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# Continuous-discrete time observers for homogeneous nonlinear systems with sampled-data outputs

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## Abstract

In this article, we propose a new observer design for systems whose output is sampled in an aperiodic and asynchronous way and for which a continuous-time homogeneous observer of negative degree is available. The proposed method consists in adapting the existing continuous-time observer in order to cope with the fact that only sampled measurements are available instead of continuous ones. The obtained observer error is shown to be globally uniformly ultimately bounded for any upper bound on the sampling periods. Furthermore, the ultimate bound decreases as the upper bound on the sampling periods decreases. The stability analysis is based on a Lyapunov approach and the performances of the proposed observer are illustrated with simulations.

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## 1 Introduction

The study of nonlinear observers has been an active research area for several decades [1]. Indeed, they are powerful tools with a large number of possible applications since they allow to reconstruct all the states of a system from the model and the measured outputs only. This can be useful when all the states cannot be measured, for example when sensors do not exist which is the case in robotics to reconstruct stiffness [2] or in process control for some specific variables [3]. Another reason can be that the sensors are too expensive in some cases such as an industry production for example. Another use can be to check that the available sensors are working correctly by using such observers for fault detection and isolation [4].

Many approaches have then been developed to design such observers. One of the first ideas has been to transform nonlinear systems into a linear ones so that observers can be constructed [5]. Transformations

into more general forms have also been considered such as in [6] for example. General classes of systems such that observers can be designed have also been considered. For Single-Input Single-Output (SISO) uniformly observable systems, a high gain observer can always be constructed [7], though the change of coordinates can be difficult to find. General classes of Multi-Input Multi-Output (MIMO) uniformly observable systems have also been considered in [8]. Some other approaches rely on the specific structure of the system, such as backstepping [9] or adaptive observers [10].

Homogeneity has been considered more recently for observer design [11–13], though it has been introduced early in [14] and [15]. Homogeneity is an interesting property since it is well suited to approximation or domination design [16,17]. Other properties can also be obtained from homogeneity of systems, such as finite-time or fixed-time stability for example [18,19]. Several structures of observers have been developed. One way is to use an inductive approach such as backstepping [20]. One can also rely on the fact that the considered homogeneous observer is close to an observer whose convergence is already established [21]. Another possible approach is to consider an Implicit Lyapunov Function (ILF) [22].

In the aforementioned works, the outputs are supposed to be available continuously, but this is rarely the case

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in practice. Indeed, most observers are implemented digitally on computers or micro controllers, the output is then usually sampled and the sampling times may be aperiodic [23]. This may be the case when there are packet dropouts in Networked Control Systems (NCS) [24] or in order to lower the load of a network when it is used by many agents at the same time [25]. One way is to use a fully discrete-time approach [26], but one loses the physical meaning of the states and only semi-global stability might result. Another way is to use the emulation method [27], which means that a continuous observer is first synthesized and discretized afterward. A third way is to adapt the continuous observer to obtain a so called continuous-discrete time observer which uses the discrete-time measurements and provide a continuous estimation of the states. Several designs have been proposed following this way. A first idea is to use observers whose state is reset whenever a new measurement is received. A high gain approach has been used in [28] for a class of linear systems up to input injections and for a class of uniformly observer systems whose nonlinear part is input dependant in [29]. LMIs have been used to guarantee the stability in [30] for a class of nonlinear systems using reachability analysis, in [31] using an hybrid approach, for a class of systems whose dynamics is assumed to belong to a polytopic set of matrices in [32], for a class of linear systems with noise analysis in [33] and for the same class of linear systems but with a less conservative approach in [34]. Another idea is to use continuous observers based on an output error predictor which is reset when a new measurement is received. This approach has been first used in [35] and then in [36] using a high-gain approach and in [37] by using LMIs. Both continuous and discontinuous observers have been combined in [38].

It has been shown recently that sampled homogeneous systems enjoy good properties. Indeed, depending on the degree of homogeneity either global practical stability or semi-global asymptotic stability can be obtained [39]. Furthermore, it has been shown in [40] that robustness with respect to exogenous disturbance in the dynamics is also obtained. Such results have also been obtained for time-delay systems which can be seen as a more general framework than sampled systems, such as in [41,42].

In this paper, we are particularly interested in the adaptation of homogeneous observers of negative degree (the precise meaning of homogeneous observer will be detailed in the following) in order to cope with the sampling of the output. Several features of the proposed design are worth being emphasized:

- i) the proposed observer error is *globally* uniformly ultimately bounded.
- ii) a Lyapunov function is explicitly constructed for the continuous-discrete time observer.

- iii) the vector field representing the original system may be non-homogeneous, which is not the case of [39].
- iv) simulations show that the performances of the proposed scheme are better than the one proposed in [39].

The paper is organized as follows. The class of systems and the different assumptions are stated in section 2. Section 3 is devoted to the presentation of the proposed continuous-discrete time observer. Some simulations are provided in section 4 in order to show the performances of the proposed scheme. Finally, section 5 ends the paper.

## 2 Class of considered systems

Let us consider the following system

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \quad (1)$$

with  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}$ ,  $y \in \mathbb{R}$ . The function  $f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$  is assumed to be continuous and  $h : \mathbb{R}^n \rightarrow \mathbb{R}$  is assumed to be  $C^1$  and homogeneous of degree  $d_h > 0$  with respect to some weights  $(\alpha_1, \dots, \alpha_n) \in \mathbb{R}_+^n$ .

One assumes that a continuous observer, meaning that it has access to the output continuously for all  $t \geq 0$ , exists for system (1), which is in the following form

$$\begin{cases} \dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) - G(\hat{y}(t) - y(t)) \\ \hat{y}(t) = h(\hat{x}(t)) \end{cases} \quad (2)$$

where the function  $G : \mathbb{R} \rightarrow \mathbb{R}^n$  is assumed to be homogeneous of degree  $\alpha < 0$  with respect to the weight  $d_h$ . One assumes that the states of observer (2) converge to the states of system (1). More precisely, one assumes the following.

