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# Robust sensor faults observer design for continuous-time switched systems using descriptor approach

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**Abstract**—This paper considers the problem of designing a robust descriptor observer for simultaneous state and sensor faults estimation of continuous-time switched linear systems in the presence of unknown external disturbances. An augmented singular switched system is constructed from the original switched systems by considering the sensor faults term as a part of a new extended state vector. Stability conditions are established and formulated into  $H_\infty$  performances by minimizing the effect of external disturbance from the residual and a Lyapunov stability theory is utilized for switched system stability combined with the Average Dwell Time (ADT) concept. The observer parameter matrices are also determined by solving a convex optimization problem under Linear Matrix Inequalities (LMI) constraints. Finally, a numerical example and a Highly MAneuverable Technology (HiMAT) vehicle are investigated to prove the efficacy of the obtained results.

**Index Terms**—Switched systems, Sensor fault estimation, Descriptor observer, Average dwell time, Linear matrix inequalities,  $H_\infty$  control

## I. INTRODUCTION

Switched systems are a special class of hybrid dynamical systems. They are constituted of a finite number of subsystems combined with a switching signal that a logical law orchestrates transitions between them and indicates the active mode [1]. In fact, switched systems can be used to efficiently model many real-world physical process such as the electrical system dynamics, the mechanical system dynamics, a the chemical process industries [2-4].

Fault diagnosis techniques are used to enhance the reliability of practical engineering systems. Diagnosis methods can be divided into two main categories that are based on a priori process knowledge: model-based and non model-based. Observer-based fault diagnosis method is the most popular analytical scheme from model-based fault detection. By using an analytical redundancy scheme, a comparison between the measured signals and their estimated from the mathematical model is done and the filtered difference constitutes a fault indicating signal which is referred to as a residual signal. When the system is in normal operation, the residual should be zero-valued and nonzero when a fault occurs in the system

[5]. For the fault reconstruction or estimation task, various advanced observer-based approaches have been proposed in the literature, such as proportional and integral observers [6], unknown input observers [7], adaptive observers [8], fuzzy observers [9], sliding mode observers [10] and descriptor observers [11].

More recently, the problem of Fault Diagnosis and Identification has attracted more and more attention in both industry and academic communities due to the increasing demand for safety and reliability in a wider engineering application and several research papers on this subject have been published. [12-20]. For example, in [12] a robust state estimation and actuator fault detection has been developed for a class of uncertain switched systems using the Average Dwell Time (ADT) concept. Authors in [13] have dealt with the problem of state and sensor fault estimation for a class of switched linear systems with unknown inputs. In [14], two types of observer approaches are developed for switched systems that investigate their state and fault estimation problems. The first one was based on a linear descriptor reduced-order observer to generate the exact estimation of state and sensor faults simultaneously. The second one was based on a descriptor sliding mode observer approach to reconstruct actuator and sensor faults. The sensor fault estimation issue for switched systems is investigated in [15,16]. These papers present switched observers for simultaneously estimating system states and sensor faults. In [17], for switched power electronics systems, the authors proposed a Sliding Mode Observer (SMO). Based on the switched Lyapunov function technique, the sufficient conditions for the stability of the developed observer with a prescribed  $H_\infty$  performance level are formulated in terms of linear matrix inequalities (LMIs). The authors in [18], proposed an Unknown Input Observer (UIO) design framework based on robust fault estimation observer for a class of continuous-time switched systems. By using the ADT approach and the switched Lyapunov function technique sufficient conditions for the  $H_\infty$  fault estimation observer are obtained in the form of LMIs. In [19], the

adaptive actuator and sensor fault estimation observer issue has been investigated for continuous-time switched systems despite the measurement disturbances. A robust sliding mode observer-based fault detection for a boost converter was considered in [20] by using the ADT concept with respect  $H_\infty$  performance index to minimize disturbance effect from residual signal and a Lyapunov functional technique is also used for switched systems stability.

Motivated by the aforementioned discussions, in this work, we address the state/sensor fault estimation issue for a class of switched linear systems with unknown bounded disturbances. The main contributions of this paper are outlined as follows:

- (i) An augmented singular switched system is created by including the sensor faults vector as part of a new state variable.
- (ii) A switched descriptor observer is developed for the augmented switched systems to reconstruct the augmented state vector.
- (iii) Under the ADT method and the Lyapunov stability theory, the switching signal is designed, and sufficient conditions are provided to guarantee the globally uniformly asymptotically stable (GUAS) of the observer error system with a prescribed  $H_\infty$  performance index.
- (iv) The designed fault estimation algorithm can reconstruct any type of fault.

The remainder of this paper is summarized as follows: Section 2, briefly describes the problem statement and preliminaries. The main results of designing the descriptor observer are presented in Section 3. In Section 4, simulation results are shown with two examples to validate the correctness and performance of the proposed design techniques. Finally, the conclusion and future work are given in Section 5.

