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# Flocking in One Dimension: Asters and Reversals

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We study the one-dimensional active Ising model in which aligning particles undergo diffusion biased by the signs of their spins. The phase diagram obtained varying the density of particles, their hopping rate and the temperature controlling the alignment shows a homogeneous disordered phase but no homogeneous ordered one, as well as two phases with localized dense structures. In the flocking phase, large ordered aggregates move ballistically and stochastically reverse their direction of motion. In what we termed the “aster” phase, static aggregates of opposite magnetization face each other. Using a combination of numerical simulations and mean-field theory, we study in particular the shape of the flocks and its evolution in time, the statistics of their reversal times, and their coarsening dynamics. Solving exactly for the zero-temperature dynamics of an aster allows us to understand their coarsening, which shows extremal dynamics, while mean-field equations account for their shape.

## CONTENTS

## I. INTRODUCTION

I. Introduction	1
II. Model and its phases	2
A. 1D active Ising model	2
B. The three phases	3
C. Fate of the liquid-gas transition scenario	4
III. Flocks and their reversals	4
A. Shape of a flock	4
B. Statistics of reversal times	6
C. Role of parameters $D$ , $\varepsilon$ and $\beta$	8
D. Coarsening	9
IV. Asters	10
A. Lifetime of an aster at zero temperature	10
B. Coarsening at small temperature	12
C. Shape of asters	13
V. Conclusion	13
Acknowledgments	14
A. Numerical simulations	14
1. Discrete-time simulations	15
2. Continuous-time simulations	15
3. Numerical procedure to spatially align flocks.	16
B. Another zero-temperature limit	16
References	17

Active matter often consists of large assemblies of self-propelled particles. The variety of collective behaviors that they exhibit has come under intense scrutiny [1–4]. This interest stems from both the pervasiveness of active matter in nature, from molecular motors [5, 6] at subcellular scale to bacterial swarms [7–11] and animal groups [12–15], and from the possibility to synthesize various kinds of self-propelled particles, such as chemically propelled Janus particles [16–19], rolling colloids [20–23] or vibrated macroscopic objects [24–26].

A challenge for physicists is to understand the fundamental differences between active matter, on the one hand, and passive systems or other classes of nonequilibrium systems, on the other hand. The role of fluctuations offers a striking example of such differences. Indeed, it was realized by Toner and Tu [27], following the introduction of the Vicsek model [28], that self-propelled particles with a local ferromagnetic alignment of their direction of motion, a “flying XY model”, exhibit an ordered phase of collective motion even in two-dimensions (2D), whereas such a phase would be destroyed by fluctuations in a passive system, as requested by the Mermin-Wagner theorem [29]. Although they do not destroy the ordered state, density fluctuations are measured to be anomalously large [30–33]. The nature of the transition to the ordered state of collective motion from a disordered system shows another role of fluctuations. At mean-field level the transition is predicted to be continuous for many systems [34–37] but becomes discontinuous once fluctuations are accounted for [38]. This translates into large finite-size effects in simulations: The transition seems continuous in small systems [28] but appears discontinuous [31, 39], akin to a liquid-gas transition, for large-enough system sizes [37, 38, 40].

The effect of fluctuations is most prominent in 1D.

For a passive system with short-range interactions, the Van Hove theorem forbids a phase transition at non-zero temperature in absence of long-ranged interactions [41]. Nonequilibrium systems can evade this requirement: For example driven diffusive systems show boundary-induced phase transitions [42–44]. One-dimensional active systems also order and they do so in a peculiar way. It was shown in several models of aligning self-propelled particles that when the noise on alignment is reduced, the system transitions from a disordered state to a state where a finite fraction of all particles is contained in a single ordered aggregate [37, 45–50]. This “flock” propagates ballistically while spreading and spontaneously reverses its direction of motion under the action of fluctuations. Because the time between two reversals increases only logarithmically with system size [46], there is no symmetry breaking asymptotically [37]. The situation is that of a stochastically switching global order, even in infinite systems.

One-dimensional flocking also has some experimental relevance. Locusts marching in a quasi-1D ring-shaped arena order if they are sufficiently dense and show spontaneous reversals of the global direction of motion [51, 52]. Recently, the collective motion of cells confined to 1D racetracks has also been studied in detail [53].

Here, we analyze the 1D version of the active Ising model (AIM), first introduced in Ref. [37]. This lattice model only involves ferromagnetic alignment and self-propulsion, with no excluded volume or other interaction. We explore the phase diagram in terms of the temperature controlling the aligning interaction, the hopping rate, and the average density of particles. Consistently with previous studies considering different implementations of the same ingredients [45, 46, 48–50], we observe a transition from a disordered state to a flocking state featuring a single moving aggregate, which stochastically reverses its direction of motion. Although it was reported to be continuous in early papers [45, 46] the transition was found to be discontinuous in Ref. [37], a fact that is somewhat hidden by reversals. Here we clarify the connection with the liquid-gas transition happening in 2D [37, 54] by showing that the flocks tend to relax to a phase-separated state but are prevented to do so by reversals. Moreover, we confirm the logarithmic scaling with system size of the reversal time found in Ref. [46] for a different 1D flocking model and track down its origin. At low temperature and/or hopping rate, when alignment dominates over self-propulsion, we encounter a different type of structure that we term “asters”. These asters are static structures composed of two highly localized peaks of opposite magnetization. We first account for the remarkable stability of these structures by solving exactly the dynamics of an single aster at zero temperature (which is the asymptotic configuration then). We then discuss the finite-temperature coarsening of asters and show the existence of two possible scenarios leading either to macroscopic condensation or to an extensive number of asters with a characteristic size. Finally, we

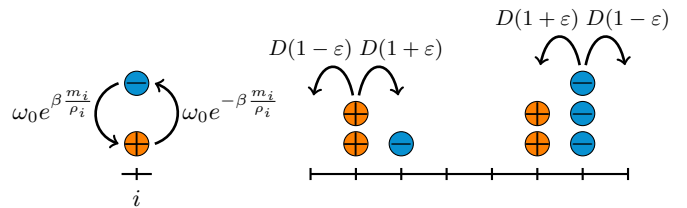


FIG. 1. Sketch of the two possible actions, spin-flipping and hopping, and their associated rates.

derive the steady-state profile of the asters in the mean-field approximation, which turns out to be very accurate.

The paper is organized as follows: In Sec. II we first define the AIM and present the three phases observed in our simulations (disordered, flocking and aster phases) with phase diagrams for the main parameters of the model. We then investigate the two non-trivial phases. First, the flocks in Sec. III where we look at their shapes and how they evolve in time (Sec. III A), the reversal statistics (Sec. III B), and the two distinct regimes observed at small and large velocity and/or temperature (Sec. III C). We then investigate the aster phase in Sec. IV, where we derive an exact solution for the dynamics of an aster at zero temperature in Sec. IV A, before looking at the coarsening dynamics at small-but-finite temperature in Sec. IV B and investigating the aster shape in Sec. IV C.

## II. MODEL AND ITS PHASES

### A. 1D active Ising model

Following Refs. [37, 54], we consider  $N$  particles on a ring of  $L$  sites. There is no excluding-volume interaction so that each site can accommodate an arbitrary number of particles. Each particle carries an Ising spin  $s = \pm 1$  and undergoes diffusion biased by its spin: A particle with spin  $s$  hops to the next site on its right (resp. left) at rate  $D(1 + s\varepsilon)$  (resp.  $D(1 - s\varepsilon)$ ). Parameter  $\varepsilon \in [0, 1]$  controls the asymmetry between the passive limit  $\varepsilon = 0$  and the fully asymmetric hopping  $\varepsilon = 1$ , while  $D$  controls the overall hopping frequency. In average, a particle thus moves at a speed  $v = 2D\varepsilon a$  in the direction set by the sign of its spin, ( $a$  is the lattice spacing). Finally, on a site  $i$  occupied by  $n_i^+ + n_i^-$  particles with spin respectively  $+1$  and  $-1$ , a particle of spin  $s$  flips it at a rate

$$W(s \rightarrow -s) = \omega_0 e^{-\beta s \frac{m_i}{\rho_i}}, \quad (1)$$

where  $\rho_i = n_i^+ + n_i^-$  and  $m_i = n_i^+ - n_i^-$  are the local density and magnetization. This rate is chosen such that the flipping dynamics of each site  $i$  is independent and corresponds to that of a fully-connected on-site Ising model with Hamiltonian  $-m_i^2 / (2\rho_i)$  undergoing an equilibrium dynamics at inverse temperature  $\beta = 1/T$ . The two actions (hopping and flipping spin) and the associated rates are depicted in Fig. 1. Note that the model

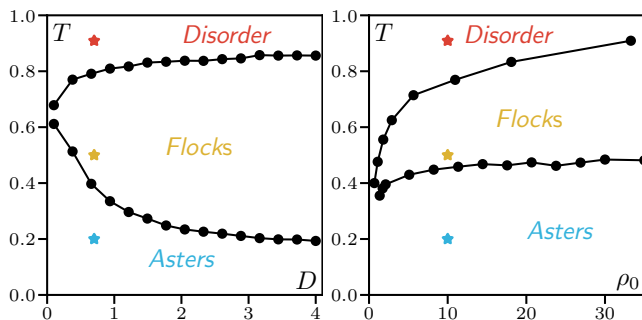


FIG. 2. Phase diagrams in the (hopping-rate, temperature) plane at  $\rho_0 = 10$  and  $\varepsilon = 0.7$  (left) and (density, temperature) plane at  $D = 0.5$  and  $\varepsilon = 0.7$  (right). The stars indicate the parameters used in Fig. 3 to illustrate the phases. The Disorder-flocks line in the  $(\rho_0, T)$  plane is set by the density  $\rho_g(T)$  of the gas in the flocking phase (see Sec. II C). The other lines are determined using the threshold  $\langle |\tilde{m}| \rangle = 0.05$  on the time-average magnetization per particle (which is non-zero only in the flocks phase). Data obtained in a system of size  $L = 500$ ; increasing system size displaces the lines only within the size of the symbols.

is out of equilibrium even at  $\varepsilon = 0$  since the symmetric hopping dynamics is insensitive to the changes of the total Hamiltonian  $H = -\sum_i m_i^2 / (2\rho_i)$  [55].

In the rest of the paper, we choose without loss of generality  $\omega_0 = a = 1$  thus fixing the time and space units. We study the system as a function of the parameters  $\beta$  (or  $T$ ),  $D$ ,  $\varepsilon$  and the average density  $\rho_0 = N/L$ . Our simulations relied either on discrete time steps with random sequential updates or on an exact continuous-time Monte Carlo algorithm. The efficiencies of both schemes depend on the temperature and phases under study, as detailed in Appendix A.

### B. The three phases

Looking at the phase diagrams in the  $D-T$  and  $\rho_0-T$  planes shown in Fig. 2 (and in the qualitatively similar  $\varepsilon-T$  diagram not shown), we see that, as expected, at high temperature the system is disordered. Density and magnetization are homogeneous and fluctuate, respectively, around  $\rho_0$  and 0. Small aggregates can be observed to move ballistically (as seen in Fig. 3e in the gas region surrounding the main aggregate) but they are transient and of microscopic size, independent of system size.