**Assumption 1** *There exists a class  $C^1$  homogeneous Lyapunov function  $U : \mathbb{R}^n \rightarrow \mathbb{R}$  of degree  $d_U > \max\{-\alpha, d_h, \alpha_1, \dots, \alpha_n\}$  with respect to the weights  $(\alpha_1, \dots, \alpha_n)$ , whose derivative along the trajectories of the following error system*

$$\dot{e}(t) = f(\hat{x}(t), u(t)) - f(x(t), u(t)) - G(\hat{y}(t) - y(t)) \quad (3)$$

*is negative definite, where  $e = \hat{x} - x$ , i.e. there exists  $\varepsilon > 0$  such that*

$$\dot{U}|_{(3)}(e(t)) \leq -\varepsilon (U(e(t)))^{\frac{d_U + \alpha}{d_U}}. \quad (4)$$

**Remark 1** *Since the function  $U$  is homogeneous of degree  $d_U$  with respect to the weights  $(\alpha_1, \dots, \alpha_n)$ , several useful properties can be derived using Lemma 4.2 in [43]. Firstly, the function  $\partial U / \partial e_j$  is homogeneous of degree*

$d_U - \alpha_j$  and continuous, hence there exists  $\kappa_j \geq 0$  such that

$$\left| \frac{\partial U}{\partial e_j}(e) \right| \leq \kappa_j \sum_{l=1}^n |e_l|^{\frac{d_U - \alpha_j}{\alpha_l}}. \quad (5)$$

Secondly, using the fact that  $U$  is positive definite gives the existence of a constant  $\varrho > 0$  such that

$$\left( \sum_{l=1}^n |e_l|^{\frac{d_U + \alpha}{\alpha_l}} \right) \leq \varrho(U(e))^{\frac{d_U + \alpha}{d_U}}. \quad (6)$$

**Remark 2** It can be noted that the vector field  $f$  is not assumed to be homogeneous. Nevertheless, in general, some homogeneous properties will be required to obtain a homogeneous Lyapunov function  $U$  for the error system. For example, such Lyapunov functions can be found for systems where  $f$  is composed of an homogeneous drift term  $f_1(x)$ , which is homogeneous of degree  $\alpha < 0$  with respect to the weights  $(\alpha_1, \dots, \alpha_n)$ , and a term  $f_2(x, u)$  which will be dominated in the stability analysis, such as in [20] for example.

**Remark 3** While the form of observer (2) seems restrictive, it should be noted that it is typically obtained in other coordinates than those of the system and an estimate of the state can be obtained by inverting the change of coordinates.

### 3 Main results

The aim of this work is to construct a continuous-discrete time observer for system (1), that is an observer that provides a continuous estimate of the state of system (1) when the output  $y$  is only available at some sampling times  $(t_k)_{k \in \mathbb{N}}$ . The sampling times are assumed to verify:

$$0 = t_0 < t_1 < \dots < t_k < \dots \quad (7)$$

$$0 < T_{\min} < t_{k+1} - t_k < T_{\max}, \quad \forall k \in \mathbb{N} \quad (8)$$

The proposed observer is given by

$$\begin{cases} \dot{\hat{z}}(t) = -\frac{\partial h}{\partial x}(\hat{x}(t))G(\hat{z}(t)) \\ \dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) - G(\hat{z}(t)) \end{cases} \quad (9)$$

when  $t \in [t_k, t_{k+1})$ , and

$$\begin{cases} \hat{z}(t_k) = \hat{y}(t_k) - y(t_k) \\ \hat{x}(t_k) = \hat{x}(t_k^-) \end{cases} \quad (10)$$

Let us now write the error dynamics for observer (9). One denotes  $e_0(t) = \hat{z}(t) - (\hat{y}(t) - y(t))$ ,  $e = \hat{x} - x = (e_1, \dots, e_n)$ , then, for  $t \in [t_k, t_{k+1})$ , one has

$$\begin{aligned} \dot{e}_0(t) &= \frac{\partial h}{\partial x}(x(t))f(x(t), u(t)) - \frac{\partial h}{\partial x}(\hat{x}(t))f(\hat{x}(t), u(t)) \\ \dot{e}(t) &= (f(\hat{x}(t), u(t)) - f(x(t), u(t)) - G(\hat{y}(t) - y(t))) \\ &\quad - (G(\hat{z}(t)) - G(\hat{y}(t) - y(t))) \end{aligned} \quad (11)$$

The whole error state is denoted  $E = (e_0, \dots, e_n) \in \mathbb{R}^{n+1}$  and the considered candidate Lyapunov function  $V(E, t)$  is define for a given set of sampling instant verifying (7)–(8) as follows. Let  $E \in \mathbb{R}^{n+1}$  and  $t \geq 0$ , then there exists a unique  $k \in \mathbb{N}$  such that  $t \in [t_k, t_{k+1})$  and  $V(E, t)$  is defined as

$$V(E, t) = U(e) + \exp(\delta(t_{k+1} - t))|e_0|^{\frac{d_U}{d_h}}, \quad (12)$$

with  $\delta > 0$ . Note that the function  $V$  is a valid candidate Lyapunov function, indeed, it is positive definite since  $\exp(\delta(t_{k+1} - t)) \in [1, \exp(\delta T_{\max})]$  for all  $t \in [t_k, t_{k+1})$ .