## II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the continuous-time switched system described by:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + R_{\sigma(t)}d(t) \\ y(t) = C_{\sigma(t)}x(t) + F_s f_s(t) \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the the state vector,  $u(t) \in \mathbb{R}^p$  is the control input vector,  $y(t) \in \mathbb{R}^m$  is the measurable output vector,  $f_s(t) \in \mathbb{R}^l$  is the sensor fault vector and  $d(t) \in \mathbb{R}^q$  represents the unknown bounded disturbance vector. The switching signal  $\sigma(t) : \mathbb{R}^+ \rightarrow \Theta$  is assumed to be a piecewise continuous from the right function of time. Where  $\Theta$  is a finite set described by  $\Theta = \{1, 2, \dots, N\}$  and  $N$  denotes the number of subsystems. When  $\sigma(t) = i$ , the  $i$ -th subsystem is activated for some  $i \in \Theta$ .

In this case, the switched system (1) can be represented by:

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + R_i d(t) \\ y(t) = C_i x(t) + F_s f_s(t) \end{cases} \quad (2)$$

$A_i$ ,  $B_i$ ,  $R_i$ ,  $C_i$  and  $F_s$  are all known constant matrices with appropriate dimensions.

In this framework, we consider the state/sensor fault estimation problem for the switched system. These issues

are investigated using the average dwell time concept. The following definition serves as an introduction to this concept.

**Definition 1.** [21] For any switching signal  $\sigma(t)$  and any  $\tau_2 \geq \tau_1 \geq 0$ , let  $N_{\sigma(t)}(\tau_1, \tau_2)$  denote the number of switchings  $\sigma(t)$  on an interval  $(\tau_1, \tau_2)$ . If

$$N_{\sigma(t)}(\tau_1, \tau_2) \leq N_0 + \frac{\tau_2 - \tau_1}{\tau_a} \quad (3)$$

holds for a given two positive numbers  $N_0$  and  $\tau_a$ , then the constant  $\tau_a$  is called the average dwell time and  $N_0$  is the chattering bound.

For the main object of this paper, we need the following lemma and assumption:

**Lemma 1.** [22] Suppose that there exist  $C^1$  functions  $V_{\sigma_i(t)} : \mathbb{R}^n \rightarrow \mathbb{R}$  and two class  $K_\infty$  functions  $\kappa_1$  and  $\kappa_2$  and two positive numbers  $\alpha > 0$ ,  $\mu > 1$  such that we have

$$\kappa_1(|x(t)|) \leq V_{\sigma_i(t)}(x(t)) \leq \kappa_2(|x(t)|) \quad (4)$$

$$\dot{V}_{\sigma_i(t)}(x(t)) \leq -\alpha V_{\sigma_i(t)}(x(t)) \quad (5)$$

and  $\forall (\sigma_i(t_l) = i, \sigma_i(t_l^-) = j) \in N \times N, i \neq j$ ,

$$V_i(x(t)) \leq \mu V_j(x(t)) \quad (6)$$

then the switched linear system  $\dot{x}(t) = f_{\sigma_i(t)}(x(t))$  is Globally Uniformly Asymptotically Stable (GUAS) for any switching signal with average dwell time:

$$\tau_a \geq \tau_a^* = \frac{\ln \mu}{\alpha} \quad (7)$$

**Assumption 1.** [23] The matrix  $F_s$  is full column rank.

Using the descriptor approach, an augmented system is built to estimate both the system states and the sensor fault at the same time. Then the switched linear system (2) with sensor fault is given by:

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = \bar{A}_i\bar{x}(t) + \bar{B}_i u(t) + \bar{R}_i d(t) + \bar{D}_s x_s(t) \\ y(t) = \bar{C}_i \bar{x}(t) \end{cases} \quad (8)$$

where

$$\bar{E} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & -I_m \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$$

$$\bar{R}_i = \begin{bmatrix} R_i \\ 0 \end{bmatrix}, \bar{D}_s = \begin{bmatrix} 0 \\ I_m \end{bmatrix}, \bar{C}_i = [C_i \quad I_m]$$

$$x_s(t) = F_s f_s(t), \bar{x}(t) = \begin{bmatrix} x(t) \\ x_s(t) \end{bmatrix}$$

$x_s(t)$  are assumed as an auxiliary state of the system (2).

In this study, we construct a robust switched descriptor observer that can estimate states and sensor faults simultaneously. Hence, consider the following switching descriptor

observer:

$$\begin{cases} E\dot{z}(t) = N_i z(t) + G_i u(t) + L_i y(t) \\ \dot{\hat{x}}(t) = z(t) + Ty(t) \\ r(t) = y(t) - \hat{y}(t) \\ \hat{f}_s(t) = (F_s^T F_s)^{-1} F_s^T [0 \quad I_m] \hat{x}(t) \end{cases} \quad (9)$$

where  $z(t) \in \mathbb{R}^{n+m}$  is the auxiliary state of the observer,  $\hat{x}(t) \in \mathbb{R}^{n+m}$  is the estimation vector of augmented system (4),  $\hat{y}(t) \in \mathbb{R}^m$  is the estimated output vector,  $\hat{f}_s(t) \in \mathbb{R}^l$  is the estimated vector of sensor faults and  $E \in \mathbb{R}^{(n+m) \times (n+m)}$ ,  $N_i \in \mathbb{R}^{(n+m) \times (n+m)}$ ,  $G_i \in \mathbb{R}^{(n+m) \times p}$ ,  $L_i \in \mathbb{R}^{(n+m) \times m}$  and  $T \in \mathbb{R}^{(n+m) \times m}$  are the observer parameter matrices to be determined later.

