



# Properties of the transmission line matrix model for outdoor sound propagation: Numerical dispersion effects

Quentin Goestchel<sup>1</sup>, Gwenaël Guillaume<sup>1</sup>, David Ecotièrè<sup>1</sup>, Benoit Gauvreau<sup>2</sup>

24<sup>th</sup> International Congress on Acoustics  
October 10, 2022

<sup>1</sup>UMRAE, CEREMA, Univ Gustave Eiffel, F-67035 Strasbourg, France

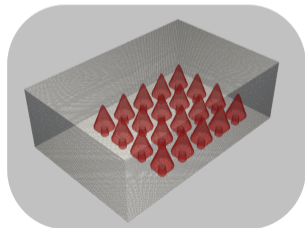
<sup>2</sup>UMRAE, Univ Gustave Eiffel, CEREMA, F-44344 Bouguenais, France

- Introduction & context
- *Transmission Line Matrix* (TLM) numerical model
- Numerical experiments
- Results
- Conclusion & perspectives

- Acoustic propagation in forest environments

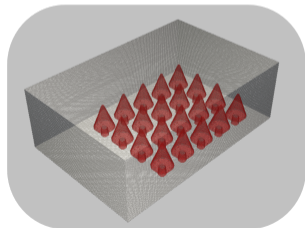


- Acoustic propagation in forest environments
  - Multi-scale
  - Multiple scatterers
  - Impedance boundary conditions

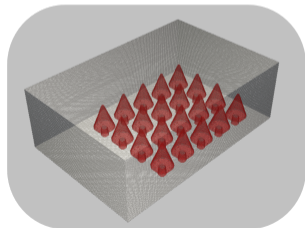




- Acoustic propagation in forest environments
  - Multi-scale
  - Multiple scatterers
  - Impedance boundary conditions
- Various sources



- Acoustic propagation in forest environments
  - Multi-scale
  - Multiple scatterers
  - Impedance boundary conditions
- Various sources
- European directive [2002/49/EC]: protection of quiet areas



## History

### *Transmission Line Matrix model (TLM)*

Introduced to model high frequency magnetic fields  
[Johns and Beurle, 1971].

- Time-domain method

## History

### *Transmission Line Matrix model (TLM)*

Introduced to model high frequency magnetic fields  
[Johns and Beurle, 1971].

- Time-domain method
- Validated and/or studied for

## History

### *Transmission Line Matrix model (TLM)*

Introduced to model high frequency magnetic fields  
[Johns and Beurle, 1971].

- Time-domain method
- Validated and/or studied for
  - Room acoustics [Guillaume, 2009]
  - Propagation in urban areas [Guillaume et al., 2008]
  - Propagation through parallel cylinders above a horizontal plane [ChobEAU et al., 2017]

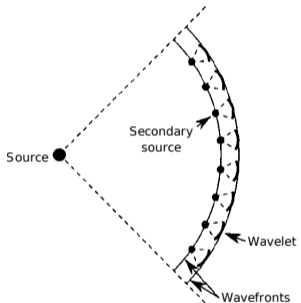
## History

### *Transmission Line Matrix model (TLM)*

Introduced to model high frequency magnetic fields  
[Johns and Beurle, 1971].

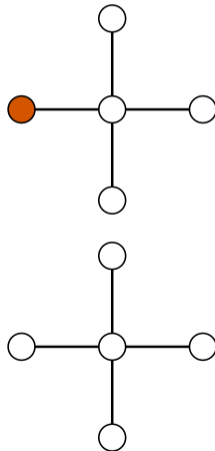
- Time-domain method
- Validated and/or studied for
  - Room acoustics [Guillaume, 2009]
  - Propagation in urban areas [Guillaume et al., 2008]
  - Propagation through parallel cylinders above a horizontal plane [ChobEAU et al., 2017]
- Python/OpenCL parallelized on GPUs [Guillaume and Fortin, 2014]

## Basis

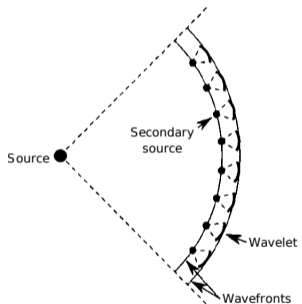


Huygens principle [Guillaume, 2009].

