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# Inverse problem to estimate lips parameters values of outward-striking trumpet model for successive playing registers

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1 The objective of this work is to estimate by inverse problem lip parameters values of  
2 trumpet model so that the oscillation thresholds for successive playing registers occur  
3 for the same blowing pressure as the one measured on several trumpet players. The  
4 lips vibration is modeled through an oscillator including unknown parameters such  
5 as: resonance frequency, quality factor, surface mass, stiffness and opening at rest  
6 of the lips. The oscillation threshold is calculated through linear stability analysis  
7 of the outward-striking model including the nonlinear coupling with the bore of the  
8 trumpet.

9 It appears that many combinations of parameter values are suitable to obtain the  
10 same blowing pressure at threshold as in the experiments. According to the analysis  
11 of the possible parameter values, some hypotheses are formulated about the playing  
12 strategies used by the trumpeter to select the different registers of the instrument.  
13 In addition to the lips resonance frequency, the control of the lips opening at rest  
14 appears as a viable strategy to match experimental oscillation thresholds in terms of  
15 blowing pressure. A set of numerical values for the lip parameters is given and allows  
16 through sound synthesis to play the successive registers of the trumpet.

## 17 I. INTRODUCTION

18 Digital models of brass instruments are commonly used to simulate<sup>1-4</sup> the sound pro-  
19 duced or to study the dynamic behaviour of the instrument<sup>5-8</sup>. Models used to describe  
20 the lips motion may seem oversimplified regarding the complexity of a human lip. However,  
21 taking into account biomechanical details is probably unreachable. Moreover, even a sim-  
22 plistic modeling of this vibration mechanism involves a large number of parameters<sup>9,10</sup>. The  
23 simplest model for the lips vibration relies on a single degree of freedom oscillator including  
24 several unknowns parameters: resonance frequency, quality factor, surfacic mass, stiffness  
25 and opening at rest of the lips<sup>11,12</sup>.

26 Limited knowledge is available concerning the values of these parameters. A bibliograph-  
27 ical study is carried out in<sup>12</sup>. In most studies, these values are chosen a posteriori so that  
28 the model considered produces self-sustained oscillation. Moreover they are given for a note,  
29 but no information is given on how to play the different registers (i.e. the notes playable  
30 with this fingering). This is due to the fact that direct estimation during the playing is  
31 not achievable through the experimental methods available. Some results based on exper-  
32 imental observations are however given for a two-mode lip's model by analyzing frequency  
33 responses of an artificial lip<sup>6,13</sup> or by using a high-speed camera to record the motion of a  
34 brass player's lips during playing<sup>14</sup>. Considering in addition to high-speed imaging an in-  
35 strumented mouthpiece, many parameters are estimated and discussed in<sup>15</sup>. More recently,  
36 in the case of the trumpet<sup>7</sup>, some lip parameters, like the lip position at rest, are tuned by  
37 hand so that for the note  $B\flat_4$ , the blowing pressure at the oscillation threshold is not unre-

38 alistically high while keeping a low value for the quality factor, as estimated by<sup>16</sup> on human  
39 lips. Most often in the previous studies, it seems difficult whatever the model considered to  
40 provide a complete set of parameters values.

41 In this paper, a methodology is proposed to choose parameter values according to an  
42 expected behavior of the model. More precisely, an inverse problem is posed to tune the  
43 parameters of the lip model so that the minimal blowing pressure that allows the emergence  
44 of sound is the same as the minimal blowing pressure measured on trumpet player to play  
45 the same note.

46 On the experimental side, the minimal blowing pressures are estimated through in-vivo  
47 measurements with trumpeters playing an instrumented mouthpiece (see section II). On  
48 the numerical side, the model considered is presented in section III. For a given set of lip  
49 parameter values, the minimal blowing pressure is defined as the value at which a Hopf  
50 bifurcation occurs. The following inverse problem is then studied in the rest of section IV:  
51 for different registers of the open-valve fingering, find the parameters value of the lip model  
52 so that the minimal blowing pressure is the same numerically and experimentally. Results  
53 are analyzed in regard to possible playing strategies used by trumpeters to select the different  
54 registers of the instrument. A summary table is given that provides convenient lip parameters  
55 value estimated by the inverse problem. It is demonstrated in section V using time domain  
56 simulation<sup>17</sup> that these parameters can be used to make the model play on the expected  
57 registers even far from the oscillation threshold.

## 58 II. MEASUREMENT OF BLOWING PRESSURES THRESHOLDS

59 The minimal blowing pressure used by musicians to produce a sound for a given note is  
60 measured. This is done for the registers 2 to 6 of the open valve position of the trumpet.  
61 Register 1, which corresponds to the pedal tone is rarely used by trumpet players and is not  
62 considered in this work. The objective of these measurements is to provide input data for  
63 the inverse method of estimating lip parameters. Therefore, these measurements are only  
64 intended to provide a realistic order of magnitude for the threshold blowing pressures for  
65 each register considered.

### 66 A. Experimental device

67 The trumpet used for this experiment is a Yamaha YTR 1335 and the mouthpiece is  
68 a Bach 1-1/2 C. The instrumented mouthpiece developed in<sup>18</sup> for a more general purpose  
69 has been used. A short and thin tube is inserted in the mouth of trumpet players. This  
70 tube is connected to an Endevco 8510B-5 pressure sensor outside the mouth (see figure 1).  
71 Since we are only interested in quasi-static blowing pressure, this tube does not influence the  
72 measurement. Acoustic pressure inside the mouthpiece is also recorded thanks to a pressure  
73 sensor of the same model.

### 74 B. Experiment Protocol

75 Before each measurement, an unsupervised free play warm-up phase takes place so that  
76 the musicians can get used to the instrumented mouthpiece and bring the instrument up to



Figure 1. (Color online) Measurement of the mouth pressure through a thin tube inserted at the corner of the mouth.

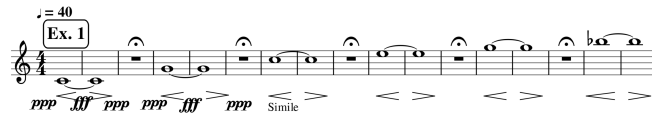


Figure 2. Instructions given to trumpeters in the form of a score. The "ppp"/"fff" indications correspond to the lowest/highest playing nuance that the musician can achieve. The notes to play correspond to registers 2 to 6.

77 temperature. The participants in the study are experienced amateur musicians. Trumpet  
 78 players are asked to play crescendo and decrescendo on regimes 2 to 6 of a B $\flat$  trumpet,  
 79 which correspond to notes C3 to B $\flat$ 4 (written note). More precisely they are given the score  
 80 which is shown in figure 2.

81 The "ppp"/"fff" indications correspond to the lowest/highest playing nuance that the  
 82 musician can achieve. The metronome is set to 40 light pulses per minute, and each note  
 83 should last 8 pulses. This exercise can be repeated many times in order to have several  
 84 threshold pressures values per musician and per register in the estimation of the final pressure  
 85 threshold. Trumpet players are informed that musicality is not an objective to be achieved  
 86 during this exercise.