Decreasing temperature, the system reaches a flocking phase which consists, at long times, of a dense ordered aggregate moving ballistically in a disordered gas, as illustrated on the snapshot shown in Fig. 3c. These “flocks” have a rather complex dynamics: They slowly but regularly widen as they travel, and undergo stochastic reversals during which they “regroup” into a very thin condensate and their magnetization changes sign. These dynamics together with the coarsening leading to a sin-

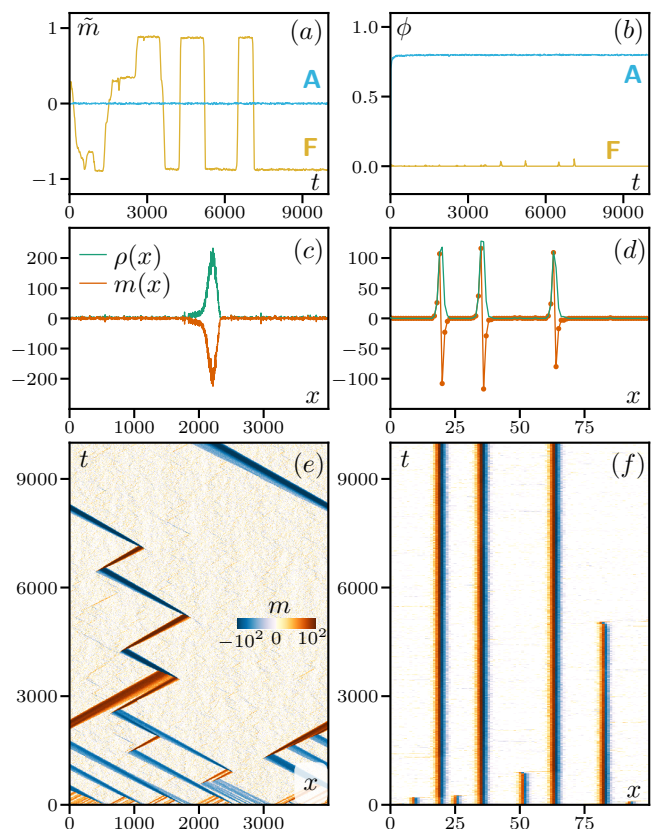


FIG. 3. **(a)**: The magnetization per spin  $\tilde{m}$ , is non-vanishing only in the flocking phase. **(b)**: The fraction of particles  $\phi$  contained in asters is non-vanishing only in the aster phase. **(c-d)**: Instantaneous density and magnetization fields at the final time of the space time diagrams shown in (e-f). **(e-f)**: Space time diagrams in the flocking (e) and aster (f) phases. The parameters, indicated by stars in the phase diagrams of Fig. 2, are  $\rho_0 = 10$ ,  $\varepsilon = 0.7$ ,  $D = 0.5$  and  $\beta = 0.8, 2, 5$  for the D, F and A phases, respectively. The system sizes  $L = 4000$  (F phase) and  $L = 100$  (A phase) were chosen for legibility. Simulations are started from a homogeneous disordered initial conditions. The runs in (a,b) for the A and F phases are the same as in (e,f). The profiles (c,d) correspond to the final state at  $t = 10^4$ .

gle aggregate, is displayed in the space-time diagram of Fig. 3e, and is the subject of Sec. III.

Finally, at lower temperatures, the system exhibits what we have called asters, that are illustrated in the snapshot shown in Fig. 3d: Sharp peaks of positive and negative magnetizations, spread over a few sites, face each other. These structures are long-lived, despite the absence of repulsive interactions. As shown on the space-time plot Fig. 3f, asters can dissolve on long time scales, leading to coarsening. Our study of this phase is in Sec. IV.

To distinguish between the three phases, we introduce two order parameters, the magnetization per particle  $\tilde{m} = \sum_{i=1}^N s_i / N$ , which is non-zero at long times only in the flocking phase where it alternates between a positive

and a negative value because of reversals, and the fraction  $\phi$  of particles contained in asters. Our criterion to detect an aster is to find two adjacent sites with magnetization of opposite sign and density larger than  $2\rho_0$ . (This value is chosen such that asters are robustly detected while not erroneously counting fluctuations as asters. Our results do not depend on the precise criterion used.). The two order parameters are shown as a function of time in the flocking and aster phases (Fig. 3a-b).

### C. Fate of the liquid-gas transition scenario

In 2D, the transition to collective motion in polar flocking models has been shown to be akin to a liquid-gas transition between a disordered gas and a polar ordered liquid [37, 54]. The form of the phase coexistence depends on the symmetry of the spins: for the Vicsek model with continuous spins one observes microphase separation with an extensive number of dense ordered traveling bands having a characteristic size, while in the AIM, which has a discrete spin symmetry, one obtains phase separation between two macroscopic domains [40].

In the same way, in 1D one observes the coexistence of an ordered flock with a disordered gas. Although, because of fluctuations it can easily be mistaken for a continuous transition [45, 46], the transition between the disordered phase and the flocking state also shows a phase-separation scenario. In particular, the transition exhibits metastability and the telltale negative peaks in the Binder fourth-order cumulant for large (but finite) systems which arise from discontinuous nucleation events [37, 56]. In addition, the density in the gas is independent of the average density  $\rho_0$  in the system.

However, looking at the phase diagram in the  $(\rho_0, T)$  plane (Fig. 2, right), a second transition line to a homogeneous ordered liquid is conspicuously absent. This is because, contrary to what happens in 2D, fluctuations destroy the homogeneous ordered phase in 1D. Indeed, as in Ref. [47], we see that, if we prepare the system in this state, it is metastable but eventually gets destabilized by a fluctuation. As shown in the left panel of Fig. 4 (see also the supplementary movie XXX), a local fluctuation of the opposite magnetization propagates through the entire system, until only a localized flock remains. Thus, even if the probability of such a fluctuation may be rare, it increases linearly with system size since it can happen anywhere in the system. Therefore, we expect that the lifetime of the ordered phase decreases as  $L^{-1}$ , which is what we measure (see Fig. 4, right). At large enough system size, the ordered phase is thus destabilized very quickly.

Thus, the form of the phase coexistence is also altered by fluctuations in 1D. Instead of the separation into two liquid and gas domains at densities  $\rho_g$  and  $\rho_\ell$  independent of the average density  $\rho_0$ , one sees the dynamical flocking phase described in the previous section. Only the gas density  $\rho_g$  is easily defined and separates the disordered

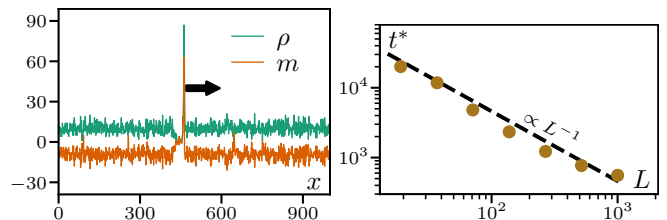


FIG. 4. **Left:** Instantaneous profile showing a fluctuation propagating through the homogeneous ordered phase. **Right:** Average life time  $t^*$  of the metastable homogeneous ordered phase. Parameters:  $L = 1000$  (left),  $\beta = 2$ ,  $\varepsilon = 1$ ,  $D = 0.5$  and  $\rho_0 = 10$ .

and flocking state in the phase diagram Fig. 2 (right). We will see in Sec. III that the flocks actually tend to relax to the phase-separated state at constant density observed in 2D but are prevented to do so by the reversals.

## III. FLOCKS AND THEIR REVERSALS

In the flocking phase, the system presents a single ordered aggregate in the steady state. As already visible in Fig. 3(e) and illustrated in Fig. 5 (see also supplementary movie XXX), this flock moves ballistically but its shape evolves continuously: initially narrow and sharp it progressively spreads. After some time, a protrusion with a well-defined density corresponding to the (metastable) homogeneous liquid phase begins to develop at the leading edge, growing out of the main peak. Once the flock is sufficiently spread out, a fluctuation that flips the first few sites at the front can become large enough that it flips systematically all the particles in the flock. This leads to a full reversal of the flock, whose dynamics then resets, starting with the same initial sharply peaked shape but with the opposite magnetization (compare Fig. 5 (a) and (f)).

In this section, we study quantitatively this dynamics. We first look at the evolution of the shape of the flock (Sec. III A), which provides insight into how the time between two reversals varies with system size (Sec. III B) and with other parameters (Sec. III C). Finally we study the coarsening process following a quench from the disordered state (Sec. III D).

### A. Shape of a flock

To separate the trend from fluctuations in the evolution of the flock shape, we average them over many realizations and construct the average flock shape at time  $t$  after their last reversal. To do so, we define an origin of time for each reversal and spatially align flocks of similar age  $t$ . As shown in Fig. 3(a), the magnetisation per spin  $\tilde{m}$  flips from a well defined value  $\tilde{m}_0$  to  $-\tilde{m}_0$  during a reversal. We define the time origin of each reversal as the

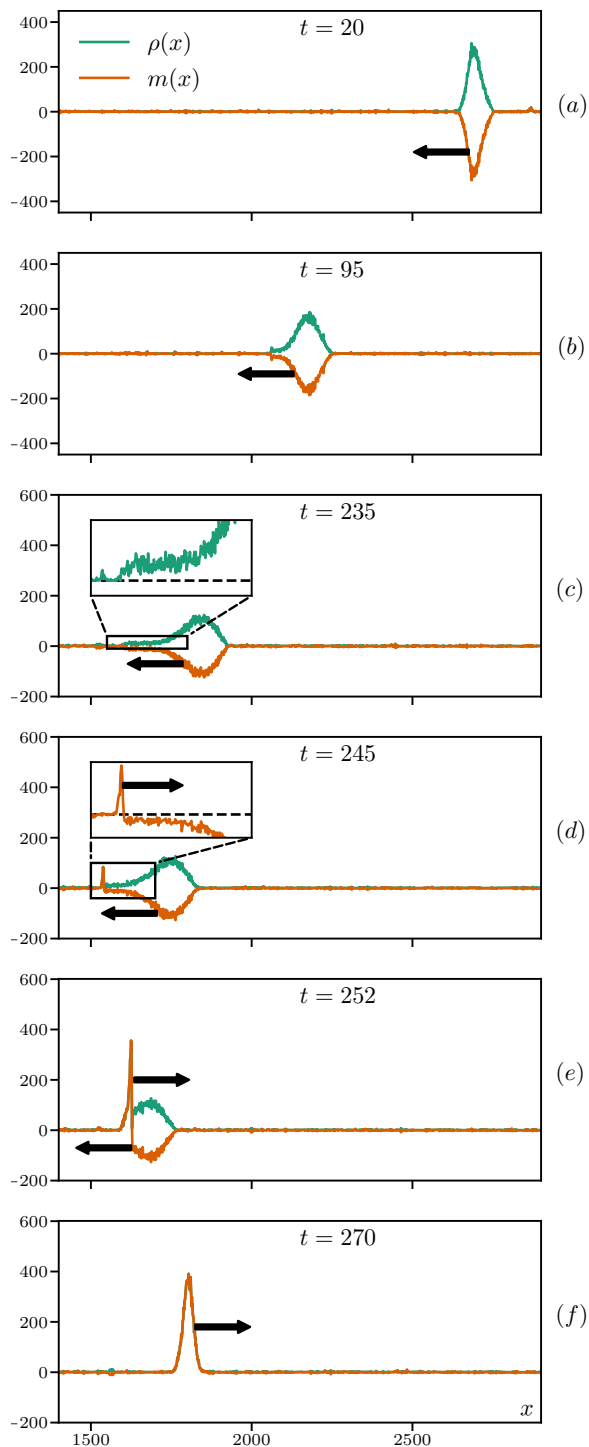


FIG. 5. Instantaneous density (green) and magnetization (orange) profiles during the spreading and the reversal of a flock. An initially peaked aggregate (a) spreads while propagating (b). After some time, it develops a protrusion at the leading edge (c). A spontaneous fluctuation at the front (d) can propagate inside the flock while flipping all the particles (e). Finally, just after the reversal, one is left with a flock with the same shape as in (a) (which was taken just after the previous reversal), albeit with a reversed magnetization. Time is counted from the previous reversal. Parameters:  $L = 4000$ ,  $\beta = 2$ ,  $\varepsilon = 1$ ,  $D = 0.5$ ,  $\rho_0 = 10$ .

time when  $\tilde{m}(t) = -0.7\tilde{m}_0$ . (The 0.7 factor works well, but other values are of course possible.) Flocks of the same age are of course localized at different positions on the lattice. A simple idea to align them so as to be able to average their shape would be to use their peak density, but this is very noisy and leads to large fluctuations that artificially smear out the flock shape. Instead, we use the rear edge of flocks, which is always very sharp since the liquid protrusion grows at the leading edge. (The detailed aligning procedure is described in Appendix A 3.) This leads to satisfying results: As shown in Fig. 6(a), profiles superimpose with little spread in most of the profiles, allowing to extract meaningful average shapes. Note however that the precise position of the leading edge fluctuates, leading to an average profile whose leading edge is smoother than that of the instantaneous ones. This confirms clearly the two features mentioned earlier, as shown in Fig. 6(b): the flock contains a main peak which spreads continuously and, after some time, a protrusion with a constant height develops at its leading edge. We now discuss both of these features.

As shown in Fig. 6(c), the main peak of the flock, at a given age, is proportional to the system size and thus contains a macroscopic fraction of the particles. The top of the peak is well approximated by a Gaussian with a variance  $\sigma^2$  that is independent of system size and evolves linearly in time (Fig. 6(d)). In addition, the peak propagates at a constant speed  $v_p$ , defined as the speed of the maximum of the density profile, which is smaller than the speed  $v$  of individual particles (Fig. 6(e)).