(+) detailed : [Goestchel et al., 2022]

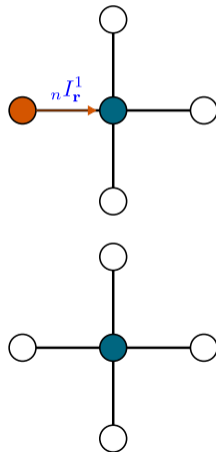


## Basis



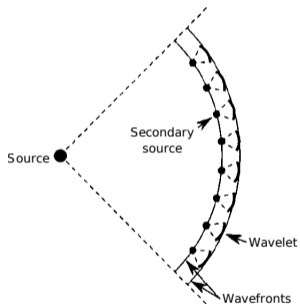
Huygens principle [Guillaume, 2009].

(+) detailed : [Goestchel et al., 2022]



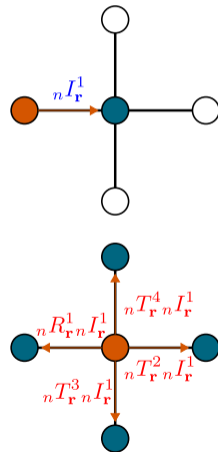


## Basis



Huygens principle [Guillaume, 2009].

(+) detailed : [Goestchel et al., 2022]



## Stability study

## Homogeneous case

$${}_{n+1}P_r + {}_{n-1}P_r = \frac{1}{d} \sum_{m=1}^d \left[ {}_n P_{(j_1+\delta_{m1}, \dots, j_d+\delta_{md})} + {}_n P_{(j_1-\delta_{m1}, \dots, j_d-\delta_{md})} \right], \quad (1)$$

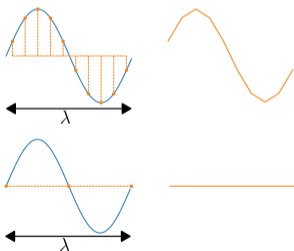
$$\frac{\partial^2 p}{\partial t^2} - c_{\text{TLM}}^2 \nabla^2 p = \mathcal{O}(\Delta t^2) + \mathcal{O}\left(\frac{\Delta \ell^4}{\Delta t^2}\right), \quad \text{with } c_{\text{TLM}} = \frac{\Delta \ell}{\sqrt{d} \Delta t}. \quad (2)$$

## Axial dispersion

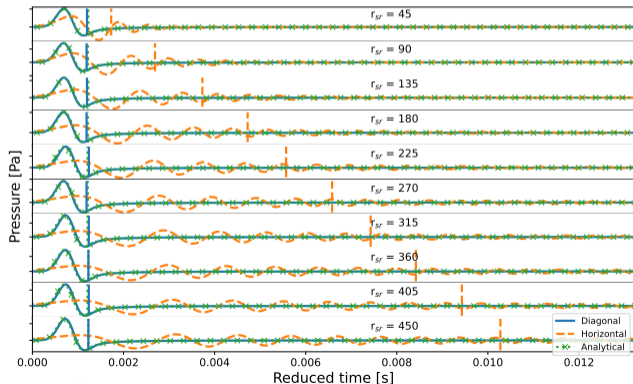
$$\cos(\omega \Delta t) = \frac{1}{d} \sum_{m=1}^d \cos(k_{x_m} \Delta \ell) \quad \forall \Delta t, \quad \forall \Delta \ell. \quad (3)$$

## Dispersion effect on results

Number of points per wavelength ( $N_{ppw}$ )

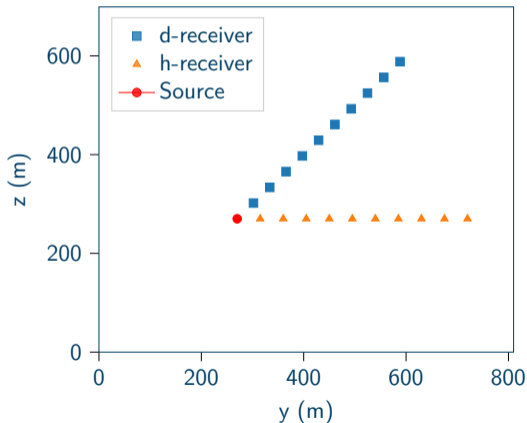


## Cartesian mesh numerical dispersion



## Free field setup

Calculation domain

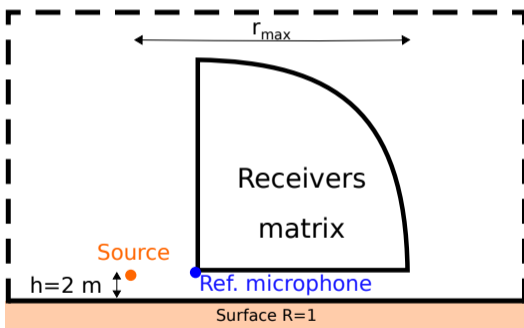


Configuration for  $f_{\max} = 2400$  Hz,  
 $\Delta\ell = \lambda/10$ ,  $r_{\max} = 450$  m:

