



# Transverse Doppler effect due to Gaussian beams

O. Emile, J. Emile

## ► To cite this version:

O. Emile, J. Emile. Transverse Doppler effect due to Gaussian beams. Optics Letters, 2023, 48 (3), pp.546-549. 10.1364/OL.480549 . hal-04016715

**HAL Id: hal-04016715**

**<https://hal.science/hal-04016715>**

Submitted on 14 Mar 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

# Transverse Doppler effect with Gaussian beams

O. Emile\*

*Université de Rennes 1, Campus de Beaulieu, F-35000 Rennes, France*

J. Emile

*Université de Rennes 1, CNRS IPR UMR 6251, F-35000 Rennes, France*

We report on the observation of a transverse Doppler shift in the optical domain. It occurs when a receiving system travels perpendicularly to the propagation direction of a Gaussian beam. Shifts of a few tens of Hz have been evidence for a detector moving in the mm/s range. The shift increases as the detector is far from the beam axis. The observations fully agree with theoretical calculations on the propagation of Gaussian beams. It can be observed for any kind of waves, including radio and acoustic waves. Practical consequences are then discussed, especially for technics using Doppler measurements in microsystems.

The so-called Doppler effect is the apparent change in frequency or pitch of a wave when the emitter moves either toward or away from the receiver, or vice versa [1–3]. It has found many applications, such as in astrophysics [4], biology [5, 6], meteorology [7], medicine [8] or radars systems [9] to name a few. It is deeply rooted in the double spatial and temporal periodicity of the wave. A correlation between space and time dependence, such as the relation between time and position via the velocity, then leads to a Doppler effect. Whereas it has mainly dealt with longitudinal velocities, a rotational Doppler effect has also been recently evidenced [10–12] for beams with an azimuthal phase dependence. Nevertheless, other kinds of beams such as Gaussian beams have not only a longitudinal phase variation, but also a transverse one [13]. One may then wonder whereas a transverse Doppler effect could be evidenced in this case. The aim of this letter is thus to investigate the signal detected by a detector moving perpendicularly to a Gaussian beam.

A Gaussian laser beam is expressed as [13]

$$E(r, z) = E_0 \frac{w_0}{w(z)} e^{-\frac{r^2}{w^2(z)}} e^{i \frac{k r^2}{2R(z)}} e^{-i(kz - \omega t + \varphi(z))} \quad (1)$$

$E_0$  is the field amplitude,  $r$  is the radial distance from the beam axis,  $z$  is the distance from the beam waist  $w_0$ ,  $w(z)$  is the size of the beam at distance  $z$ ,  $k$  is the wavevector,  $\omega$  is the pulsation of light,  $R(z)$  is the radius of

curvature of the beam, and  $\varphi(z)$  is the Gouy phase. The term with a phase with a  $z$  dependence is responsible for the usual Doppler effect. It scales as  $k v_z$ ,  $v_z$  being the longitudinal velocity. Let us focus here on the term with a transverse phase dependence associated with the term with a time dependence of the Gaussian beam

$$\Delta\varphi = \frac{k r^2}{2R(z)} + \omega t \quad (2)$$

and let us consider a linear variation of the radial distance with time at a distance  $r_0$  ( $r = r_0 + vt$ ,  $v$  being the transverse velocity). The phase term then reads

$$\Delta\varphi = k \frac{r_0^2}{2R(z)} + k \frac{r_0 v t}{R(z)} + k \frac{(vt)^2}{2R(z)} + \omega t \quad (3)$$

Assuming a small displacement ( $vt \ll r_0$ ) this thus leads to a transverse Doppler shift

$$\delta\omega_d = 2\pi\delta v_d = k \frac{r_0 v}{R(z)} \quad (4)$$

According to this equation, there should be a transverse Doppler shift as soon as a receiving system travels at a given velocity, perpendicular to the direction of propagation of a Gaussian beam. The effect must increase as the system travels far from the beam axis. Since it depends on the transverse velocity, we thus call it a transverse Doppler effect. It has not to be confused with the relativistic Doppler shift sometimes also called the transverse Doppler

