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## Properties of the transmission line matrix model for outdoor sound propagation: Numerical dispersion effects

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### ABSTRACT

Time-domain numerical methods are attractive tools for the prediction of outdoor sound propagation as they are able to handle most of the physical phenomena involved in environmental acoustics. The transmission line matrix (TLM) method is considered for modelling sound fields in complex outdoor environments but it remains relatively poorly documented. In this paper, the space-time integration scheme of the method and a stability analysis are provided. The review of the model highlights similarities with a finite difference scheme and leads to a more robust understanding of its link with the wave equation using Taylor expansions. The stability analysis allows the numerical dispersion relation of the model to be formulated and the associated errors to be quantified. Two different numerical experiments are presented to characterize precisely the limitations of the method applied to outdoor sound propagation. To conclude, the results are analyzed, highlighting the effects of the propagation distance and the spectral distribution of the simulated sources.

Keywords: Numerical Methods, Outdoor Sound Propagation, Numerical Dispersion

### 1 INTRODUCTION

Human-induced noise disturbances have a negative impact on public health and biodiversity [1]. Sound pollution is not limited to urban environments and reaches natural areas. However, the impact of noise pollution on living beings and their environment is not well quantified and this lack of knowledge could be overcome by predicting long-range outdoor sound propagation. From frequency domain methods to time domain methods, the study and improvement of numerical acoustic model have recently been developed with the available computing power.

Compared to other time-domain numerical methods, the transmission line matrix method (TLM) has not been documented extensively [2]. Initially introduced to model electromagnetic fields, the TLM model and its formalism appeared in the early 1970s [3]. Subsequently, the flexibility of the method has been demonstrated by applying it to various fields [4].

In acoustics, the TLM model successfully simulates indoor propagation and the theoretical basis of the method has been relatively well studied [5]. For outdoor scenarios, the solver seems to tackle with most of the difficulties usually involved in modelling inhomogeneous media. Application cases for urban areas or even forest-like environments have been developed and compared to the experimental results [6, 7, 8]. From a computational point of view, an OpenCL™ implementation has been optimized to run on parallel graphic processing units [9]. Finally, complex features such as impedance boundary conditions or meteorological effects have been introduced into the TLM model in order to model more realistic environments [10].

However, no dispersion error analysis of the method has been undertaken yet. Therefore, this article aims to explore the potential effects of the numerical dispersion inherent in the TLM and the limitations it may

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imply. The following study focuses on long-range outdoor noise predictions and the simulation of reflective boundary conditions. They are complex situations to model and therefore the most likely to expose limitations of numerical models.

A  $d$ -dimensional formalism of the numerical integration scheme under the TLM is given in the next section, together with a short stability analysis theoretically introducing the dispersion error. The second section of this paper presents the virtual experiments performed to demonstrate the effect of numerical dispersion on the modelled sound pressure fields. Finally, the results are partially presented to give an overview of the limitations of the TLM model when applied to long-range outdoor acoustics.

## 2 NUMERICAL METHOD

### 2.1 Origin

The particularity of the TLM is that it is derived from Huygens' principle for wave propagation and from an electro-acoustic analogy [11]. The property of a wavefront to be recursively discretized in secondary sources is exploited and applied to a regular structured mesh. The sound propagation is then modelled as pressure pulses, propagating along transmission lines. This division of the propagation mechanism is the TLM basis. It allows the analogy between the progression of a sound wave and the diffusion of pulses between the nodes of a mesh.