87 The measurements presented hereafter are therefore marred by an uncertainty linked  
88 to the measurement protocol and the musician's instrumentation. This does not call into  
89 question the inverse method of estimating the lip parameters presented in this article. The  
90 influence of the uncertainty of the blowing pressure thresholds on the final results of the  
91 method are discussed in Section [VB](#).

### 92 C. Results

93 A typical output of the experiment is displayed in Fig. 3, with the blowing pressure and  
94 the mouthpiece pressure synchronized. Playing at threshold is not a musical task. Therefore  
95 a musician can't be ask to tell when he/she plays at threshold. Hence, the blowing pressure  
96 threshold is determined by an analysis of the recordings displayed in Fig. 3: it is defined as  
97 the minimum air pressure measured in the mouth of the musician that allows to sustain a  
98 sound. It is represented by a red point in Fig. 3.

99 The results obtained by gathering the measurements of four amateur trumpeters are  
100 considered. For each register, some outliers points are eliminated. They correspond to  
101 unreliable threshold values due to material problems or failed musical attempts. The boxplot  
102 representation of the threshold pressures is shown in Fig. 4. The red "+" signs correspond  
103 to the median value for each register. The numerical values are indicated in table [I](#).

104 Fig. 4 shows for each register the significant variation in threshold pressure produced by  
105 the musicians. To provide input data for the inverse method, the median of all the values  
106 retained is calculated.



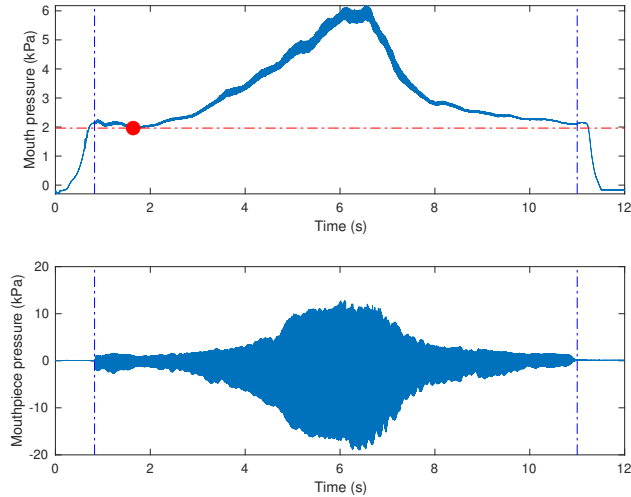


Figure 3. (Color online) A typical example of raw data obtained for one trumpeter playing the note G3 (second note in the exercise shown in figure 2) with respect to time. Top : mouth pressure; Bottom: mouthpiece pressure. The time interval where sound is produced is delimited by the two vertical dot-dashed lines. The minimum blowing pressure used within this interval is represented by the horizontal dot-dashed line.

Note	C3	G3	C4	E4	G4
$p_{m_{th}}^{exp}$ (Pa)	1047	1788	2477	3158	4109

Table I. Minimal mouth pressures used by trumpet players to play regimes 2 to 6 of the trumpet (open-valve position). Median values obtained with 4 trumpet players.

107 Generally a steady increase of the blowing pressure threshold is observed with the register  
 108 index. This is consistent with the experience of trumpet players. Very few experimental  
 109 data are available in the litterature that can be directly compared with. However, in<sup>19</sup>,

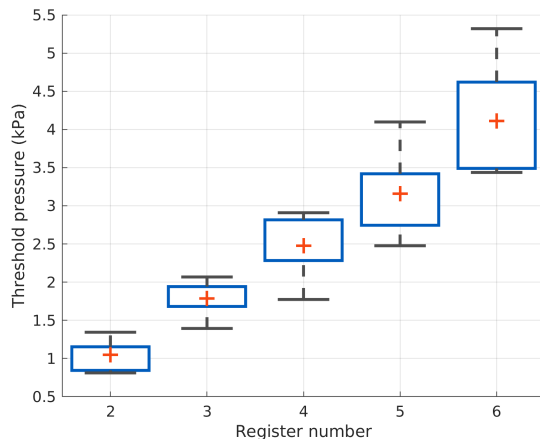


Figure 4. (Color online) Blowing pressure thresholds measured on trumpet players and represented as box plots for registers 2 to 6. Signs "+" point the median values for each register, and the corresponding values are given in table I. The blue boxes delimit the interval between the first and third quartile. The horizontal short black segments represent the min/max values.

110 Fletcher and Tarnopolsky measured the threshold blowing pressures for three players and  
 111 give values of the same order.

112 The question explored throughout this paper is: to what extent is it possible to tune  
 113 the unknown lip's parameters of a physical model of trumpet so that the minimal blowing  
 114 pressure required to play the oscillation regimes 2 to 6 is the same as the values shown in  
 115 table I? The physical model considered is described in the next section.

### 116 III. TRUMPET PHYSICAL MODEL CONSIDERED

117 Sound production in brass instruments relies on the coupling of a mechanical oscillator  
 118 (the lips of the player) and an acoustical resonator (the air column inside the instrument).

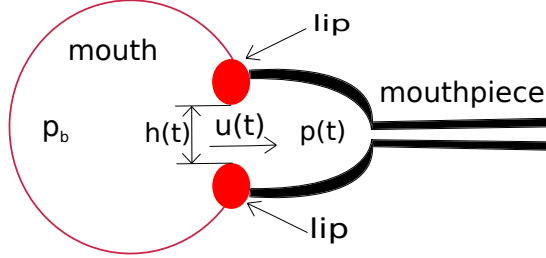


Figure 5. (Color online) Sketch of the mouth and lips of the musician and the instrument mouthpiece. The mouth (left) is considered as a volume under a static pressure  $p_b$  imposed by the musician. The height between the lips is  $h(t)$ , the airflow between the lips is  $u(t)$  and the pressure in the cup of the mouthpiece is  $p(t)$ .

119 The air flow blown into the instrument through the lips results the coupling mechanism  
 120 between these two oscillators<sup>20–22</sup>.

### 121 A. Lips model

122 A common modelling approach consists in considering a single mechanical mode for the  
 123 lips<sup>23,24</sup>. The lips are supposed in this paper to open when the pressure difference across  
 124 the lips is positive, which corresponds to the so-called outward-striking valve model (as  
 125 opposed to the inward-striking model widely used to model single-reed instruments<sup>25,26</sup>).  
 126 The relevance of this choice has been demonstrated for common playing situations<sup>12</sup>.