Before stating the main result, some additional assumptions have to be made in order to obtain the stability of the continuous-discrete time observer. In particular, it is required that some Hölder properties hold:

**Assumption 2** (i) For all  $x^1 = (x_1^1, \dots, x_n^1)$ ,  $x^2 = (x_1^2, \dots, x_n^2) \in \mathbb{R}^n$  and  $u \in \mathbb{R}$ , there exists  $\nu_1 \geq 0$  such that

$$\begin{aligned} \left| \frac{\partial h}{\partial x}(x^1)f(x^1, u) - \frac{\partial h}{\partial x}(x^2)f(x^2, u) \right| \\ \leq \nu_1 \sum_{i=1}^n |x_i^1 - x_i^2|^{\frac{d_h + \alpha}{\alpha_i}}. \end{aligned} \quad (13)$$

(ii) For all  $z_1, z_2 \in \mathbb{R}$ ,  $i = 1, \dots, n$ , there exists  $\nu_2^i$  such that

$$|G_i(z_1) - G_i(z_2)| \leq \nu_2^i |z_1 - z_2|^{\frac{\alpha_i + \alpha}{d_h}}. \quad (14)$$

**Theorem 1 (Ultimate Boundedness)** Consider system (1) and observer (9)–(10) subject to Assumption 2, then for any  $T_{\max} \geq T_{\min} > 0$  the error system (11) is globally stable and there exist  $K \geq 0$ , independent of  $T_{\max}$ , such that the set

$$\left\{ E = \begin{pmatrix} e_0 \\ e_1 \\ \vdots \\ e_n \end{pmatrix} \in \mathbb{R}^{n+1} : U(e) + |e_0|^{\frac{d_U}{d_h}} \leq K(T_{\max})^{\frac{d_U}{\alpha}} \right\} \quad (15)$$

is globally attractive.

**Remark 4** Theorem 1 states that the proposed observer cannot explode in finite-time even if the upper bound on the sampling periods period increases. This is not the case for most of the existing observers since if the sampling periods increases too much, the observer usually becomes unstable.

This of course does not mean that the observation error stay small since it increases as  $T_{\max}$  increases. Nevertheless, this is a very interesting feature since it allows

to keep satisfactory performances even when approaching the limit where other observers become unstable as it can be seen in the example section. This can also be useful to allow a very fast recovery of the estimation for example when the sampling instants are irregular and the sampling periods become large at some moments.

**Proof.** The proof is divided into three steps. In Step 1, one shows that the Lyapunov function candidate  $V$  is decreasing when new measurements are available, that is for  $t = t_k$  with  $k \in \mathbb{N}$ . In Step 2, one is interested in the behavior of the error dynamics when the dynamics is continuous, that is when  $t \in [t_k, t_{k+1})$  with  $k \in \mathbb{N}$ . An over-valuation of the derivative of  $V$  is then obtained. In Step 3, the results of Steps 1 and 2 are combined to show that the observer error is globally uniformly ultimately bounded.

Step 1. Step 1 is rather direct, indeed, since the dynamics of  $e$  are continuous ( $e(t_k^-) = e(t_k)$ ) and since  $e_0(t_k) = 0$ , one has

$$\begin{aligned} V(E(t_k^-), t_k^-) &= U(e(t_k^-)) + \exp(\delta(t_k - t_k)) |e_0(t_k^-)|^{\frac{d_U}{d_h}} \\ &= U(e(t_k)) + \underbrace{|e_0(t_k^-)|^{\frac{d_U}{d_h}}}_{\geq 0} \\ &\geq U(e(t_k)) + \exp(\delta(t_{k+1} - t_k)) \underbrace{|e_0(t_k)|^{\frac{d_U}{d_h}}}_{=0} \\ &\geq V(E(t_k), t_k) \end{aligned} \quad (16)$$

Step 2. The derivative of the function  $V$  along the trajectories of system (11), for  $t \in [t_k, t_{k+1})$ , is given by

$$\begin{aligned} \dot{V}(E(t), t)|_{(11)} &= \quad (17) \\ \dot{U}(e(t))|_{(3)} &- \delta \exp(\delta(t_{k+1} - t)) |e_0(t)|^{\frac{d_U}{d_h}} \\ &+ \sum_{j=1}^n \frac{\partial U}{\partial e_j}(e(t)) (G_j(\hat{z}(t)) - G_j(\hat{y}(t) - y(t))) \\ &+ \frac{d_U}{d_h} \exp(\delta(t_{k+1} - t)) |e_0(t)|^{\frac{d_U - d_h}{d_h}} \\ &\times \left( \frac{\partial h}{\partial x}(x(t)) f(x(t), u(t)) - \frac{\partial h}{\partial x}(\hat{x}(t)) f(\hat{x}(t), u(t)) \right) \end{aligned}$$

One now states two facts whose proof can be found in the appendix.

**Fact 1.** For any  $\gamma_1 > 0$ , there exists  $\Gamma_1 \geq 0$  such that

$$\begin{aligned} \sum_{j=1}^n \frac{\partial U}{\partial e_j}(e) (G_j(\hat{z}) - G_j(\hat{y} - y)) \quad (18) \\ \leq \gamma_1 (U(e))^{\frac{d_U + \alpha}{d_U}} + \Gamma_1 |e_0|^{\frac{d_U + \alpha}{d_h}} \end{aligned}$$

holds for all  $E \in \mathbb{R}^{n+1}$ .

**Fact 2.** For any  $\gamma_2 > 0$ , there exists  $\Gamma_2 \geq 0$  such that

$$\begin{aligned} \frac{d_U}{d_h} \exp(\delta(t_{k+1} - t)) |e_0|^{\frac{d_U - d_h}{d_h}} \quad (19) \\ \times \left( \frac{\partial h}{\partial x}(x) f(x, u) - \frac{\partial h}{\partial x}(\hat{x}) f(\hat{x}, u) \right) \\ \leq \gamma_2 (U(e))^{\frac{d_U + \alpha}{d_U}} + \Gamma_2 (\exp(\delta(t_{k+1} - t)))^{\frac{d_U + \alpha}{d_U - d_h}} |e_0|^{\frac{d_U + \alpha}{d_h}} \end{aligned}$$

holds for all  $t \in [t_k, t_{k+1})$  and  $E \in \mathbb{R}^{n+1}$ .