Since the peak contains a high density of particles, we expect it to be well described by a mean-field theory that we now construct starting from the microscopic dynamics, following Ref. [54]. To do so, we first write exact equations for the average density and magnetization  $\langle \rho_i \rangle$  and  $\langle m_i \rangle$  on site  $i$ , the average being over realizations of the stochastic microscopic dynamics. This gives

$$\partial_t \langle \rho_i \rangle = D \langle \rho_{i+1} + \rho_{i-1} - 2\rho_i \rangle - \frac{v}{2} \langle m_{i+1} - m_{i-1} \rangle \quad (2)$$

$$\begin{aligned} \partial_t \langle m_i \rangle = & D \langle m_{i+1} + m_{i-1} - 2m_i \rangle - \frac{v}{2} \langle \rho_{i+1} - \rho_{i-1} \rangle \\ & + 2 \left\langle \rho_i \sinh \left[ \beta \frac{m_i}{\rho_i} \right] - m_i \cosh \left[ \beta \frac{m_i}{\rho_i} \right] \right\rangle \end{aligned} \quad (3)$$

These equations are not closed since Eq. (3) involves the average of a nonlinear term. However, in the mean-field approximation where fluctuations and correlations are neglected,  $\langle f(x) \rangle = f(\langle x \rangle)$  for any function  $f$ . Taking in addition the continuous limit, Eqs. (2)-(3) rewrite

$$\partial_t \rho = D \partial_x^2 \rho - v \partial_x m \quad (4)$$

$$\partial_t m = D \partial_x^2 m - v \partial_x \rho - 2m \cosh \left[ \beta \frac{m}{\rho} \right] + 2\rho \sinh \left[ \beta \frac{m}{\rho} \right]. \quad (5)$$

with  $\rho(x=ia) = \langle \rho_i \rangle$  and similarly for  $m$ .

Contrary to the density field,  $m(x, t)$  is a fast mode which relaxes rapidly to a value that makes the right-hand-side of Eq. (5) vanish. To leading order in a gradient expansion, this amounts to requiring that the interaction term in Eq. (5) vanishes, i.e. that  $m/\rho =$

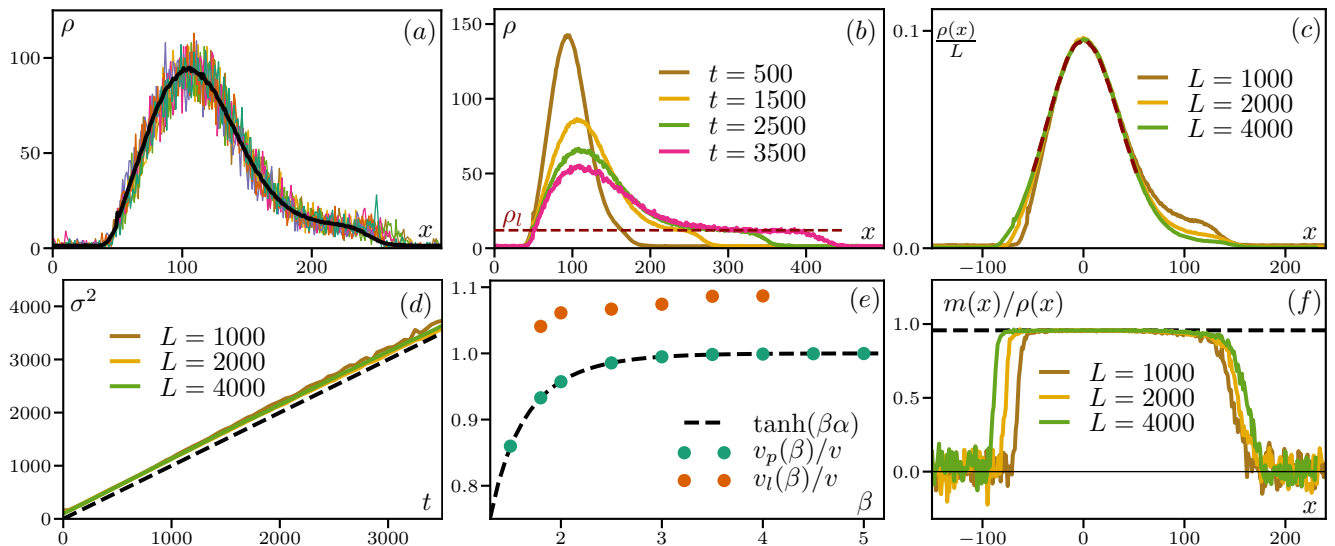


FIG. 6. **(a)**: Seven representative instantaneous density profiles (colors) superimposed to the average profile (thick black line). **(b)**: Evolution of the averaged density profile as a function of time. The red dashed line is a fit to the density  $\rho_\ell$  of the protrusion. **(c)**: Average density profiles at  $t = 1200$  showing that the main peak is extensive in system size. The red dashed line is a Gaussian fit used to compute the variance  $\sigma^2$  in (d). **(d)**: Variance of the main peak. The dashed line is the mean-field prediction  $\sigma^2 = 2Dt$ . **(e)**: Velocity  $v_p$  of the main peak (computed as the velocity of the maximum) and  $v_\ell$  of the protrusion (computed as the velocity of the point at  $\rho = \rho_\ell/2$  in the average profiles) divided by the particles velocity  $v = 2D\varepsilon$ . The dashed line is the mean-field prediction. **(f)**: Average profile for  $m/\rho$ . The dashed line is the mean-field prediction, solution of  $m/\rho = \tanh(\beta m/\rho)$ . Parameters:  $\rho_0 = 10$ ,  $\beta = 2$ ,  $\varepsilon = 1$ ,  $D = 0.5$ ,  $t = 1000$  (a,c,e,f),  $L = 1000$  (a,b),  $L = 5000$  (e).

$\tanh(\beta m/\rho)$ . This thus predicts  $\alpha = m/\rho$  to be the solution of  $\alpha = \tanh(\beta\alpha)$ , which is verified numerically in Fig. 6(f). Using  $m(x) = \alpha\rho(x)$  in Eq. (4), we obtain a diffusion-drift approximation for the density field:

$$\partial_t \rho = D \partial_x^2 \rho - v \tanh(\beta\alpha) \partial_x \rho. \quad (6)$$

The diffusion and drift coefficients read from Eq. (6) both match numerical measurements as seen in Fig. 6(d,e).

Let us now turn to the protrusion appearing at the front of the flock. Since reversals start at the leading edge, this protrusion is expected to play an important role in the dynamics. Contrary to the main peak, it has a fixed height (in density and magnetization) independent of time and system size (not shown). Its velocity results both from the persistent hop of the particles and from the recruitment of new sites at the leading edge due to the aligning dynamics. This explains why the leading edge of the protrusion grows with a speed larger than the individual particle speed  $v$  (Fig. 6(e)), much like for the polar bands of the coexistence region in 2D [54]. The front speed is thus also larger than the drift velocity of the peak, which explains the increase of the protrusion length as time passes. All this suggests that the flock is trying to relax to the fixed density  $\rho_\ell$  of the homogeneous ordered liquid but is prevented to do so by the reversals. This is confirmed by choosing parameters such that the full phase separation can be observed before a reversal happens. This is achieved in Fig. 7 by increasing  $D$ ; one then observes a relatively long-lived phase-separated profile.

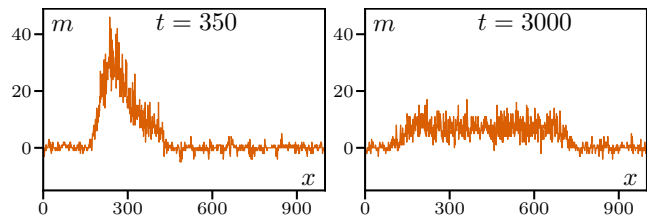


FIG. 7. Instantaneous density profile at times  $t = 350$  (left) and  $t = 3000$  (right) after the last reversal. The profile relaxes to a long-lived phase-separated state, whose shape is stationary (until the next reversal occurs). Parameters:  $\rho_0 = 5$ ,  $L = 1000$ ,  $\beta = 2$ ,  $\varepsilon = 1$  and  $D = 2$ .

## B. Statistics of reversal times

The average time between two reversals,  $\langle \tau \rangle$ , shows a logarithmic increase with system size  $\langle \tau \rangle \propto \log(L)$  (Fig. 8(a)). This is in line with previous results obtained by O'Loan and Evans for a different model [46], and confirms that no true symmetry-breaking arises.

To understand the physical origin of this logarithmic scaling, we consider  $P(\tau)$ , the distribution of inter-reversal times. Shown in Fig. 8(b), it has a peak and an approximately exponential tail with a decay rate that is roughly independent of  $L$ . Increasing  $L$ , the distribution shifts slightly to the right. As shown in Fig. 8(c), a horizontal shift by  $\alpha \log L$ , with  $\alpha$  a constant, collapses reasonably well the distributions obtained at several values of  $L$ .

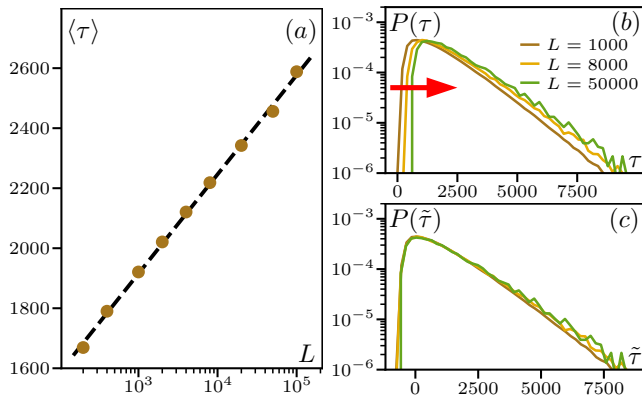


FIG. 8. **(a)**: Mean time between reversals with a fit to a logarithmic function  $f(L) = a \log L + b$ , leading to  $a = 139$  and  $b = 964$  (dashed line). **(b)**: Probability density function of the reversal time  $\tau$ . The red arrow emphasizes the shift of the distributions at small  $\tau$  responsible for the logarithmic scaling of the mean. **(c)**: Same as in (b) with an horizontal shift  $\tilde{\tau} = \tau - \alpha \log L$  with  $\alpha = 110$ . Parameters:  $\rho_0 = 10$ ,  $\beta = 2$ ,  $\varepsilon = 1$ ,  $D = 0.5$ .

To better characterize the various processes at play, we compute the reversal rate  $\lambda(t)$ , defined via the probability  $\lambda(t)dt$  that a flock of age  $t$  reverses within  $[t, t + dt]$ . This rate is related to the distribution  $P(\tau)$  via the number of flocks  $N_f(t)$  that have survived until time  $t$  from an initial population of  $N_f(0)$ :  $\dot{N}_f(t) = -\lambda(t)N_f(t)$  so that  $\lambda = -\dot{N}_f/N_f$ . In addition  $N_f$  itself is related to  $P$  through  $N_f(t)/N_f(0) = \int_t^\infty P(\tau)d\tau$ , which yields

$$\lambda(t) = \frac{P(t)}{\int_t^\infty P(\tau)d\tau}. \quad (7)$$

In physical terms, Eq. (7) simply states that the probability that an event happens at time  $t$  is the rate of occurrence of this event at time  $t$  multiplied by the probability that it did not happen before:  $P(t) = \lambda(t)[1 - \int_0^t P(\tau)d\tau]$ . Note that for an exponential distribution,  $\lambda$  equals the decay rate of the exponential. In Fig. 9(a), we show  $\lambda(t)$  computed using Eq. (7) for several system sizes. We see an initial rapid increase in  $\lambda$ , with a characteristic time that increases with system size, followed by an approximately flat plateau corresponding to the exponential tail of  $P(\tau)$ . As shown in Fig. 9(b), shifting the time by  $\alpha \log L$  provides a good collapse of the curves  $\lambda(t - \alpha \log L)$  measured for different system sizes, consistent with what was reported for  $P(\tau - \alpha \log L)$  in Fig. 8.

