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**Abstract**

The critical condition for the deflagration to detonation transition (DDT) on the tip of elongated flames in tubes is revisited. Outstanding experiments and numerics, performed in 2010, have shown that a train of successive shock waves are produced by the self-accelerating flame-front. Just prior to the DDT, a pre-conditioned state is thus generated in a slice of unreacted gas adjacent to the flame front with a temperature too low for self-ignition, leaving unexplained the abrupt transition. Using a simplified one-dimensional model, the objective of the theoretical analysis of this short note is to identify the pre-conditioned state just prior to the spontaneous formation of a flow singularity on the flame front. This finite-time singularity responsible for the DDT is produced by the least further increase in propagation velocity beyond a critical value. The attention is focused on the unsteady compression waves emitted in the unreacted gas by the self-accelerating flame front. The unsteady effects were overlooked by the previous theoretical analyses.

*Keywords:* Deflagration-to-detonation transition. Laminar flow. Finite-time singularity.

## 1. Introduction

Various mechanisms of the deflagration-to-detonation transition (DDT) in gaseous mixtures are mentioned in the modern literature. The attention will be limited here to a tentative explanation of the DDT on the tip of a self-accelerating curved flame propagating in a radius tube larger than the laminar flame thickness but small enough for the flow to be laminar. The real problem is multidimensional. However the DDT mechanism can be enlightened in plane geometry with a flame treated as a discontinuity pushed from behind by a flow of burned gas perpendicular to the

flame front [1] [2]. In the real process, this burned gas flow, called back flow in the following, is generated by the lateral wings of the elongated flame and is thus proportional to the elongation of the curved front. The attention is focused on a small rate of elongation, smaller than the inverse of the transit time across the laminar flame speed. In one centimeter radius tubes, this occurs typically some time after the formation of the tulip-shaped flame. Extending the pioneering analysis of Deshaies-Joulin (1989) [3] (DJ), strong arguments have been recently provided in [1] and [2] indicating that the abrupt transition ($1\mu s$) observed in the outstanding 2010 experiments and numerics [4] [5] could correspond to a critical condition similar to the one identified long ago by DJ for the self-similar solutions. The DJ analysis describes a thermal feed back which can be summarized as follows; the laminar flame speed is an increasing function of the temperature which is controlled by the lead shock, the strength of which increases with the flame speed. Considering a plane turbulent flame in the wrinkled regime with a propagation velocity larger than the laminar flame speed by a wrinkling factor $\sigma > 1$, DJ showed that, because of a nonlinear effect associated with the high thermal sensitivity of the laminar flame speed, the self-similar solutions present a turning point for a critical value of $\sigma$ about 10, beyond which no solution exists. Such a turning point exists also for the laminar flame at the tip of the curved front for a critical elongation [1]. Moreover a finite-time singularity of the flow was shown to develop on the flame when the increasing elongation reaches the critical value [1] [2]. Unfortunately, these analyses are based on self-similar solutions that are quasi-static in the sense that the flow is homogeneous and steady between the flame and the lead shock, each of them propagating with a constant velocity, subsonic and supersonic respectively. A key element of the sudden transition is overlooked by the self-similar solutions, namely the unsteadiness and non-homogeneity of the compression waves that are generated by a self-accelerating flame front. The purpose of this short note is precisely to take into account the strong unsteady effects of the compression waves emitted by the self-accelerating flame front. The objective is to find out a generic expression for the critical propagation velocity of the tip of the curved flame front at which the DDT is produced quasi-spontaneously by the least further increase on flame elongation.

As mentioned by Liberman and co-workers [4] [5], a train of successive shockwaves is issued from the self-accelerating flame and generates a critical pre-conditioned state for the DDT. The

attention is focused here on the theoretical identification of this pre-conditioned state just prior to the transition. Focusing the attention on the unsteady process in the one-dimensional flow of unreacted gas, a finite-time singularity of the solution of the Euler's equations is shown to occur systematically on the self-accelerating flame front. This divergence of the flow gradient on the flame front is expected to be the basic explanation of the DDT observed in experiments and numerics [4] [5].