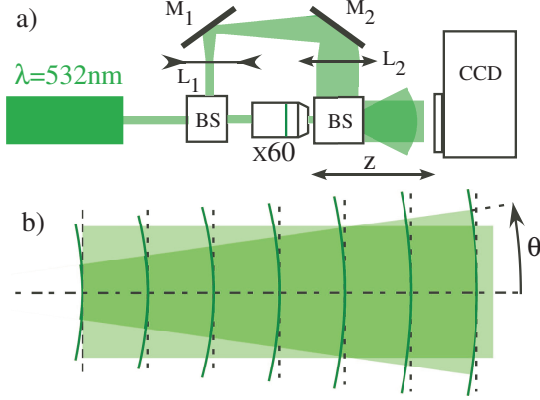


FIG. 1. a) Experimental set-up: interference between a Gaussian beam and a plane wave. BS: beam splitter;  $L_1$  ( $f_1 = -5$  cm) and  $L_2$  ( $f_2 = 40$  cm): lenses (distance between the lenses 35 cm);  $M_1$  and  $M_2$ : mirrors. The interferences are detected either by a CCD camera or by a point like moving detector.  $z$ : distance between the position of the waist and the detector, taking into account the index of the BS. Distances on the drawing are not to scale. b) Phase front of a Gaussian beam (full line) compared with the phase front of a plane wave (dotted line).

shift [14, 15], since it is also sensitive to the transverse velocity. However, it is a problem of time dilatation when the source and the receiver are at their closest point. It scales as  $v^2/c^2$ ,  $c$  being the velocity of light. It is purely a problem of relativity and has nothing to do with a phase variation. It plays no role here.

In order to evidence this effect, we have performed the following experiment (see Fig.1a). The fundamental Gaussian beam from a laser source (Luminbird,  $P = 100$  mW,  $\lambda = 532$  nm) is transformed into a strongly diverging Gaussian beam by a microscope objective with a high magnification (X60). It corresponds to the source in the transverse Doppler effect. At a given distance  $z$  from the focus, we measure the beam size  $w(z)$ . We deduce the divergence  $\theta$  of the beam  $\tan \theta = w(z)/z$  and estimate the waist to be  $w_0 = \lambda/(\pi \tan \theta) = 1.7 \mu\text{m}$ . This corresponds to a Rayleigh range  $z_0 = \pi w_0^2/\lambda = 17 \mu\text{m}$ . Far from the beam waist ( $z \gg z_0$ ), the equiphase surfaces are spherical caps (see Fig.1b). They are compared

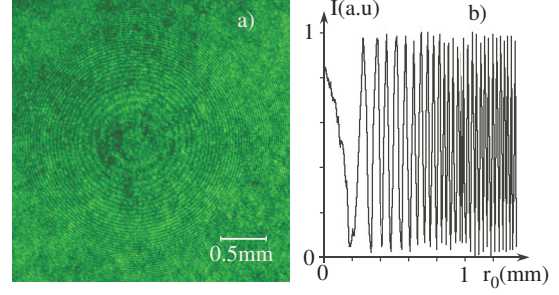


FIG. 2. a) Picture of the interference pattern between a strongly focused Gaussian laser beam (radius of curvature  $R = 5.9$  cm) and a plane wave. b) Plot of the intensity variation of the interference pattern versus the distance from the center of the Gaussian beam ( $r_0$ ).

with the equiphase planes of a plane wave that originates from the same laser source. To perform this, part of the laser light is enlarged by two lenses ( $L_1$ , and  $L_2$ ) that makes the beam nearly parallel. It is then equivalent to a plane wave. The phase variation is measured by the interference fringes registered on a CCD camera (Thorlabs DCU 224C).

Figure 2a shows the picture of the two-waves interference pattern between the diverging beam and the quasi plane wave at a given distance  $z = 5.9$  mm (including the path through the beam splitter) from the beam waist. The system is static here ( $v = 0$ ). Since  $z$  is far from the Rayleigh zone, the beam radius of curvature  $R$  equals  $z$  [13]. One can notice regular rings corresponding to positions where the focused laser beam has the same phase as the plane wave. As one moves far from the beam center ( $r_0$  increases), the rings get closer to each other, as expected from Eq. 2. One can then register the intensity maxima and minima versus the distance from the beam center  $r_0$  (see Fig. 2b), and deduce the corresponding phase variation (see Fig. 3). We have plotted on the same figure the theoretical phase variation corresponding to Eq. 2 and the experimental phase variation measured for three different distances from the waist, corresponding to radius of curvature  $R = 5.9$  cm,  $R = 8.4$  cm and  $R = 12.0$  cm, respectively

$$\Delta\varphi = \frac{kr_0^2}{2R} \quad (5)$$

There is a very good agreement between the theoretical curve and the experimental measurement, for the three radius of curvature. The phase variation is smaller as the beam radius increases in agreement with Eq. 2 and Eq. 5. Please note that there are no adjustable parameters in the theoretical curves. They directly originate from Eq. 5.