The evolution of  $\lambda(t)$  can be related to that of the shape of the flock. Reversals are initiated at the leading edge of flocks. At short times, the leading edge is very stiff so that there is little chance that a spontaneous fluctuation in the gas ahead, whose typical density is very low, triggers a reversal (Fig. 5(a)). Correspondingly,  $\lambda(\tau) \rightarrow 0$  as  $\tau \rightarrow 0$ . At longer times two processes take place that make reversals more likely. First, flocks spread

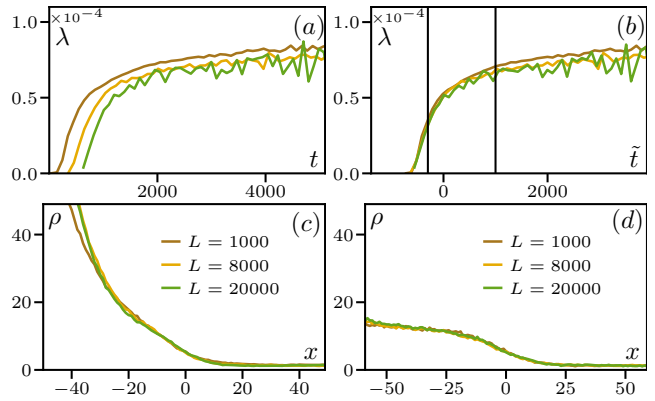


FIG. 9. **Top**: Rate of reversal  $\lambda(t)$  as a function of time since the last reversal **(a)** and with time shifted as  $\tilde{t} = t - \alpha \log L$  **(b)**,  $\alpha = 110$  as in Fig. 8. **Bottom**: Average profiles at fixed  $\tilde{t}$ , at  $\tilde{t} = -300$  when the protrusion just begins to appear **(c)**, and at  $\tilde{t} = 1000$  when it has developed **(d)**. The corresponding times are indicated by vertical lines in panel (b). The profiles are aligned on the point where  $\rho = 5$ . Parameters:  $\rho_0 = 10$ ,  $\beta = 2$ ,  $\varepsilon = 1$ ,  $D = 0.5$ .

diffusively, due to the stochastic hopping of the particles. As the leading edge smoothen, it becomes more and more susceptible to fluctuations. Once the leading edge has sufficiently spread out, its shape becomes compatible with the development of a liquid phase [57]. As the liquid protrusion develops, the shape of its leading front becomes constant in time (Fig. 5(b)). The probability that a fluctuation flips the protrusion becomes time-independent, leading to a plateau value of  $\lambda(t)$  at late times, independent of  $L$ .

Let us now try to rationalize the scaling form  $P(\tau - \alpha \log L)$  and  $\lambda(t - \alpha \log L)$  reported in Figs. 8 and 9. We see in Figs. 9(c,d) that the leading edge of the density profiles also collapse under the same shift. To account for this, we compute the time  $t^*$  it takes for the profile to reach a given slope  $-k^*$  at a given density  $\rho_\ell$ :

$$\rho(x, t^*) = \rho_\ell; \quad \partial_x \rho(x, t^*) = -k^*. \quad (8)$$

We first focus on the early-time dynamics, where we expect a Gaussian spreading in the co-moving frame. There, the density profiles can be approximated as

$$\rho(x, t) = \frac{N_0}{\sqrt{2\pi Dt}} e^{-\frac{x^2}{4Dt}}, \quad (9)$$

where  $D$  is an effective diffusion coefficient and  $N_0 = L\rho_0$  is the number of particles in the aggregate. Denoting by  $x^*$  the position at which the density profiles equals  $\rho_\ell$ , we find that the slope satisfies  $k^* = x^* \rho_\ell / (2Dt)$  while  $x^*$  is given by

$$x^* = 2\sqrt{Dt} \sqrt{\log \frac{N_0}{\rho_\ell} - \frac{1}{2} \log(2\pi Dt)}. \quad (10)$$

At early times and large system sizes,  $\log(2\pi Dt) \ll \log \frac{N_0}{\rho_\ell} \propto \log L$ , so that  $x^* \propto 2\sqrt{Dt} \log L$ , and thus



$k^* \propto \rho_\ell \sqrt{(\log L)/Dt}$ . In this limit,  $k^*$  is a function of  $t/\log L$ . The slope as a function of time then satisfies

$$k(t) = \rho_\ell \sqrt{\frac{\log \frac{N_0}{\rho_\ell}}{Dt}} \sqrt{1 - \frac{1}{2} \frac{\log 2\pi Dt}{\log \frac{N_0}{\rho_\ell}}}. \quad (11)$$

At early times and large system sizes,  $\log(2\pi Dt) \ll \log \frac{N_0}{\rho_\ell} \propto \log L$ , so that  $k$  is a function of  $t/\log L$ . Note that assuming a rate of reversals controlled by the slope of the front would lead to scaling forms  $P(\tau/\log L)$  and  $\lambda(t/\log L)$  during the early Gaussian spreading of the flock.

After a time  $t_{\text{prot}}$  that can be estimated as the inflexion point in Fig. 9(a), the protrusion grows out of the main peak and the diffusive spreading of the main peak does not affect the leading edge anymore. At this stage,  $\lambda(t)$  keeps increasing for some time, since the larger the protrusion, the more likely its flipping will generate a peak with a mass that is sufficient to revert the whole flock. (In addition,  $\lambda(t)$  can also increase after  $t_{\text{prot}}$  because of the small (as seen in Fig. 6) but nonzero dispersal in flock shape so that the protrusion grows at a slightly larger time for part of the flocks.) For  $t > t_{\text{prot}}$ , the  $\tau/\log L$  scaling stops since the leading edge converges to a well-defined steady profile. The time  $t_{\text{prot}}$  can then be estimated by the time it takes for the front to reach the corresponding slope, so that  $t_{\text{prot}} \propto \log L$ .

We thus expect two different scaling regimes: an initial Gaussian spreading leading to  $\lambda(t/\log L)$  and a late-time scaling form  $\lambda(t - \alpha \log L)$ , once the protrusion has grown out of the main peak. As shown in Fig. 9(c,d), the protrusion grows out quite early so that the first regime is never quantitatively observed in our simulations: most reversals take place after the protrusion has grown out. Thus the  $\lambda(t - \alpha \log L)$  and  $P(\tau - \alpha \log L)$  scalings work satisfactorily.

### C. Role of parameters $D$ , $\varepsilon$ and $\beta$

Most results presented so far were obtained at fixed, rather typical, parameter values. We now report on the effect of changing the three parameters  $D$ ,  $\varepsilon$  and  $\beta$ . Figure 10 shows how  $\langle \tau \rangle$  and  $\lambda$ , the reversal rate extracted from the exponential tail of  $P(\tau)$ , vary with each one of these parameters, keeping the other two constant.

Comparing  $\langle \tau \rangle$  with  $\lambda^{-1}$  (which is the mean of a normalized exponential distribution with rate  $\lambda$ ), we first see that  $\lambda^{-1}$  ‘underestimates’  $\langle \tau \rangle$  but both quantities essentially vary in the same manner. (Their difference is due to the fact that early reversals are rare, as they require a perturbation to overcome the sharp leading edge of the diffusively spreading flock, as explained in the previous subsection.) To account for the variations of  $\langle \tau \rangle$ , we can thus focus on the reversals at late times, which take place after the protrusion has developed at the front of the flocks.

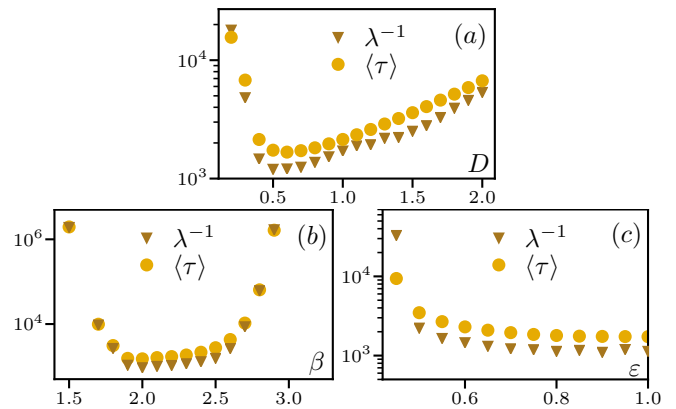


FIG. 10. Mean reversal time  $\langle \tau \rangle$  as a function of  $D$  for  $\beta = 1.8$  and  $\varepsilon = 0.8$  (left), of  $\beta$  with  $D = 0.5$  and  $\varepsilon = 0.8$  (right), and of  $\varepsilon$  with  $\beta = 2$  and  $D = 0.5$  (bottom). The data are compared with the prediction  $\lambda^{-1}$  that corresponds to a purely exponential distribution  $P_{\text{exp}}(\tau) = \lambda e^{-\lambda\tau}$ , with  $\lambda$  measured from the tail of the true distribution  $P(\tau)$ . Parameters:  $\rho_0 = 3$ ,  $L = 1000$ .

We find that the divergences of  $\langle \tau \rangle$  visible on Fig. 10 can be explained by two different mechanisms. First, decreasing  $D$  or  $\varepsilon$  or increasing  $\beta$  while keeping the other parameters fixed brings the system from the flocking phase to the aster phase (see the phase diagrams in Fig. 2). Close to the transition, transient asters appear at the beginning of an attempted reversal, as in Fig. 11e. Once the aster is formed, the propagation of the fluctuation stops until the aster dissolves. (See section Sec. IV for a detailed discussion on aster dynamics.) In the mean time, a large amount of particles arrive from the flock, which tends to destroy the aster and resume the forward motion of the flock. This process thus tends to suppress reversals: transient asters protect the flock against fluctuations, hence increasing their lifetime.

The other divergences of  $\langle \tau \rangle$ , in the large  $D$  and small  $\beta$  limits, can be accounted for by comparing the roles of diffusion and alignment during a reversal. Aligning interactions tend to flip particles from the flock, hence strengthening the fluctuation. On the contrary, diffusion damps the fluctuation as it propagates. Diffusion dominates when  $D/e^\beta$ —the ratio of hopping to alignment rates—increases, i.e. when  $D$  increases or  $\beta$  decreases. Figure 10 shows a steeper divergence when varying  $\beta$  compared to varying  $D$ , as expected from this reasoning.

To support these two scenarios, we now evaluate how likely a fluctuation is to reverse a flock in an idealized situation. We consider the deterministic mean-field evolution of an initial fluctuation of tunable size that encounters an ordered phase mimicking the protrusion of a flock at density  $\rho_\ell$  and magnetization  $m_\ell$  such that  $m_\ell/\rho_\ell = \tanh(\beta m_\ell/\rho_\ell)$  (its mean-field value).

The initial condition is depicted in Fig. 11a and the evolution is that of Eqs. (2)-(3) after one takes a mean-field approximation for the nonlinear terms. We choose  $\rho_\ell = 1$  without loss of generality, thus fixing the unit of

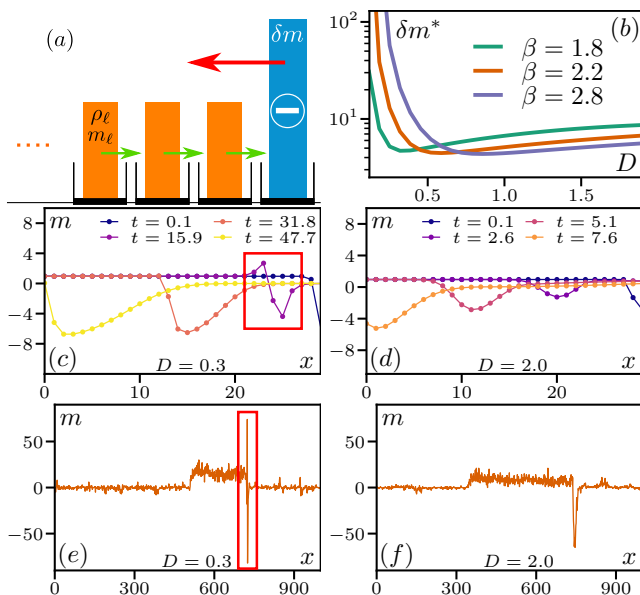


FIG. 11. **(a)**: Sketch of the initial condition in the reduced mean-field model. **(b)**: Minimal fluctuation  $\delta m^*$  necessary to reverse the flock in the reduced model. **(c,d)**: Evolution of the density profile in the reduced model at low  $D = 0.3$  (c) and high  $D = 2.0$  (d) for a fluctuation  $\delta m = 8$  (c) and  $\delta m = 6$  (d) slightly bigger than  $\delta m^*$ . Other parameters:  $\varepsilon = 1$  and  $\beta = 2$ . **(e,f)** Instantaneous profile in the microscopic model at the beginning of a reversal at low  $D = 0.3$  (e) and high  $D = 2$  (f). The transient peak (red boxes) observed at low  $D$  in the reduced model also appears in the full AIM.  $\rho_0 = 5$ ,  $L = 1000$ ,  $\varepsilon = 1$  and  $\beta = 2$ .

density, and take the initial fluctuation to be fully ordered with magnetization  $-\delta m$  and density  $\delta m$ , propagating to the left. We observe that there is a value  $\delta m^*$  such that small fluctuations  $\delta m < \delta m^*$  do not propagate and the system remains ordered, whereas large fluctuations  $\delta m > \delta m^*$  propagate and flip the entire initial flock [58]. This critical fluctuation size varies with the parameters, as shown in Fig. 11b where we vary  $D$  for several values of  $\beta$ . At small  $D$  we see the transient asters observed in the microscopic model whereas they are not observed for larger  $D$  (see Fig. 11c,d). The variations of  $\delta m^*$  are consistent with the variations of  $\langle \tau \rangle$  in the microscopic model as we see a sharp increase at small  $D$  corresponding to the appearance of transient asters and a slower increase at high  $D$  with no asters. This minimal model of reversals, despite its simplicity, thus reproduces the basic features of the mean reversal time and supports the two scenarios outlined above for the divergences of  $\langle \tau \rangle$ .