From (9),  $z(t)$  can be written as:

$$z(t) = \hat{x}(t) - Ty(t) \quad (10)$$

so

$$E\dot{z}(t) = E\dot{\hat{x}}(t) - ET\dot{C}\hat{x}(t) \quad (11)$$

and then,

$$\bar{E}\dot{\hat{x}}(t) - E\dot{z}(t) = (ET\bar{C} + \bar{E})\dot{\hat{x}}(t) - E\dot{\hat{x}}(t) \quad (12)$$

Based on reference [24], equation (12) is equivalent to:

$$\bar{E}\dot{\hat{x}}(t) - E\dot{z}(t) = E(\dot{\hat{x}}(t) - \dot{\hat{x}}(t)) \quad (13)$$

such that:

$$E = ET\bar{C} + \bar{E} \quad (14)$$

We define  $\bar{e}_s(t) = \bar{x}(t) - \hat{x}(t)$  and according to (8) and (9), equation (13) will be expressed as follows:

$$\begin{aligned} E\dot{\bar{e}}_s(t) &= \bar{A}_i \bar{x}(t) + \bar{B}_i u(t) + \bar{R}_i d(t) + \bar{D}_s x_s(t) \\ &\quad - N_i z(t) - G_i u(t) - L_i y(t) \end{aligned} \quad (15)$$

The above equation (15) becomes after some calculations

$$\begin{aligned} E\dot{\bar{e}}_s(t) &= (\bar{A}_i + (N_i T - L_i)\bar{C}_i - N_i)\bar{x}(t) + N_i \bar{e}_s(t) \\ &\quad + \bar{R}_i d(t) + \bar{D}_s x_s(t) + (\bar{B}_i - G_i)u(t) \end{aligned} \quad (16)$$

If the following conditions is satisfied  $\forall i \in \Theta$ :

$$G_i = \bar{B}_i \quad (17)$$

$$N_i = \bar{A}_i + M_i \bar{C}_i \quad (18)$$

with,

$$M_i = N_i T - L_i \quad (19)$$

Thus, the dynamic of augmented state estimation error can be reduced as:

$$E\dot{\bar{e}}_s(t) = N_i \bar{e}_s(t) + \bar{R}_i d(t) + \bar{D}_s x_s(t) \quad (20)$$

Considering that the matrix  $E$  nonsingular and if exists  $E_1 = E^+$ , the error dynamic can be given by:

$$\dot{\bar{e}}_s(t) = E_1 N_i \bar{e}_s(t) + E_1 \bar{R}_i d(t) + E_1 \bar{D}_s x_s(t) \quad (21)$$

To find simultaneously gain matrices  $E_1$  and  $T$  from equations (14) and (21), one can define the following augmented matrix:

$$\begin{bmatrix} E_1 & T \end{bmatrix} \begin{bmatrix} \bar{E} & \bar{D}_s \\ \bar{C} & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix} \quad (22)$$

A solution  $\begin{bmatrix} E_1 & T \end{bmatrix}$  exists if [24]:

$$\text{rank} \begin{bmatrix} \bar{E} & \bar{D}_s \\ \bar{C} & 0 \end{bmatrix} = n + m + \text{rank}(\bar{D}_s) \quad (23)$$

Then,  $E_1$  and  $T$  can be expressed by the following system:

$$\begin{bmatrix} E_1 & T \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} \bar{E} & \bar{D}_s \\ \bar{C} & 0 \end{bmatrix}^+ \quad (24)$$

We define  $S_i = E_1 N_i$  and  $H_i = E_1 \bar{R}_i$ , then based on (21) and (22), we have

$$\dot{\bar{e}}_s(t) = S_i \bar{e}_s(t) + H_i d(t) \quad (25)$$

Our goal here is to ensure the observer robustness, so the dynamics of the observation error and the generated residual signal can be expressed as:

$$\begin{cases} \dot{\bar{e}}_s(t) = S_i \bar{e}_s(t) + H_i d(t) \\ r(t) = \bar{C}_i \bar{e}_s(t) \end{cases} \quad (26)$$

### III. MAIN RESULTS

This section discusses how to design a robust sensor fault observer for continuous-time switched systems. The sufficient conditions about the existence of the considered observer are derived using the Lyapunov stability theory and ADT concept which allows formalizing as an LMI optimization problem. The observer parameter can be obtained by solving a set of LMIs presented by the following theorem.

