

Supporting Information for ”A statistical method to model non-stationarity in precipitation records changes”

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Introduction First, this document illustrates with an example that checking $p_{1,2}(t)$ is not sufficient to capture changes on record probabilities associated to other r . In a second time, it details the equations used to create Figure 2. In a third time, it presents the estimation algorithm for $p_{1,r}(t)$. Next, it defines the asymptotic confidence intervals of our parametric estimator $\widehat{p}_{1,r}(t)$. Later, we evaluate the validity of the W-class assumption in our application. Finally we develop a supplementary analysis of centennial records.

1. Example: $p_{1,2}(t)$ is not sufficient for detecting changes on record probabilities

Let X and Z_t follow Generalized Extreme Value (GEV) distributions, characterized by the parameters (μ_x, σ_x, ξ_x) and $(\mu_{z_t}, \sigma_{z_t}, \xi_{z_t})$. The parameters λ_t and k_t encompass the relative behavior between X and Z_t and can be expressed as $k_t = \frac{\xi_x}{\xi_{z_t}}$ and $\lambda_t = (k_t \frac{\sigma_{z_t}}{\sigma_x})^{-1/\xi_x}$ (see Lemma 1 in Worms & Naveau; 2022). In the stationary scenario where $Z_t \stackrel{d}{=} X$, the relative behavior is characterized by $\lambda_t = 1$ and $k_t = 1$, and record probabilities for $r = 2$ and $r = 3$ are $p_{1,2}(t) = 1/2$ and $p_{1,3}(t) = 1/3$ respectively. To define the second hypothetical scenario (factual and counterfactual are different), we set $\xi_x = 1/4$, $\xi_{z_t} = 1/2$, $\sigma_x = 1$ and $\sigma_{z_t} \sim 1.86$, which yields $\lambda_t = 1.33$ and $k_t = 1/2$. In this case, the record probability for $r = 2$ remains unchanged with $p_{1,2}(t) = 1/2$, but we point out that $p_{1,3}(t) = 0.39$. So testing $P(X \leq Z_t) = .5$ is not sufficient to differentiate record probabilities for $r > 2$. Furthermore, as r increases record probability $p_{1,r}(t)$ continues to diverge from $1/r$. This brings to mind the scenario where two random variables can share the same median but have different upper quantiles. Conversely, having $k_t \neq 1$ and/or $\lambda_t \neq 1$ only tells us that the random variables X and Z_t follow different probability

distributions. It does not imply that $\forall r, p_{1,r}(t) \neq 1/r$. For a given record length r , it is still possible to have $p_{1,r} = 1/r$ when $k_t \neq 1$ and/or $\lambda_t \neq 1$.

As mentioned in the previous paragraph, for a given year t , record probabilities can be deduced from a Weibull distribution with parameters (k_t, λ_t) . This is a key element of our approach. If this hypothesis is not rejected, (k_t, λ_t) fully characterizes record probability $p_{1,r}(t)$ for any year t and record length r . Our inference strategy (see Section 3 of the supplementary material) is to use the statistics $p_{1,2}(t)$ and $p_{1,3}(t)$ to estimate λ_t and k_t . Yet, the extent of changes in record probabilities does not directly unravel from (k_t, λ_t) . Plugging these Weibull distribution parameters into equation (6) (in the article) translates to communicating in terms of return periods.