#### D. Coarsening

So far we only considered the steady state regime when the system exhibits a single flock. We now study the coarsening dynamics that bring the system to such a steady state from a disordered initial condition. As

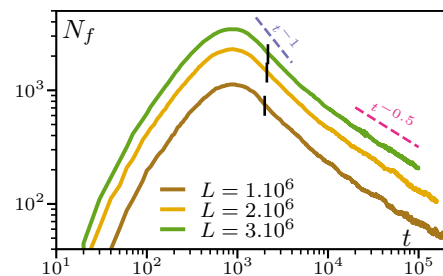


FIG. 12. Number of flocks  $N_f$  as a function of time, starting from an homogeneous disordered initial conditions. The small black lines indicates the crossover time  $t_c$  as given in Eq. (14).  $\rho_0 = 10$ ,  $\beta = 2$ ,  $D = 0.5$ ,  $\varepsilon = 1$ .

shown on the space-time diagram of Fig. 3e, many small flocks form at early times and merge when they encounter until only one remains. The evolution of  $N_f$ , the number of flocks, can be written as

$$\dot{N}_f(t) = -\frac{N_f(t)}{t_{\text{coll}}(t)} \quad (12)$$

with a function  $t_{\text{coll}}(t)$  that may depend on  $N_f$  and which we can interpret as the mean time before a flock collisions with a neighboring one.

We can anticipate two regimes for this collision time. At early times, when flocks are close to one another, they typically encounter a neighbouring flock before reversing their direction. The collision time will then be the ballistic time  $t_{\text{coll}}^B = \ell/v$  with  $\ell = L/N_f$  the mean distance between flocks. At later times, flocks are further apart and thus reverse their direction of motion before colliding. The collision time then has a diffusive scaling  $t_{\text{coll}}^D = \ell^2/(2D_{\text{eff}})$  with the effective diffusion coefficient  $D_{\text{eff}} = v^2\langle \tau \rangle$ . The crossover between these two regimes is expected at a time  $t_c$  such that  $t_{\text{coll}}^B(t_c) = \langle \tau \rangle$ . Solving Eq. (12) in the ballistic regime gives

$$N_f(t) = \frac{L}{\frac{L}{N_f(0)} + vt} \quad (13)$$

from which we deduce that the crossover time is

$$t_c = \langle \tau \rangle - \frac{L}{vN_f(0)}. \quad (14)$$

Neglecting the dependence of  $\langle \tau \rangle$  on  $\log L$ , which would give subdominant corrections, the number of flocks in the diffusive regime then follows  $\dot{N}_f \propto -N_f^3$  and thus  $N_f(t) \sim 1/\sqrt{t}$ .

Comparing with simulations in Fig. 12, we see that the late-time coarsening is indeed clearly diffusive. At short time, since we start from a homogeneous disordered configuration, we first see an increase of  $N_f$  due to the formation of many flocks. Before the diffusive regime sets in we see a short time window with a faster-than-diffusive coarsening, consistent with the analysis above. We see that the crossover time  $t_c$  Eq. (14), indicated by black lines in Fig. 12, gives a reasonable estimate for the crossover between the ballistic and diffusive regimes.

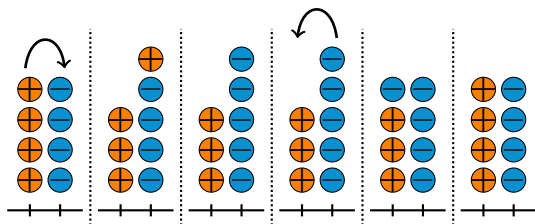


FIG. 13. A typical sequence of configurations that make asters stable at low temperature.

#### IV. ASTERS

At low temperature or small velocity, a phase develops in which static structures coexist with a dilute fluid. (See the phase diagrams in Fig. 2 and the snapshot in Fig. 3d.) Each of these ‘asters’ consists in two peaks of right and left-moving particles which apparently ‘block’ each other. Remember, however, that there is no exclusion in our model so that the underlying mechanism is necessarily more complex and calls for an explanation. We termed these structures asters, by analogy to the star-shaped defects observed in 2D active systems [59, 60].

A closer look at the microscopic dynamics of an aster reveals that its stability arises from periodic orbits in configuration space, as illustrated in Fig. 13. Starting from two peaks of opposite magnetization, one particle—say with positive spin—hops forward and lands onto the second site where it belongs to the minority phase. It then flips and aligns with its new environment. The most likely move is then that one of the minus particles hop forward onto the site populated by plus particles. Once again, this particle belongs to the minority phase and flips, so that the system returns to its original state. At low temperatures, such trajectories are much more likely than trajectories leading to the evaporation of the aster. To quantify the stability of asters, we first consider in Sec. IV A a zero-temperature fully-asymmetric model in which the lifetime of an aster can be computed exactly. We then discuss the stability of asters at finite temperature and the corresponding coarsening dynamics in Sec. IV B. Finally, we show in Sec. IV C that, for  $0 < \varepsilon < 1$ , asters typically have a richer shape than suggested by Fig. 13: they are not perfectly localized and admit an exponential tail with a characteristic size. Using a mean-field approximation, we compute the corresponding decay length and determine a necessary condition for the existence of asters that captures qualitatively, albeit not quantitatively, the phase boundary between flocks and asters shown in Fig 2.

##### A. Lifetime of an aster at zero temperature

To make progress analytically, we first consider a  $T = 0$ , fully asymmetric version of the AIM. Particles hop at rate  $v = 2D$  and the aligning interaction is resolved

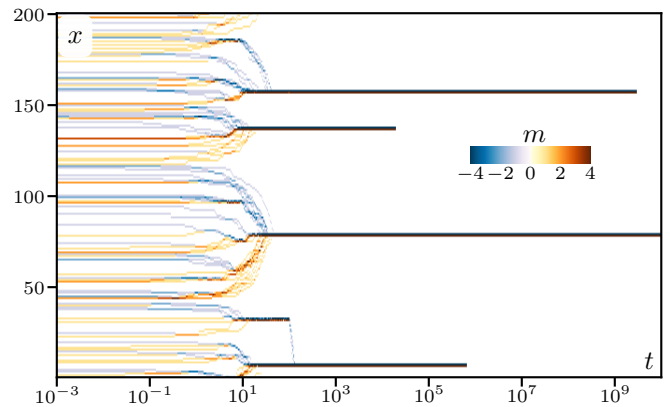


FIG. 14. Starting from a random configuration at  $t = 0$  in the zero-temperature model, the system rapidly evolves into a large number of small asters. These coarsen on much longer time scales, as smaller condensates evaporate and are redistributed into larger ones. Parameters:  $L = 200$ ,  $N = 100$ ,  $v = 1$ ,  $\varepsilon = 1$ .

instantaneously, due to the  $\beta \rightarrow \infty$  limit of the flipping rate (1). In practice, when a particle hops onto a new site with two particles or more, it immediately acquires the magnetization of the target site if the latter is not empty. The limit is ill defined when one has both  $m_i = 0$  and  $\beta = \infty$ . For simplicity, we here assume that when a particle of spin  $s$  arrives on a site occupied by a particle of spin  $-s$ , it flips and acquires a  $-s$  spin. As a result, the sole microscopic time-scale of the system is  $v^{-1}$ . The case in which both particles keep their current spins and flip at rate  $\omega_0$  is qualitatively similar—albeit more involved due to the presence of the second time scale  $\omega_0^{-1}$ —as discussed in Appendix B.

Consider an initial condition in which  $N$  particles are randomly placed on the lattice and each site is given a magnetization at random. The dynamics is such that all sites remain fully ordered at all times, with  $m_i = \pm \rho_i$ , and the total escape rate is always  $Nv$ . A typical trajectory is shown in Fig. 14. It rapidly leads to the emergence of an extensive number of asters, whose rare—but sudden—evaporation leads to a coarsening dynamics. To characterize the latter, we compute the lifetime of an aster comprising  $M$  particles.

Let us consider the situation depicted in Fig. 15 in which an aster is formed with  $n$  particles with  $+1$  spins at site  $i$  and  $M - n$  particles with  $-1$  spins at site  $i + 1$ . We denote this configuration as  $\mathbf{n} \equiv (n, M - n)$ . The system evolves at rate  $q_n = vn$  towards the configuration  $\mathbf{n} - \mathbf{1}$  and  $p_n = v(M - n)$  towards  $\mathbf{n} + \mathbf{1}$ . We denote by  $r_n = p_n + q_n$  the escape rate from configuration  $\mathbf{n}$ , which is here given by  $r_n = vM$ . Given the expressions of  $q_n$  and  $p_n$ , there is a linear drift that takes the system towards the most likely configuration  $\mathbf{n} = \mathbf{M}/2$ . Note that the configurations  $\mathbf{M}$  and  $\mathbf{0}$  are limiting cases that correspond to the evaporation of the aster. To get an intuitive understanding of the aster dynamics, we consider

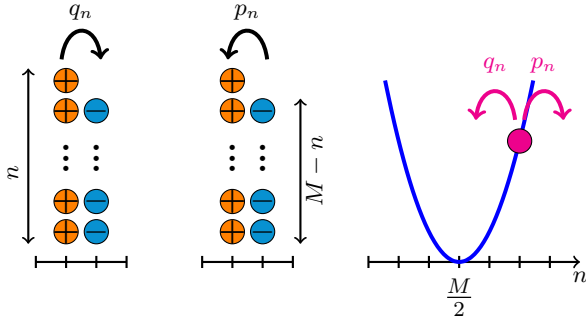


FIG. 15. **Left:** A particle hops out of a site with  $n$  particles at a rate  $q_n = nv$ . This takes the aster from  $(n, M - n)$  to  $(n - 1, M - n + 1)$ . **Center:** A particle hops out of a site with  $M - n$  particles at a rate  $p_n = (M - n)v$ . This takes the aster from  $(n, M - n)$  to  $(n + 1, M - n - 1)$ . **Right:** This dynamics is equivalent to the Brownian dynamics of a single particle at position  $n$ , confined in a harmonic potential  $V(n) = \frac{v}{2}(\frac{n}{N_a} - \frac{1}{2})^2$ , with  $n = 0$  and  $n = M$  being absorbing boundaries corresponding to the evaporation of the aster.

the master equation of this process:

$$\partial_t P(n) = q_{n+1}P(n+1) + p_{n-1}P(n-1) - (q_n + p_n)P(n). \quad (15)$$

Small Gaussian fluctuations close to the most likely configuration  $n = M/2$  are well described by introducing  $P(x = n/M) = MP(n)$  and expanding (15) to first order in  $dx = M^{-1}$ . Doing so yields the Fokker-Planck equation

$$\partial_t P(x) = \frac{\partial}{\partial x} \left[ V'(x)P(x) + \frac{v}{2M} \frac{\partial}{\partial x} P(x) \right], \quad (16)$$

which corresponds to the dynamics of a Brownian particle in a harmonic potential  $V(x) = v(x - \frac{1}{2})^2$  at temperature  $\frac{v}{2M}$ , as illustrated on Fig. 15. The mean-first passage time  $\tau$  until the evaporation of an aster with  $M$  particles can then be estimated using the Arrhenius scaling  $\log \tau \propto M/2$ . This scaling is not expected to hold quantitatively, since the diffusive approximation (15) is expected to fail in the large-deviation regime where  $x \gg \frac{1}{\sqrt{M}}$ , but it captures the physics that makes the aster long-lived at zero temperature.