**Theorem 1.** Consider system (26), for given constants scalars  $\gamma > 0$ ,  $\mu > 1$  and  $\alpha > 0$ , if there exist definite matrices  $P_i = P_i^T > 0$ , matrices  $Q_i$  and matrices  $M_i = (P_i E_1)^{-1} Q_i$  such that the following LMIs are satisfied  $\forall (i, j) \in \Theta \times \Theta, i \neq j$ :

$$P_i \leq \mu P_j \quad (27)$$

$$\Psi_i = \begin{bmatrix} \Psi_{11i} & P_i E_1 \bar{R}_i & \bar{C}_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (28)$$

where

$$\Psi_{11i} = (E_1 \bar{A}_i)^T P_i + P_i (E_1 \bar{A}_i) + \bar{C}_i^T Q_i^T + Q_i \bar{C}_i + \alpha P_i \quad (29)$$

Then, the observer error dynamics is globally uniformly asymptotically stable with an  $H_\infty$  disturbance attenuation level  $\gamma$  if the ADT satisfies the following condition  $\tau_a \geq \tau_a^* = \frac{\ln \mu}{\alpha}$ . The observer gain matrix of (9) can be given by:

$$N_i = \bar{A}_i + M_i \bar{C}_i \quad (30)$$

$$L_i = N_i T_i - M_i \quad (31)$$

**Proof.** Consider the switched Lyapunov function for the error system (25) as

$$V_i(\bar{e}_s(t)) = \bar{e}_s^T(t) P_i \bar{e}_s(t) \quad (32)$$

where  $P_i = P_i^T > 0$ .

The derivate of  $V_i(\bar{e}_s(t))$  is

$$\dot{V}_i(\bar{e}_s(t)) = \dot{\bar{e}}_s^T(t) P_i \bar{e}_s(t) + \bar{e}_s^T(t) P_i \dot{\bar{e}}_s(t) \quad (33)$$

Using the differentiel equation (25) generating  $\bar{e}_s(t)$  we obtain

$$\begin{aligned} \dot{V}_i(\bar{e}_s(t)) &= \bar{e}_s^T(t) (S_i^T P_i + P_i S_i) \bar{e}_s(t) \\ &+ \bar{e}_s^T(t) P_i H_i d(t) + d^T(t) H_i^T P_i \bar{e}_s(t) \end{aligned} \quad (34)$$

Satisfying the robustness of the residual signal to disturbance, the following LMI condition must be respected for given positive scalar  $\gamma$

$$\|r(t)\|_2^2 - \gamma^2 \|d(t)\|_2^2 \leq 0 \quad (35)$$

For the sake of guaranteeing observer robustness according to formula (35), we impose the following performance index:

$$J = \int_0^t r^T(t) r(t) - \gamma^2 \int_0^t d^T(t) d(t) \quad (36)$$

By (35), which means

$$r^T(t) r(t) - \gamma^2 d^T(t) d(t) < 0 \quad (37)$$

For any nonzero  $d(t) \in l_2[0, \infty)$  and under zero initial conditions, the optimization algorithm of  $H_\infty$  performance index for the fault detection observer is combined with our switched system.

By (5) and (37), we get

$$r^T(t) r(t) - \gamma^2 d^T(t) d(t) + \dot{V}_i(\bar{e}_s(t)) + \alpha V_i(\bar{e}_s(t)) < 0 \quad (38)$$

Substituting  $r(t)$ ,  $V_i(\bar{e}_s(t))$  and  $\dot{V}_i(\bar{e}_s(t))$  from (26), (32) and (34) in (38), one can get the following inequality:

$$\begin{aligned} &\bar{e}_s^T(t) \bar{C}_i^T \bar{C}_i \bar{e}_s(t) - \gamma^2 d^T(t) d(t) + \alpha \bar{e}_s^T(t) P_i \bar{e}_s(t) \\ &+ \bar{e}_s^T(t) (S_i^T P_i + P_i S_i) \bar{e}_s(t) + d^T(t) H_i^T P_i \bar{e}_s(t) \\ &+ \bar{e}_s^T(t) P_i H_i d(t) < 0 \end{aligned} \quad (39)$$

The inequality (39) can be written as

$$\begin{bmatrix} \bar{e}_s(t) \\ d(t) \end{bmatrix}^T \Psi_i \begin{bmatrix} \bar{e}_s(t) \\ d(t) \end{bmatrix} < 0 \quad (40)$$

Then condition (40) is satisfied if the following inequalities are met:

$$\Psi_i = \begin{bmatrix} S_i^T P_i + P_i S_i + \bar{C}_i^T \bar{C}_i + \alpha P_i & P_i H_i \\ H_i^T P_i & -\gamma^2 I \end{bmatrix} < 0 \quad (41)$$

Using Schur complement, the following inequality can be

obtained:

$$\Psi_i = \begin{bmatrix} S_i^T P_i + P_i S_i + \alpha P_i & P_i H_i & \bar{C}_i^T \\ H_i^T P_i & -\gamma^2 I & 0 \\ \bar{C}_i & 0 & -I \end{bmatrix} < 0 \quad (42)$$

Finally, to convert the nonlinear matrix inequality (42) in the form of LMIs, we first substitute the values of define  $S_i = E_1 N_i$ ,  $N_i = \bar{A}_i + M_i \bar{C}_i$  and  $H_i = E_1 \bar{R}_i$  and taking the change of variables  $Q_i = P_i E_1 M_i$ , we can obtain the inequality (28) given in **Theorem 1** which achieves the proof.