## 2. One-dimensional model

The model is the same as in [1] [2]. The flame acts as a semi-transparent piston with a velocity $U_P(t)$ proportional to the laminar flame velocity $U_b(T_u)$ relative to the burned gas for the same temperature of unburned gas $T_u(t)$ as on the tip of the elongated flame. Assuming an internal structure of the flame in quasi-steady state, the piston velocity in the laboratory frame is, according to the mass conservation, $U_P = u_b + U_b = u_u + U_L$ where $u_b$ is the back flow of burned gas, $u_u$ is the flow of unburned gas and $U_L$ is the laminar flame speed relative to the unreacted gas. The lateral skirt of the flame front being quasi-parallel to the lateral wall of the tube, a radial flow of burned gas is emitted with a velocity $U_b$ from the lateral wall. Following [6], the mass conservation in the turning flow of burned gas leads to a longitudinal back flow $u_b$ impinging on the flame tip from behind, taking the form $u_b \approx [2L(t)/R]U_b(T_u)$. The elongation of the curved front $2L(t)/R > 1$ increasing like the length of the flame skirt $L(t)$ ($R$ denotes the radius of the tube), the piston velocity $U_P$ takes the form

$$u_b = [2L(t)/R]U_b(T_u) \quad \Rightarrow \quad U_P(t) = S(t)U_b(T_u(t)), \quad S(t) \equiv 1 + 2L(t)/R \approx 2L(t)/R, \qquad (1)$$

where $S$ is the elongation parameter. The flow velocity of the fresh mixture just ahead of the flame is $u_u = U_P - (T_u/T_b)U_b$ where $T_u$ is the local temperature of the unburned gas and $T_b = T_u + q/c_p$ the flame temperature, $q$ and $c_p$ being respectively the chemical energy and the specific heat per unit mass, $U_L = (T_u/T_b)U_b$. If the elongation parameter is sufficiently larger than unity, the unreacted gas flow on the piston is approximately equal to the piston velocity and the piston acts like an

impermeable piston,

$$S(t) \gg 1 : \quad u_u \approx U_P, \quad U_P(t) = S(t) U_b(T_u(t)), \quad S(t) \approx 2L(t)/R. \tag{2}$$

This approximation simplifies the algebra and can easily be removed without modifying qualitatively the results. An impermeable piston is the simplest one-dimensional model for pointing out a fundamental mechanism of DDT on the tip of elongated flames.

A second ingredient is that the laminar flame speed is an increasing function of the temperature, characterized by a parameter of thermal sensitivity[1] $\tilde{b} > 1$,

$$\tilde{b} \equiv \frac{T_u}{U_b(T_u)} \frac{\mathrm{d}U_b}{\mathrm{d}T_u} > 1, \tag{3}$$

with, typically $\tilde{b} \gtrsim 2$ for energetic mixtures [1]. For simplicity, we will use the exponential form associated with an Arrhenius law in the limit of large activation energy,

$$\frac{U_b(T_u(t))}{U_b(T_u(0))} = \exp\left[\tilde{b}\left(\frac{T_u(t)}{T_u(0)} - 1\right)\right]. \tag{4}$$

The analysis can be performed with a temperature dependence different from (4) to the price of a more complex algebra. The essential ingredient is a function $U_b(T_u)$ increasing strongly enough with the temperature. The detailed temperature-dependence is not important and the exponential form (4) is convenient for an analytically description.

## 3. Analysis of the critical dynamics

### *3.1. Preliminary considerations*

Starting from a small elongation of the finger flame, a train of successive shock waves is created in the unburned mixture at finite distance from the piston (flame front). Just before the abrupt transition observed in [4] [5], the rate of increase of $U_P$ is not larger than the rate of increase of the

[1] The thermal sensitivity of the laminar flame velocity of energetic mixtures is mainly due to the pre-factor of the Arrhenius law, as explained in [1]

elongation $1/t_{evol}$ which is typically smaller than the inverse of the transit time of a fluid particle across the flame. Moreover, the gradient of the unreacted flow is small near the piston in the fresh gas between the flame front and the last shock wave formed ahead of the flame. Therefore the flow velocity near the piston is nearly equal to the piston velocity and the temperature is quasi-uniform. However, this temperature depends on the past history of the piston acceleration. When a train of successive shock waves is produced, the temperature on the piston is not the same as in a self-similar solution characterized by a single lead shock propagating in the unburned gas at rest. This is a consequence of the nonlinearity of the Rankine-Hugoniot relations. What can be said about the critical state adjacent to the flame just prior to the transition ? This is the first question addressed in this note, the second one concerns the sharp flow gradient occurring suddenly on the flame front.