We now wish to compute exactly the mean-first passage time to reach either  $\mathbf{M}$  or  $\mathbf{0}$ , which will trigger the redistribution of the aster particles into a neighboring aster, hence driving the coarsening process. Let us note  $T_n$  the average adsorption time, starting from configuration  $\mathbf{n}$ . In an average time  $r_n^{-1}$ , the system jumps to  $\mathbf{n} - 1$  or  $\mathbf{n} + 1$  with probabilities  $q_n/r_n$  and  $p_n/r_n$ , respectively. One thus has the recursive relation:

$$T_n = \frac{1}{r_n} + \frac{p_n}{r_n} T_{n+1} + \frac{q_n}{r_n} T_{n-1} \quad (17)$$

that has to be solved with the boundary conditions

$$T_0 = T_M = 0. \quad (18)$$

To do so, we follow standard methods [61, 62] and introduce  $U_n = T_{n-1} - T_n$ . Equation (17) can then be rewritten as

$$p_n U_{n+1} = q_n U_n + 1. \quad (19)$$

Introducing  $\pi_0 = 1$  and, for  $i \geq 0$ ,

$$\pi_i \equiv \prod_{k=1}^i \frac{q_k}{p_k} \quad \text{and, further,} \quad \sigma_i = \sum_{k=1}^i \frac{1}{p_k \pi_k}, \quad (20)$$

Eq. (19) is readily solved as

$$U_{n+1} = \pi_n U_1 + \pi_n \sigma_n. \quad (21)$$

Using  $T_0 = 0$ , we first get  $T_1 = -U_1$ . The definition of  $U_n$  then recursively leads to

$$T_{n \geq 2} = -U_1 \sum_{i=0}^{n-1} \pi_i - \sum_{i=1}^{n-1} [\pi_i \sigma_i]. \quad (22)$$

Finally,  $U_1$  is determined by imposing the boundary condition  $T_M = 0$ , which gives:

$$U_1 = -\frac{\sum_{i=1}^{M-1} [\pi_i \sigma_i]}{\sum_{i=0}^{M-1} \pi_i}. \quad (23)$$

All in all, the mean time to absorption starting from the configuration  $\mathbf{n}$  is given by

$$T_n = \frac{\sum_{i=0}^{n-1} \pi_i}{\sum_{i=0}^{M-1} \pi_i} \sum_{i=1}^{M-1} [\pi_i \sigma_i] - \sum_{i=1}^{n-1} [\pi_i \sigma_i] \quad (24)$$

where the second sum vanishes for  $n = 1$ . The above results are valid for generic random walks with non-vanishing rates  $p_n$  and  $q_n$ . In the case at hand, using  $p_n = v(M - n)$  and  $q_n = vn$ , we find

$$\pi_i = \binom{M-1}{i}^{-1}, \quad (25)$$

which allows to compute  $T_n$ .

The mean-first passage times to absorption starting from all possible configurations are plotted in Fig. 16 (left), for  $M$  up to 14. Interestingly,  $T_n$  becomes rapidly independent of  $n$ . This can be understood as follows: due to the asymmetry between  $p_n$  and  $q_n$ , the most likely path to absorption is to fall from  $n$  to  $M/2$  and then to get absorbed. Furthermore, Fig. 16 (left) suggests an exponential increase of  $T_n$  with  $M$ . To compute the leading order of  $T_n$ , consider the various terms of Eq. (24) and their scaling as  $M \rightarrow \infty$ . We first note, using Eq. (25), that  $\pi_0 = \pi_{M-1} = 1$ ,  $\pi_1 = \pi_{M-2} = (M-1)^{-1}$ , and  $\pi_{1 < i < M-2} = \mathcal{O}(M^{-1})$ . Consequently,  $\pi_1 + \pi_2 + \dots + \pi_{n-1} \rightarrow 0$  for  $n < M$  and only  $\pi_0$  and  $\pi_{M-1}$  contribute to the sums. Noting also that  $(\pi_k p_k)^{-1}$  simplifies into

$$\frac{1}{\pi_k p_k} = \frac{(M-1)!}{vk!(M-k)!} = \frac{1}{vM} \binom{M}{k}, \quad (26)$$

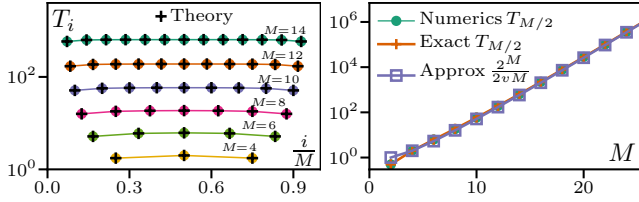


FIG. 16. **Left:** Mean-first-passage times from site  $i$  to sites 0 or  $M$ , as  $M$  varies from 4 to 14. The circles correspond to numerical simulations whereas the crosses correspond to Eq. (24). **Right:** Mean-first passage times to absorption for an aster with  $M$  particles. Green discs are results of numerical simulations, averaged over  $10^4$  realizations. Orange crosses correspond to the exact formula (24), and purple squares to approximation (30).

we see that Eq. (24) can be approximated as

$$T_n \sim \frac{1}{2vM} \sum_{i=1}^{M-1} \left[ \pi_i \sum_{k=1}^i \binom{M}{k} \right] - \frac{1}{vM} \sum_{i=1}^{n-1} \left[ \pi_i \sum_{k=1}^i \binom{M}{k} \right]. \quad (27)$$

Finally, we note that, for  $n < M - 1$ ,

$$\sum_{i=1}^n \left[ \pi_i \sum_{k=1}^i \binom{M}{k} \right] < \left[ \sum_{i=1}^n \pi_i \right] \left[ \sum_{k=1}^{M-2} \binom{M}{k} \right] \quad (28)$$

$$= o \left( \sum_{k=1}^{M-1} \binom{M}{k} \right), \quad (29)$$

so that

$$T_n \sim \frac{1}{2vM} \pi_{M-1} \sum_{k=1}^{M-1} \binom{M}{k} \sim \frac{2^M}{2vM} \quad (30)$$

Figure 16 (right) shows the comparison between numerical measurements of  $T_{M/2}$ , its exact expression (24) and the asymptotic estimate (30), which is remarkably close to the exact values. The stability of asters and their coarsening at zero temperature thus stems from an evaporation rate that vanishes exponentially with their density.

## B. Coarsening at small temperature

At zero temperature, coarsening of asters occurs via an extremal dynamics: smallest asters evaporate first and are redistributed among their neighbors. Since the lifetime of an aster diverges exponentially with its height, this leads to an extremely slow dynamics, as apparent from Fig. 14.

At finite temperature, a new phenomenon sets in: when a particle hops forward, as in the second configuration of Fig. 13, it now has a finite probability  $P_{\text{hop}}$  of hopping further forward and leave the aster before flipping it spin, with

$$P_{\text{hop}} = \frac{p}{p + W(s \rightarrow -s)}. \quad (31)$$

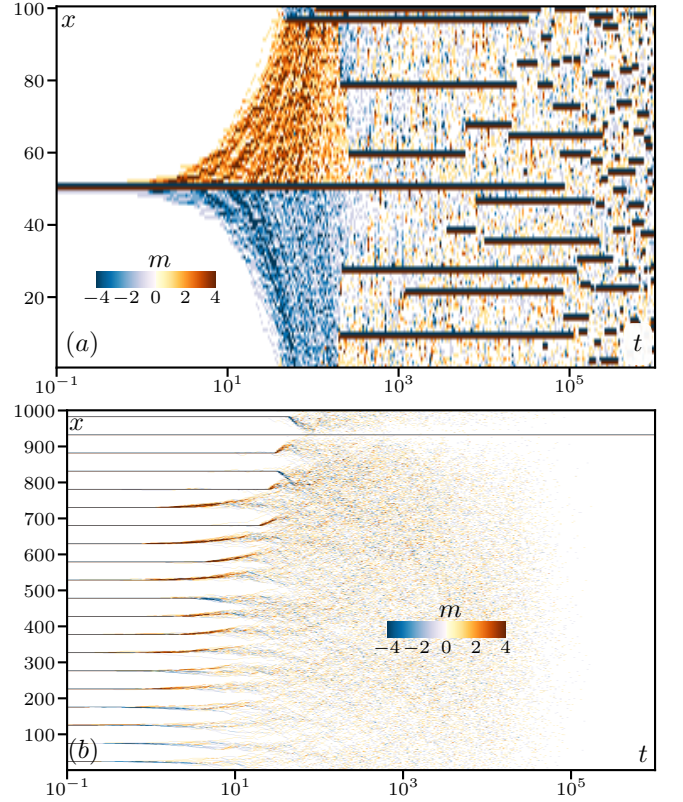


FIG. 17. **(a)** Spatiotemporal plot of the relaxation dynamics from a single aster to the fluctuating aster phase of the AIM with rates (1) ( $T = 0.2$ ,  $\varepsilon = 1$ ,  $L = 100$ ,  $\rho_0 = 10$ ,  $D = 0.5$ ). **(b):** same as (a) but for the AIM with rates (33), starting from 20 asters whose initial density increases linearly from 6 to 25 as  $x$  increases; coarsening leaves a single macroscopic aster ( $D = 1$ ,  $L = 1000$ ,  $\omega_0 = 1$ ,  $\beta = 0.375$ ,  $\rho_0 = 0.29$ ).

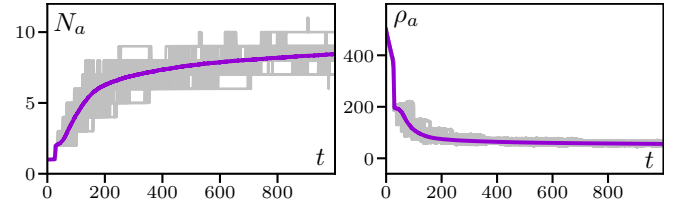


FIG. 18. Relaxation dynamics to the aster phase for the AIM with rates (1). **(a):** Number of asters as a function of time, averaged over 1000 simulations (magenta). Fifty representative trajectories appear in gray. **(b):** Same as (a), but for the average density contained in one aster. Parameters as in Fig. 17a.

In an aster comprising two sites with  $\rho$  particles, this leads to a flux of particle  $j_{\text{leak}}(\rho)$  leaving the aster, where

$$j_{\text{leak}}(\rho) \propto p\rho P_{\text{hop}} = \frac{p^2}{p + W(s \rightarrow -s)} \rho. \quad (32)$$

The variations of  $j_{\text{leak}}(\rho)$  with  $\rho$  then determine the late-stage dynamics. For the flipping rates (1),  $j_{\text{leak}}(\rho) \propto \rho$

at large densities since the hopping rate is bounded,  $W(s \rightarrow -s) < \exp(\beta)$ : large asters leak particles faster than smaller ones, which arrests the coarsening and lead to a steady-state with an extended number of finite-size asters, as illustrated in Fig. 17a and Fig. 18a-b. (It would be interesting to generalize approaches developed in the past to predict the size of competing finite condensates [63] but this is beyond the scope of this study.) For the unbounded rates studied in [64]:

$$W(s \rightarrow -s) = \omega_0 e^{-\beta s m_i}, \quad (33)$$

larger asters leak slower, leading to a single macroscopic aster, as illustrated in Fig. (17)b. Finally, in a related 1D flocking model, a coarsening to a single structure reminiscent of an aster was observed in the presence of “centering” interactions, which bias the motion of particles towards dense regions [47]. The generality of the mechanisms leading to aster-like structures and their stability is thus an interesting question beyond the sole case of the AIM.