Using inequality (4), with Facts 1 and 2 in (17) gives

$$\begin{aligned} \dot{V}(E(t), t) &\leq -\varepsilon (U(e(t)))^{\frac{d_U + \alpha}{d_U}} \quad (20) \\ &- \delta \exp(\delta(t_{k+1} - t)) |e_0|^{\frac{d_U}{d_h}} \\ &+ \gamma_1 (U(e(t)))^{\frac{d_U + \alpha}{d_U}} + \Gamma_1 |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \\ &+ \gamma_2 (U(e))^{\frac{d_U + \alpha}{d_U}} \\ &+ \Gamma_2 (\exp(\delta(t_{k+1} - t)))^{\frac{d_U + \alpha}{d_U - d_h}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \\ &\leq -(\varepsilon - \gamma_1 - \gamma_2) (U(e(t)))^{\frac{d_U + \alpha}{d_U}} \quad (21) \\ &- \delta \exp(\delta(t_{k+1} - t)) |e_0(t)|^{\frac{d_U}{d_h}} \\ &+ (\Gamma_1 + \Gamma_2) (\exp(\delta(t_{k+1} - t)))^{\frac{d_U + \alpha}{d_U - d_h}} \\ &\quad \times |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \end{aligned}$$

One now states another fact whose proof can be found in the appendix.

**Fact 3** For  $\delta = 1/T_{\max}$ , there exists  $K_1 \geq 0$  independent of  $T_{\max}$  such that the following inequality holds true

$$\begin{aligned} -\delta \exp(\delta(t_{k+1} - t)) |e_0(t)|^{\frac{d_U}{d_h}} \quad (22) \\ + (\Gamma_1 + \Gamma_2) (\exp(\delta(t_{k+1} - t)))^{\frac{d_U + \alpha}{d_U - d_h}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \\ \leq -\frac{\varepsilon}{2} (\exp(\delta(t_{k+1} - t)))^{\frac{d_U + \alpha}{d_U}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} + K_1 T_{\max}^{\frac{d_U + \alpha}{d_h}} \end{aligned}$$

for all  $t \in [t_k, t_{k+1})$ .

Using Fact 3 in equation (21) and taking  $\gamma_1, \gamma_2$  such that

$\varepsilon - \gamma_1 - \gamma_2 = \varepsilon/2$  gives

$$\begin{aligned}
& \dot{V}(E(t), t) \\
& \leq -\frac{\varepsilon}{2} (U(e(t)))^{\frac{d_U + \alpha}{d_U}} \\
& \quad - \frac{\varepsilon}{2} \exp(\delta(t_{k+1} - t))^{\frac{d_U + \alpha}{d_U}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} + K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}} \\
& \leq -\frac{\varepsilon}{2} (U(e(t)))^{\frac{d_U + \alpha}{d_U}} \\
& \quad - \frac{\varepsilon}{2} \left( \exp(\delta(t_{k+1} - t)) |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \right)^{\frac{d_U + \alpha}{d_U}} + K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}} \\
& \leq -\frac{\varepsilon}{2} \left( U(e(t)) + \exp(\delta(t_{k+1} - t)) |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \right)^{\frac{d_U + \alpha}{d_U}} \\
& \quad + K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}} \\
& \leq -\nu (V(E(t), t))^{\frac{d_U + \alpha}{d_U}} + \Gamma \tag{23}
\end{aligned}$$

by applying Lemma 1-(iii) with  $p = \frac{d_U + \alpha}{d_U}$  and where  $\Gamma = K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}}$  and  $\nu = \frac{\varepsilon}{2}$ .

**Step 3.** From the results of Steps 1 and 2, it is clear that  $\bar{V}(E(t), t)$  is decreasing outside of the interval  $\left[0, \left(\frac{\Gamma}{\nu}\right)^{\frac{d_U}{d_U + \alpha}}\right]$ , then either  $V(E(t), t)$  enters the interval  $\left[0, \left(\frac{\Gamma}{\nu}\right)^{\frac{d_U}{d_U + \alpha}}\right]$  and stay in there or it converges to a finite value  $a > \left(\frac{\Gamma}{\nu}\right)^{\frac{d_U}{d_U + \alpha}}$ . It is direct to see that the second option is not possible because of inequality (23) and the results of Step 1.

Noting in addition that  $\left(\frac{\Gamma}{\nu}\right)^{\frac{d_U}{d_U + \alpha}} = K(T_{\max})^{\frac{d_U}{-\alpha}}$  and

$$U(e) + |e_0|^{\frac{d_U}{d_h}} \leq V(E, t), \quad \forall E \in \mathbb{R}^{n+1}, \quad \forall t \geq 0 \tag{24}$$

ends Step 3. ■

#### 4 Example

One considers here the following system

$$\begin{cases}
\dot{x}_1(t) = x_2(t) - 0.1[x_1(t)]^{0.8} \\
\dot{x}_2(t) = x_3(t) + 0.1[x_1(t)]^{0.6} + 0.1[x_2(t)]^{0.75} \\
\dot{x}_3(t) = -0.5[x_1(t)]^{0.4} - 0.5[x_2(t)]^{0.5} \\
\quad - [x_3(t)]^{2/3} + u(t) \\
y(t) = x_1(t)
\end{cases} \tag{25}$$

with  $u(t) = 5 \cos(t) + \sin(2t)$  and where  $[z]^\alpha = \text{sign}(z)|z|^\alpha$  for all  $z \in \mathbb{R}$  and  $\alpha \geq 0$ . The system is initialized as  $x(0) = [-10, -5, 2]^T$ . The sampling times belong to the interval  $[T_{\min}, T_{\max}]$  which will be defined