2. Equations for Figure 2

Let X and Z follow Generalized Extreme Value (GEV) distributions, with G and F their cumulative distribution functions

$$G(x; \mu_G, \sigma_G, \xi_G) = \exp \left(- \left(1 + \xi_G \left(\frac{x - \mu_G}{\sigma_G} \right) \right)^{-1/\xi_G} \right)$$

$$F(z; \mu_F, \sigma_F, \xi_F) = \exp \left(- \left(1 + \xi_F \left(\frac{z - \mu_F}{\sigma_F} \right) \right)^{-1/\xi_F} \right).$$

Figure 2 illustrates the joint probability density between X or Z and the maxima of a number d of random variables X , i.e $\max(X_1, \dots, X_d)$. This joint density is defined by

$$f(x, z) = \frac{d}{\sigma_G \sigma_F} \left(1 + \xi_G \left(\frac{x - \mu_G}{\sigma_G} \right) \right)^{-\alpha/\xi_G - 1} \left(1 + \xi_F \left(\frac{z - \mu_F}{\sigma_F} \right) \right)^{-\alpha/\xi_F - 1}$$

$$V^{1-\alpha} (V^{1-\alpha} + (1-\alpha)V^{-\alpha}) e^{-V},$$

where α measures correlation between X and Z and V is the dependence structure, here the logistic model

$$V(x, y) = \left(\frac{1}{x^\alpha} + \frac{1}{y^\alpha} \right)^{1/\alpha}.$$

Densities of figure 2 where obtained using $\alpha = 1$. In panel a) and c) we visualize the joint probability densities of X and $\max(X_1, \dots, X_d)$, with $\mu_G = 0$, $\sigma_G = 1$ and $\xi_G = 0.2$, in b) and d) the ones of Z and $\max(X_1, \dots, X_d)$, with $\mu_G = 0$, $\sigma_G = 1$, $\xi_G = 0.2$, $\mu_F = 1$, $\sigma_G = 1$ and $\xi_G = 0.2$.

3. Estimation algorithm for $p_{1,r}(t)$

Defining I and J as the lengths of the counterfactual and factual trajectories, t_j is the time step j with $j \in \{1, \dots, J\}$ and X_{t_j} and Z_{t_j} the random variables associated with the counterfactual and factual worlds at time t_j .

The estimation process of $p_{1,r}(t)$, when we assume the W-class assumption for all t , is a three step algorithm. The first step is to estimate $p_{1,2}(t)$ and $p_{1,3}(t)$, for this we use the non-parametric estimation method developed in Naveau et al. (2018); Naveau and Thao (2022), which locally averages $G(Z_t)$ and $G^2(Z_t)$ by using the Nadaraya-Watson kernel regression method (Härdle, 1991), where G is the CDF of X_t . Hence we use \mathbb{G}_I , the empirical estimator of G and K_h , the renormalized Epanechnikov kernel function of bandwidth h , to estimate $p_{1,2}(t)$ and $p_{1,3}(t)$ as

$$\begin{aligned} \hat{p}_{1,2}(t) &= \sum_{j=1}^J K_h(t - t_j) \mathbb{G}_I(Z_{t_j}) \\ \hat{p}_{1,3}(t) &= \sum_{j=1}^J K_h(t - t_j) \mathbb{G}_I^2(Z_{t_j}), \end{aligned}$$

where,

$$K_h(t - t_j) = \frac{k((t - t_j)/h)}{\frac{1}{J} \sum_{l=1}^J k((t - t_l)/h)},$$

The second step is to use $(\widehat{p}_{1,2}(t), \widehat{p}_{1,3}(t))$ and the method of moments to estimate $\widehat{\lambda}_t$ and \widehat{k}_t , for a chosen t , via the following estimating equations, issued from Equation (6) (on the article) and the W-class assumption :

$$\begin{cases} \widehat{p}_{1,2}(t) = \int_0^1 \exp(-\widehat{\lambda}_t(-\log x)^{1/\widehat{k}_t}) dx \\ \widehat{p}_{1,3}(t) = \int_0^1 \exp(-2\widehat{\lambda}_t(-\log x)^{1/\widehat{k}_t}) dx. \end{cases}$$

Finally, plugging the parameters $\widehat{\lambda}_t$ and \widehat{k}_t in Equation (6) (on the article) we can compute $\widehat{p}_{1,r}(t)$ for any chosen value of r . In summary, starting from a counterfactual and factual trajectory and a given r , this methodology allows us to estimate the record probability $p_{1,r}(t)$ at any chosen time t , taking into account the non-stationarity of Z_t .