### *3.2. Formulation*

Starting at time $t = 0$ after the last formation of a shock at finite distance from the flame, the flow in a slice of the fresh mixture adjacent to the piston is quasi-uniform nearby the flame front and equal to the piston velocity $u(x, t = 0) = U_P(0)$ with a gas temperature also quasi-uniform, $T \approx T_P(0)$. The subscript u has been removed for saving the notation. This situation is similar locally to a self-similar solution but the thermodynamic state of the unburned gas adjacent to the flame depends now on the past history of the piston acceleration. Compression waves are then launched from the self-accelerating piston. Before the formation of a new shock on the leading edge of the compression wave, the dissipative processes (heat conduction and viscosity) are negligible so that the compression waves are quasi-isentropic. More precisely, the fluctuation of entropy generated by the shock are assumed small without noticeable effect on the state near the piston. This is the case before the leading edge of the compression wave reaches the last formed shock. According to the classical theory of simple compression waves in a perfect gas (Riemannn 1860), the temperature of the gas is related to the unsteady flow by a quadratic relation,

$$\frac{T}{T(0)} = \left[1 + \frac{1}{2}(\gamma - 1)\frac{u - U_P(0)}{a(0)}\right]^2 . \tag{5}$$

This relation can be linearized as long as the flow velocity varies by an amount smaller than the speed of sound

$$0 \leqslant \frac{u - U_P(0}{a(0)} \ll 1 : \quad \frac{T}{T(0)} - 1 \approx (\gamma - 1)\frac{u - U_P(0)}{a(0)}. \tag{6}$$

Using (6) on the piston, a relation between the temperature of the gas on the piston and the piston velocity is obtained,

$$\text{on the piston:} \quad u = U_P \quad \text{and} \quad \frac{T}{T(0)} - 1 = (\gamma - 1)\frac{U_P(t) - U_P(0)}{a(0)}. \tag{7}$$

According to (2) and (4), the chemical kinetics provides us with an additional relation between $T$ and $U_P$ which involves the thermal sensitivity of the laminar flame speed $\tilde{b}$, defined in (3),

$$\frac{U_P - U_P(0)}{U_P(0)} = \frac{S}{S(0)} \exp\left[\tilde{b}\left(\frac{T}{T(0)} - 1\right)\right] - 1. \tag{8}$$

A key point is that the variation of the laminar flame speed $U_b(T)$ with the temperature in (4) is stronger than the temperature dependence (5) of the isentropic flow in a compression wave. From an asymptotic point of view, the association of a strongly nonlinear relation (8) with a linear approximation (7) is consistent for a large thermal sensitivity $\tilde{b} \gg 1$ and a small propagation Mach number of the laminar flame $U_b/a \ll 1$, in the distinguished limit $\lim_{\tilde{b}\to\infty} \tilde{b}(\gamma - 1)U_b(0)/a(0) = O(1)$.

### *3.3. Critical elongation and pre-conditioned state*

Introducing the notation $\Theta_P \equiv \tilde{b}\,[T_P/T(0) - 1] = O(1)$ for the temperature $T_P$ on the piston and the parameter $B \equiv \tilde{b}\,(\gamma - 1)U_b(0)/a(0) < 1$ depending on both thermal sensitivity and initial state, equations (7)-(8) lead to a nonlinear relation between the gas temperature on the piston $\Theta_P$ and the elongation parameter $S$ ,

$$\Theta_P(t) = BS(0)\left[\frac{S(t)}{S(0)} \exp\Theta_P(t) - 1\right] \quad \text{with} \quad S(0) \leqslant S_{max}(0) \quad \text{where} \quad S_{max}(0) \equiv \frac{1}{B} > 1, \tag{9}$$

the last expression is discussed few lines below.