In order to investigate the transverse Doppler shift, we have replaced the CCD camera by a point like detector. This corresponds to the detector in the transverse Doppler effect. It is a usual photodiode (FND 100), with a 10  $\mu\text{m}$  pinhole in front of it. As this photodiode is displaced at a constant velocity in a direction perpendicular to the direction of propagation of the Gaussian beam ( $v \neq 0$ ), it experiences an intensity variation that corresponds to the interference fringes shown in Fig. 2, like in every Doppler experiment. However, we concentrate here, on the one hand, on the transverse velocity of the detector, and on the other hand, on the Gaussian nature of the light beam. These fringes (see insert of Fig. 4a) are a manifestation of the beat frequency between the reference plane wave and the Gaussian laser beam measured by the detector. Since the reference beam is not Doppler shifted, it implies that the Gaussian beam detected by the photodiode is frequency shifted, as expected. From Eq. 4, the frequency of the detected beam increases as the system is moving toward the beam axis and decreases as the system moves away from the beam axis.

Actually, Eq. 5 can be derived from a more general formula, as the one established in [16]. A Doppler effect can be expected from a phase gradient whatever its direction. For example, if one adds a phase with an azimuthal dependence in Eq. 1, as for Laguerre-Gaussian beams [13], it might induce a rotational Doppler effect. A structured polarization distribution may also lead to a kind of Doppler effect. In the case of Laguerre-Gaussian beam, it has then been proposed to

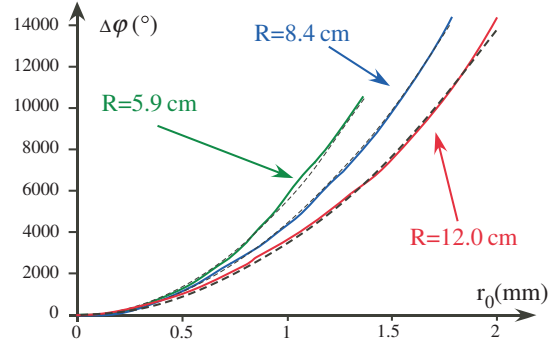


FIG. 3. Phase variation of the Gaussian laser beam perpendicular to the beam propagation for three different radii of curvature  $R = 5.9$  cm,  $8.4$  cm and  $12.0$  cm, versus  $r_0$ . Solid line: experimental data, phase uncertainty  $\pm 20^\circ$ . The uncertainty comes from the uncertainty in the determination of the maxima and minima of the fringes. Dotted lines: theoretical curves corresponding to Eq. 5.

be used for the detection of transverse velocities, and applied to structured light and the detection of the rotational Doppler effect via the orbital angular momentum of light [16, 17]. In our case, we focus on a very simple radial phase gradient that can be found in usual Gaussian beam. It is worth noting that an intensity variation may lead to a modulation of the interference pattern and then may induce an error in the phase variation identification. This is not the case here.

We can then plot the transverse Doppler shift, either versus the distance from the beam axis for a given velocity (Fig. 4a), or versus the transverse velocity at a given position (see Fig. 4b). We have also plotted the theoretical variations of this shift according to Eq. 4. The agreement between the theoretical curves and the experimental ones is very good. Please note again that there are no adjustable parameters in these curves. Obviously, according to these figures, it exists a transverse Doppler effect for a system that travels through a Gaussian beam, perpendicular to the beam axis. It could be easily observed. This is also true for a moving system that reflects light. However, contrarily to the usual longitudinal shift, the effect is not doubled in reflection. One has