4. Asymptotic confidence intervals

The following theorem is a non-stationary generalization of Worms and Naveau (2022) (Proposition 4).

Theorem 1 *When I and J go to infinity, if $\sqrt{J/I}$ converges to some finite constant, then for any X_t and Z_t belonging to the W-class and any fixed $r \geq 3$, the parametric estimator $\widehat{p}_{1,r}(t)$ satisfies*

$$\sqrt{J} \frac{\widehat{p}_{1,r}(t) - p_{1,r}(t)}{\sigma_{rt}} \stackrel{d}{\approx} \mathcal{N}(0, 1)$$

with

$$\sigma_{rt} = \sqrt{J_{r-1}(\theta_t)(J_{1,2}(\theta_t))^{-1} \Sigma_t (J_{1,2}^T(\theta_t))^{-1} (J_{r-1}(\theta_t))^T},$$

where $\theta_t = (\lambda_t, k_t)$ is a vector containing the parameters of the Weibull distribution at time t . $J_j(\theta_t)$ is the Jacobian matrix of $g_j(\theta_t) = \int_0^1 \exp(-j\lambda_t(-\log x)^{1/k_t})dx$ at time t for any integer $j \geq 1$, $J_{1,2}(\theta_t)$ the Jacobian matrix associated to $\theta_t \mapsto (g_1(\theta_t), g_2(\theta_t))^T$ at time t and Σ_t the asymptotic covariance matrix of $(\widehat{p}_{1,2}(t) - p_{1,2}(t), \widehat{p}_{1,3}(t) - p_{1,3}(t))^T$.

We can then compute the confidence intervals of significance level $1 - \alpha$ as follows

$$[\widehat{p}_{1,r}(t) \pm z_\alpha \widehat{\sigma}_{rt}],$$

simply issued from the replacement of σ_{rt} by $\widehat{\sigma}_{rt}$ and with z_α the Gaussian threshold associated to significance level $1 - \alpha$.

Proof. Recalling that I and J are the lengths of our counterfactual and factual trajectories, the first step is to compute the variance-covariance matrix of the bivariate vector $N_{I,J} = (\widehat{p}_{1,2}(t) - p_{1,2}(t), \widehat{p}_{1,3}(t) - p_{1,3}(t))^T$, where $\widehat{p}_{1,2}(t)$ and $\widehat{p}_{1,3}(t)$ are the non-parametric estimators of $p_{1,2}(t)$ and $p_{1,3}(t)$ obtained using the method developped in Naveau et al. (2018); Naveau and Thao (2022).

For this first step we divide $N_{I,J}$ in the two following independent vectors

$$N_{I,J} = \begin{pmatrix} \widehat{p}_{1,2}(t) - p_{1,2}(t) \\ \widehat{p}_{1,3}(t) - p_{1,3}(t) \end{pmatrix} = \begin{pmatrix} \widehat{p}_{1,2}(t) - \widetilde{p}_{1,2}(t) \\ \widehat{p}_{1,3}(t) - \widetilde{p}_{1,3}(t) \end{pmatrix} + \begin{pmatrix} \widetilde{p}_{1,2}(t) - p_{1,2}(t) \\ \widetilde{p}_{1,3}(t) - p_{1,3}(t) \end{pmatrix} =: N_A + N_B$$

where

$$\begin{aligned} p_{1,r}(t) &= \mathbb{E}(G^{r-1}(Z_t)) \\ \widetilde{p}_{1,r}(t) &= \frac{1}{J} \sum_{j=1}^J K_h(t - t_j) G^{r-1}(Z_{t_j}) \\ \widehat{p}_{1,r}(t) &= \frac{1}{J} \sum_{j=1}^J K_h(t - t_j) \mathbb{G}_I^{r-1}(Z_{t_j}) \end{aligned}$$

with K_h a renormalized kernel function of bandwidth h (see Section 3 of the supplementary material) and $\widetilde{p}_{1,r}(t)$ a version of $\widehat{p}_{1,r}(t)$ where the cdf G is considered known. Thereby,

N_A is a vector that represents the fluctuations between \mathbb{G}_I and G and N_B is a vector that contains the error generated by the estimation of the expectation. We study the behaviours of each of these vectors separately.