A first outcome of (9) is that the temperature on the piston depends on the time only through the elongation $S(t)$. The parameter $B \equiv \tilde{b}\,(\gamma-1)U_b(0)/a(0) = O(1)$ of a unity order in the limit $\tilde{b} \to \infty$ is smaller than unity ($B < 1$) because the Mach number of the laminar flame speed $U_b(0)/a(0)$ of a very energetic mixture is typically ten times smaller than the inverse of the parameter $\tilde{b}\,(\gamma - 1)$ controlling the thermal sensitivity, $(\gamma - 1)\tilde{b}U_b/a \approx 10^{-1}$. The nonlinear relation linking the temperature and the elongation can be obtained numerically using an expression of the laminar flame speed more general than (4). For example, this was done in [1] for the self-similar problem. The simplicity of the nonlinear equation (9) allows a simple analytical description leading to an accurate order of magnitude of the solution.

A second outcome of (9) is the existence of an upper bound of the initial elongation $S(0) \leqslant S_{max}(0) = 1/B$. For an initial elongation $S(0) \leqslant S_{max}(0)$, the flame temperature and the flame velocity increases with the time up to a critical value $S^*$, as it should be on a physical branch of solutions. The solution of (9) shows effectively the existence of a critical elongation $S^*$ larger than its initial value but smaller than $S_{max}(0)$, $S(0) \leqslant S^* \leqslant S_{max}(0)$,

$$\frac{S^*}{S_{max}(0)} = \exp\left[\frac{S(0)}{S_{max}(0)} - 1\right] \leqslant 1 \quad \text{where} \quad S_{max}(0) = \frac{1}{\tilde{b}\,(\gamma-1)U_b(0)/a(0)} \tag{10}$$

$$0 < \frac{S(0)}{S_{max}(0)} \leqslant 1 \quad \Rightarrow \quad S(0) \leqslant S^* \leqslant S_{max}(0), \quad \frac{1}{\mathrm{e}} < \frac{S^*}{S_{max}(0)} \leqslant 1 \tag{11}$$

$$\text{and} \qquad \Theta_P^* \equiv b\left[\frac{T_P^*}{T(0)} - 1\right] = 1 - \frac{S(0)}{S_{max}(0)} \geqslant 0, \tag{12}$$

$T_P^*$ is the critical gas temperature on the piston, $T_P^* = T_P(S^*)$. The critical elongation $S^*$ in (10) corresponds to a turning point on the C-shaped curve (9) "*temperature versus elongation*" $\Theta_P(S)$, reminiscent of the critical wrinkling factor in the pioneering analysis [3] above which self-similar solutions no longer exist. Here, according to (9), isentropic compression waves can no longer be launched from the self-accelerating flame if the elongation is larger than its critical value ($S > S^*$) since the boundary condition on the piston can no longer be satisfied. Before discussing in more detail the C-shaped curve $T_P(S)$ in (9), it is worth mentioning that a singularity of the flow will be shown in § 3.4 to be formed on the flame front when $S$ reaches $S^*$, enlightening the underlying dynamical nature of the critical condition near the turning point $S = S^*$.

A single parameter characterizing the unburned gas at rest ahead of the lead shock was involved in the C-shaped curve of the self-similar solutions, see [3] and (19) in [1], so that the critical condition was the same for any initial point on the C-shaped curve. In contrast to the self-similar solutions, the C-shaped curve $T_P(S)$ in (9) and also the critical elongation $S^*$ in (10) both depend on the initial condition $S(0)$ pointing out the limited interest of the self-similar solutions. However, for an initial elongation $S(0)$ closer and closer to its maximum value $S_{max}(0)$, the critical elongation $S^*$ approaches quickly $S_{max}(0)$ from below, as shown by an expansion of $S(0)$ around $S_{max}(0)$ in (10)

$$0 < \frac{S_{max}(0) - S(0)}{S_{max}(0)} \ll 1 \quad \Rightarrow \quad \frac{S_{max}(0) - S^*}{S_{max}(0)} \approx \left( \frac{S_{max}(0) - S(0)}{S_{max}(0)} \right)^2 \ll 1. \tag{13}$$