$\Delta\ell$ [m]	$\Delta t$ [s]	$\Delta\ell_{\text{mic}}$ [m]	$N_y \times N_z$ [-]
0.014	2.95E-05	45	56476*48746

Memory [Gb] (float32 numpy array)	$t_{\text{sim}}$ [s]	$r_{\max}/\lambda_{\min}$ [-]
10.22	11143	3176

## Reflective ground setup



## Normalisation &amp; results

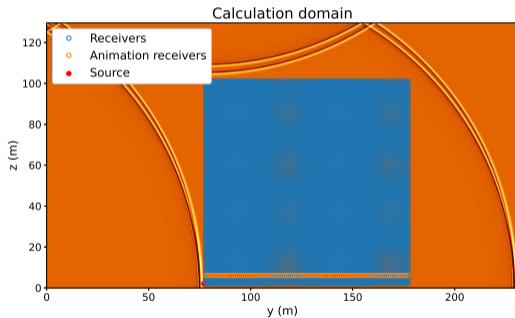
- Normalisation: standard deviation ratio

$$m_{ij}^n = \frac{m_{ij}^n \times \sigma(p_{0j}^n)}{\sigma(m_{0j}^n)}$$

- Attenuation // reference microphone:

$$Att(r) = 10 \times \log \left( \frac{\sum_n p_{ij}^2[n]}{\sum_n p_{0j}^2[n]} \right)$$

## Reflective ground setup



## Normalisation & results

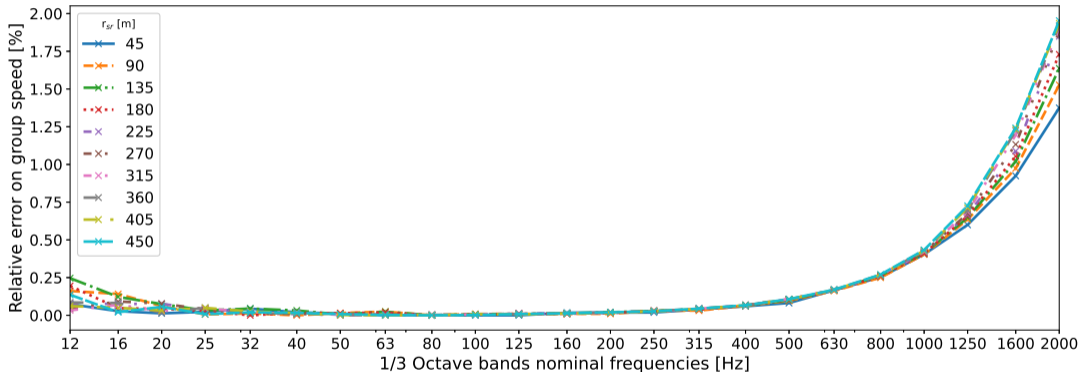
- Normalisation: standard deviation ratio

$$m_{ij}^n = \frac{m_{ij}^n \times \sigma(p_{0j}^n)}{\sigma(m_{0j}^n)}$$

- Attenuation // reference microphone:

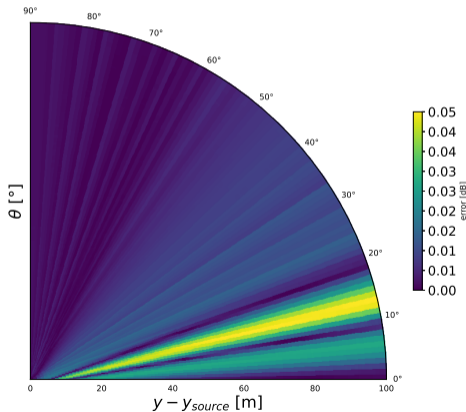
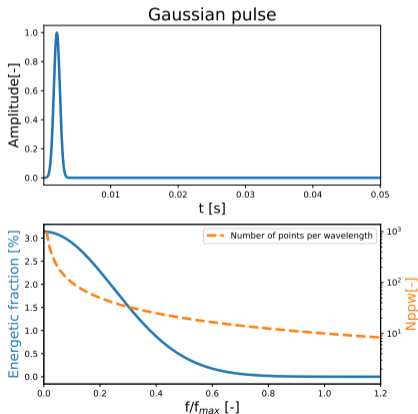
$$Att(r) = 10 \times \log \left( \frac{\sum p_{ij}^2[n]}{n} \right) / \left( \frac{\sum p_{0j}^2[n]}{n} \right)$$