Let us first deal with N_A . This first component can be rewritten approximately as

$$\tilde{N}_A \approx \frac{1}{\sqrt{IJ}} \sum_{j=1}^J K_h(t - t_j) \begin{pmatrix} B(G(Z_{t_j})) \\ 2G(Z_{t_j})B(G(Z_{t_j})) \end{pmatrix},$$

where $s \mapsto B(s)$ is a brownian bridge on $[0, 1]$. Thanks to the independence between the brownian bridge B and the factual world observations Z_{t_j} , \tilde{N}_A follows a Gaussian distribution. Then, asymptotically

$$N_A = \begin{pmatrix} \hat{p}_{1,2}(t) - \tilde{p}_{1,2}(t) \\ \hat{p}_{1,3}(t) - \tilde{p}_{1,3}(t) \end{pmatrix} \approx \mathcal{N}(\mathbb{E}(\tilde{N}_A), \Sigma_A).$$

Using brownian bridge's properties, we find the expectation of $N_A = (N_{A,1}, N_{A,2})^T$

$$\mathbb{E}(N_{A,1}) \approx \mathbb{E} \left(\frac{1}{\sqrt{IJ}} \sum_{j=1}^J K_h(t - t_j) \mathbb{E}(B(G(Z_{t_j})|Z_{t_j})) \right) = 0$$

and

$$\mathbb{E}(N_{A,2}) \approx \mathbb{E} \left(2 \frac{1}{\sqrt{IJ}} \sum_{j=1}^J K_h(t - t_j) G(Z_{t_j}) \mathbb{E}(B(G(Z_{t_j})|Z_{t_j})) \right) = 0.$$

Before starting the computations of its covariance matrix, we define the vector

$$V_{t_j} = \begin{pmatrix} B(G(Z_{t_j})) \\ 2G(Z_{t_j})B(G(Z_{t_j})) \end{pmatrix}.$$

With this notation, the covariance matrix of N_A can be rewritten as follows

$$\text{Cov}(N_A) \approx \frac{1}{IJ^2} \sum_{j=1}^J \sum_{i=1}^J K_h(t - t_j) K_h(t - t_i) \text{Cov}(V_{t_j}, V_{t_i}),$$

where the Cov on the left-hand side refers to the variance-covariance matrix of a random vector, and the Cov on the right-hand side refers to the covariance matrix between the two random vectors V_{t_j} and V_{t_i} .

Similarly to $E(N_A) \approx 0$, we can prove that $E(V_{t_j}) = 0$. Hence, the covariance can be rewritten as an expectation

$$\text{Cov}(V_{t_j}, V_{t_i}) = E(V_{t_j} V_{t_i}^t) = \begin{pmatrix} A_{ji} & C_{ji} \\ C_{ij} & B_{ji} \end{pmatrix}.$$

The components of the matrix can be written as

$$\begin{aligned} A_{ji} &= E \left(E(B(G(Z_{t_j}))B(G(Z_{t_i}))|Z_{t_j}, Z_{t_i}) \right) \\ &= E \left(\min(G(Z_{t_j}), G(Z_{t_i})) - G(Z_{t_j})G(Z_{t_i}) \right) \end{aligned}$$

$$\begin{aligned} B_{ji} &= E \left(E(4G(Z_{t_j})G(Z_{t_i})B(G(Z_{t_j}))B(G(Z_{t_i}))|Z_{t_j}, Z_{t_i}) \right) \\ &= 4 E \left(G(Z_{t_j})G(Z_{t_i}) [\min(G(Z_{t_j}), G(Z_{t_i})) - G(Z_{t_j})G(Z_{t_i})] \right) \end{aligned}$$