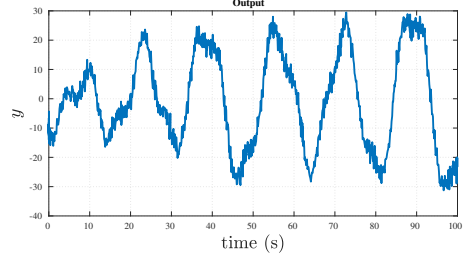


Fig. 1. Measured output

later depending on the simulations. The output is assumed to be measured with a Gaussian white noise with a standard deviation equal to 2, it is illustrated on Figure 1. It can be seen that system (25) is homogeneous of degree  $-0.2$  with respect to the weights  $(1, 0.8, 0.6)$  and an homogeneous observer is given by

$$\begin{cases}
\dot{\hat{x}}_1(t) = \hat{x}_2(t) - 0.1[\hat{x}_1(t)]^{0.8} - k_1[x_1(t) - \hat{x}_1(t)]^{0.8} \\
\dot{\hat{x}}_2(t) = \hat{x}_3(t) + 0.1[\hat{x}_1(t)]^{0.6} + 0.1[\hat{x}_2(t)]^{0.75} \\
\quad - k_2[x_1(t) - \hat{x}_1(t)]^{0.6} \\
\dot{\hat{x}}_3(t) = -0.5[\hat{x}_1(t)]^{0.4} - 0.5[\hat{x}_2(t)]^{0.5} \\
\quad - [\hat{x}_3(t)]^{2/3} + u(t) - k_3[x_1(t) - \hat{x}_1(t)]^{0.4}
\end{cases} \tag{26}$$

with the gains chosen as  $k_1 = 3$ ,  $k_2 = 3$  and  $k_3 = 1$ . The adapted continuous-discrete time observer as proposed in this paper is then given by

$$\begin{cases}
\dot{\hat{z}}(t) = -k_1[\hat{z}(t)]^{0.8} \\
\dot{\hat{x}}_1(t) = \hat{x}_2(t) - 0.1[\hat{x}_1(t)]^{0.8} - k_1[\hat{z}(t)]^{0.8} \\
\dot{\hat{x}}_2(t) = \hat{x}_3(t) + 0.1[\hat{x}_1(t)]^{0.6} + 0.1[\hat{x}_2(t)]^{0.75} \\
\quad - k_2[\hat{z}(t)]^{0.6} \\
\dot{\hat{x}}_3(t) = -0.5[\hat{x}_1(t)]^{0.4} - 0.5[\hat{x}_2(t)]^{0.5} \\
\quad - [\hat{x}_3(t)]^{2/3} + u(t) - k_3[\hat{z}(t)]^{0.4} \\
\hat{z}(t_k) = x_1(t_k) - \hat{x}_1(t_k)
\end{cases} \tag{27}$$

The stability of the continuous observer and a homogeneous Lyapunov function can be obtained by following the approaches developed in [20] or [44] but are rather involved and then not reported here.

Assumption 2-(i) is verified, indeed one has

$$\frac{\partial h}{\partial x}(x)f(x, u) = x_2 - 0.1[x_1]^{0.8} \tag{28}$$

then

$$\begin{aligned}
& \frac{\partial h}{\partial x}(x^1)f(x^1, u) - \frac{\partial h}{\partial x}(x^2)f(x^2, u) \\
& \leq |x_2^1 - x_2^2| + 0.1|[x_1^1]^{0.8} - [x_1^2]^{0.8}| \\
& \leq |x_2^1 - x_2^2| + 0.1 \times 2^{-0.2}|x_1^1 - x_1^2|^{0.8}
\end{aligned}$$

by applying Lemma 1-(i) and the assumption holds since  $d_h = 1$ ,  $\alpha = -0.2$ ,  $\alpha_1 = 1$  and  $\alpha_2 = 0.8$ .

Assumption 2-(ii) is also verified, indeed one has

$$\begin{aligned} |G_i(z_1) - G_i(z_2)| &= k_i | \lceil z_1 \rceil^{i\alpha+1} - \lceil z_2 \rceil^{i\alpha+1} | \\ &\leq k_i 2^{-i\alpha} |z_1 - z_2|^{i\alpha+1} \end{aligned}$$

by applying Lemma 1-(i) since  $(i\alpha + 1) \in (0, 1)$  for  $i = 1, 2, 3$ .

For the considered example, there is a convenient way to explain the ultimate boundedness of the proposed observer. Indeed, the predictor  $\hat{z}$  for the output error is finite-time stable and so it becomes zero after a finite time and the observer dynamics is then a copy of the system dynamics. This explains why the observer does not become unstable with a large  $T_{\max}$  contrarily to most existing approaches.

The observer proposed in this paper is compared with the ones proposed in [39, Proposition 15] and [45] with the same gains  $k_1, k_2, k_3$ . All the observers are initialized as  $\hat{x}_1(0) = \hat{x}_2(0) = \hat{x}_3(0) = 0$ . The first simulation is done with relatively small sampling times, that is  $T_{\min} = 0.05\text{s}$  and  $T_{\max} = 0.1\text{s}$ . As it can be seen on Figure 2 the three observers perform well. The quadratic errors are respectively equal to 0.77 for the proposed observer, 0.88 for the observer from [39] and 0.93 for the observer from [45].