## Free Field - pulse



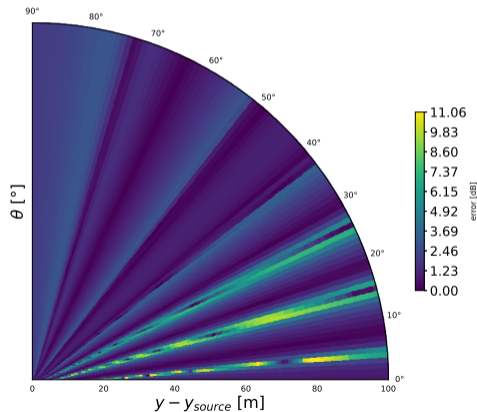
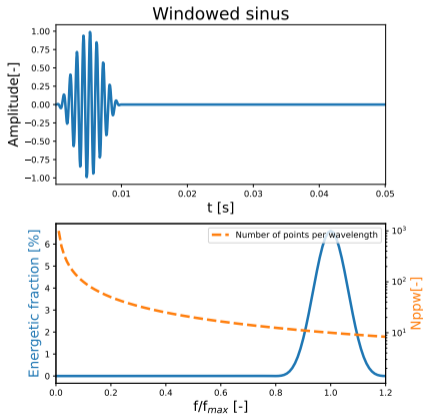
Error on group speed,  $\Delta \ell = \lambda/10$ ,  $f_{\max} = 2400$  Hz.