$$\begin{aligned} C_{ji} &= E \left(E(2G(Z_{t_j})B(G(Z_{t_j}))B(G(Z_{t_i}))|Z_{t_j}, Z_{t_i}) \right) \\ &= 2 E \left(G(Z_{t_j}) [\min(G(Z_{t_j}), G(Z_{t_i})) - G(Z_{t_j})G(Z_{t_i})] \right), \end{aligned}$$

and we summarize these formulas by

$$A_{ji} = M_{2,j,i} - p_{1,2}(t_j) p_{1,2}(t_i)$$

$$B_{ji} = 4(M_{3,j,i} - p_{1,3}(t_j) p_{1,3}(t_i))$$

$$C_{ji} = 2(E_{ji} - p_{1,3}(t_j) p_{1,2}(t_i)),$$

where we define

$$M_{r,j,i} = E[G^{r-2}(Z_{t_j})G^{r-2}(Z_{t_i}) \min(G(Z_{t_j}), G(Z_{t_i}))]$$

and

$$E_{ji} = E[G(Z_{t_j}) \min(G(Z_{t_j}), G(Z_{t_i}))].$$

Now we deal with the second vector, N_B , which can be written as follows

$$N_B = \begin{pmatrix} \tilde{p}_{1,2}(t) - p_{1,2}(t) \\ \tilde{p}_{1,3}(t) - p_{1,3}(t) \end{pmatrix} = \frac{1}{J} \sum_{j=1}^J K_h(t - t_j) \begin{pmatrix} G(Z_{t_j}) - p_{1,2}(t) \\ G^2(Z_{t_j}) - p_{1,3}(t) \end{pmatrix}.$$

For large I and J , N_B approximately follows a bivariate Gaussian distribution. Introducing

$$\begin{aligned} \bar{p}_{1,2}(t) &= E(\tilde{p}_{1,2}(t)) = \frac{1}{J} \sum_{j=1}^J K_h(t - t_j) p_{1,2}(t) \\ \bar{p}_{1,3}(t) &= E(\tilde{p}_{1,3}(t)) = \frac{1}{J} \sum_{j=1}^J K_h(t - t_j) p_{1,3}(t), \end{aligned}$$

the covariance matrix of N_B can then be calculated as

$$\text{Cov}(N_B) = \frac{1}{J^2} \sum_{j=1}^J K_h(t - t_j)^2 \begin{pmatrix} p_{1,3}(t_j) - p_{1,2}(t_j)^2 & p_{1,4}(t_j) - p_{1,2}(t_j)p_{1,3}(t_j) \\ p_{1,4}(t_j) - p_{1,2}(t_j)p_{1,3}(t_j) & p_{1,5}(t_j) - p_{1,3}(t_j)^2 \end{pmatrix}.$$

Now we can calculate the approximate variance-covariance matrix of $N_{I,J}$. Starting from Σ_t , the standard M-estimation theory allows us to find the variance-covariance matrix of our estimators $(\hat{\lambda}_t, \hat{k}_t)$. Additionally, the delta method allows us to find the asymptotic variance of our the parametric estimator $\hat{p}_{1,r}(t)$. We first notice that

$$\sqrt{J} \left((\hat{\lambda}_t, \hat{k}_t) - (\lambda_t, k_t) \right) = \sqrt{J} \left(g^{-1}(\hat{p}_{1,2}(t), \hat{p}_{1,3}(t)) - g^{-1}(p_{1,2}(t), p_{1,3}(t)) \right),$$