127 The outward-striking valve model<sup>9</sup> gives a relation between the height  $h$  of the channel  
 128 between the lips, its velocity  $\partial_t h$ , its acceleration  $\partial_t^2 h$  and the pressure difference between  
 129 the mouth  $p_b$  and the cup of the mouthpiece  $p$  (see figure 5):

$$\partial_t^2 h(t) + \frac{\omega_l}{Q_l} \partial_t h(t) + \omega_l^2 (h(t) - h_0) = \frac{1}{\mu} (p_b - p(t)) \quad (1)$$

130 where  $\omega_l = 2\pi f_l$  (rad.s<sup>-1</sup>) is the lip resonance angular frequency;  $Q_l$  the dimensionless  
 131 quality factor of the lips;  $h_0$  the value of  $h(t)$  at rest;  $\mu$  a lip surface mass (kg.m<sup>-2</sup>). These  
 132 parameters are considered independent.

### 133 B. Model for the air column

134 As far as the air column is concerned, a few acoustical modes are considered. At least  
 135 one mode is necessary to model a resonance peak of the input impedance of the air column.

136 Nonlinear effects in the resonator, responsible for brassy sounds at medium/high playing  
 137 levels<sup>27</sup>, should be taken into account for a more accurate modelling<sup>28</sup>. However, the primary  
 138 goal of this work is to adjust the model parameters according to measurements carried out  
 139 at low levels. Therefore the acoustic propagation along the bore is considered linear in this  
 140 study.

141 The input impedance of the B♭ trumpet used in this study is measured with the sensor  
 142 presented in<sup>29</sup>. The result is shown in Fig. 6 and is fitted by a sum of complex modes:

$$Z(\omega) = Z_c \sum_{n=1}^N \frac{C_n}{j\omega - s_n} + \frac{C_n^*}{j\omega - s_n^*}, \quad (2)$$

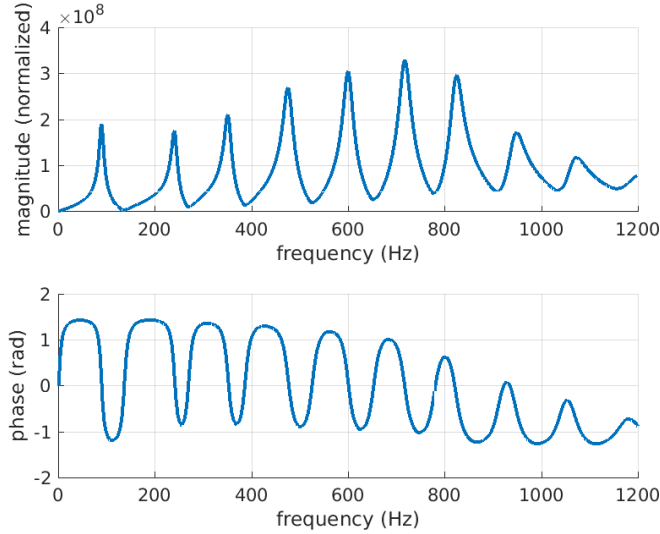


Figure 6. (Color online) Measured input impedance (modulus and phase) of the Bb trumpet with mouthpiece when no valves are pressed so as to play the score shown in fig. 2.

143  $s_n$  and  $C_n$  (resp.  $s_n^*$  and  $C_n^*$ ) being respectively the poles and the residues of the  $n^{th}$   
 144 complex mode (\* denotes the complex conjugate). The characteristic impedance of the  
 145 resonator is  $Z_c = \rho c/S$ ,  $S$  being the input cross section of the bore at the mouthpiece rim.  
 146 Transformation of Eq. (2) in the time domain and decomposition of  $p(t)$  into its modal  
 147 components  $p_n$ , such as  $p(t) = 2 \sum_{n=1}^N \text{Re}(p_n)$  results in an ordinary differential equation for  
 148 each  $p_n$ :

$$\frac{dp_n}{dt} = Z_c C_n u(t) + s_n p_n, \quad \forall n \in [1, N]. \quad (3)$$

### 149 C. Model for the air flow blown into the instrument

150 The two mechanical and acoustical oscillators presented above are nonlinearly coupled by  
 151 the airflow through the lip channel. The air flow is assumed laminar in the lip channel, but

152 turbulent in the mouthpiece. Moreover, it is supposed that the turbulent mixing dissipates  
 153 all the kinetic energy of the jet without pressure recovery<sup>30,31</sup>. The following expression of  
 154 the flow between the lips is then obtained and commonly used in models of brass and reed  
 155 instruments<sup>20,25,32</sup>:

$$u(t) = \sqrt{\frac{2}{\rho}} L \max(0, h(t)) \sqrt{|p_b - p(t)|} \text{sign}(p_b - p(t)), \quad (4)$$

156 with  $u(t)$  being the airflow rate ( $\text{m}^3\text{s}^{-1}$ ),  $\rho = 1.19\text{kg}\cdot\text{m}^{-3}$  the density of the air at  $20^\circ\text{C}$  and  
 157  $L$  the width of the lip channel (m).

#### 158 **IV. INVERSE PROBLEM BY LINEAR STABILITY ANALYSIS OF THE PRES-** 159 **SURE THRESHOLD**

160 The aim of this study is to estimate values of the lip parameters for which the blowing  
 161 pressure at threshold is the same as the pressure threshold measurements given in Table I.

162 This inverse problem is based on systematic linear stability analysis for a large range of  
 163 values of the outward-striking valve model parameters. The range of parameters studied  
 164 are arbitrary and do not take into account the practical aspect of the musician's playing  
 165 (morphology, playing confort, air noise...).

##### 166 **A. Linear stability analysis method**

167 The well-known Linear Stability Analysis method (LSA) is commonly used to analyse the  
 168 behaviour of dynamical systems around equilibrium points and has already been applied to

169 physical models of woodwind and brass instruments. It provides the stability information of  
 170 the equilibrium (non oscillating solution). Although limited to the equilibrium points, LSA  
 171 can predict important features of periodic regimes produced by brass instruments<sup>12</sup>. LSA  
 172 method works on the nonlinear coupled-system, described by Eqs (1),(3) and (4), rewritten  
 173 into a state-space representation  $\dot{X} = F(X)$ , where  $F$  is a nonlinear vector function and  $X$   
 174 the state vector:

$$X = \left[ h(t), \frac{dh}{dt}, \{p_n(t), n \in [1, N]\} \right]^t. \quad (5)$$

175 Note that  $F$  contains the parameters of the lips model and the poles and residues result  
 176 from the modal decomposition of the input impedance of the trumpet.

177 For a given set of lips parameters, the LSA method is used to determine the minimum  
 178 blowing pressure required to produce sound at small amplitude on the different registers of  
 179 the trumpet. To do this, the equilibrium solution of the coupled system is calculated and  
 180 the system is linearized in the vicinity of this solution. The loss of stability of the equilib-  
 181 rium solution, through the crossing of the imaginary axis by a pair of complex conjugate  
 182 eigenvalues of the linearized system, implies the emergence of a periodic solution. Under  
 183 the hypothesis of a direct bifurcation, the emergent periodic oscillation is stable. Thus, the  
 184 threshold pressure value corresponds to the blowing pressure above which the equilibrium  
 185 solution is unstable and the system admits a self-sustaining oscillation regime (at least in a  
 186 neighborhood of the bifurcation point). When the equilibrium solution becomes unstable,  
 187 it is also possible to calculate the frequency  $f_{th}$  of the solutions of the linearized system and  
 188 thus to estimate the pulsation of the emerging self-oscillation.

