



- ▶ WAVES 2019
- ▶ 14th International Conference on Mathematical and Numerical Aspects of Wave Propagation
- ▶ Vienna University of Technology (TU Wien), Vienna, Austria

DE LA RECHERCHE À L'INDUSTRIE

A high order impedance boundary condition with unique solution for the time harmonic Maxwell's equations

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- ▶ **The industrial context**
- ▶ **High Order Impedance Boundary Condition (HOIBC)**
- ▶ **Sufficient Uniqueness Conditions (SUC)**
- ▶ **Constrained optimisation problem**
- ▶ **Integrals Equations (IE) with HOIBC**
- ▶ **Numerical results**
- ▶ **Outlooks & conclusion**

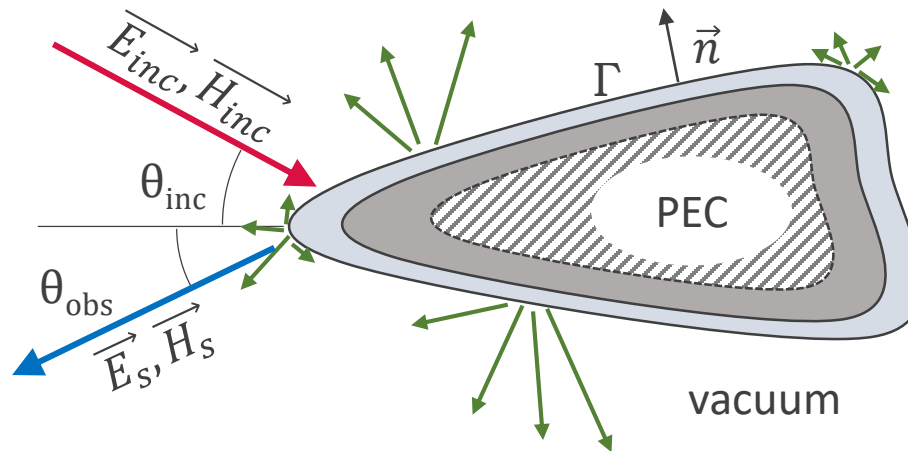
- ▶ \vec{u} is a vector of \mathbb{R}^2 or \mathbb{R}^3
- ▶ A, B (capital) are matrices
- ▶ Vectorial product is denoted by \times
- ▶ Let $\vec{\mathcal{H}} = \sqrt{\mu_0/\epsilon_0} \vec{H}$
- ▶ Time harmonic Maxwell ($e^{i\omega t}$) equations read ($k_0 = \omega\sqrt{\mu_0\epsilon_0}$)

$$\begin{cases} \vec{\nabla} \times \vec{E} + ik_0\mu_r \vec{\mathcal{H}} = 0 \\ \vec{\nabla} \times \vec{\mathcal{H}} - ik_0\epsilon_r \vec{E} = 0 \end{cases}$$

The industrial context

Commissariat à l'énergie atomique et aux énergies alternatives - www.cea.fr

Object : a coating of arbitrary materials over a smooth perfect electric conductor (PEC)

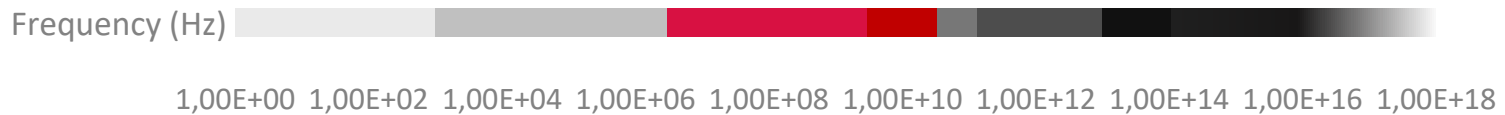


Goal: compute accurately the Radar Cross Section (RCS) of the coated object

- For any incidence angle
- For various materials
- For the high frequency spectrum and microwaves

$$\text{RCS} = \lim_{R \rightarrow \infty} 4\pi R^2 \left(\frac{|\vec{E}_s|^2}{|\vec{E}_{inc}|^2} \right)$$

■ Low
 ■ Radio
 ■ High
 ■ Microwave
 ■ Infrared
 ■ Visible light
 ■ Ultraviolet
 ■ X ray ...