This limiting case $S^* \approx S_{max}(0)$ for $S(0) \approx S_{max}(0)$ leads to a universal critical Mach number $U_P^*/a_u^*$ of the piston velocity (i.e. the critical velocity of the tip of the elongated flame) depending only on the reactive mixture through the thermal sensitivity $\tilde{b}$ of the laminar flame speed,

$$2\frac{L^*}{R} = \frac{1}{(\gamma - 1)\tilde{b}\, U_b^*/a_u^*} \quad \Rightarrow \quad \frac{U_P^*}{a_u^*} = \frac{1}{(\gamma - 1)\tilde{b}} \tag{14}$$

yielding $U_P^*/a_u^* \approx 1.25$ for $\tilde{b} \approx 2$, in good agreement with the experiments [4] [5]. In the last relation in (14) yielding the critical value of the piston velocity $U_P^*$, the temperature of the unburned gas just ahead of the flame occurs only through the speed of sound which increases weakly with the temperature like $\sqrt{T}$. Therefore, according to (2) and (14), the upper bound of the piston velocity (velocity of the flame tip) $U_P^* = 2(L^*/R)U_b^* = a_u^*/[(\gamma - 1)\tilde{b}]$ varies as the inverse of the thermal sensitivity $\tilde{b}$ with a coefficient of proportionality which does not change much. The smaller the Mach number $U_b^*/a_u^*$, the greater the critical elongation $L^*/R$.

The result (14) differs from the self-similar solution recalled in § 3.2 of [1] by the fact that the critical condition is related to the state of the fresh gas adjacent to the flame front and not to the state of the initial mixture at rest in front of the lead shock. The thermodynamic state of this pre-conditioned state is not fully determined by the first equation (14) relating the elongation to the flame temperature through (4). The critical state depends on the past history of the self-

accelerating flame. A self-similar solution is relevant to describe the DDT only if the flame speed is quasi-constant from the initial condition so that no secondary shocks are formed. This is not the case in [4] [5] showing the formation of a train of successive shock waves near the flame front before the DDT on the flame tip. Most of the time this occurs in tubes during a phase of re-acceleration after a transitory stage of deceleration associated with the formation of a tulip-shaped flame. Numerical simulations have also observed transition in the fold of a tulip flame [7] [8]. This multidimensional phenomenon is not considered in this short note in which the attention is focused on the DDT occurring before or after the formation of a tulip-shaped flame. In any case, no tulip flame is formed in very narrow tubes in which the DDT was nevertheless observed [9] [10].

If the initial elongation $S(0)$ is not close enough to its upper bound $S_{max}(0)$, more precisely if the time delay for reaching the critical condition is longer than the time delay for a shock wave to be created on the leading edge of the isentropic rarefaction wave, the condition on the piston will not correspond well to the critical condition of spontaneous transition. Consequently, the pre-conditioned state of the slice of unburned mixture adjacent to the flame front just prior to the transition [4] [5], should correspond roughly to (14). In any case, according to (14), the Mach number $U_P^*/a_u^*$ cannot be larger than $1/[(\gamma-1)\tilde{b}]$. This confirms a conclusion obtained previously with the self-similar solution: the DDT is produced for a flame velocity much smaller than the one which would be required for self-igniting the reactive gaseous mixture ahead of the flame front. According to (14), the critical flow velocity corresponds to a Mach number of the shock propagating in the gas at rest smaller than 2.5, which is effectively too small for self-ignition of the compressed reactive gaseous mixture, in full agreement with the experiments.

The solution of the unsteady flow presented in the following subsection enlightens the DDT mechanism by exhibiting the singular character of the underlying dynamics of the flow near the turning point of the C-shaped curve [2].

### *3.4. Finite-time singularity of the solution of the Euler's equation*

If $S(t)$ is a prescribed function increasing with the time without bound, a singularity of the solution of the Euler's equations appears spontaneously on the flame front at the critical time $t^*$,

$S(t^*) = S^*$. This is why isentropic compression waves can no longer be launched from the flame front for $t > t^*$ ($S(t) > S^*$). Such a scenario was mentioned in [1] within the context of self-similar solutions, hoping that the inconsistency of the static nature of the solutions could be overcome in some way, see the foot note 3 p. 9 in [1]. The proper way is to solve the fully unsteady problem by considering the compression waves, instead of the self-similar solutions.