189 This work is based on the formulation of the LSA method proposed in article<sup>12</sup> which has  
190 demonstrated its relevance in highlighting several behaviours of an outward-striking valve  
191 model. People interested in the details of the calculation steps can refer to<sup>12</sup>.

192 Here is a summary of the computation method of threshold pressures and frequencies at  
193 the pressure threshold with the LSA method. For a given set of lips parameters ( $\omega_l$ ,  $Q_l$ ,  $\mu$ ,  
194  $h_0$  and  $L$ ), the equilibrium solutions of the model are calculated iteratively for an increasing  
195 blowing pressure  $p_b$  from 0Pa to 15kPa with an interval of 400Pa. A first approximation of the  
196 threshold pressure is calculated as the first pressure value that makes the equilibrium solution  
197 unstable. To ensure a more accurate estimate, the equilibrium solutions are recalculated on a  
198 blowing pressure grid with a much finer step of 0.1Pa around the first approximate threshold.

199 Fig. 7 shows an example of threshold pressures (top) and frequencies at the pressure  
200 threshold (bottom) obtained using the LSA method. The upper curve corresponds to the  
201 blowing pressure limit between non-oscillating and oscillating regimes. Hence each point of  
202 this curve is a Hopf bifurcation point. Each U-shaped portion is related to a register. The  
203 identification of the register number is done by comparing the oscillation frequency at the  
204 threshold with the resonance frequencies of the instrument (represented in horizontal dotted  
205 lines on the bottom Fig. 7). For the lip parameter values chosen in this calculation, the first  
206 eight registers of the trumpet can be produced over a wide lip resonance range. Between  
207 the 1<sup>st</sup> and 2<sup>nd</sup> register the curve is interrupted because the equilibrium solution is always  
208 stable whatever  $p_b$  for these lip resonance frequency values.

209 The evolution of the oscillation frequencies at threshold as a function of the lip resonance  
210 frequency also shows that the oscillation frequencies thresholds are always higher than the



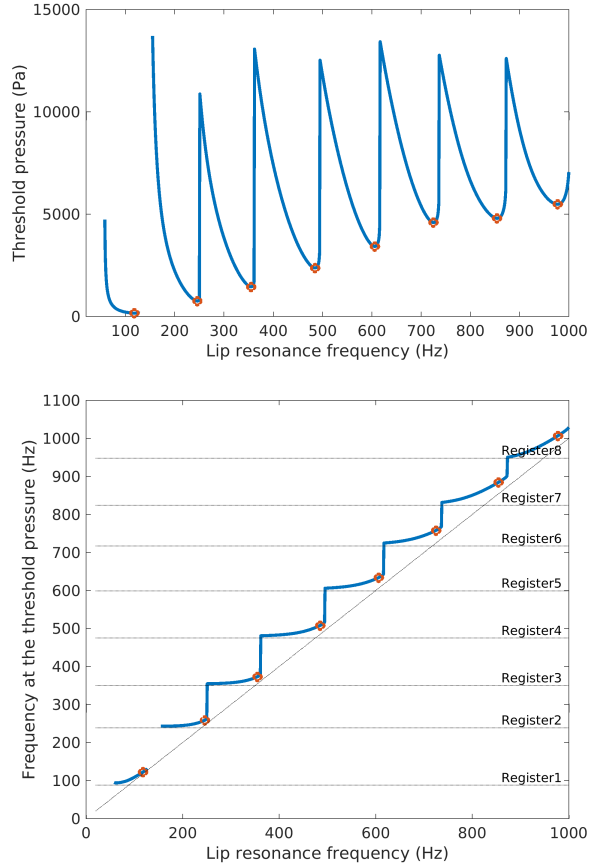


Figure 7. (Color online) Threshold pressures (top) and frequencies at the pressure threshold (bottom) according to the lip resonance frequency  $f_l$ , calculated with the LSA method with the following lip parameters:  $Q_l = 40$ ,  $\mu = 6\text{kg}\cdot\text{m}^{-2}$ ,  $h_0 = 0.5\text{mm}$  and  $L = 6\text{mm}$ . The red circles are located at the  $f_l$  corresponding to the minimum threshold pressures for each register of the instrument. The resonance frequencies of the input impedance of the instrument are plotted as horizontal dotted lines. A dotted diagonal line  $f_{th} = f_l$  is also plotted.

211 instrument's resonance frequencies. This behaviour has been classically reported for the  
 212 outward-striking valve<sup>12</sup>. It is also observed that for each register, the minimum of the  
 213 related U-shaped portion (marked by a red point) corresponds to an oscillation frequency at

214 threshold close to the resonance frequency of the lips (dotted line of unit slope on bottom  
215 Fig. 7).

## 216 B. The least effort strategy

217 Fig. 7 shows that when  $Q_l, \mu, h_0$  and  $L$  are fixed, the oscillation threshold for each register  
218 is not defined uniquely: on the contrary, each pair of values  $(p_b, f_l)$  lying on the U-shape  
219 related to a register corresponds to a possible oscillation threshold. In this work we follow  
220 Velut et al.<sup>12</sup> who consider that for each register “the optimal playing configuration for a  
221 human performer” corresponds to the minimum of the related U-shape portion. This defines  
222 a unique pair of value  $(p_b^*, f_l^*)$  for each register. As commented in<sup>12</sup>, “this hypothesis is in line  
223 with what musicians report, i.e. they develop a strategy to minimize the effort to produce  
224 a sound on a given regime”.

225 Fig. 7 indicates by the red circles the points  $(p_b^*, f_l^*)$  for each register. From a practical  
226 point of view, these points are identified by detecting peaks of the inverse of the threshold  
227 pressure curve. Therefore the accuracy of threshold identification  $(p_b^*, f_l^*)$  depends on the  
228 frequency resolution  $f_l$  of the stability analysis. This is why a step of 1Hz is used for the  
229 resolution of the LSA method.

230 Considering the successive points  $(p_b^*, f_l^*)$  for the different registers reveals a progressive  
231 increase of the blowing pressure at threshold, which is coherent with pressure thresholds  
232 measured on trumpet players with the instrumented mouthpiece. Unfortunately the exper-  
233 imental set-up is not capable of estimating the lips resonance frequencies, which prevents  
234 any comparison.