### 2.2 Formalism in a nutshell

In the following, a syntax corresponding to a  $d$ -dimension spatial generalization of the TLM model is adopted to be consistent with previous works [12]. The theoretical aspects discussed below intentionally avoid the iterative process of the TLM pulses since it is already detailed in the literature [13]. The spatial domain is subdivided into a Cartesian mesh with a step  $\Delta\ell$  such as  $x_d = j_d\Delta\ell$ ,  $j_d \in \mathbb{Z}$  and the time is decomposed into steps such as  $t_n = n\Delta t$ ,  $n \in \mathbb{N}$ . The nodes on the grid are located by the index vector:  $\mathbf{r} = (j_1, \dots, j_d)$ . The core of the model is that each node in the volume receives and emits incident and scattered pulses instantaneously, at each time step  $n$ . These pressure pulses are travelling along transmission lines  $m$  connecting the nodes. From their modelling, it is possible to calculate  ${}_n P_{\mathbf{r}}$ , an approximated value of the exact acoustic pressure  $p(x_1, \dots, x_d, t_n)$  taken at the point  $(x_1, \dots, x_d)$  at time  $t_n$ .

### 2.3 Link with the wave equation

Starting from the pressure pulses equations of the method, the corresponding numerical pressure scheme can be expressed [13]:

$${}_{n+1}P_{\mathbf{r}} + {}_{n-1}P_{\mathbf{r}} = \frac{1}{d} \sum_{m=1}^d [{}_n P_{(j_1+\delta_{m1}, \dots, j_d+\delta_{md})} + {}_n P_{(j_1-\delta_{m1}, \dots, j_d-\delta_{md})}], \quad (1)$$

$\delta$  being used here as the Kronecker delta. To find out the order of approximation of this scheme, and to retrieve the wave equation, Taylor expansions can be used [11] and Eq. (1) becomes:

$$\frac{\partial^2 p}{\partial t^2} - c_{\text{TLM}}^2 \nabla^2 p = \mathcal{O}(\Delta t^2) + \mathcal{O}\left(\frac{\Delta\ell^4}{\Delta t^2}\right), \quad (2)$$

with  $c_{\text{TLM}} = \frac{\Delta\ell}{\sqrt{d}\Delta t}$ . It is now possible to recognize the wave equation and to observe that the TLM model is a second-order approximation method in time, and space. Another interesting observation is that the TLM solves the wave equation only if the condition  $c_{\text{TLM}}^2 = c_0^2$  is fulfilled. Developing this condition gives:

$$\frac{c_0\Delta t}{\Delta\ell} = \frac{1}{\sqrt{d}}, \quad (3)$$

which corresponds with the CFL criterion of the so-called finite difference Leap-Frog scheme [2].

## 2.4 Numerical dispersion

To evaluate the model stability, the numerical dispersion relation of the method can be written:

$$\cos(\omega\Delta t) = \frac{1}{d} \sum_{m=1}^d \cos(k_m\Delta\ell), \quad \forall\Delta t, \forall\Delta\ell. \quad (4)$$

This equation implies that the TLM method is unconditionally stable in the homogeneous non-dissipative case. However, this relation also shows that the model presents numerical dispersion in the main directions of the grid (for instance in the 2D case  $\theta = \beta\frac{\pi}{2}$ ,  $\beta \in \mathbb{Z}$ ). To illustrate this phenomenon, the dispersion error is represented as a function of the angle  $\theta$  between the plane wave vector  $\mathbf{k}$  and the horizontal direction of the mesh. The same reasoning was done for an inhomogeneous environment and an overview of the results is given for a case with linear vertical gradients of temperature ( $\partial T/\partial z = \pm 0,35 \text{ C}^\circ.\text{m}^{-1}$ ) and wind speed ( $\partial w/\partial z = \pm 0,2 \text{ s}^{-1}$ ) [13]. Figure 1 shows that the dispersion error is maximal at the bottom part of the grid in the downward case.

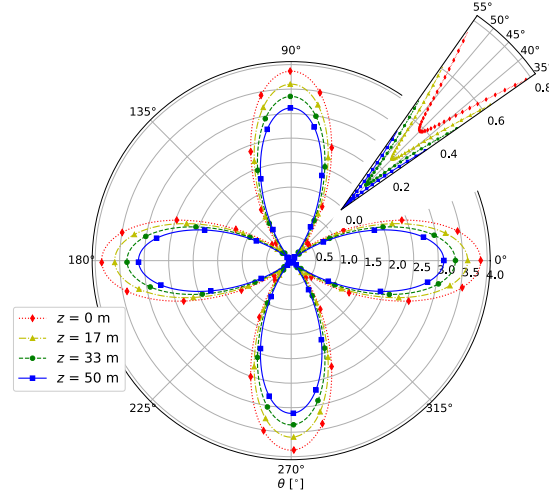


Figure 1. Modeling of downward refraction conditions for  $N = 5$ : relative errors on the phase speed [%] according to  $z$  and  $\theta$ .