#### IV. AN ILLUSTRATIVE EXAMPLES

In this section, two examples will be considered to demonstrate the validity and applicability of the obtained theoretical results. The first one is a numerical example and the second one is a Highly Maneuverable Technology (HiMAT) vehicle with various sensor fault shapes.

##### A. Example 1: A numerical example

Suppose that the continuous-time switched system (2) with unknown bounded disturbance and sensor faults are borrowed from [13], which consist of two subsystems and the parameters are given by

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ -2 & -1 & 0 \\ -2 & 1 & -8 \end{bmatrix}, A_2 = \begin{bmatrix} -3 & 2 & 2 \\ -1 & -3 & 0.2 \\ 0.5 & 2 & -5 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -1 & -3 \\ 1 & 2.5 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} -0.15 & 0 \\ 1 & 0.05 \\ 1 & 0.5 \end{bmatrix}$$

$$R_1 = R_2 = [1 \ 1 \ 1]^T$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, F_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By using MATLAB LMI Toolbox and solving the optimization problems (27) and (28) derived in **Theorem 1**, we can obtain the parameters  $P_i$ ,  $Q_i$  and the switched descriptor observer gains matrices can be calculated with the  $H_\infty$  performance level  $\gamma = 0.15$ .

For the switching signal, by choosing the scalars  $\mu = 2.5$  and  $\alpha = 0.8$ , it is easy to calculate the average dwell time  $\tau_a^* = \frac{\ln \mu}{\alpha} = 1.1454$ . Thus, let us generate a possible switching signal sequence  $\sigma(t)$  with ADT  $\tau_a = 1.5 > 1.1454$  as shown in Fig. 1 specifies the switching between the two subsystems.

The known input vector  $u(t)$  is assigned as:

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 2 \sin(t + 1.8) \\ 1.5 \cos(2t + 3.6) \end{bmatrix}$$

To illustrate the robustness of the developed method, the unknown disturbance input  $d(t)$  is assumed to be Gaussian

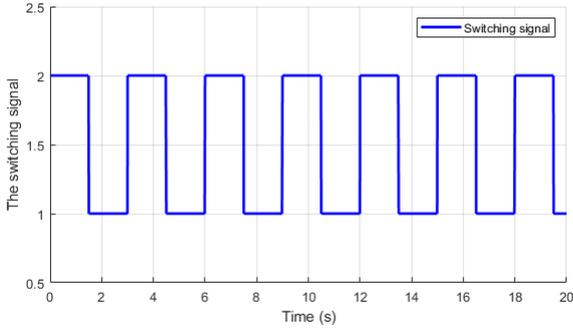


Fig. 1. Switching signal evolution

noise.

Let's consider the first-time-varying sensor fault as follows:

$$f_1(t) = \begin{cases} 0, & 0s \leq t < 11s \\ 1.8 \cos(1.6t + 3.25), & 11s \leq t < 18s \\ 0, & 18s \leq t \leq 20s \end{cases}$$

In this case, the sensor fault  $f_1(t)$  and its estimated  $\hat{f}_1(t)$  are described in Fig. 2.

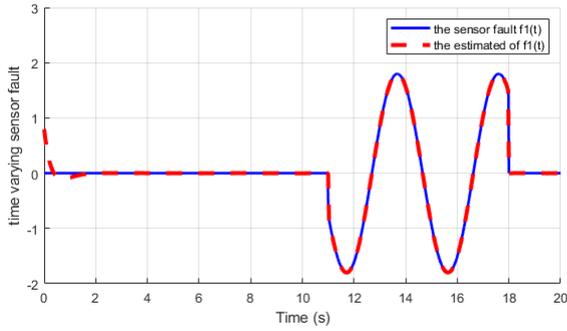


Fig. 2. The sensor fault  $f_1(t)$  and its estimate  $\hat{f}_1(t)$

Consider now the second time-varying sensor fault described as:

$$f_2(t) = \begin{cases} 0, & 0s \leq t < 5s \\ -0.2t + 2.5, & 5s \leq t < 10s \\ 0, & 10s \leq t \leq 20s \end{cases}$$

In this case, the sensor fault  $f_2(t)$  and its estimated  $\hat{f}_2(t)$  are described in Fig. 3.

As shown in Figs. 2 and 3, the two faults which are considered in this part are rapidly and accurately estimated.

Figs. 4-6 express the trajectories of the original system states and their estimates. It can be seen from these figures that the developed switched descriptor observer can reconstruct the states very accurately, even after the occurrence of a sensor fault.