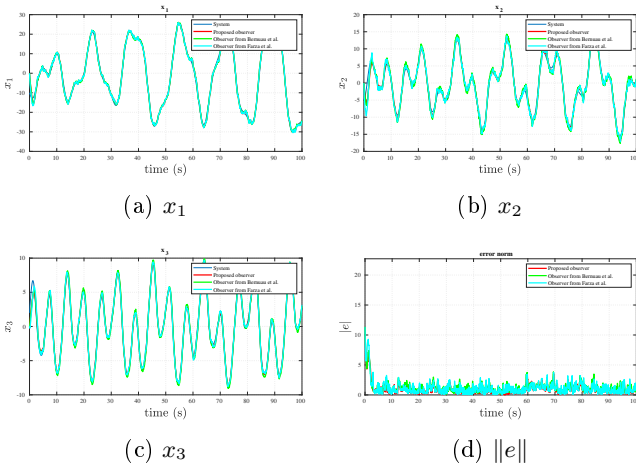


Fig. 2. States and estimations for  $T_{\min} = 0.05\text{s}$  and  $T_{\max} = 0.1\text{s}$

The second simulation is done with  $T_{\min} = T_{\max} = 1\text{s}$ . In that case, it can be seen on Figure 3 that the proposed observer and the one from [45] perform well while the one from [39] is not very accurate. The quadratic errors are respectively equal to 2.28 for the proposed observer, 661.25 for the observer from [39] and 2.88 for the observer from [45].

The last simulation is done with  $T_{\min} = T_{\max} = 2.65\text{s}$  which corresponds to the maximum allowable sampling time for which the observer from [45] is stable. It can be

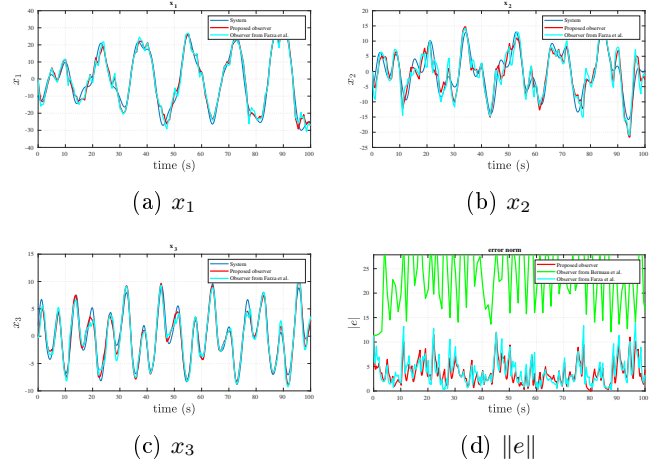


Fig. 3. States and estimations for  $T_{\min} = T_{\max} = 1\text{s}$

seen that the proposed observer is the only one that performs well. The quadratic errors are respectively equal to 128.9 for the proposed observer,  $2.5 \times 10^5$  for the observer from [39] and  $3.9 \times 10^3$  for the observer from [45].

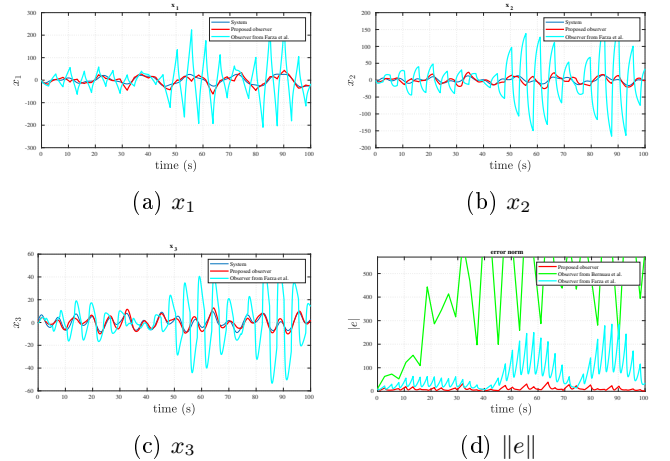


Fig. 4. States and estimations for  $T_{\min} = T_{\max} = 2.65\text{s}$ .

## 5 Conclusion

In this paper, we have proposed a new observer for homogeneous systems of negative degree whose output is available only at some discrete instants. It has been proved that the proposed observer error is globally uniformly ultimately bounded for any maximum sampling time. The proposed observer has been compared with a classical continuous-discrete time observer which has a maximum allowable sampling period and it has been shown in simulation that the proposed observer still performs well while the classical continuous-discrete time observer is at its limit. While the proposed observer performs well for small sampling times, it is only ultimately

bounded, so future works will be concerned with the extension of the proposed observer so that it is exponentially stable for small maximum sampling periods and still practically stable for larger sampling periods.

## A Technical lemmas

### Lemma 1 [46,20]

(i) Let  $p \in (0,1)$  and  $x, y \in \mathbb{R}$ , then the following inequality holds

$$||x|^p - |y|^p| \leq 2^{1-p}|x - y|^p \quad (\text{A.1})$$

(ii) Let  $c, d, \gamma \geq 0$ , then for all  $x, y \in \mathbb{R}$ , one has

$$|x|^c |y|^d \leq \gamma |x|^{c+d} + \frac{d}{c+d} \left( \frac{c}{\gamma(c+d)} \right)^{\frac{c}{d}} |y|^{c+d} \quad (\text{A.2})$$

(iii) Let  $p \in (0,1)$  and  $x, y \in \mathbb{R}$ , then the following inequality holds

$$(|x| + |y|)^p \leq |x|^p + |y|^p \quad (\text{A.3})$$

**Lemma 2** Let  $\alpha_1, \alpha_2 > 0$  be such that  $\alpha_1 > \alpha_2$  and let  $\gamma > 0$  be any constant parameter, then for all  $x \geq 0$  the following inequality holds

$$x^{\alpha_1} + \Gamma \geq \gamma x^{\alpha_2} \quad (\text{A.4})$$

with  $\Gamma = \gamma(x_*)^{\alpha_2} - (x_*)^{\alpha_1} \geq 0$  and  $x_* = \left( \frac{\alpha_2 \gamma}{\alpha_1} \right)^{\frac{1}{\alpha_1 - \alpha_2}}$ . Furthermore, the inequality is optimal in the sense that the left hand-side and right hand-side of Inequality A.4 are equal for  $x = x_*$ .