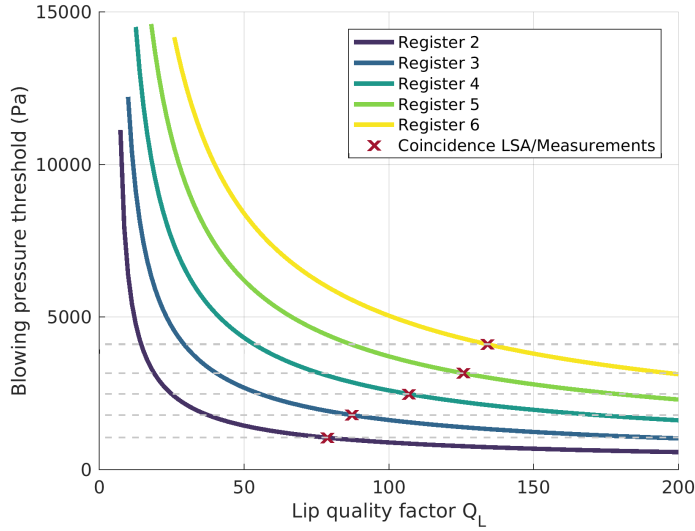


Figure 8. (Color online) Computed blowing pressure thresholds  $p_b^*$  with respect to the dimensionless lip quality factor  $Q_l$  for registers 2 to 6 of the instrument (plain lines). The other lip parameters are set to :  $\mu = 10.5\text{kg.m}^{-2}$ ,  $h_0 = 0.5\text{mm}$  and  $L = 12\text{mm}$ . The experimental values of the pressure thresholds are represented by horizontal dotted lines. For each register, the intersection between the curve of computed thresholds  $p_b^*$  and the horizontal line is marked with a red cross. The corresponding  $Q_l$  gives the value for which numerical and experimental thresholds are equal.

### 235 C. Coincidence of model pressure thresholds with targets values

236 For each set of values of the lips parameters  $Q_l$ ,  $\mu$ ,  $h_0$  and  $L$ , the linear stability analysis  
 237 (see section IV A) completed by the assumption that the player seeks for least effort at  
 238 threshold (see section IV B) allows to identify for each register a pair of values  $(p_b^*, f_l^*)$   
 239 corresponding to the oscillation threshold. From a practical point of view, in order to  
 240 accelerate the computation, LSA is parallelized on several processors.

241 This procedure is repeated for many different values of lips parameters among  $Q_l$ ,  $\mu$ ,  $h_0$   
 242 and  $L$ . The aim is to identify which sets of values, if any, allow to obtain for registers 2  
 243 to 6 the same blowing pressure at threshold  $p_b^*$  as the ones measured with musicians  $p_{m_{th}}^{exp}$ .  
 244 We focus on registers 2 to 6, since register 1 is peculiar (pedal note) and registers above 6  
 245 correspond to high-pitches note out of the scope of this study. Fig. 8 shows a first example  
 246 of all the values obtained for  $p_b^*$  when the lip quality factor  $Q_l$  is varied. Each plain line  
 247 corresponds to a different playing register. For a given curve each point corresponds to the  
 248 red circle (as defined in figure 7) of the corresponding register for a LSA achieved with the  
 249 value of  $Q_l$  corresponding to the abscissa of that point. Each point also corresponds to a  
 250 precise lips resonance frequency  $f_l^*$ . Therefore, for a given register, the curve represents the  
 251 evolution with  $Q_l$  of the minimum blowing pressure threshold  $p_b^*$  of the model. On the same  
 252 figure, horizontal dotted lines are plotted that correspond to the minimal pressure threshold  
 253 obtained experimentally  $p_{m_{th}}^{exp}$ . The intersection point with the curve corresponding to the  
 254 same register is marked with a cross symbol. For each playing register, this provides the value  
 255 of  $Q_l$  which ensures equal experimental and numerical oscillation thresholds. A practical  
 256 conclusion is that in this case, i.e. given the values of the others parameters, it is possible to  
 257 pick, for each register, the value of  $Q_l$  for which numerical results and experimental results  
 258 coincide. Put differently, the outward model can have the same oscillation thresholds for  
 259 registers 2 to 6 as trumpet players, provided that it is fed by the adequate lips parameters  
 260 values. However, for the chosen parameters  $\mu$ ,  $h_0$  and  $L$ , it appears that a unique value  
 261 of  $Q_l$  does not allow the model to reproduce the measured blowing pressures at threshold  
 262 for the registers considered. This would correspond to vertically aligned cross symbols on

263 Fig. 8. The value of  $Q_L$  reproducing experimental thresholds have a monotonous evolution  
264 from the lowest to the highest registers.

265 To expand the study and highlight the versatility of the approach, the values of  $p_b^*$  are now  
266 computed by varying two parameters:  $Q_l$  (as above) and the lips surfacic mass  $\mu$ . The aim  
267 is then to find, if any, sets of values of the pair  $(Q_l, \mu)$  for which numerical and experimental  
268 thresholds are equal (same blowing pressure). Results can be presented in a two-dimensional  
269 lip parameter space like in Fig. 9. For each register a curve with a different color represents  
270 values of  $(Q_l, \mu)$  for which numerical and experimental thresholds match, i.e. occur for the  
271 same blowing pressure.

272 To draw the curve for each register, the identification of admissible values of  $(Q_l, \mu)$  is  
273 carried out on a regular mesh. The range of both parameters  $Q_l$  and  $\mu$  is discretized on  
274 30 values, which means that 900 LSA are calculated to draw the figure. Then, in order to  
275 avoid a higher computational cost, an interpolation of the computed thresholds is performed  
276 to determine precisely pairs  $(Q_l, \mu)$  ensuring the coincidence between the experimental and  
277 numerical thresholds. The main observation is that there is no intersection between the  
278 curves obtained in Fig. 9, which means that there is no value of the pair of parameters  $(Q_l,$   
279  $\mu)$  which can be used to reproduce with the model the experimental thresholds for more than  
280 one register. The linear evolution of the curves of Fig. 9 shows that a realistic production  
281 of the pressure thresholds with the model imposes a balance between stiffness and vibrating  
282 mass of the lips. Thus, choosing a large lip mass  $\mu$  in the model implies a high quality factor  
283  $Q_l$ .

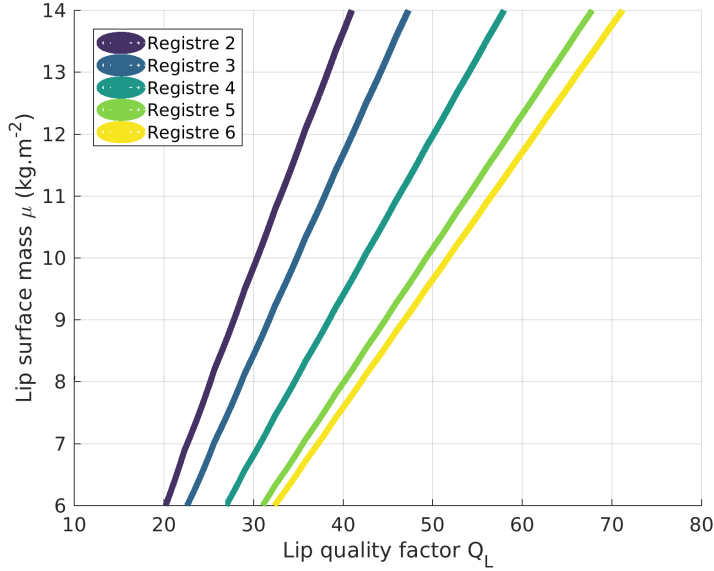


Figure 9. (Color online) Values of  $(Q_L, \mu)$  for which the blowing pressure  $p_b^*$  at oscillation threshold given by the model through LSA is the same as the experimental value  $p_{m_{th}}^{exp}$  obtained with trumpet players. The other lip parameters are set to :  $h_0 = 0.3\text{mm}$  and  $L = 12\text{mm}$ .