### C. Shape of asters

Let us now consider the case  $\varepsilon < 1$ , where hopping is not fully asymmetric. Thanks to diffusion, particles can hop backward and asters are then spread over several sites. Since they typically comprise many particles, we expect to be able to describe their shape using the mean-field equations (4,5) introduced in Sec. III A. It is straightforward to check that these equations admit stationary solutions of the form

$$\rho(x) = k e^{-\mu_c x}; \quad m(x) = k \phi e^{-\mu_c x} \quad (34)$$

with  $k$  a normalization constant and  $\phi$  and  $\mu_c$  (the subscript “c” stands for “continuous” for a reason that will become apparent shortly) solutions of

$$\frac{v^2}{2D} \phi (\phi^2 - 1) = \phi \cosh(\beta \phi) - \sinh(\beta \phi) \quad (35)$$

$$\mu_c = -v\phi/D. \quad (36)$$

Furthermore, these are the only stationary solutions of Eqs. (4,5) such that  $\rho(x)$  and  $m(x)$  are proportional to each other. Note that Eq. (34) does not describe the full aster but only half of it, for  $x > 0$  or  $x < 0$ . We are not aware of any analytic solution describing the full aster solution, including its singularity at  $x = 0$ . Finally, the competition between the asymmetric hops leading particles towards the aster and the diffusive dynamics allowing them to explore neighboring sites is reminiscent of the diffusive motion of colloids under gravity. The solutions (34) can thus be seen as two exponential atmospheres pointing towards the core of the aster.

We now compare these solutions with the aster profiles measured in the microscopic model (Fig. 19(a)). The profiles of density and magnetization indeed exhibit an exponential decay on both sides of the aster. In Fig. 19(b),

we see that the measured decay exponent  $\mu$  agrees well with the solution  $\mu_c$  of Eq. (36) at small  $\varepsilon$  but deviates for larger  $\varepsilon$ . Note that we plot only the curve  $\mu_c(\beta = \infty)$  in Fig. 19b since the curves for different values of  $\beta$  would be indistinguishable on the scale of the figure. Setting  $\beta = \infty$  also simplifies the calculation since it means fixing  $\phi = \pm 1$  in Eq. (35) so that  $\mu_c(\beta = \infty) = \pm v/D = \pm 2\varepsilon$ .

Increasing  $\varepsilon$ , we see that the discrepancies between  $\mu_c$  and the measured  $\mu$  increases. This deviation can be attributed to the continuous limit used to derive Eqs. (4,5). If instead we retain the full discrete equations Eqs. (2,3) in the mean-field approximation, and look in the same way for exponential solutions

$$\rho_i = k \kappa^i; \quad m_i = k \phi \kappa^i, \quad (37)$$

we find the conditions

$$\frac{4D(\kappa - 1)^2 \phi - v(\kappa^2 - 1)}{4\kappa} = \phi \cosh[\beta \phi] - \sinh[\beta \phi] \quad (38)$$

$$\left(\kappa + \frac{1}{\kappa} - 2\right) = \frac{v\phi}{2D} \left(\kappa - \frac{1}{\kappa}\right); \quad \mu_d = -\log \kappa \quad (39)$$

The decay exponent  $\mu_d$  (subscript “d” for “discrete”) predicted by Eqs. (38,39) is now in near-perfect agreement with microscopic simulations as shown in Fig. 19b.

Interestingly, we find that there is a maximum value of  $\varepsilon$  that we denote  $\varepsilon_c$  above which no physical exponential solution is found (we require that  $\phi \in [-1, 1]$  and that  $\mu_d$  be of opposite sign to  $\phi$  for a physical solution). In Fig. 19c, we compare this limit value with the transition line between the flock and aster phases in the microscopic model. Both lines show the same trend as a function of  $\beta$  but are quantitatively different. This indicates that, unsurprisingly, exponential solutions can exist at the mean-field level while asters are not observed because either the full aster solutions do not exist at the mean-field level or because they are unstable to fluctuations.

## V. CONCLUSION

We have provided a detailed study of the active Ising model in one spatial dimension. Despite its simplicity, the AIM shows two non-trivial phases characterized by flocks and asters. We found the flocking phase to exhibit the same properties as in other one-dimensional flocking models [37, 45–50]: Large ordered aggregates spread while propagating, merge when they collide, and regroup when stochastically reversing their direction of motion. As reported before [46], the time between two reversals of a flock increases logarithmically with system size. We went further and explained this dependence as well as the variations with the velocity of particles and the temperature. In addition, we have also analyzed in detail the shape of flocks and how it evolves in time. The global picture is that of a liquid-gas phase separation which cannot relax to steady state because of reversal events. Since this is expected to be generic for 1D flocking models, we

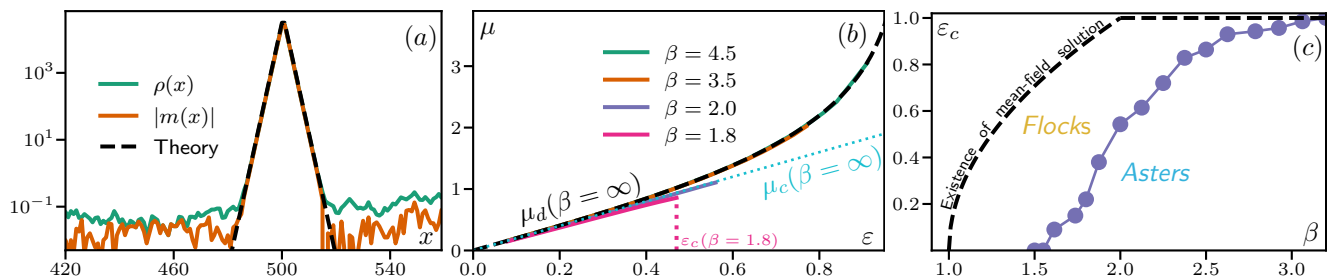


FIG. 19. **(a)**: Profile of an aster in the microscopic model. The dashed lines are the exponential decays predicted by Eqs. (38)-(39).  $L = 1000$ ,  $\rho_0 = 100$ ,  $\beta = 4$ ,  $\epsilon = 0.5$ ,  $D = 0.5$ . **(b)**: Measured value of  $\mu$  in the microscopic model at different  $\beta$  compared with the prediction from the continuous mean-field equation (blue dotted line) and the discrete ones (black dashed line).  $L = 1000$ ,  $\rho_0 = 100$ ,  $D = 0.5$ . **(c)**: Phase boundary  $\epsilon_c(\beta)$  such that asters are observed for  $\epsilon < \epsilon_c$  in microscopic simulations (symbols), compared to the limit of existence of aster solutions in the mean-field equations (dashed line).  $L = 500$ ,  $\rho_0 = 10$ ,  $D = 0.5$ .

thus also expect our analysis of the reversal times based on the shape of the flocks to be relevant to other models.

We reported a new phase of the AIM. It is populated by “asters” which are composed of two static peaks of magnetization facing each other. These asters appear here only because of the interplay of alignment and self-propulsion and are thus qualitatively different from traffic jams due to steric interactions or from the “dipoles” observed in Ref. [47] which are produced by centering interactions that favor motion towards high-density regions. We have provided an exact solution of the zero-temperature dynamics of an aster and showed that they dissolve in a time that is exponential in their size. At infinite time, this leads to a single aster in the zero-temperature limit but to a finite density of them at small but finite temperature. Finally, we have shown that an aster has an exponential profile with a decay exponent which is well predicted by a mean-field theory.

Common statistical mechanics wisdom state that fluctuations become more important as the dimension of space becomes lower. In equilibrium, this goes as far as preventing the existence of an ordered state in a model with only short-ranged interactions. In our active system, the flocking phase shows that ordering is possible but fluctuations still prevent true symmetry breaking by inducing reversals of the flock direction. Fluctuations play a somewhat different but also essential role in the dynamics of asters: we found that the dissolution of asters and thus their coarsening is due to rare fluctuations, giving rise to extremal dynamics. In contrast, we found that some features of flocks and asters are accounted for to a good accuracy by a mean-field approach that neglects fluctuations.

#### ACKNOWLEDGMENTS

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#### Appendix A: Numerical simulations

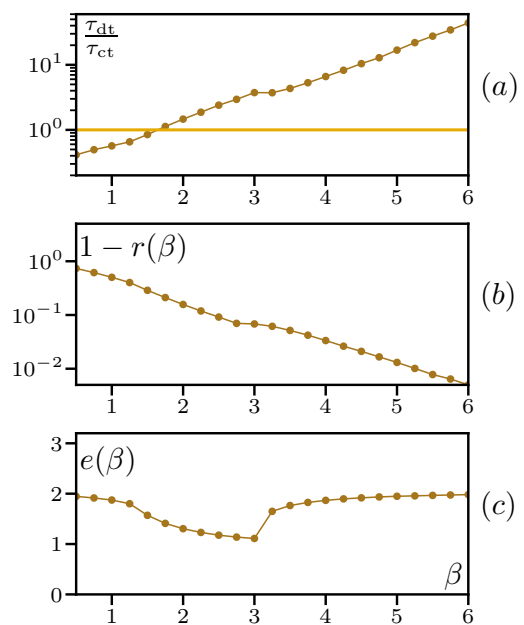


FIG. 20. Comparison between discrete- and continuous-time algorithms. **(a)**: Ratio between the cpu time used for a discrete-time simulation,  $\tau_{dt}$ , and a continuous-time simulations,  $\tau_{ct}$ . The discrete-time simulation is faster at large temperature but its efficiency decreases exponentially with  $\beta$ . **(b)**: The rejection rate of the discrete-time algorithm,  $r(\beta)$ , converges exponentially to  $r(\beta) = 1$  as  $\beta$  increases. **(c)**: The average escape rate per particle,  $e(\beta)$ , shows little variation as  $\beta$  is varied and remains close to  $r \simeq 2D + \omega_0$ . The dip for intermediate  $\beta$  corresponds to the flocking phase where most particles are aligned. Parameters:  $D = 0.5$ ,  $\epsilon = 1$ ,  $\omega_0 = 1$ ,  $L = 500$ ,  $\rho_0 = 10$ . In panel (a), the system is simulated up to a final time  $t = 110\,000$ .

## 1. Discrete-time simulations

Our discrete-time simulations are implemented using random sequential updates: Particles are chosen at random and updated sequentially. For each update, the time is increased by  $dt/N$ . Denoting by  $i$  the site of the chosen particle and by  $s$  its spin, the particle either hops, flips its spin, or does nothing with probabilities:

$$\begin{cases} \text{Proba(hop to site } i+1): & D(1+s\varepsilon)dt \\ \text{Proba(hop to site } i-1): & D(1-s\varepsilon)dt \\ \text{Proba(spin flips):} & \omega_0 e^{-\beta s \frac{m_i}{\rho_i}} dt \\ \text{Proba(no update):} & 1 - 2Ddt - \omega_0 e^{-\beta s \frac{m_i}{\rho_i}} dt \end{cases} \quad (\text{A1})$$

In the last case, we say that a move has been “rejected”.

Let us note  $\mathcal{C}_i$  the state of particle  $i$ , defined by its spin and position. A configuration  $\mathcal{C}$  is then given by  $\mathcal{C} = \{\mathcal{C}_i\}$ . Consider the discrete-time Markov process defined by the above algorithm. Since there is a  $1/N$  probability of choosing a given particle, the corresponding discrete-time master equation reads

$$P\left(\mathcal{C}, t + \frac{dt}{N}\right) = \frac{1}{N} \left[ \sum_i \sum_{\mathcal{C}'_i \neq \mathcal{C}_i} P(\mathcal{C}', t) W(\mathcal{C}' \rightarrow \mathcal{C}) dt + P(\mathcal{C}, t) \sum_i \left( 1 - \sum_{\mathcal{C}'_i \neq \mathcal{C}_i} W(\mathcal{C} \rightarrow \mathcal{C}') dt \right) \right] \quad (\text{A2})$$

where  $\mathcal{C}'$  is identical to  $\mathcal{C}$  but with particle  $i$  in state  $\mathcal{C}'_i$ , and the rates  $W$  are given by  $W = D(1 \pm \varepsilon)$  or by Eq. (1), depending on the transition. It is then straightforward to check that the limit of (A2) as  $N \rightarrow \infty$  is the continuous-time master equation of the AIM:

$$\partial_t P(\mathcal{C}, t) = \sum_{\mathcal{C}'_i \neq \mathcal{C}_i} [P(\mathcal{C}', t) W(\mathcal{C}' \rightarrow \mathcal{C}) - P(\mathcal{C}, t) W(\mathcal{C} \rightarrow \mathcal{C}')] ]$$

This algorithm has two main limitations. First, it is only exact in the large  $N$  limit and it is thus not adapted to study small systems. This precludes using it for the study of the finite-size asters of section IV A. Then, its efficiency relies on the fact that there are as few proposed updates as possible that lead to rejection. Since the only constraint on the algorithm is that the probabilities in (A1) are between 0 and 1, we take the largest possible  $dt$ , given by  $dt = (2D + \omega_0 e^{\beta})^{-1}$ , to minimize the probability of a particle doing nothing during an update.