### B. Example 2: A case study of HiMAT vehicle

Now, we consider the HiMAT vehicle system proposed in [25]. This vehicle can be modeled as a switched system

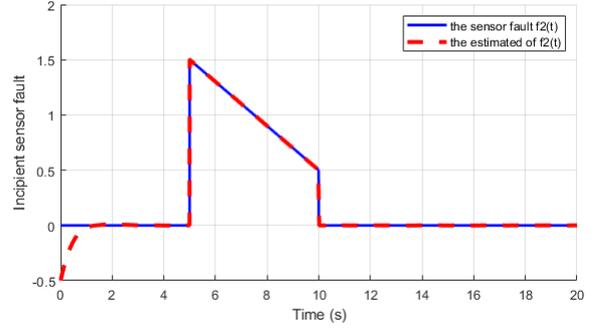


Fig. 3. The sensor fault  $f_2(t)$  and its estimate  $\hat{f}_2(t)$

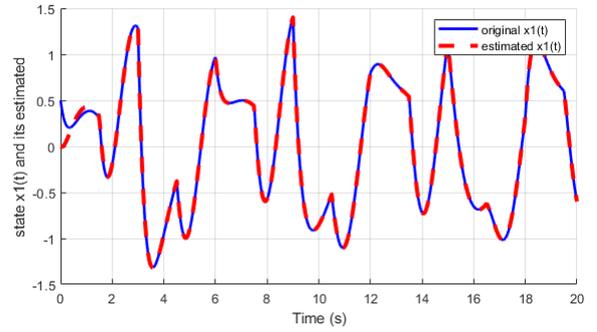


Fig. 4. The state  $x_1(t)$  and its estimate  $\hat{x}_1(t)$

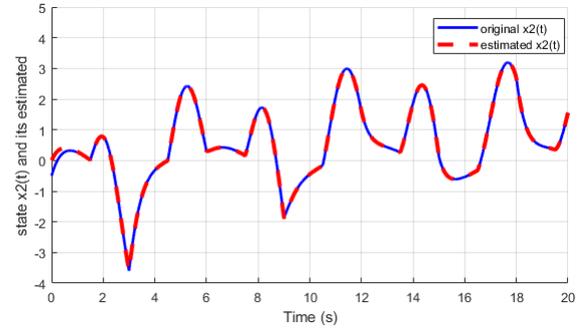


Fig. 5. The state  $x_2(t)$  and its estimate  $\hat{x}_2(t)$

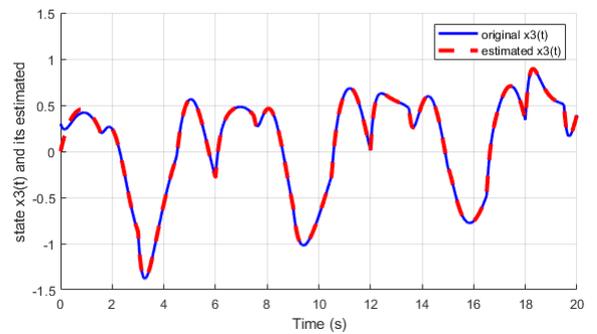


Fig. 6. The state  $x_3(t)$  and its estimate  $\hat{x}_3(t)$

with two modes. Let  $x(t) = [x_1(t) \ x_2(t)]^T$  be the state vector, where the states  $x_1(t)$  is the angle of attack and  $x_2(t)$  represents the pitch rate. Then, the HiMAT vehicle system can be described by (2) with:

$$A_1 = \begin{bmatrix} -1.35 & -0.98 \\ 17.1 & -1.85 \end{bmatrix}, A_2 = \begin{bmatrix} -1.87 & -0.98 \\ 12.6 & -2.63 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1.7 \\ 0.9 \end{bmatrix}, B_2 = \begin{bmatrix} 1.9 \\ 3.8 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_1 = R_2 = \begin{bmatrix} 0.8 \\ 0.7 \end{bmatrix}, F_s = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The proposed switched descriptor observer (9) is designed by solving the LMIs inequalities from Theorem 1. The parameters values are chosen to:  $\mu = 4$ ,  $\alpha = 11$  and  $\gamma = 0.67$ , one has  $\tau_a^* = \frac{\ln \mu}{\alpha} = 0.126$ . For simulation purposes, a possible switching signal with ADT of  $\tau_a = 0.5$  is generated and it is shown as in Fig. 7.

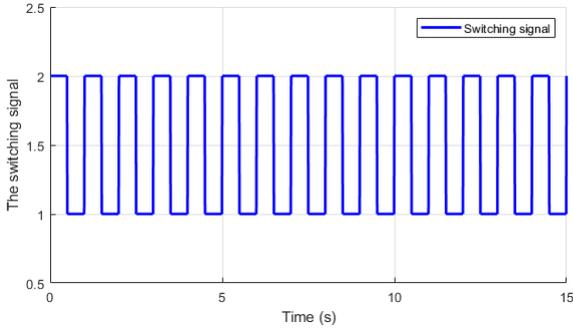


Fig. 7. Switching signal evolution

The known input vector  $u(t)$  is assigned as  $u(t) = 1.5 \sin(2.3t + 0.8)$ . To illustrate the robustness of the designed method, the disturbance input  $d(t)$  is assumed as a Gaussian distributed noise signal with a null average and variance equal to  $10^{-2}$ .

In this example, we consider two cases with different sensor fault types. In the first case, we consider the following constant sensor fault:

$$f_1(t) = \begin{cases} 0, & 0s \leq t < 4s \\ 1, & 4s \leq t \leq 15s \end{cases}$$

Fig. 8 gives the constant sensor fault and its estimation.