284 **V. ESTIMATION OF LIPS PARAMETERS ASSURING THE REGISTERS SE-**  
 285 **LECTION**

286 The previous section highlighted the importance of the values of the lip damping and  
 287 surfacic mass parameters  $(Q_L, \mu)$  to produce sound on the different trumpet registers. In fact,  
 288 two other parameters are known to vary to play successive registers in brass instruments<sup>14</sup>:  
 289 the width  $L$  and height  $h_0$  of the lips opening at rest. These parameters are also included  
 290 in the outward-striking valve lip model. The estimation by inverse problem of admissible  
 291 values of lips parameters in order to produce the trumpet registers is now extended to a 3D  
 292 parameter space. The methodology remains the same, but the exploration of coincidence

293 between simulated and measured pressure thresholds is extended to the width  $L$  or the  
294 height of the lips opening at rest  $h_0$ .

### 295 **A. Coincidence of simulated and measured threshold pressures in a 3D lips pa-** 296 **rameter space**

297 The correspondence between the measured and simulated blowing pressure thresholds  
298 is represented in a 3D parameter space by a hyperplane for each register. These hyper-  
299 planes indicate the lips parameter values that allow the measured threshold pressures to be  
300 reproduced with the outward-striking valve model. As with 2D plans (see Section [IV C](#)),  
301 interpolation between experimental and simulated data is necessary to obtain their coin-  
302 cidence with a reduced discretization of the parameter space. It still takes about 22000  
303 parameter combinations to converge to the solution of the inverse problem (about 12 hours  
304 with a multiprocessor desktop computer).

305 Fig. [10](#) shows that the opening at rest of the lips  $h_0$  has a great importance on the  
306 selection on the produced registers. The hyperplanes are overlaid on each other. The  
307 registers are ordered from the lowest to the highest when  $h_0$  decreases. This is relevant  
308 with the experimental observations of playing strategies employed by musicians<sup>14</sup>. It is also  
309 observed that the spacing between the hyperplanes gradually tightens as  $h_0$  decreases.

310 Fig. [10](#) illustrates several scenarios for selecting instrument registers only by modifying  
311 the lip parameters within the model. A simplistic scenario would consist in fixing a couple  
312 of parameters  $(Q_l, \mu)$  and selecting the register only by adjusting the height of the lips  
313 opening at rest  $h_0$ . This playing strategy with the model is tested by sound synthesis in

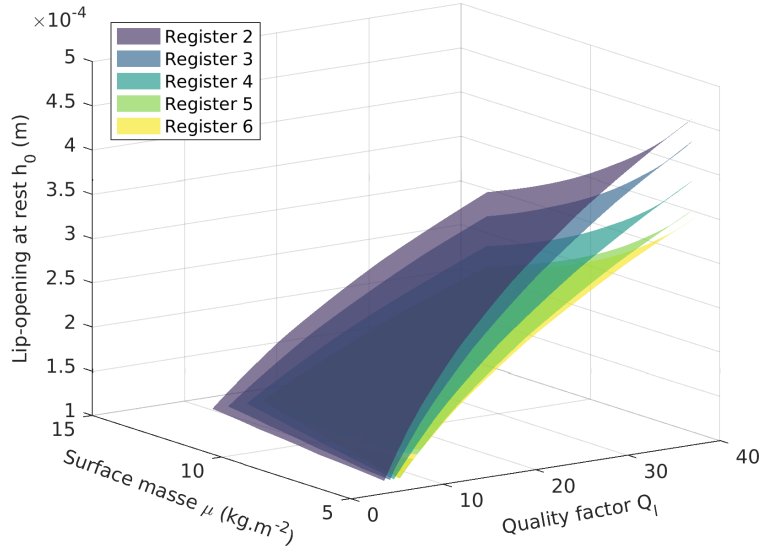


Figure 10. (Color online) Coincidence between blowing pressure thresholds measured  $p_{m_{th}}^{exp}$  and calculated  $p_b^*$  by the LSA method in the space of parameters  $(Q_l, \mu, h_0)$  for the different registers of the instrument ( $L = 12\text{mm}$ ).

314 part VC. In practice, the musician having to increase the resonance frequency of their lips,  
 315 it is probably oversimplified to consider a constant vibrating mass of the lips regardless of  
 316 the register played. Many playing strategy - possibly corresponding to different players -  
 317 can be proposed as a progression through one hyperplane to another in Fig. 10. So, the  
 318 successive selection of registers can be done with multiple combinations of parameters.

319 Another way to represent the evolution of the lip parameters in order to select a particular  
 320 register of the instrument is to explore the coincidences between simulation and threshold  
 321 pressure measurements by varying the width of the lips opening  $L$  (the height at rest is then  
 322 constant). Fig. 11 presents the result of the inverse method in the space of the parameters  
 323  $(Q_L, \mu$  and  $L)$  for an intermediate value of opening height of the lips at rest  $h_0 = 0.5$  mm.



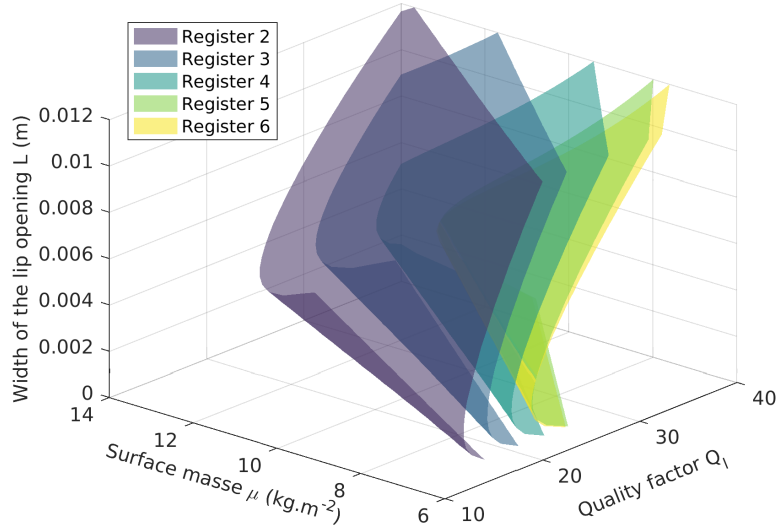


Figure 11. (Color online) Coincidence between pressure thresholds measured  $p_{m_{th}}^{exp}$  and calculated  $p_b^*$  by the LSA method in the space of parameters  $(Q_L, \mu, L)$  for the different registers of the instrument ( $h_0 = 0.3\text{mm}$ ).

