^+$ , a negative interference between bound and continuum states is instead found from H25, while in the low- and medium-energy region of the spectrum bound and continuum states contribute at the same extent to the full HHG spectrum.

The analysis of each fragment and the interplay between geometry, electron dynamics and rotational average in the HHG spectrum is necessary to finally characterise the HHG spectrum of the sample made by the all nine fragments studied in this work (see Figure 1). We calculate the HHG spectrum from Equation 8 by calculating the weighted-rotationally-averaged time-dependent dipole moment from Equation 7.

We considered two different sets of weights  $\omega_F$  in Equation 7. Both sets have been extracted from the intensity of peaks of the mass spectrum of uracil fragmentation. In particular, the first set was obtained from the work of Hutchison *et al.* [36] while the second set was obtained from the work of Lopez *et al.* [37] We reported the normalised sets of weights in Table 1 and in Figure 8 the sample rotationally-averaged HHG spectra together with the rotationally-averaged HHG spectrum of the neutral uracil molecule. Using two different sets of weights does not significantly modify the shape of the weighted-averaged HHG spectrum, even though weights from Lopez *et al.* Ref. [37] produce higher harmonics in the range H19-H31.

Between all the fragments, the  $\text{CH}_2\text{N}^+$  is the one which is mostly proba-



ble. However, the sample rotationally-averaged HHG spectra in Figure 8 do not resemble that of  $\text{CH}_2\text{N}^+$  reported in Figure 2. The largest contribution to the sample rotationally-averaged HHG is given by  $\text{C}_3\text{H}_4\text{NO}^+$  reported in Figure 5, as one can observe in Figure S6 of SI, where time-dependent dipoles of the fragments are reported.

It is interesting to compare the sample and the single-fragment rotationally-averaged HHG spectra, which are all charged, with the rotationally-averaged HHG spectrum of the neutral uracil molecule [51] (Figure 8). The intensity of the uracil HHG peaks are weaker starting from H20. The charged fragments are characterised by an ionisation potential which is in average 5 eV larger than that of the neutral uracil; for this reason, the high-energy part of the sample HHG spectrum develops towards larger harmonic orders.

## 5. Conclusions

In the present work we have applied Gaussian-based RT-TD-CIS to fragments of the uracil cation to compute rotationally-averaged HHG spectra in different conditions. We first have grouped the results in terms of structural arrangement of the fragments. For both low and high pulse intensities, similarities in the HHG spectra have been found in the pseudo-diatomics fragments  $\text{CH}_2\text{N}^+$  and  $\text{CH}_3\text{N}^+$ , in the linear and nearly-linear moieties  $\text{C}_2\text{H}_4\text{N}^+$ ,  $\text{C}_2\text{H}_2\text{O}^+$ ,  $\text{OCN}^+$  and  $\text{C}_2\text{NO}_2^+$  (with  $\text{C}_2\text{NO}_2^+$  being a kind of “twofold”  $\text{OCN}^+$ ), and in nonlinear fragments  $\text{C}_3\text{H}_4\text{NO}^+$ ,  $\text{C}_3\text{H}_3\text{NO}^+$  and  $\text{C}_3\text{H}_3\text{NO}_2^+$ .

	$\omega_F$ from Hutchison <i>et al.</i> [36]	$\omega_F$ from Lopez-Quintas <i>et al.</i> [37]
$\text{CH}_2\text{N}^+$	0.390	0.321
$\text{CH}_3\text{N}^+$	0.117	0.053
$\text{C}_2\text{H}_4\text{N}^+$	0.130	0.107
$\text{C}_2\text{H}_2\text{O}^+$	0.130	0.107
$\text{OCN}^+$	0.130	0.107
$\text{C}_2\text{NO}_2^+$	0.013	0.080
$\text{C}_3\text{H}_4\text{NO}^+$	0.013	0.080
$\text{C}_3\text{H}_3\text{NO}^+$	0.039	0.080
$\text{C}_3\text{H}_3\text{NO}_2^+$	0.039	0.064

Table 1: Normalised weights  $\omega_F$  extracted from the mass spectrum of the uracil fragmentation from Hutchison *et al.* [36] and from Lopez-Quintas *et al.* [37]

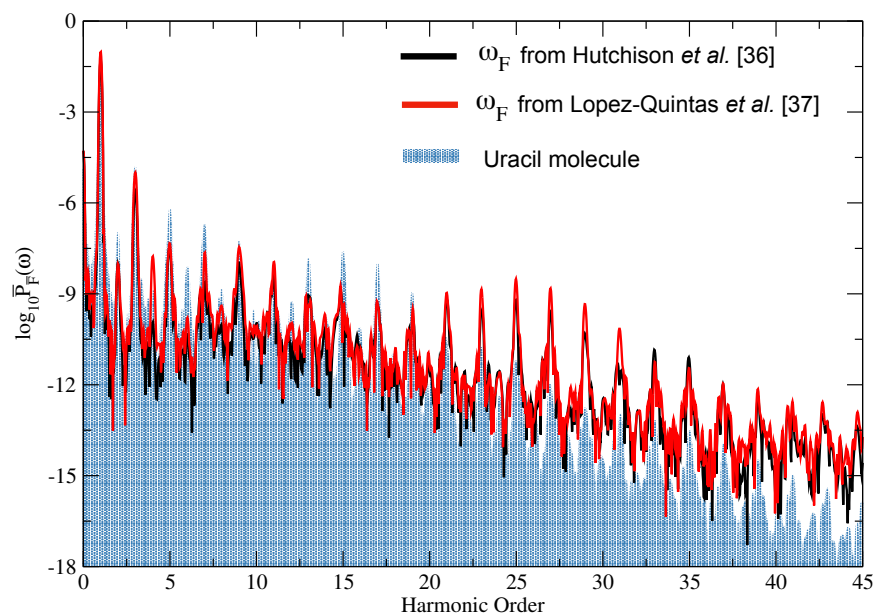


Figure 8: Weighted-rotationally-averaged HHG spectrum for uracil fragments using weights from Hutchison *et al.* [36] and from Lopez-Quintas *et al.* [37] together with the rotationally-averaged HHG spectrum of the neutral uracil molecule.

305 By decomposing the total time-dependent dipole moment in terms of  
bound- and continuum-state contributions, we have seen a general behaviour  
in the analysed fragments, regardless of their geometry, charge or spin state:  
low-energy (high-energy) harmonics are mainly produced by bound (contin-  
uum) states. Differences among the fragments are also observed, especially  
310 in the presence for  $\text{OCN}^+$  of destructive interference effects between bound-  
and continuum states.

In order to reduce the gap with the experimental findings we have com-  
puted the rotationally-averaged HHG spectrum of the sample, by weighting  
the fragment contribution according to the peak height in the experimental  
315 mass spectrum. What is still missing in our approach is modeling the inter-  
action among fragments and macroscopic propagation effects in the sample.

Our results show that Gaussian-based time-resolved methods can be suc-  
cessfully applied to the investigation of the strong-field electron dynamics  
on many-electron molecular systems, with different charge and spin states.

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## **Supporting Information**

Rotationally-averaged HHG spectra of the fragments at  $I = 3 \times 10^{13}$   
325  $\text{W}/\text{cm}^2$ . Rotationally-averaged time-dependent dipoles of the fragments at  
 $I = 10^{14} \text{ W}/\text{cm}^2$ .

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