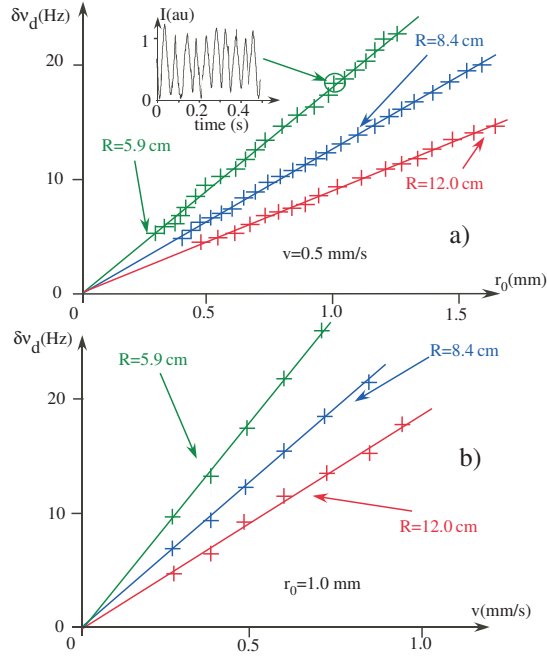


FIG. 4. Doppler shift versus a) the distance from the beam center for a velocity  $v = 0.5$  mm/s and b) the velocity of the system for a distance from the beam center of  $r = 1$  mm. Crosses: experimental data. The uncertainty in the frequency is  $\pm 0.2$  Hz, due to the uncertainty on the determination of the beat frequency. Solid line: theoretical curve corresponding to Eq. 4. Inset: beat frequency for  $v = 0.5$  mm/s and  $r_0 = 1$  mm.

to take into account the radius of curvature of the reflected beam at the position of detection. This could also be the case when the reflecting moving system is not plane. The induced extra curvature in the beam propagation has then also to be taken into account.

Such effect must exist for every non-strictly plane waves. Besides, it is not limited to optics but also includes in particular radars and acoustics waves where the Doppler effect has many applications. Aside from fundamental aspects, it may thus have important consequences in many experimental set-ups using the Doppler effect. Nevertheless, there are situations where the effect described here is far below the current sensitivity of the experiment, considering the wavefront curvatures that are used. For example, this is the case

in the probing of movements in the ionosphere using radiowaves in the megahertz range [18], or in laser anemometry [19]. The transverse Doppler effect is negligible in those cases.

However, there are situations where the transverse Doppler effect may affect the experimental data, for example in laser Doppler vibrometry [20], especially in the microscopic domain [21, 22]. Actually in such experiments, the incoming beam can be considered as a plane wave with a very good approximation. Nevertheless, the reflecting objects are barely flat. They thus introduce a non plane-wave reflection with a radius of curvature that depends on the curvature of the object. Since the light is collected after propagation on micro to millimeter scales, the transverse doppler effect may then induce extra or lacking interference fringes as the object is moving perpendicular to the beam propagation. For example, for a 2 mm radius of the curved reflecting object, after a 1 mm propagation, at a 0.1 distance from the beam center and for a 0.1 mm/s velocity, the transverse Doppler shift is of the order of 20 Hz. This can be extremely penalizing especially for calibrations or precise measurements.

As a conclusion, we have experimentally evidenced a transverse Doppler shift when a system is travelling perpendicular to a probing Gaussian laser beam. The light is blue shifted when it travels towards the beam axis and red shifted as it moves away from the axis. The shift is in the tens of Hertz range for a velocity in the millimeter range and depends on the position of the system within the beam. The observations are in very good agreement with theoretical expectations. Since any Doppler effect is associated with a work and thus with a force [23], this may have dramatic consequences in optical tweezers that use strongly focused laser beams [24, 25]. Actually, many other phase gradients such as polarization gradients, intensity gradients as well as Gouy phase gradients may also exist for tightly focused beams. They may induce specific Doppler effects. However, the existence of a transverse Doppler effect as the one described here, may provide a new intuitive insight in the complicated forces involved in optical twee-

ers. In particular it may slow down more effectively particles as they enter the trapping beam region and accelerate them as they leave the beam.

*Acknowledgments* We gratefully acknowledge fruitful discussions with Erwann Carvou (IPR Rennes) and technical assistance from Jean-René Thébault (Université de Rennes 1).