where $g : (\lambda_t, k_t) \mapsto (g_1(\lambda_t, k_t), g_2(\lambda_t, k_t))$, with $g_j(\lambda_t, k_t) = \int_0^1 \exp(-j\lambda_t(-\log x)^{1/k_t}) dx$ a locally one-to-one function. Then, for (X_t, Z_t) belonging to the W-class, the MOM estimators $(\hat{\lambda}_t, \hat{k}_t)$ asymptotically follow a normal distribution

$$\sqrt{J} \left((\hat{\lambda}_t, \hat{k}_t)^T - (\lambda_t, k_t)^T \right) \sim \mathcal{N} \left(0, (J_{1,2}(\lambda_t, k_t))^{-1} \Sigma_t (J_{1,2}^T(\lambda_t, k_t))^{-1} \right),$$

$J_{1,2}(\lambda_t, k_t)$ denotes the Jacobian matrix associated to $(g_1(\lambda_t, k_t), g_2(\lambda_t, k_t))^T$ at time t .

Finally, via the delta method we find that the parametric estimator $\widehat{p}_{1,r}(t)$ asymptotically satisfies

$$\sqrt{J} \frac{\widehat{p}_{1,r}(t) - p_{1,r}(t)}{\sigma_{rt}} \stackrel{d}{\approx} \mathcal{N}(0, 1)$$

with

$$\sigma_{rt} = \sqrt{J_{r-1}(\lambda_t, k_t)(J_{1,2}(\lambda_t, k_t))^{-1} \Sigma_t (J_{1,2}^T(\lambda_t, k_t))^{-1} (J_{r-1}(\lambda_t, k_t))^T},$$

where $J_{r-1}(\lambda_t, k_t)$ is the transposed gradient of $g_{r-1}(\lambda_t, k_t)$ and $J_{1,2}(\lambda_t, k_t)$ and Σ_t the same as before. This because $\sqrt{J}(\widehat{p}_{1,r}(t) - p_{1,r}(t)) = \sqrt{J}(g_{r-1}(\widehat{\lambda}_t, \widehat{k}_t) - g_{r-1}(\lambda_t, k_t))$.

5. W-class assumption

In the field of statistical climatology, the most commonly used framework when dealing with block maxima is to assume that our variables follow Generalized Extreme Value (GEV) distributions with distinct parameters, as outlined by Li et al. (2021). Building upon this, Worms and Naveau (2022) showed that the Weibullity hypothesis of W is equivalent to the hypothesis of the same support for X and Z . In particular, for extreme precipitation it is common to have $\xi_x > 0$ and $\xi_z > 0$, indicating that both X and Z follow distributions with infinite upper bound, which reassures the viability of your assumption. The W-class assumption still relies on the assumption that lower bounds won't diverge significantly.

In this section we verify that the Weibullity of the variable $W_t = -\log \mathbb{G}(Z_t)$ is a proper assumption. We recall that Z_t is the variable associated to the factual world at time t and \mathbb{G} the empirical cumulative distribution function of the counterfactual world X .

To assess the validity of the Weibullity assumption, we use the following procedure. We re-scaled our data into a Weibull(1,1) distribution by applying the transformation

$$T_t(w) = (w/\lambda_t)^{k_t}$$

on \widehat{W}_t , where λ_t and k_t are approximated by the estimates $\hat{\lambda}_t$ and \hat{k}_t . Next we visualize the qqplot between the rescaled \widehat{W}_t and the quantiles of a Weibull(1,1) distribution (which is also an exponential). For the sake of conciseness, we have decided to only present our assessments for 16 randomly chosen grid-points, Figure S1 indicates that the Weibullity of \widehat{W}_t is not an inappropriate assumption.

6. Application on centennial records

To complement the application of Section 3, in this supplementary section we analyze the yearly maxima of daily precipitation from IPSL-CM6A-LR climate model with scenario SSP5-8.5 by looking at centennial records probability in 2050 and the time of emergence of centennial records, rather than decadal as in the main paper.