## Ground reflection - pulse

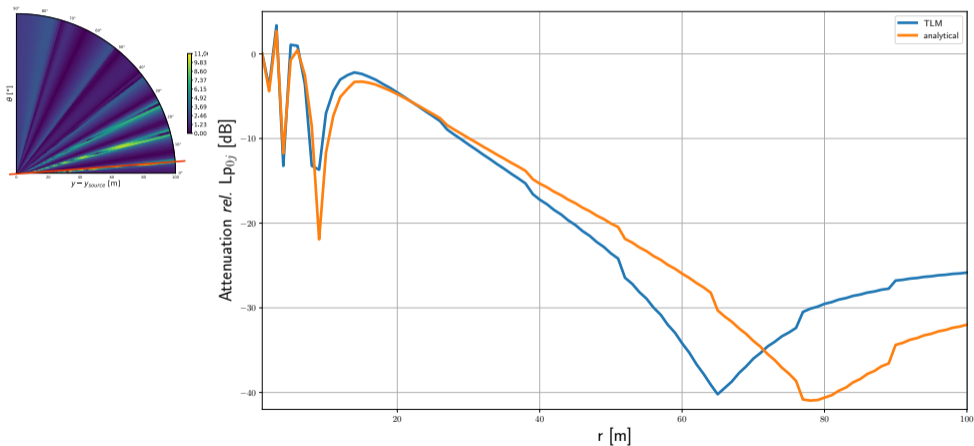




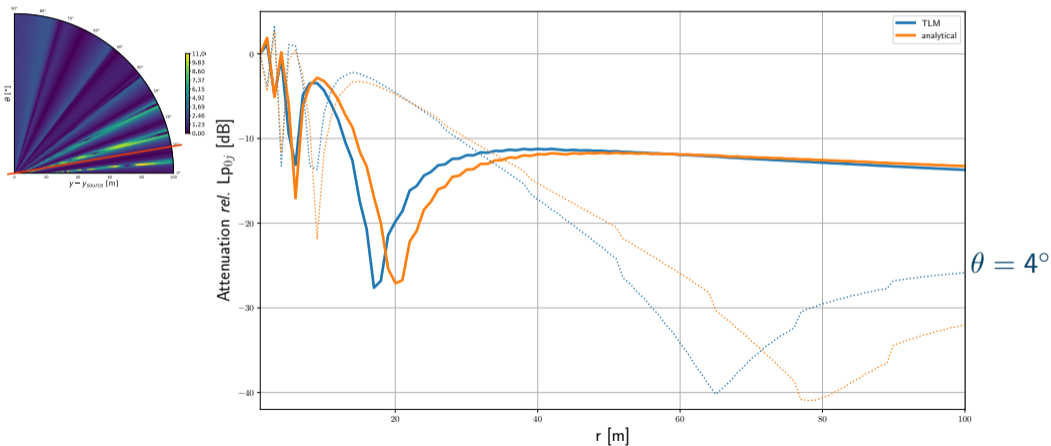
## Ground reflection - windowed sinus



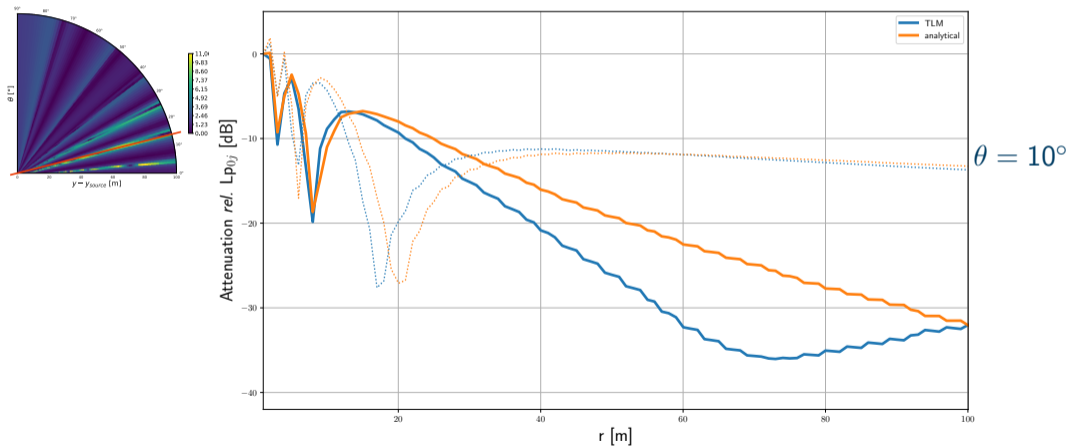
## Ground reflection - windowed sinus, $\theta = 4^\circ$



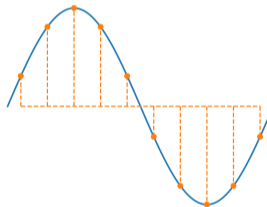
## Ground reflection - windowed sinus, $\theta = 10^\circ$



## Ground reflection - windowed sinus, $\theta = 15^\circ$



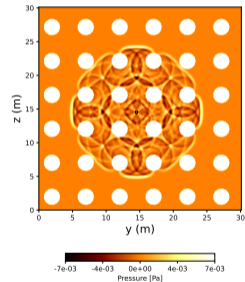
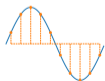
- 10 points per wavelength criterion not sufficient



- 10 points per wavelength criterion not sufficient

---

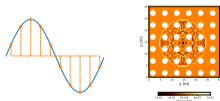
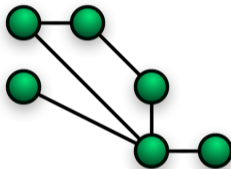
- Compare complex modelling with measurements



- 10 points per wavelength criterion not sufficient

---

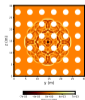
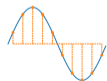
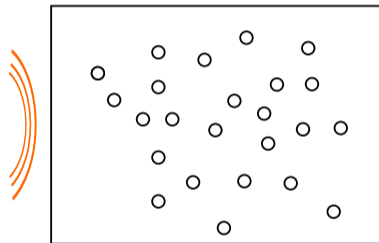
- Compare complex modelling with measurements
- Increase the scheme order or change?