**Proof.** One can directly show that the function  $f(x) = \gamma x^{\alpha_2} - x^{\alpha_1}$  has only one maximum point which is global. This maximum is reached at  $x_*$  and is equal to  $\Gamma$ . ■

## B Proof of Fact 1

Using Assumption 2-(ii), Inequality (5) and Lemma 1-(ii) with  $c = d_U - \alpha_j$  and  $d = \alpha_j + \alpha$ ,  $x = |e_l|^{\frac{1}{\alpha_l}}$  and

$y = |e_0|^{\frac{1}{d_h}}$  gives

$$\sum_{j=1}^n \frac{\partial U}{\partial e_j}(e) (G_j(\hat{z}) - G_j(\hat{y} - y)) \quad (\text{B.1})$$

$$\leq \sum_{j=1}^n \kappa_j \left( \sum_{l=1}^n |e_l|^{\frac{d_U - \alpha_j}{\alpha_l}} \right) |G_j(\hat{z}) - G_j(\hat{y} - y)| \quad (\text{B.2})$$

$$\leq \sum_{j=1}^n \kappa_j \left( \sum_{l=1}^n |e_l|^{\frac{d_U - \alpha_j}{\alpha_l}} \right) \nu_2^j |e_0|^{\frac{\alpha_j + \alpha}{d_h}} \quad (\text{B.3})$$

$$\leq \sum_{j=1}^n \sum_{l=1}^n \kappa_j \nu_2^j \left( |e_l|^{\frac{1}{\alpha_l}} \right)^{d_U - \alpha_j} \left( |e_0|^{\frac{1}{d_h}} \right)^{\alpha_j + \alpha} \quad (\text{B.4})$$

$$\leq \sum_{j=1}^n \sum_{l=1}^n \kappa_j \nu_2^j \quad (\text{B.5})$$

$$\times \left[ \bar{\gamma}_1 \left( |e_l|^{\frac{1}{\alpha_l}} \right)^{d_U + \alpha} + \rho_1^j(\bar{\gamma}_1) \left( |e_0|^{\frac{1}{d_h}} \right)^{d_U + \alpha} \right] \leq \bar{\gamma}_1 \left( \sum_{j=1}^n \kappa_j \nu_2^j \right) |e_l|^{\frac{d_U + \alpha}{\alpha_l}} \quad (\text{B.6})$$

$$+ \left( \sum_{j=1}^n n \kappa_j \nu_2^j \rho_1^j(\bar{\gamma}_1) \right) |e_0|^{\frac{d_U + \alpha}{d_h}}$$

where  $\bar{\gamma}_1$  can be chosen arbitrarily small, and  $\rho_1^j(\bar{\gamma}_1) = \frac{\alpha_j + \alpha}{d_U + \alpha} \left( \frac{d_U - \alpha_j}{\bar{\gamma}_1 (d_U + \alpha)} \right)^{\frac{d_U - \alpha_j}{\alpha_j + \alpha}}$ . Using (6), one obtains

$$\sum_{j=1}^n \frac{\partial U}{\partial e_j}(e) (G_j(\hat{z}) - G_j(\hat{y} - y)) \quad (\text{B.7})$$

$$\leq \bar{\gamma}_1 \left( \sum_{j=1}^n \kappa_j \nu_2^j \right) \varrho(U(e))^{\frac{d_U + \alpha}{d_U}}$$

$$+ \left( \sum_{j=1}^n n \kappa_j \nu_2^j \rho_1^j(\bar{\gamma}_1) \right) |e_0|^{\frac{d_U + \alpha}{d_h}}$$

Fact 1 is then proved by taking  $\gamma_1 = \bar{\gamma}_1 \left( \sum_{j=1}^n \kappa_j \nu_2^j \right) \varrho$  and  $\Gamma_1 = \left( \sum_{j=1}^n n \kappa_j \nu_2^j \rho_1^j(\bar{\gamma}_1) \right)$ .



## C Proof of Fact 2

Similarly to the proof of Fact 1, using Assumption 2-(i) and Lemma 1-(ii), one gets

$$\begin{aligned} & \frac{d_U}{d_h} \exp(\delta(t_{k+1} - t)) |e_0|^{\frac{d_U - d_h}{d_h}} \\ & \times \left( \frac{\partial h}{\partial x}(x(t))f(x(t), u(t)) - \frac{\partial h}{\partial x}(\hat{x}(t))f(\hat{x}(t), u(t)) \right) \\ & \leq \frac{d_U}{d_h} \left( \exp(\delta(t_{k+1} - t))^{\frac{1}{d_U - d_h}} |e_0|^{\frac{1}{d_h}} \right)^{d_U - d_h} \end{aligned} \quad (C.1)$$

$$\begin{aligned} & \times \left( \nu_1 \sum_{l=1}^n |e_l|^{\frac{d_h + \alpha}{\alpha_l}} \right) \\ & \leq \frac{d_U}{d_h} \nu_1 \sum_{l=1}^n \left( |e_l|^{\frac{1}{\alpha_l}} \right)^{d_h + \alpha} \end{aligned} \quad (C.2)$$

$$\begin{aligned} & \times \left( \exp(\delta(t_{k+1} - t))^{\frac{1}{d_U - d_h}} |e_0|^{\frac{1}{d_h}} \right)^{d_U - d_h} \\ & \leq \frac{d_U}{d_h} \nu_1 \sum_{l=1}^n \left[ \bar{\gamma}_2 \left( |e_l|^{\frac{1}{\alpha_l}} \right)^{d_U + \alpha} \right. \\ & \left. + \rho_2^l(\bar{\gamma}_2) \left( \exp(\delta(t_{k+1} - t))^{\frac{1}{d_U - d_h}} |e_0|^{\frac{1}{d_h}} \right)^{d_U + \alpha} \right] \end{aligned} \quad (C.3)$$