This talk :

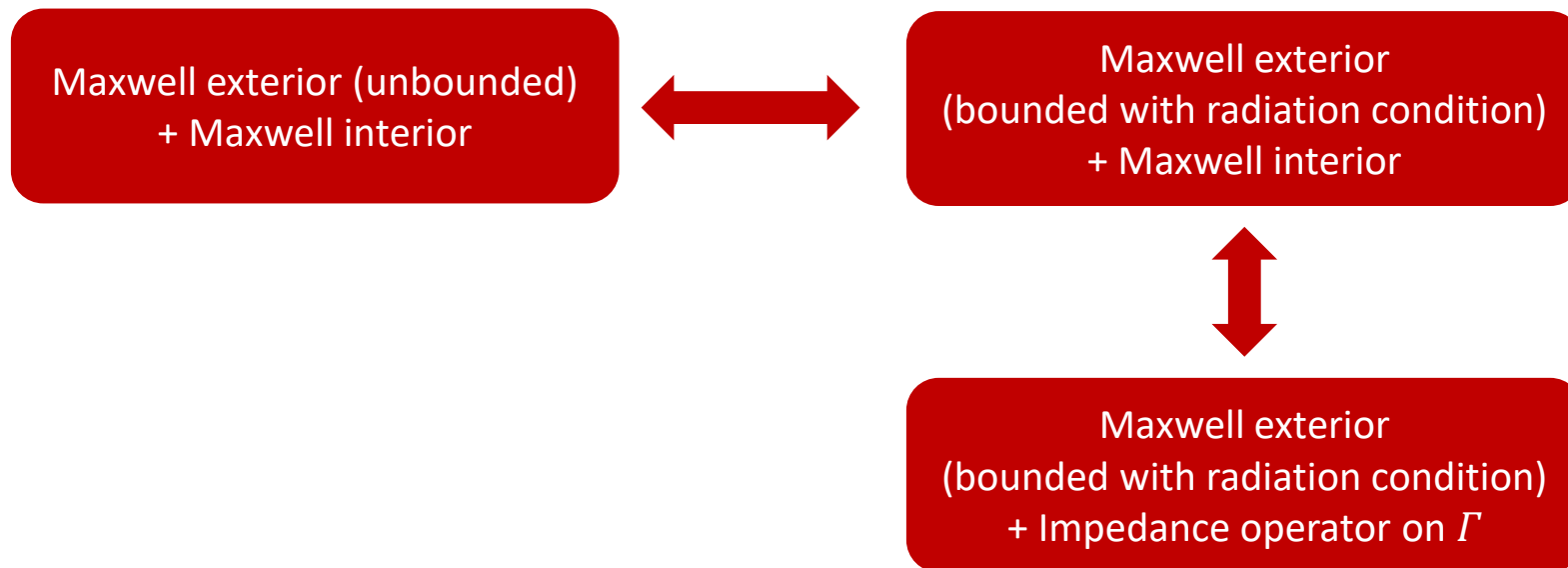
- ▶ Integral Equation + Impedance Boundary Condition (IBC)
 - ▶ Need to mesh only the exterior surface of the object
 - ▶ Accuracy depends essentially on the impedance boundary condition
 - ▶ Once integral equation solutions are obtained, RCS can be deduced

Another talk :

- ▶ *High order transmission conditions for a Domain Decomposition Method applied to an efficient and accurate solution of EM scattering problems.*
B. Stupfel, M. Lecouvez, CEA, WAVES, Vienna, 29/08/19

High Order Impedance Boundary Condition

From **M. Cessenat**, *Mathematical Methods in Electromagnetism: Linear Theory and Applications*, vol. 41. WORLD SCIENTIFIC, 1996, it is known that



A Calderon operator exists such that

$$\vec{E}_t(\vec{x}, \omega) = Op(\hat{Z})(\vec{n} \times \vec{H})(\vec{x}, \omega)$$

There are few cases where this operator can be used numerically

Under the following hypothesis

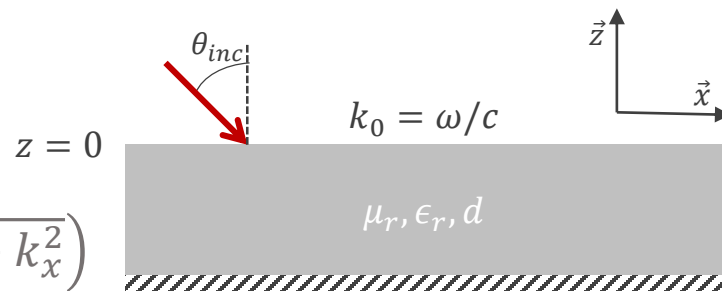
- Simple geometry: infinite plane, infinite cylinder, sphere
- Each layer has spatially uniform parameters (ϵ_r, μ_r , thickness)

One can deduce the expression of the Fourier multiplier \hat{Z} .