- 10 points per wavelength criterion not sufficient

---

- Compare complex modelling with measurements
- Increase the scheme order or change?
- Waves in complex media theory

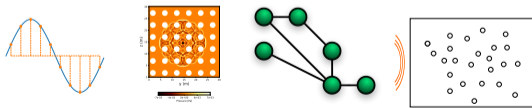
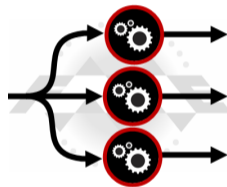










- 10 points per wavelength criterion not sufficient

---

- Compare complex modelling with measurements
- Increase the scheme order or change?
- Waves in complex media theory
- MPI parallelization



- 
- Chobeau, P., Guillaume, G., Picaut, J., Ecotière, D., and Dutilleux, G. (2017).  
A Transmission Line Matrix model for sound propagation in arrays of cylinders normal to an impedance plane.  
*Journal of Sound and Vibration*, 389:454–467.
- 
- Goestchel, Q., Guillaume, G., Ecotière, D., and Gauvreau, B. (2022).  
Analysis of the numerical properties of the transmission line matrix model for outdoor sound propagation.  
*Journal of Sound and Vibration*, 531:116974.
- 
- Guillaume, G. (2009).  
*Application de La Méthode TLM à La Modélisation de La Propagation Acoustique En Milieu Urbain*.  
PhD thesis.
- 
- Guillaume, G. and Fortin, N. (2014).  
Optimized transmission line matrix model implementation for graphics processing units computing in built-up environment.  
*Journal of Building Performance Simulation*, 7(6):445–456.
- 
- Guillaume, G., Picaut, J., and Dutilleux, G. (2008).  
Use of the transmission line matrix method for the sound propagation modeling in urban area.  
*The Journal of the Acoustical Society of America*, 123(5):3924–3924.
- 
- Johns, P. and Beurle, R. (1971).  
Numerical solution of 2-dimensional scattering problems using a transmission-line matrix.  
*Proceedings of the Institution of Electrical Engineers*, 118(9):1203–1203.

- Contacts :
  - [quentin.goestchel@cerema.fr](mailto:quentin.goestchel@cerema.fr)
  - [gwenael.guillaume@cerema.fr](mailto:gwenael.guillaume@cerema.fr)
  - [david.ecotiere@cerema.fr](mailto:david.ecotiere@cerema.fr)
  - [benoit.gauvreau@univ-eiffel.fr](mailto:benoit.gauvreau@univ-eiffel.fr)
- links :
  - <http://www.umrae.fr/>



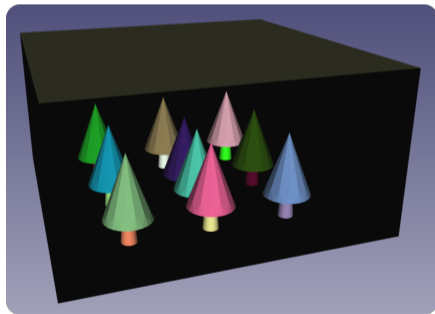
*Unité Mixte de Recherche en Acoustique  
Environnementale (UMRAE) is a joint research unit in  
environmental acoustics between Gustave Eiffel University  
& Cerema.*



# Appendix

## Meshing & implementation

- FreeCAD API automatic scene generation
  - (Supervision of an IT student's internship)



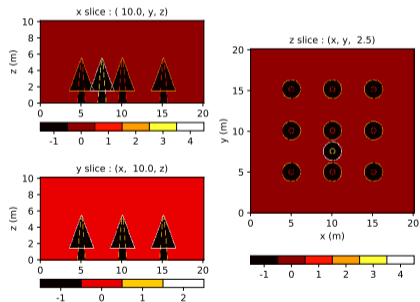
## Meshing & implementation

- FreeCAD API automatic scene generation

- (Supervision of an IT student's internship)

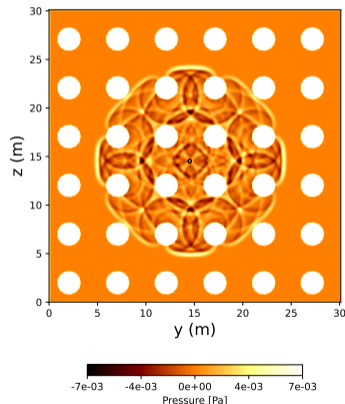
- Fast voxelization process

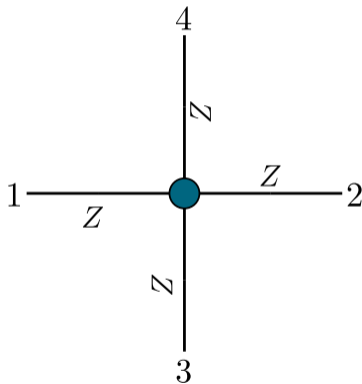
<https://github.com/nicolas-f/FastVoxel>