The values of the local errors are acceptable for  $N=5$  and negligible for  $N=10$  and they are similar in the upward case. However, it is necessary to study the impact of local errors on a larger scale to analyse the effect of the propagation of dispersion errors. Moreover, it is known that dispersion affects the group speed more than the phase speed [14], so the following section aims to analyze its effects on this quantity through a numerical long-range propagation experiment.

## 3 VIRTUAL EXPERIMENTS

The numerical dispersion error is inhomogeneous in a 2D-Cartesian mesh and decreases from the axial direction to the diagonal direction (Figure 1). To visualise this effect in terms of sound pressure level predictions, the setups illustrated in Figure 2 are simulated. The most important parameter for the analysis is the number of points per wavelength ( $N_{ppw}$ ) which is set at the maximal frequency  $f_{max}$  of the source emission.

### 3.1 Free-field setup

Visible on Figure 2(a), it is composed of a unique sound source surrounded by two arrays of ten receivers located along the horizontal and diagonal ( $\theta = \frac{\pi}{4}$ ) directions of the mesh.

### 3.2 Specular reflection setup

Visible on Figure 2(b), it consists in a unique sound source next to a polar matrix of microphones in a domain with a perfectly reflective ground. It is implemented to emphasize two geometrical parameters: the inclination angle  $\theta$  and the distance  $r$  between a receiver and the source.

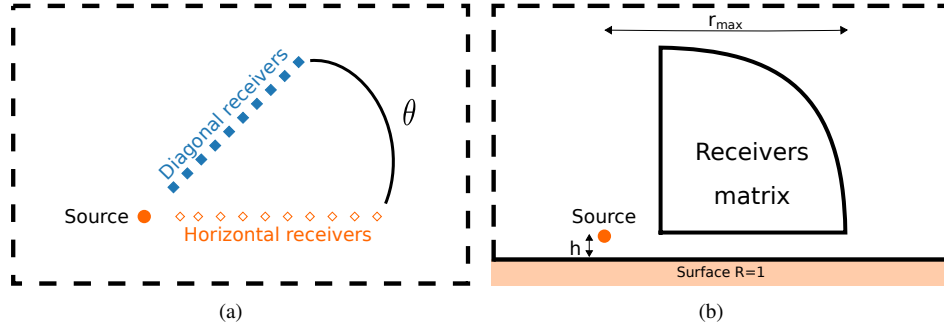


Figure 2. (a) Dispersion analysis setup. (a) Free-field, source in the center and microphones positioned along the horizontal and diagonal ( $\theta = \frac{\pi}{4}$ ) directions. (b) Specular reflection, source placed at distance  $h$  from the ground and microphones placed to form a polar matrix.

## 4 RESULTS

This section presents the analysis of the results for experiment detailed in Section 3 by comparing them to analytical solutions[13]. For both experiments, the criterion  $N_{ppw}$  points per wavelength is respected.

### 4.1 Free-field

For this experiment, a sound source with a Gaussian temporal profile ( $f_{\max} = 2400$  Hz) has been used to approximate a pulse. In Figure 3, the pressure signals at the receivers are displayed according to the reduced time ( $t_n - \frac{r}{c_0}$ ) which allows to observe the delayed high frequencies along the propagation distance. The effect of the dispersion along the horizontal array of microphones is directly noticeable: the results displayed on Figure 3 show an apparent decrease of the group speed while the propagation distance increases. For a better visualisation of this delay, the vertical lines on Figure 3 indicate the moment when 95 % of the signal energy has reached the microphone.