$$\begin{aligned} & \leq \bar{\gamma}_2 \frac{d_U}{d_h} \nu_1 \sum_{l=1}^n |e_l|^{\frac{d_U + \alpha}{\alpha_l}} \\ & + \left( \frac{d_U}{d_h} \nu_1 \sum_{l=1}^n \rho_2^l(\bar{\gamma}_2) \right) \exp(\delta(t_{k+1} - t))^{\frac{d_U + \alpha}{d_U - d_h}} |e_0|^{\frac{d_U + \alpha}{d_h}} \end{aligned} \quad (C.4)$$

$$\begin{aligned} & \leq \bar{\gamma}_2 \frac{d_U}{d_h} \nu_1 \varrho(U(e))^{\frac{d_U + \alpha}{d_U}} \\ & + \left( \frac{d_U}{d_h} \nu_1 \sum_{l=1}^n \rho_2^l(\bar{\gamma}_2) \right) \exp(\delta(t_{k+1} - t))^{\frac{d_U + \alpha}{d_U - d_h}} |e_0|^{\frac{d_U + \alpha}{d_h}} \end{aligned} \quad (C.5)$$

where  $\bar{\gamma}_2 \geq 0$  can be chosen as small as desired and  $\rho_2(\bar{\gamma}_2) = \frac{d_U - d_h}{d_U + \alpha} \left( \frac{d_h + \alpha}{\bar{\gamma}_2(d_U + \alpha)} \right)^{\frac{d_h + \alpha}{d_U - d_h}}$ . Fact 2 is then proved by taking  $\gamma_2 = \bar{\gamma}_2 \frac{d_U}{d_h} \nu_1 \varrho$  and  $\Gamma_2 = \left( \frac{d_U}{d_h} \nu_1 \sum_{l=1}^n \rho_2^l(\bar{\gamma}_2) \right)$ .

## D Proof of Fact 3

One can notice that Equation (22) is valid if the following inequality is valid

$$\begin{aligned} & -\delta |e_0(t)|^{\frac{d_U}{d_h}} + (\Gamma_1 + \Gamma_2) (\exp(\delta T_{\max}))^{\frac{d_U + \alpha}{d_U - d_h}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \\ & \leq -\frac{\varepsilon}{2} (\exp(\delta T_{\max}))^{\frac{d_U + \alpha}{d_U}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} + K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}} \end{aligned} \quad (D.1)$$

since  $1 \leq \exp(\delta(t_{k+1} - t)) \leq \exp(\delta T_{\max})$  for all  $t \in [t_k, t_{k+1})$ . This is equivalent to

$$\begin{aligned} & -\delta |e_0(t)|^{\frac{d_U}{d_h}} + (\Gamma_1 + \Gamma_2) e^{\frac{d_U + \alpha}{d_U - d_h}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \\ & \leq -\frac{\varepsilon}{2} e^{\frac{d_U + \alpha}{d_U}} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} + K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}} \end{aligned} \quad (D.2)$$

$$\begin{aligned} & \Leftrightarrow \underbrace{\left( (\Gamma_1 + \Gamma_2) e^{\frac{d_U + \alpha}{d_U - d_h}} + \frac{\varepsilon}{2} e^{\frac{d_U + \alpha}{d_U}} \right)}_{\triangleq K_2} |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \\ & \leq \delta |e_0(t)|^{\frac{d_U}{d_h}} + K_1 T_{\max}^{\frac{d_U + \alpha}{-\alpha}} \end{aligned} \quad (D.3)$$

since  $\delta = 1/T_{\max}$ . In order to prove Inequality (D.3) and find  $K_1$ , let us apply Lemma 2 with  $x = |e_0(t)|$ ,  $\alpha_1 = \frac{d_U}{d_h}$ ,  $\alpha_2 = \frac{d_U + \alpha}{d_h}$  and  $\gamma_3 = T_{\max} K_2$ , which gives us

$$x^{\alpha_1} + \Gamma_3 \geq \gamma_3 x^{\alpha_2} \quad (D.4)$$

$$\Leftrightarrow |e_0(t)|^{\frac{d_U}{d_h}} + \Gamma_3 \geq T_{\max} K_2 |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \quad (D.5)$$

$$\Leftrightarrow \frac{1}{T_{\max}} |e_0(t)|^{\frac{d_U}{d_h}} + \frac{\Gamma_3}{T_{\max}} \geq K_2 |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \quad (D.6)$$

$$\Leftrightarrow \delta |e_0(t)|^{\frac{d_U}{d_h}} + \frac{\Gamma_3}{T_{\max}} \geq K_2 |e_0(t)|^{\frac{d_U + \alpha}{d_h}} \quad (D.7)$$

with  $\Gamma_3 = \gamma_3^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \left( \left( \frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_2}{\alpha_1 - \alpha_2}} - \left( \frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \right)$ , that is

$$\begin{aligned} \frac{\Gamma_3}{T_{\max}} &= T_{\max}^{\frac{\alpha_1}{\alpha_1 - \alpha_2} - 1} K_2^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \left( \left( \frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_2}{\alpha_1 - \alpha_2}} - \left( \frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \right) \\ &= \underbrace{T_{\max}^{\frac{d_U + \alpha}{-\alpha}} K_2^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \left( \left( \frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_2}{\alpha_1 - \alpha_2}} - \left( \frac{\alpha_2}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1 - \alpha_2}} \right)}_{\triangleq K_1} \end{aligned} \quad (D.8)$$

Then Inequality (22) is valid.

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