## 2. Continuous-time simulations

Our continuous time simulations were implemented as follows. For each particle  $i$ , with spin  $s_i$ , we define an escape rate  $r_i = 2D + \omega_i$ , where  $\omega_i$  is the rate at which  $s_i$  flips into  $-s_i$ . At time 0, we sample a time  $\tau_i$  for each particle according to  $P(\tau_i) = r_i e^{-r_i \tau_i}$ . The algorithm then goes iteratively over the following loop:

1. Set the simulation time to  $t = \tau_j$  where  $\tau_j = \inf_i \tau_i$ .
2. Update particle  $j$  using tower sampling: pull a random number  $\eta \in [0, r_j]$ . If  $\eta < D(1 - s_j \varepsilon)$ , the particle hops to the left. If  $D(1 - s_j \varepsilon) < \eta < 2D$ , the particle hops to the right. Otherwise, the spin flips:  $s_j \rightarrow -s_j$ .
3. Update any  $r_k$  that has changed because of step 2 and sample new  $\tau_k$ 's according to  $P(\tau_k) = r_k e^{-r_k(\tau_k - t)} \Theta(\tau_k - t)$ . A new  $\tau_j$  is also sampled, whether or not  $r_j$  has changed.

To make step 1 as efficient as possible, we use a heap structure, which is a binary sorting tree consisting in a root and two sub heaps. The heap is such that the time stored at a root is always smaller than the times stored at the roots of the two sub heaps. Finding the smallest time then has a cost of  $\mathcal{O}(1)$  whereas the reorganization of the heap due to step [3] has a cost  $\mathcal{O}(\log_2 N)$ .

As we now discuss, the above algorithm, which is a generalization of the celebrated Gillespie algorithm, is an exact implementation of the continuous time Markov chain that defines the AIM. To show that an algorithm simulates a Markov chain defined by transition rates  $W(\mathcal{C} \rightarrow \mathcal{C}')$ , it suffices to show that trajectories are generated according to two principles. First, the system transitions out of a configuration  $\mathcal{C}$  after a time  $\tau$  distributed as

$$P(\tau) = r(\mathcal{C}) e^{-\tau r(\mathcal{C})} \quad \text{where } r(\mathcal{C}) = \sum_{\mathcal{C}' \neq \mathcal{C}} W(\mathcal{C} \rightarrow \mathcal{C}') . \quad (\text{A3})$$

Then, when a configuration change happens, the target configuration  $\mathcal{C}'$  is chosen with probability

$$P(\mathcal{C} \rightarrow \mathcal{C}') = \frac{W(\mathcal{C} \rightarrow \mathcal{C}')}{\sum_{\mathcal{C}'' \neq \mathcal{C}} W(\mathcal{C} \rightarrow \mathcal{C}'')} . \quad (\text{A4})$$

Consider a case with  $N$  possible transitions, with rates  $W_i$ , and take  $N$  transition times sampled according to  $p(\tau_i) = W_i e^{-W_i \tau_i}$ . The probability that the smallest time is larger than  $\tau_{\min}$  is

$$P(\inf_i \tau_i > \tau_{\min}) = \prod_i \int_{\tau_{\min}}^{\infty} d\tau_i W_i e^{-W_i \tau_i} = e^{-\tau_{\min} \sum_i W_i}$$

so that the probability density of the smallest time is given by

$$P(\tau_{\min}) = -\partial_{\tau_{\min}} P(\inf_i \tau_i > \tau_{\min}) = r e^{-\tau_{\min} r} ,$$

with  $r = \sum_i W_i$ , which indeed yields Eq. (A3). Tower sampling, as done in step 2, is then a simple way to choose a transition according to (A4).

Note that our algorithm 1-3 relies on two more ingredients. First, for  $N$  particles, there are  $3N$  possible moves, but we only use  $N$  times  $\tau_i$ , grouping hops and flips to get an escape rate per particle. This is largely arbitrary—and simply appropriate given our data structure—and



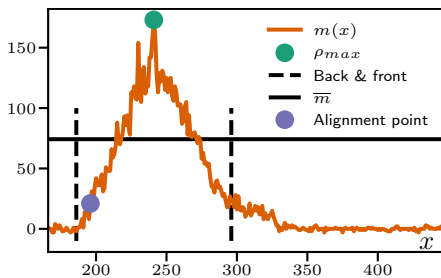


FIG. 21. Sketch of the alignment procedure and resulting sites used to align all flocks of the same age.

also leads to the exact implementation of the continuous time Markov chain because the escape rates are linear combinations of the possible rates. Further splitting the transitions into sub-transitions leaves  $P(\tau_{\min})$  unchanged since  $\sum_i W_i$  remains equal to the total escape rate. Second, we only need to sample new times  $\tau_k$  at step 3 if  $r_k$  has changed, or if the particle  $k$  has just moved. This is because the dynamics is Markovian so that any particle  $n$  whose time  $\tau_n$  is larger than  $t$  has a waiting time that is already correctly distributed. Indeed, its next configuration change was sampled at an earlier time  $\tau'_n$  with a law  $P(\tau_n) = r_n e^{-r_n(\tau_n - \tau'_n)}$ . We now know that  $\tau_n$  is larger than the current time  $t$ , which had a probability  $P(\tau_n > t) = e^{-r_n(t - \tau'_n)}$  to happen. Given that  $\tau_n$  is larger than  $t$ ,  $\tau_n$  is thus distributed according to the conditional probability  $P(\tau_n)/P(\tau_n > t) = r_n e^{-r_n(\tau_n - t)}$ , as it should at time  $t$ .

The clear advantages of this continuous-time algorithm are that it is exact and that there is no rejection. At every move, however, several new times  $\tau_k$  have to be sampled, the heap has to be reorganized, and the data structure is much heavier than for the discrete-time algorithm. As a result, the discrete-time algorithm is always more efficient when the rejection rate is low. For our simulations, the discrete-time simulations were faster at high temperatures, whereas the continuous-time simulations were faster at lower temperature (See Fig. 20) and we used them accordingly.

### 3. Numerical procedure to spatially align flocks.

We used the rear edge of the flocks to align them. To do so, consider the case of a flock with positive magnetisation shown in Fig. 21. We first localize the site  $i_{\max}$  where the density is maximal. Going backward from there,  $i_{\text{back}}$  is the first site where the magnetisation changes sign. We then calculate  $\bar{m}$  the average magnetisation per site over the region of size  $2\ell + 1$  centered on  $i_{\max}$ , where  $\ell \equiv |i_{\max} - i_{\text{back}}|$ . Starting from  $i_{\text{back}}$ , we then find the first site  $i_{\text{align}}$  such that  $m_{i_{\text{align}}} > \bar{m}/4$ . All right-going flocks are then aligned such that their site  $i_{\text{align}}$  coincide. Left-going flocks are transformed by  $m_i \rightarrow -m_i$  and  $i \rightarrow L - i$  and then run through the same algorithm.

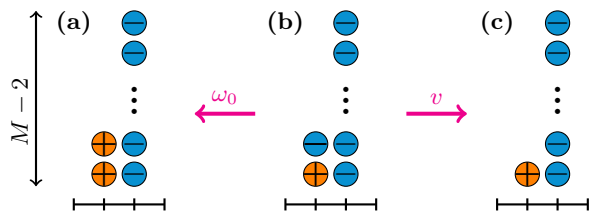


FIG. 22. The zero temperature limit of the AIM admits a new configuration with finite escape rate, shown in (b), where one site can be occupied by two particles with opposite signs, while the second site is occupied by  $M - 2$  particles. All moves lead to the evaporation of the aster but two: the minus particle on the first site can flip, at rate  $\omega_0$ , to reach the configuration  $(2, M - 2)$  depicted in (a); the minus particle on the first site can hop, at rate  $v$ , to reach the configuration  $(1, M - 2)$  depicted in (c).

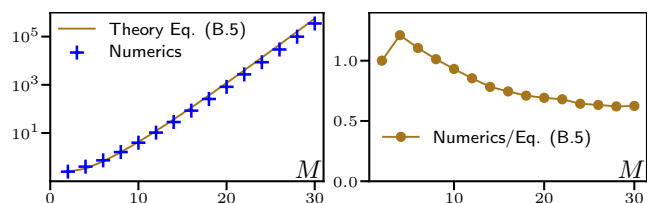


FIG. 23. **Left:** Numerical measurements of the MFPT till evaporation of an aster in the zero-temperature limit of the AIM, compared to the prediction (B5). **Right:** Taking the ratio of the data and the prediction (B5) shows the convergence to a constant slightly different from 1 as  $M \rightarrow \infty$ , due to the approximation involved in replacing  $T_{2, M-2}$  by its former expression (30).

### Appendix B: Another zero-temperature limit

Considering the AIM at finite temperature and taking the  $\beta \rightarrow \infty$  limit leads to a zero-temperature model slightly different from the one treated in Section IV. Indeed, it becomes possible to achieve configurations with vanishing magnetisation but finite density. These are such that  $n_i^+ = n_i^- \neq 0$ , so that their escape rates remain finite as  $\beta \rightarrow \infty$  since  $W(s \rightarrow -s | m_i = 0) = \omega_0$  for any  $\beta$ . Adding or removing one particle, however, leads to a non-vanishing magnetisation—and hence an immediate spin flip—so that the only new configuration that may play a role is  $n_i^+ = n_i^- = 1$ .

Let us consider the mean first-passage time starting from an aster with  $\frac{M}{2}$  spin + particles at one site and  $\frac{M}{2}$  spin - particles at the next site. The dynamics is then identical to that described in Section IV until the aster reaches the configuration  $(1, M - 1)$  (or the symmetric case). Then, at rate  $v(M - 1)$ , it reaches a new configuration, depicted in Fig. 22-b, where the magnetisation vanishes on the first site. This leads to the recurrence

$$T_{1, M-1} = \frac{1}{r} + q T_{d, M-2}, \quad (\text{B1})$$

where  $r = vM$  is the total escape rate out of the configu-

ration,  $q = \frac{M-1}{M}$  is the probability that one of the  $M-1$  spin- particles hops forward during the next configuration change, and  $T_{d,M-2}$  is the mean first passage time to evaporation starting from the configuration shown in Fig. 22-b. Inspection of the dynamics starting from this configuration then leads to

$$T_{d,M-2} = \frac{1}{r_d} + \frac{v}{r_d} T_{1,M-2} + \frac{\omega_0}{r_d} T_{2,M-2}, \quad (\text{B2})$$

where  $r_d = 2\omega_0 + vM$  is the escape rate. Injecting Eq. (B2) into Eq. (B1) then leads to:

$$T_{1,M-1} = \frac{1}{r} + \frac{q}{r_d} + \frac{qv}{r_d} T_{1,M-2} + \frac{q\omega_0}{r_d} T_{2,M-2}. \quad (\text{B3})$$

This has to be compared with the case considered in Section IV, which reads

$$T_{1,M-1} = \frac{1}{r} + qvT_{2,M-2}. \quad (\text{B4})$$

Two comments are in order. First, Eq. (B3) couples the mean first passage times to evaporation with  $M$  and

$M-1$  particles, which makes an exact solution harder to find. Second, in Eq. (B4)  $T_{1,M-1} \sim T_{2,M-2}$  as  $M \rightarrow \infty$ . On the contrary, in Eq. (B3), there is an extra factor  $r_d^{-1} \sim 1/M$  between  $T_{1,M-1}$  and  $T_{2,M-2}$ . The new configuration thus makes the evaporation happens  $M$  times faster. Since the dynamics close to  $M/2$  is insensitive to the details of the dynamics close to the evaporation, it is natural to approximate  $T_{2,M-2}$  by its original expression (30). This then leads to the self-consistent scaling

$$T_{1,M-1} \sim \frac{\omega_0}{2\omega_0 + vM} \frac{2^M}{2vM}. \quad (\text{B5})$$

As shown in Fig. 23, scaling (B5) agrees very well with our numerics. The prefactor is, unsurprisingly, incorrect, due to the approximation of  $T_{2,M-2}$  by its original expression (30). All in all, the aster still has an average life time that diverges exponentially with its number of particles, and the physics remains qualitatively unaltered from the case discussed in the main text.

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