## Meshing & implementation

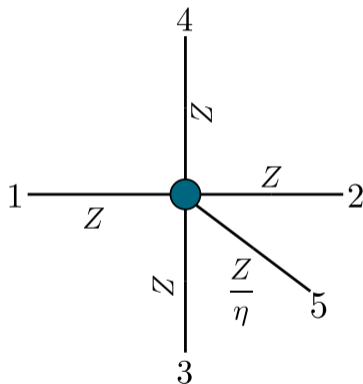
- FreeCAD API automatic scene generation
  - (Supervision of an IT student's internship)
- Fast voxelization process  
<https://github.com/nicolas-f/FastVoxel>
- TLM solver



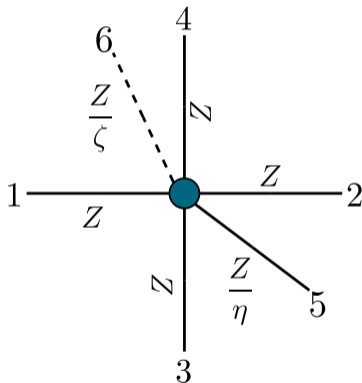


2D stencil around a node

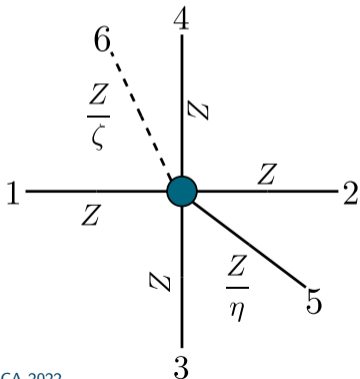




2D stencil around a node



2D stencil around a node



## Link to wave equation

$$\frac{1}{c_{\text{TLM}}^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p + \frac{\zeta_{(j,l)}}{2\Delta\ell^2} \left( 2\Delta t \frac{\partial p}{\partial t} \right) = \mathcal{O} \left( \frac{\Delta t^4}{\Delta\ell^2} \right) + \mathcal{O} \left( \frac{\Delta t^3}{\Delta\ell^2} \right) + \mathcal{O} \left( \Delta\ell^2 \right) \quad (4)$$

## Stability

$$c_{\text{TLM}}(\mathbf{x}_r) = \sqrt{\frac{2}{\eta_r + 2d} \frac{\Delta \ell}{\Delta t}} \quad (5)$$
$$\eta \geq 0$$

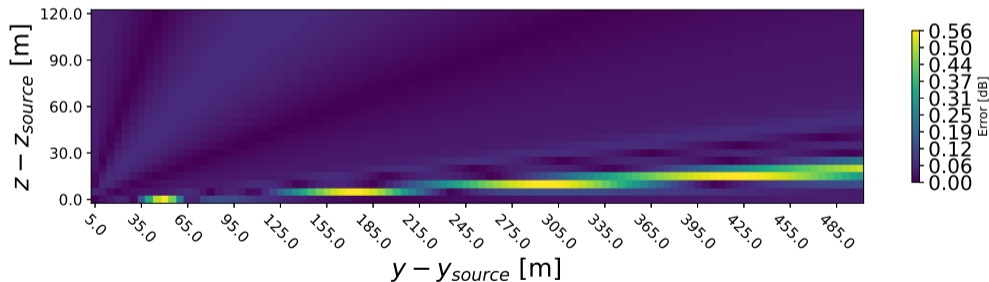
## Dispersion

$$\|\underline{k}\| \approx k_{0\text{TLM}} + i\alpha_{\text{TLM}} + \mathcal{O}(\zeta^2) \quad (6)$$

$$k_{0\text{TLM}} = \frac{\omega}{c_{\text{TLM}}} \text{ and}$$

$$\alpha_{\text{TLM}} = \frac{\zeta}{\sqrt{2(\eta + 2d)} \Delta \ell}$$

## Ground reflection : Richer Wavelet

Error map:  $f_{\max} = 2000$  Hz,  $r_{\max} = 500$  m et  $h_{\text{src}} = 2$  m

Ground reflection : windowed sinus

Error map:  $f_{\max} = 2000$  Hz,  $r_{\max} = 500$  m et  $h_{\text{src}} = 2$  m