## REFERENCES

- 
- \* emile@univ-rennes1.fr
- [1] A. Eden, The search for Christian Doppler, in *The Search for Christian Doppler* (Springer, 1992).
  - [2] P. M. Schuster, *Moving the stars: Christian Doppler, his life, his works and principle, and the world after* (Living Edition, 2005).
  - [3] D. D. Nolte, The fall and rise of the Doppler effect, *Phys. Today* **73**, 30 (2020).
  - [4] M. Mayor and D. Queloz, A jupiter-mass companion to a solar-type star, *Nature* **378**, 355 (1995).
  - [5] D. H. Evans and W. N. McDicken, *Doppler ultrasound: physics, instrumentation and signal processing* (Wiley-Blackwell, 2000).
  - [6] Z. Li, H. Sun, J. Turek, S. Jalal, M. Childress, and D. D. Nolte, Doppler fluctuation spectroscopy of intracellular dynamics in living tissue, *JOSA A* **36**, 665 (2019).
  - [7] A. Battaglia, P. Kollias, R. Dhillon, R. Roy, S. Tanelli, K. Lamer, M. Grecu, M. Lebsack, D. Watters, K. Mroz, *et al.*, Spaceborne cloud and precipitation radars: Status, challenges, and ways forward, *Rev. Geophys.* **58**, e2019RG000686 (2020).
  - [8] D. L. Franklin, W. Schlegel, and R. F. Rushmer, Blood flow measured by Doppler frequency shift of back-scattered ultrasound, *Science* **134**, 564 (1961).
  - [9] C. Hao, D. Orlando, J. Liu, and C. Yin, Introduction to radar systems, in *Advances in Adaptive Radar Detection and Range Estimation* (Springer, 2022) pp. 1–22.
  - [10] M. P. Lavery, S. M. Barnett, F. C. Speirits, and M. J. Padgett, Observation of the rotational doppler shift of a white-light, orbital-angular-momentum-carrying beam backscattered from a rotating body, *Optica* **1**, 1 (2014).
  - [11] L. Fang, M. J. Padgett, and J. Wang, Sharing a common origin between the rotational and linear doppler effects, *Laser Photon. Rev.* **11**, 1700183 (2017).
  - [12] O. Emile, J. Emile, C. Brousseau, T. Le Guennic, P. Jian, and G. Labroille, Rotational doppler shift from a rotating rod, *Opt. Lett.* **46**, 3765 (2021).
  - [13] A. E. Siegman, *Lasers* (University science books, 1986).
  - [14] A. Sommerfeld, *Lectures on theoretical physics: Optics*, Vol. 4 (Academic press, 1950).
  - [15] M. Kaivola, O. Poulsen, E. Riis, and S. A. Lee, Measurement of the relativistic Doppler shift in neon, *Phys. Rev. Lett.* **54**, 255 (1985).
  - [16] C. Rosales-Guzmán, N. Hermosa, A. Belmonte, and J. P. Torres, Experimental detection of transverse particle movement with structured light, *Sci. Rep.* **3**, 2815 (2013).
  - [17] A. Belmonte and J. P. Torres, Optical doppler shift with structured light, *Opt. Lett.* **36**, 4437 (2011).
  - [18] S. Grach, G. Komrakov, M. Yurishchev, B. Thidé, T. Leyser, and T. Carozzi, Multi-frequency Doppler radar observations of electron gyroharmonic effects during electromagnetic pumping of the ionosphere, *Phys. Rev. Lett.* **78**, 883 (1997).
  - [19] Y.-Q. Li, P. Lynam, M. Xiao, and P. J. Edwards, Sub-shot-noise laser Doppler anemometry with amplitude-squeezed light, *Phys. Rev. Lett.* **78**, 3105 (1997).
  - [20] P. Castellini, M. Martarelli, and E. P. Tomasini, Laser Doppler vibrometry: Development of advanced solutions answering to technology's needs, *Mech. Sys. Signal Process.* **20**, 1265 (2006).
  - [21] B. Ohler, Cantilever spring constant calibration using laser Doppler vibrometry, *Rev. Sci. Instrum.* **78**, 063701 (2007).
  - [22] C. Rembe, G. Siegmund, H. Steger, and M. Wörtge, Measuring mems in motion by laser Doppler vibrometry, in *Optical Inspection of Microsystems* (CRC Press, 2019) pp. 297–347.
  - [23] O. Emile and J. Emile, Energy, linear momentum, and angular momentum of light: What do we measure?, *Ann. Phys.* **530**, 1800111 (2018).
  - [24] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, and S. Chu, Observation of a single-beam gradient force optical trap for dielectric particles, *Opt. Lett.* **11**, 288 (1986).

- [25] J. R. Moffitt, Y. R. Chemla, S. B. Smith, and C. Bustamante, Recent advances in optical tweezers, *Annu. Rev. Biochem.* **77**, 205 (2008).