324 Hyperplanes have a more complex shape while still being overlaid on each other. By com-  
 325 parison between Figs. 10 and 11, it can be observed that for the opening surfaces at rest of  
 326 the lips ( $S_0 = Lh_0$ ) the shape of the hyperplanes is different. This comes from the motion  
 327 equation (1) which depends on the lip-opening at rest  $h_0$  and not on the width  $L$ . Thus,  
 328 it is possible to observe that depending on the opening section of the air flow at rest, the  
 329 parameters of the lips  $Q_L$  and  $\mu$  can be quite different for the production of the same regis-  
 330 ter. This can be seen as a way to render the wide variety of musician morphologies. Also,  
 331 on Fig. 11 we can propose a register selection scenario as a trajectory in parameter space  
 332 crossing the normal to the surface of each hyperplane.

333 The resolution of the inverse problem in a three-dimensional parameter space presents  
334 trends that are consistent with current knowledge on the mechanisms of sound production  
335 by brass instruments. We will therefore use in Section V C the results presented in Figs. 10  
336 and 11 to extract numerical values that can be used with the outward-striking trumpet  
337 model.

### 338 B. Impact of measurement uncertainty on estimated lips parameters

339 The results presented above are based on threshold blowing pressure values estimated  
340 experimentally. The instrumentation on musicians and the associated measurement protocol  
341 mar these values with uncertainty. In order to study the influence of the uncertainty on the  
342 estimation of the lip parameters, the simulations carried out in Part V A are reinterpreted  
343 with a tolerance of 10 percent on the threshold blowing pressure values  $p_{m_{th}}^{exp}$  considered in  
344 this study (see Table I). In the 3D space of the lip parameters  $(Q_l, \mu, h_0)$ , Fig. 12 shows that  
345 a variation of the threshold pressure considered on each register gives the same tendencies  
346 as those described in Fig. 10 for all registers considered. The hyperplanes remain overlaid  
347 on each other according to the lip-opening at rest  $h_0$ . A 10 percent increase in the threshold  
348 pressure corresponds to slightly greater opening values  $h_0$ . Similarly, a 10 percent decrease  
349 in the threshold pressure results in slightly lower opening values  $h_0$ . This confirms that the  
350 uncertainty relating to the measurement of the blowing pressure threshold does not call into  
351 question the methodology presented in this article.

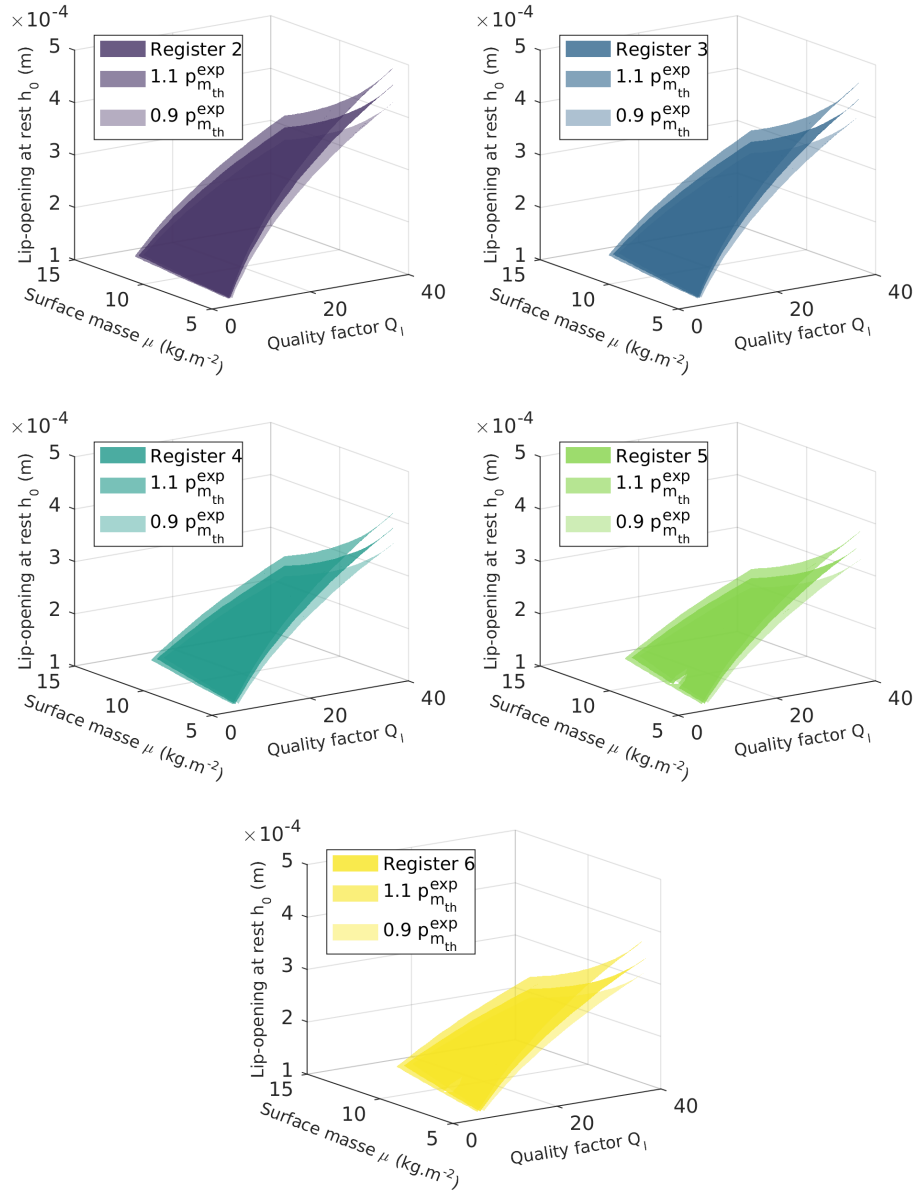


Figure 12. (Color online) Effect of an uncertainty of ten percents on the measured pressure threshold on the result of the inverse problem. Each figure corresponds to a register ( $L = 12\text{mm}$ ).

### C. On the use of estimated lip parameters for sound production.

The inverse method presented in this article is based on a static consideration of the acoustic quantities of the outward-striking trumpet model through the linear stability analysis employed (see Section IV A). In order to check that the estimated lip parameter values can be useful for the sound production simulation of brass instrument, the sound synthesis method called MoReeSC<sup>17</sup> is used to reproduce a rise of the register 2 to 6 of the trumpet. This method relies on the modal expansion of the input impedance of the trumpet used in this study associated with the outward-striking valve model (see Fig. 6). By using finite-difference scheme, MoReeSC method provides ab-initio discrete-time computation of the self-oscillations controlled by time-varying parameters, including the mouth pressure and the lips parameters. For the following simulations, a sample rate of 44100Hz is used.