We define the emergence time associated with a given record length r as the first year when $\hat{p}_{1,r}(t)$ is significantly different from its counterfactual value, i.e. different from $p_{0,r}(t) = 1/r$, with confidence level of 95%. Mathematically, this brings the following definition

$$\tau_{0.95}(r) = \min \left\{ t \text{ such that for all } t' \geq t, \frac{1}{r} \notin [\hat{p}_{1,r}(t') \pm 1.96 \hat{\sigma}_{rt'}] \right\}, \quad (1)$$

where $\hat{\sigma}_{rt}$ represents the estimation of the asymptotic standard deviation and 1.96 corresponds to the Gaussian significance level 0.95

For centennial records, a clear climate change signal emerges when the confidence intervals of our estimator no longer contain 1/100. Panel (a) of Figure S2 highlights centennial record probability ratio on 2050 in the zones where by that year there is already a significant signal of climate change. By 2050, there is a clear signal on 43% of the globe, which is more than 30% less coverage than what is observed for decadal records, where the climate change signal is present on 80% of the globe.

While the coverage of the climate change signal is less for centennial than for decadal records, the signal on significant gridpoints is stronger. For example, we expect centennial records on tropical latitudes on 2050 to be more than 10 times more likely than in a world without anthropogenic forcing. It is important to add that behavior aligns for decadal and centennial records on points where climate change signal emerges before the end of the century. For a gridpoint where decadal record probability increases, centennial record probability increases too, and the same holds for gridpoints where decadal record probability decreases. Furthermore, the centennial record emergence time map from panel (b) shows that only 12% of the climate change signal emerged between 2000 and 2023, adding up to this last year 17% of the Earth's surface. This coverage represent a 40% less globe surface than for decadal records, where by 2023 the signals already manifested over 57% of the globe.

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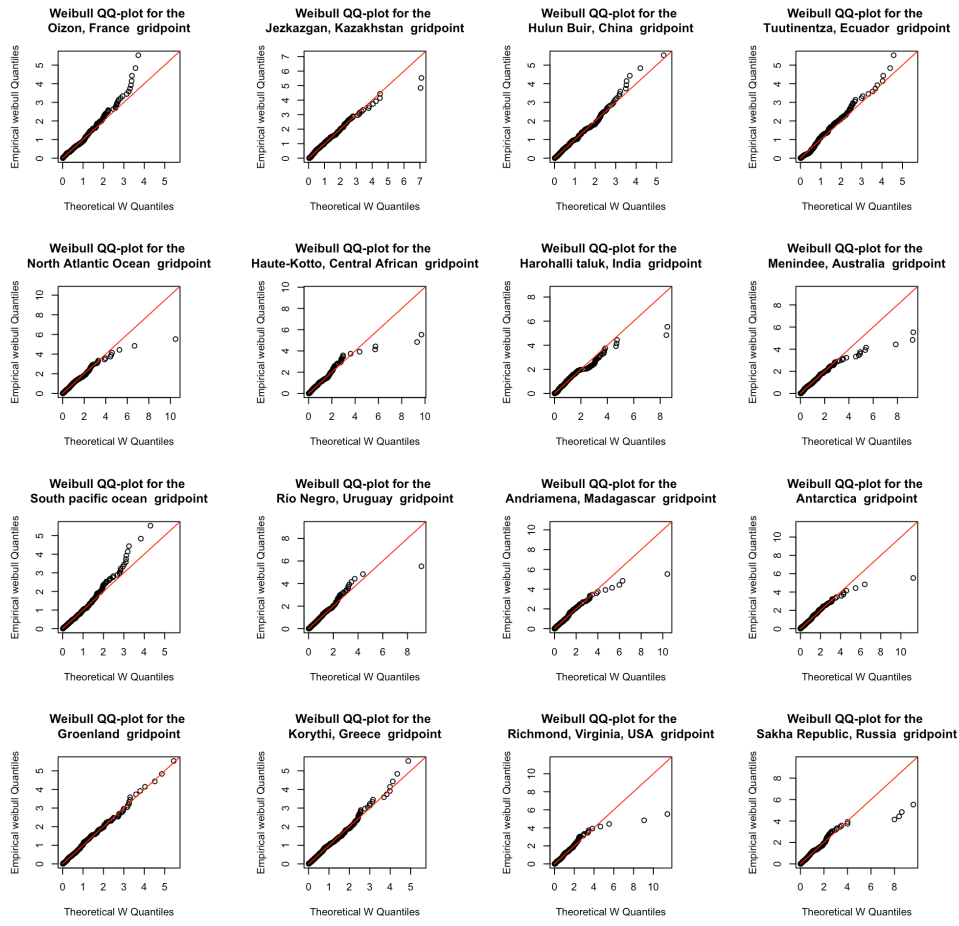