In the second scenario, we consider a time-varying sensor fault, which is described as follows:

$$f_2(t) = \begin{cases} 0, & 0s \leq t < 4s \\ \cos(1.6t + 2.25), & 4s \leq t \leq 15s \end{cases}$$

Fig. 9 depicts the time-varying sensor fault and its estimation.

It shows from Figs. 8 and 9 that the switched descriptor observer developed in this paper has good accurate estimation performance to reconstruct the sensor fault for both constant sensor faults and time-varying sensor faults.

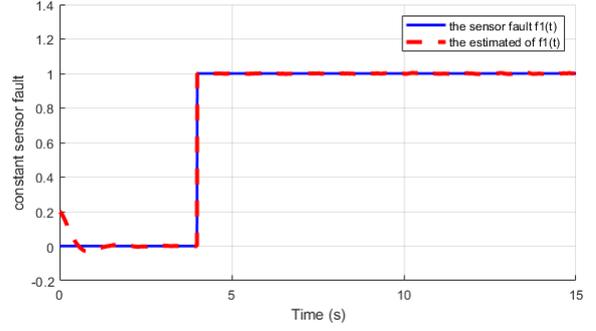


Fig. 8. The constant sensor fault  $f_1(t)$  and its estimate  $\hat{f}_1(t)$

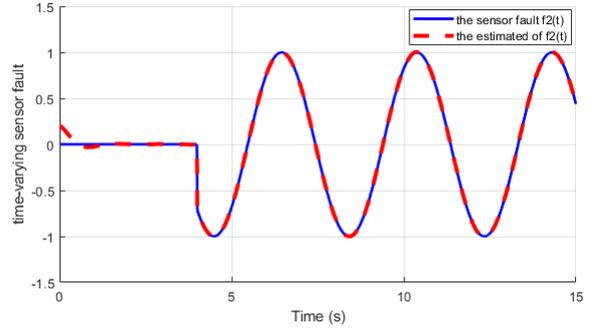


Fig. 9. The time-varying sensor fault  $f_2(t)$  and its estimate  $\hat{f}_2(t)$

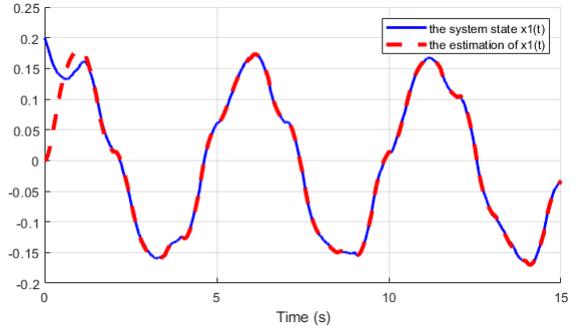


Fig. 10. The state  $x_1(t)$  and its estimate  $\hat{x}_1(t)$

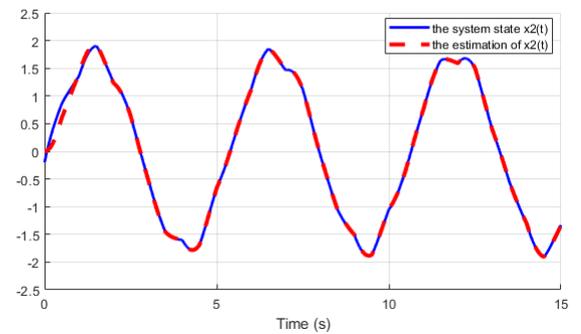


Fig. 11. The state  $x_2(t)$  and its estimate  $\hat{x}_2(t)$

The estimation curves of system states are presented in Figs. 10 and 11. we can see from these figures the designed switched descriptor observer can estimate the system states vector very accurately.

Through the above simulation, it is obvious that the designed switched descriptor observer can perfectly reconstruct the system states and the sensor fault simultaneously.

#### CONCLUSION

In this paper, the issues of state/sensor fault reconstruction has been considered for a class of continuous-time switched systems subject to external disturbances using a descriptor approach. The weighted  $H_\infty$  performance criterion is implemented to achieve the robust performance of residual signal to disturbances. LMI-based sufficient stability conditions are given to obtain the observer parameter matrices while taking the average dwell time switching into account. Finally, two simulation examples are provided to demonstrate the applicability and effectiveness of the designed approach. In our future research, the developed approach will be extended to the construction of fault-tolerant controllers.