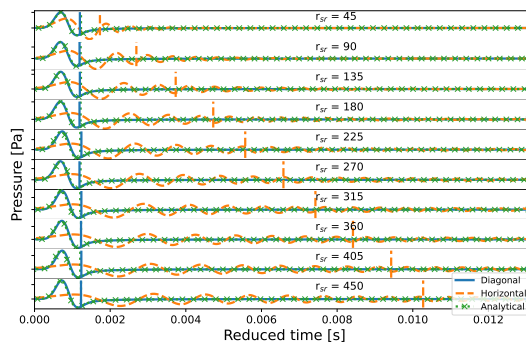


Figure 3. Normalized pressure signals at the microphones as dependent upon reduced time, for  $N=10$  grid points per wavelength. Vertical lines indicate the arrival of 95 % of the signal energy.

## 4.2 Specular reflection

A sound source with a windowed sinus temporal profile is used in this experiment as it is pseudo-harmonic. As shown on Figure 4(a), most of the source energy is focused around the frequency  $f_{\max}$  and has a 10-point spatial discretization. Thus, the incident and reflected wave are more likely to interfere and be affected by numerical dispersion. With this spectral distribution, significant interference-induced errors are generated (up to 11 dB as shown on Figure 4(b)). Indeed, at some microphones, the energy of the received signal is either increased or lost due to artificial interference patterns induced by the TLM model. The spectral distribution of the source has an impact on the spatial distribution of the error: the wider the spectral distribution of the source is, the wider the spatial distribution of the error. This is confirmed when sources with a wider distribution are used: the numerical error decreases significantly.

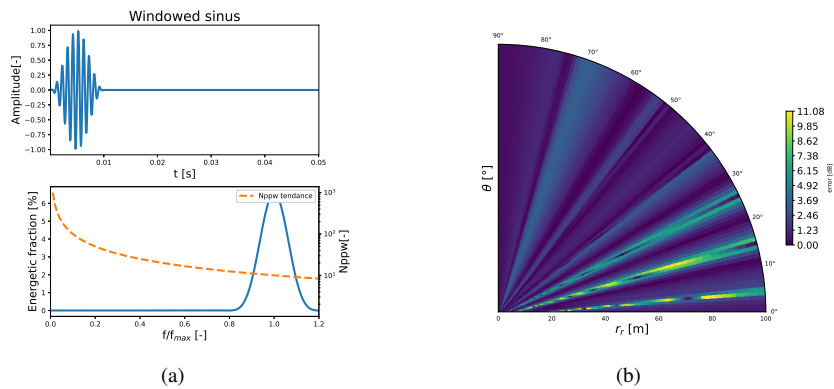


Figure 4. (a) Spectral distribution of a windowed sinus compared to  $N_{ppw}$ . (b) Map of the absolute error  $\epsilon_{j_1, j_2}$  on SPL attenuation relative to a reference microphone for a windowed sinus. Matrix of  $100 \times 100$  microphones.

## 5 CONCLUSION

Theoretical and numerical aspects of using the TLM model as a solver for long-range outdoor sound propagation were discussed. The general formulation of the pressure integration scheme was exposed, highlighting similarities with finite difference methods applied to the wave equation. Thanks to a stability analysis, the inherent anisotropic dispersion of the model has been highlighted. Finally, two numerical experiments were performed to characterize the effects of dispersion on academic cases of long-range outdoor propagation. From the results of these experiments, the observed effects on the group speed of the test signals in the main mesh directions must be taken into account when using the TLM. Indeed, this implies a careful choice of simulation parameters to avoid significant errors in the energy of the resulting signals. The effects of numerical dispersion according to the spectral distribution of the modelled sources are also noticeable with the appearance of misplaced interference patterns, impacting the sound pressure level predictions. In order to investigate these effects more thoroughly, it would be relevant to test the model applied to more complex and realistic scenarios, by comparing it with additional numerical simulations and experimental results from a measurement campaign.

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