Figure S1. QQplot comparing the theoretical Weibull(1,1) quantiles and empirical quantiles of the normalized \widehat{W}_t , for 16 random gridpoints.

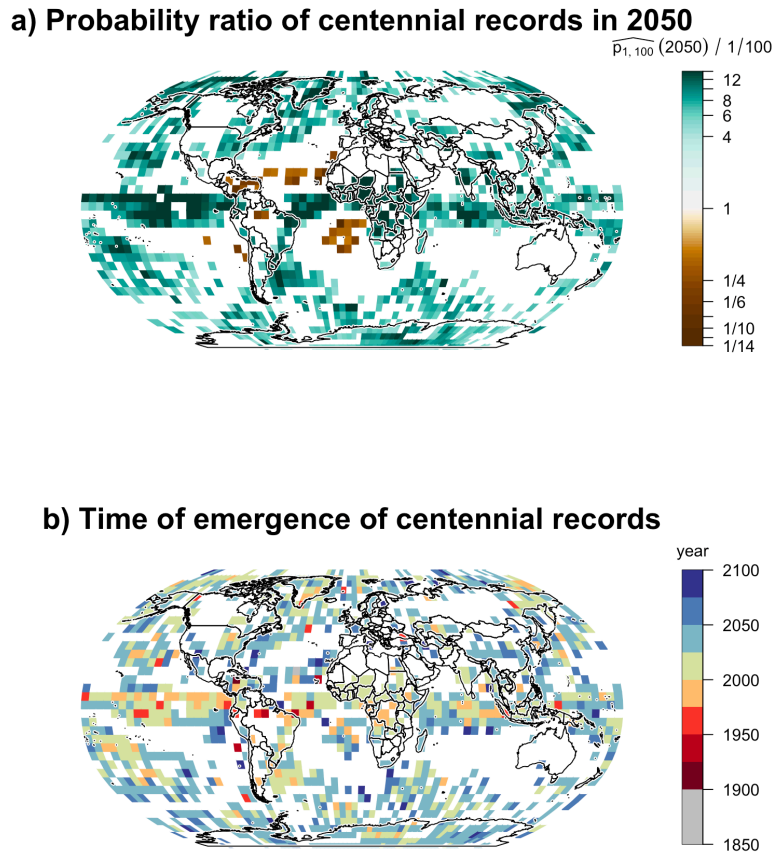


Figure S2. (a) Centennial record probability ratio on 2050 with respect to the counterfactual world, the white zones represent the gridpoints where by 2050 climate change signal has still not emerged, using (1) as criteria. (b) Emergence centennial record times defined by (1), the white zones represent the gridpoints where by 2100 climate change signal has still not emerged, the grey points represent the gridpoints where $\widehat{p}_{1,100}(t)$'s confidence interval does not contains 1/100 during the pre-industrial period, these points are left out of our analysis as considered poorly represented. Results obtained using the IPSL-CM6A-LR climate model and the scenario SSP5-8.5, with a spatial resolution of 72 x 36 grid points.

August 2, 2024, 8:56pm