The finite-time singularity of the isentropic flow generated by a self-accelerating piston reaching a finite velocity in a finite time with a divergence of the acceleration is predicted by the C-shaped curve (10) and takes a universal form near the critical condition [2], regardless of the context. The analysis is briefly outlined below for making the present short note self-contained. Additional comments are also included. Only the main steps are recalled; the reader is referred to [2] for the technical details. The critical dynamics is solved by the Riemann's method, taking full advantage of the quadratic relation $S(U_P) \propto (1 - U_P/U_P^*)^2$ near the critical point (maximum of the function $S(U_P)$ in (8)-(9)). Following Riemann, the equation for the flow velocity of a simple compression wave satisfying the downstream boundary condition $\lim_{x\to\infty} u = U_P(0)$ reads

$$\frac{\partial\left[u - U_P(0)\right]}{\partial t} + \left(\frac{\gamma+1}{2}\left[u - U_P(0)\right] + \left[a(0) + U_P(0)\right]\right)\frac{\partial\left[u - U_P(0)\right]}{\partial x} = 0. \tag{15}$$

The solution $u(x,t)$ can be written in the form[2],

$$x = \left(\frac{\gamma+1}{2}\left[u - U_P(0)\right] + \left[a(0) + U_P(0)\right]\right)t + f(u) \tag{16}$$

where the function $f(u)$ is determined by the boundary condition on the piston, $x = X_P(t) = \int_0^t U_P(t)\mathrm{d}t' : u = U_P(t)$. Thanks to the quadratic form $S(U_P) \propto (1 - U_P/U_P^*)^2$, a generic law for the piston velocity is easily obtained [1]

$$0 \leqslant \frac{(t^* - t)}{t^*_{evol}} \ll 1 : \qquad \frac{U_p^* - U_P(t)}{U_P^*} = \sqrt{\frac{t^* - t}{t^*_{evol}}}, \quad \frac{U_p^* - U_P(0)}{U_P^*} = \sqrt{\frac{t^*}{t^*_{evol}}} \tag{17}$$

[2]Equation (15) is called in the modern literature "Burger's equation" even though it was mentioned a century before; the form (16) was anticipated by Poisson (1808), see p.118 of the textbook [11]. Its derivation from the Euler's equations can be found p. 368 of [12]

where $1/t^*_{evol}$ is the gtowth rate of the elongation at the critical time $t^*$. The law (17) is the same for any value of the critical velocity $U^*_P$ and any growth rate of the elongation, as small as it could be. Coming back to the flow field, the boundary condition on the piston $x = X_P(t) = \int_0^t U_P(t)\mathrm{d}t' : u = U_P(t)$ then shows that the function $f(u)$ is a cubic polynomial, see (67)-(68) in [2], so that, according to (16), the problem reduces to solve a third-degree polynomial in $(1 - u/U^*_P)$, see (74)-(75) in [2]. Close to the critical velocity and for $(1 - u/U^*_P) \ll a^*_u/U^*_P$, the cubic term is negligible and the problem is reduced to solve a second-order algebraic equation. Moreover this equation further simplifies for $(t^* - t)/t^* \ll (a^*_u/U^*_P)^2$ yielding the following analytical expression of the flow field

$$\frac{u(x,t)}{U^*_P} = 1 - \sqrt{\frac{(t^* - t)}{t^*_{evol}} + \frac{[x - X_P(t)]}{a(0)t^*_{evol}}}. \tag{18}$$

The condition for the validity of (18) is less drastic than in [2]. More particularly, (18) is not limited to a small Mach number of the piston $U^*_P/a^*_u \ll 1$ which, according to (14), would require a too large value of $\tilde{b} \gg 1$, larger than in very energetic mixtures. The flow field (18) satisfies the boundary condition (17) on the piston, as it should be. A weak discontinuity is exhibited at the leading edge $x = x_{wd}(t)$ where the initial flow, namely the initial velocity of the piston, is recovered, $u(x,t)|_{x=x_{wd}(t)} = U_P(0)$. The point $x = x_{wd}(t)$ propagates effectively at the speed of sound relative to the gas flow $x_{wd}(t) - X_P(t) = a(0)t$, as it is checked by using (17). As already mentioned, close to the critical point, the initial sound speed $a(0)$ can be replaced by $a^*_u$. According to (14) and (18), the gradient of the flow