#### REFERENCES

- [1] R. Goebel, R. G. Sanfelice and A. R. Teel, "Hybrid dynamical systems," in *IEEE Control Systems Magazine*, vol. 29, no. 2, pp. 28-93, April 2009.
- [2] Y. Eddoukali, A. Benzaouia and M. Ouladsine, "Integrated fault detection and control design for continuous-time switched systems under asynchronous switching," *ISA transactions*, vol. 84, pp. 12-19, October 2018.
- [3] J. Li, X. Guo, C. Chen, Q. Su, "Robust fault diagnosis for switched systems based on sliding mode observer," *Applied Mathematics and Computation*, vol. 341, p. 193-203, January 2019.
- [4] Z. Sun and S. S. Ge, "Analysis and synthesis of switched linear control systems," *Automatica*, vol. 41, no 2, pp. 181-195, February 2005.
- [5] J. Chen and R. J. Patton, "Robust model-based fault diagnosis for dynamic systems;" Vol. 3, *Springer Science and Business Media*, November 2012.
- [6] K. Zhang, B. Jiang, V. Cocquempot and H. Zhang, "A framework of robust fault estimation observer design for continuous-time/discrete-time systems," *Optimal control applications and methods*, vol. 34, no 4, pp. 442-457, July 2013.
- [7] F. J. Bejarano and A. Pisano, "Switched observers for switched linear systems with unknown inputs," *IEEE Transactions on Automatic Control*, vol. 56, no 3, pp. 681-686, April 2011.
- [8] B. Jiang, M. Staroswiecki and V. Cocquempot, "Fault accommodation for nonlinear dynamic systems," *IEEE Transactions on Automatic Control*, vol. 51, no 9, pp. 1578-1583, October 2006.
- [9] K. Zhang, B. Jiang, P. Shi and J. Xu, "Analysis and Design of Robust  $H_\infty$  Fault Estimation Observer With Finite-Frequency Specifications for Discrete-Time Fuzzy Systems," *IEEE Transactions on Cybernetics*, vol. 45, no 7, pp. 1225-1235, September 2014.
- [10] X. G. Yan and C. Edwards, "Robust decentralized actuator fault detection and estimation for large-scale systems using a sliding mode observer," *International Journal of contro*, vol. 81, no 4, p. 591-606, April 2008.
- [11] J. Zhang, A. K. Swain and S. K. Nguang, "Robust  $H_\infty$  adaptive descriptor observer design for fault estimation of uncertain nonlinear systems," *Journal of the Franklin Institute*, vol. 351, no 11, pp. 5162-5181, November 2014.
- [12] Y. Chen, Y. Wang and J. Yang, "Robust state estimation and actuator fault detection for a class of uncertain switched linear systems," In : *2016 35th Chinese Control Conference (CCC)*, IEEE, pp. 1903-1908, July 2016.
- [13] J. Yang, Y. Chen, F. Zhu and F. Wang, "Simultaneous state and output disturbance estimations for a class of switched linear systems with unknown inputs. *International Journal of Systems Science*, vol. 48, no 1, pp. 22-33, February 2016.
- [14] S. Yin, H. Gao, J. Qiu and O. Kaynak, "Descriptor reduced-order sliding mode observers design for switched systems with sensor and actuator faults," *Automatica*, vol. 76, pp. 282-292, February 2017.
- [15] D. Du, B. Jiang and P. Shi, "Sensor fault estimation and compensation for time-delay switched systems," *International Journal of Systems Science*, vol. 43, no 4, pp. 629-640, April 2012.
- [16] H. Zhang, J. Han, Y. Wang and X. Liu, "Sensor fault estimation of switched fuzzy systems with unknown input," *IEEE Transactions on Fuzzy Systems*, vol. 26, no 3, pp. 1114-1124, June 2018.
- [17] J. Li, K. Pan and Q. Su, "Sensor fault detection and estimation for switched power electronics systems based on sliding mode observer," *Applied Mathematics and Computation*, vol. 353, pp. 282-294, July 2019.
- [18] D. Du, V. Cocquempot and B. Jiang, "Robust fault estimation observer design for switched systems with unknown input," *Applied Mathematics and Computation*, vol. 348, pp. 70-83, May 2019.
- [19] Y. Yang, D. Du, Y. Wu and H. Zhao, "Adaptive fault estimation observer design for switched systems with actuator and sensor faults," *2020 Chinese Control And Decision Conference (CCDC)*, pp. 3383-3388, August 2020.
- [20] S. Ali, M. T. Raza, G. Abbas, N. Ullah, S. Al Otaibi and H. Luo, "Sliding Mode Observer-Based Fault Detection in Continuous Time Linear Switched Systems," *Energies*, vol. 15, no 3, pp. 1090, February 2022.
- [21] P. Gahinet and P. Apkarian, "An linear matrix inequality approach to  $H_\infty$  control," *International journal of robust and nonlinear control*, vol. 4, no 4, pp. 421-448, July 1994.
- [22] L. Zhou, D. W. Ho and G. Zhai, "Stability analysis of switched linear singular systems," *Automatica*, vol. 49, no 5, pp. 1481-1487, May 2013.
- [23] M. Bouattour, M. Chadli, A. E. Hajjaji and M. Chaabane, " $H_\infty$  sensor faults estimation for T-S models using descriptor techniques: Application to fault diagnosis," *IEEE International Conference on Fuzzy Systems*, pp. 251-255, Aug 2009.
- [24] S. Makni, M. Bouattour, A. El Hajjaji and M. Chaabane, "Robust observer-based fault-tolerant tracking control for T-S uncertain systems subject to sensor and actuator faults," *ISA transactions*, vol. 88, pp. 1-11, November 2018.
- [25] A. A. Ladel, A. Benzaouia, R. Outbib and M. Ouladsine, "Integrated sensor fault estimation and control for continuous-time switched systems: A new separation principle," *ISA transactions*, vol. 112, pp. 224-233, June 2021.