As shown in Fig. 10, the lip-opening at rest  $h_0$  is a parameter which allows the model to select registers for fixed values of the surface mass, the quality factor and the width of the lip. Thus, we set the parameters  $L = 12\text{mm}$ ,  $\mu = 9\text{kg.m}^{-2}$  and  $Q_l = 20$  in order to extract on the hyperplanes of each register the corresponding values of lip-opening  $h_0$ . For each of these values we recover in the simulated data the associated resonance frequencies  $f_l$  of the lip. The blowing pressure values considered for successive registers are the threshold values  $p_{mth}^{exp}$  obtained experimentally increased by 20% to go well beyond the emergence of self-oscillations.

Figure 13 shows a spectrogram of the internal pressure  $p(t)$  simulated by sound synthesis. The selection of the registers is ensured by a synchronous change of the blowing pressure  $p_b$ ,

373 of the resonance frequency of the lips  $f_l^*$  and of the opening at rest  $h_0$ . For this example, it  
374 was arbitrarily chosen to vary these parameters according to a linear evolution of 20 ms to  
375 trigger a change of register. The fundamental frequencies of the simulated sounds correspond  
376 qualitatively to the notes (C3, G3, C4, E4 and G4) associated with registers 2 to 6 of the  
377 B♭ trumpet. As the tuning is not taken into account in estimating the lip parameters, the  
378 playing frequencies are slightly out of tune. The establishment of self-oscillations on register  
379 2 is a bit long (approximately 100ms) this is due to the low supply pressure too close to the  
380 threshold of this register. Once the auto-oscillation are engaged, the transition between the  
381 successive registers is carried out very quickly in reaction to the changes of the lip parameters  
382  $f_l$  and  $h_0$ .

383 The spectrogram shows that the chosen lip parameter values allow to reproduce the  
384 register selection mechanism. This validates the inverse method presented in this article  
385 to design control for sound synthesis. Of course, this provides no guarantee on the control  
386 strategies of real musicians, but gives for the outward-striking model different values of lip  
387 parameters reproducing expected behaviors. The values of lip parameters used for sound  
388 synthesis in Fig. 13 are specified in Table II.

## 389 VI. CONCLUSION

390 With the inverse problem presented in this paper, the unknown lip parameters of the  
391 outward-striking model are estimated with respect to the measured blowing pressure. To  
392 the authors' knowledge, this is the first time that the values of these lip parameters are  
393 chosen according to an experimentally observed behavior on an auto-oscillating system.

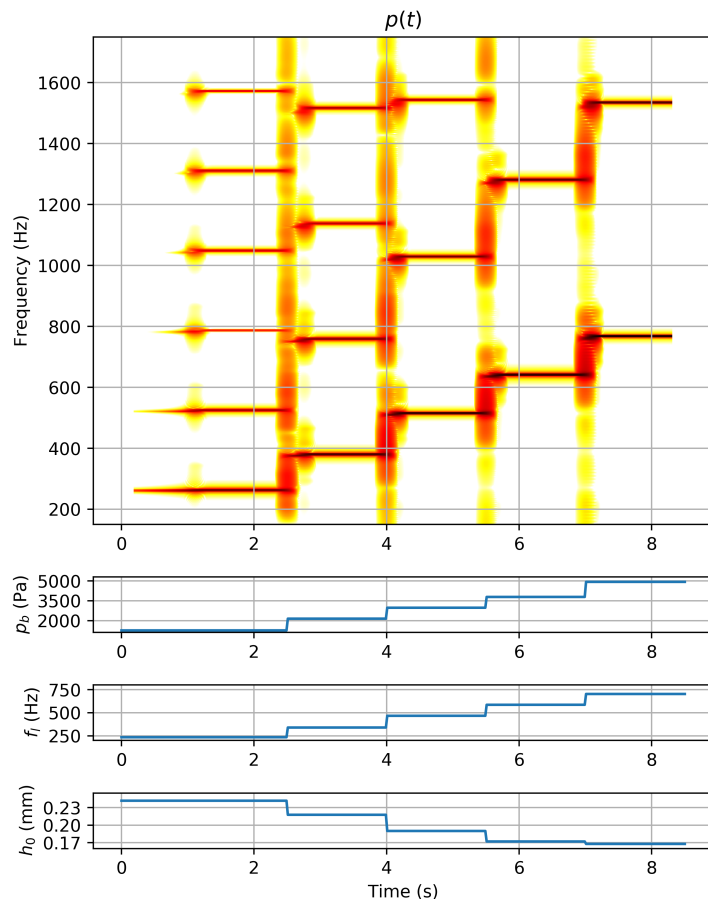


Figure 13. (Color online) Spectrogram of the internal acoustic pressure  $p(t)$  simulated by sound synthesis. The selection of the registers is ensured by a synchronous change of the blowing pressure  $p_b$ , of the resonance frequency of the lips  $f_l$  and of the opening at rest  $h_0$ . The other lip-parameters are constant over time :  $L = 12\text{mm}$ ,  $Q_l = 20$  and  $\mu = 9$ .

394 Although the method relies on a linearized analysis of the model, it is interesting that the  
 395 simulated nonlinear dynamics of the model far from the threshold confirms the robustness  
 396 of the parameter choices in terms of register selection. This opens perspectives for the  
 397 control of physical models with sound synthesis purpose. Another perspective is to use the

<b>Register</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
$p_b$ (Pa)	1256	2146	2973	3790	4931
$f_l$ (Hz)	235	340	467	586	703
$h_0$ (mm)	0.242	0.218	0.190	0.172	0.168

Table II. Values of lip parameters allowing the selection of registers 2 to 6 estimated by the inverse method and used in sound synthesis with  $L = 12\text{mm}$ ,  $Q_l = 20$  and  $\mu = 9$  (see Fig. 13).

398 experimentally based parameter values given in this article to carry out the study of the  
399 nonlinear dynamics of the model, including the calculation of its bifurcation diagram.

400 While exemplified on a simple one-degree-of-freedom outward-striking lip model, the  
401 approach proposed may be easily extended to other lip models. It can even be applied  
402 to any model of sound production, provided its linear stability analysis is carried out and  
403 experimental data at the oscillation threshold are available.

404 Concerning the experiments with trumpet players, they provide an order of magnitude of  
405 the minimum blowing pressures for playing registers 2 to 6. Of course these measurements  
406 could be generalized to a larger number of musicians in order to refine the information  
407 provided. One could also imagine, on the contrary, considering individual measurements in  
408 order to fit parameters that make the oscillation thresholds of the model correspond to those  
409 of a particular musician. A first step perhaps towards the notion of a digital twin within  
410 the context of sound synthesis.

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