$$t^*_{evol}\frac{\partial u(x,t)}{\partial x} = -\frac{U^*_P}{a^*_u}\frac{1/2}{\sqrt{\frac{(t^*-t)}{t^*_{evol}} + \frac{[x-X_P(t)]}{a^*_u t^*_{evol}}}} \quad \text{where} \quad \frac{U^*_P}{a^*_u} = \frac{1}{(\gamma-1)\,\tilde{b}} \tag{19}$$

presents a finite-time singularity on the piston at the critical time $t^*$ which, according to (17), corresponds to a divergence of the acceleration of the piston $\mathrm{d}U_P/\mathrm{d}t \propto 1/\sqrt{(t^* - t)/t^*_{evol}}$ when the piston velocity reaches the critical value $U^*_P$. The flow (18) is indeed solution of the linear version of (15) so that the wave-breaking mechanism which is responsible for the formation of shock waves, is not involved in the genesis of the finite-time singularity of the flow on the flame front.

The singularity (19) comes from the quadratic form $S(U_P) \propto (1 - U_P/U_P^*)^2$ near the turning point $S = S^*$ on the curve $U_P(S)$.

Could this finite-time singularity of the gradient of the flow on the flame front (19) be responsible for the formation of a shock wave inside the flame structure leading to a quasi-instantaneous DDT? This is likely because the temperature inside the flame structure is higher than in the unreacted gas ahead of the flame where the temperature is not large enough for self-igniting the unreacted gas in the compressed gas behind the shock waves emitted by the self-accelerating flame front. This problem requires to investigate the unsteady flame structure. In that respect, the ZFK flame model with a back flow $u_b \approx [2L(t)/R]U_b(T_u)$ applied at the reaction sheet is worth investigating by solving analytically (or numerically) a free boundary problem yielding the instantaneous velocity of the reaction sheet $U_P(t)$ in the form of an eigenvalue. A finite-time singularity of the flow gradient, associated with the singularity of the acceleration $\mathrm{d}U_P(t)/\mathrm{d}t$ at a critical value $U_P^* = U_P(t^*)$, is expected to occur on the reaction sheet. The analysis is beyond the scope of the present note. The achievement of such a difficult analysis is the price to pay for a full understanding of a fundamental problem, namely the sudden transition of a subsonic reaction-diffusion wave (the flame) into a supersonic wave (the detonation).

## 4. Discussion of the results

The results (14) and (18)-(19) predict the characteristics of the pre-conditioned state of the slab of fresh mixture adjacent to the self-accelerating flame, observed just prior to DDT in a laminar regime [4] [5]. The finite-time singularity of the solution of the Euler's equation in the laminar flow of unreacted gas ahead of the flame front could be the key of the DDT. The solution of the unsteady flame structure remains to be carried out in order to confirm this statement. The analysis is in process.

The result (14) explains why the DDT in the laminar regime can be observed only with strongly energetic mixtures, never with gaseous mixtures in air, as discussed now. The critical length in the first equation (14) is reached only if the Mach number of the laminar flame velocity $U_b/a_u$ is large enough to make the critical elongation $L/R$ accessible. The elongation of curved flames in tubes is due to hydrodynamical mechanisms. It is typically not larger than 10 in centimeter radius

tubes. According to (14) with $\gamma = 1.4$ and $\tilde{b} \approx 2$, a critical elongation $L^*/R$ about 10 requires $U_b/a_u$ not smaller than 0.06. Such a fast flame occurs in stoichiometric mixtures of hydrogen or acetylene in pure oxygen whose laminar flame speed $U_L$ (relative to the fresh mixture gas) and gas expansion parameter $T_b/T_u$ are respectively close to 10 m/s and 10. In ordinary gaseous combustible mixtures in air the Mach umber $U_b/a_u$ is between ten to a hundred times smaller. On the opposite case of highly energetic mixtures, the critical ratio $L^*/R$ can be small enough to concern the cellular structure of self-accelerating flames propagating freely in open space under the Rayleigh-Taylor instability. This could be the case in astrophysics for flames sustained by nuclear reactions.

Another interesting result in (14) is that the critical Mach number $U_P^*/a_u^*$ (measured in the laboratory frame) of the flame pushed from behind by the back flow depends only on the thermal sensitivity of the laminar flame speed (3).

## Declaration of Competing Interset

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