- **D. J. Hoppe and Y. Rahmat-Samii**, *Impedance boundary conditions in electromagnetics*. Taylor & Francis, 1995.
- **D. Cheng, Y. Zhao, and W. Lin**, *Spectral impedance boundary condition (SIBC) method for a spherical perfect electric conductor (PEC) with an uniform biisotropic coating*, J. Phys. III France, vol. 3, no. 9, pp. 1861–1870, 1993.

E.g for the following planar coating where

- $\eta_r = \sqrt{\mu_r/\epsilon_r}$, $k = k_0\sqrt{\mu_r\epsilon_r}$
- $\vec{k} = (k_x, 0, k_z) = (k_0\sin(\theta_{inc}), 0, \sqrt{k^2 - k_x^2})$



$$\hat{Z}(\vec{k}, \omega) = -\frac{i\eta_r \tan\left(\sqrt{k^2 - k_x^2} d\right)}{k\sqrt{k^2 - k_x^2}} \begin{bmatrix} k^2 - k_x^2 & 0 \\ 0 & k^2 \end{bmatrix}$$

- An impedance boundary condition (IBC) is an approximation of the impedance operator.
- The Leontovich IBC has a constant symbol ($\hat{Z}(\vec{k}, \omega) \simeq a_0$)
 - Exact for normal incidence
- HOIBC is defined as a polynomial fraction

$$(I + b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{E}_t = (a_0 I + a_1 \mathcal{L}_D - a_2 \mathcal{L}_R) (\vec{n} \times \vec{\mathcal{H}})$$

- **A. Aubakirov.** *Electromagnetic scattering problem with higher order impedance boundary conditions and integral methods*. General Mathematics. Université de Cergy Pontoise, 2014.

The following properties are used

- $\mathcal{L}_D \vec{U} = \vec{\nabla}_t \vec{\nabla}_t \cdot \vec{U} \quad \Rightarrow \quad \int_{\Gamma} \mathcal{L}_D \vec{U} \cdot \vec{U} \leq 0$
- $\mathcal{L}_R \vec{U} = \vec{\nabla}_t \times (\vec{\nabla}_t \times \vec{U}) \quad \Rightarrow \quad \int_{\Gamma} \mathcal{L}_R \vec{U} \cdot \vec{U} \geq 0$
- $\mathcal{L}_D \mathcal{L}_R = \mathcal{L}_R \mathcal{L}_D = 0$

The HOIBC gives the following Fourier multiplier

$$\hat{Z} = (I + b_1 \widehat{L}_D - b_2 \widehat{L}_R)^{-1} (a_0 I + a_1 \widehat{L}_D - a_2 \widehat{L}_R)$$

Where

$$\widehat{L}_D = - \begin{bmatrix} k_x^2 & 0 \\ 0 & 0 \end{bmatrix} \quad \widehat{L}_R = \begin{bmatrix} 0 & 0 \\ 0 & k_x^2 \end{bmatrix}$$

Our goal is to

Compute HOIBC coefficients that best fit \hat{Z} for any \vec{k}

Methods to compute HOIBC coefficients

▶ Taylor/Pade

- ▶ **O. Marceaux & B. Stupfel.** *High-order impedance boundary conditions for multilayer coated 3-D objects.* Antennas and Propagation, IEEE Transactions on. 48. 429 - 436. 2000.

▶ Collocation

- ▶ **A. Aubakirov.** *Electromagnetic scattering problem with higher order impedance boundary conditions and integral methods.* General Mathematics. Université de Cergy Pontoise, 2014.

▶ Least square minimisation

- ▶ **B. Stupfel,** *Implementation of High-Order Impedance Boundary Conditions in Some Integral Equation Formulations.* IEEE Transactions on Antennas and Propagation. (2015).

Sufficient Uniqueness Conditions

From **Cessenat**, we recall the General Sufficient Uniqueness Condition

$$\Re \left(\int_{\Gamma} \underbrace{(n(\vec{x}) \times \vec{\mathcal{H}}(\vec{x}))}_{\vec{j}} \cdot \overline{\vec{E}_t(\vec{x})} ds(\vec{x}) \right) \geq 0$$

This condition enforces uniqueness of Maxwell solution.

Using the HOIBC with the properties of $\mathcal{L}_D, \mathcal{L}_R$ operators, we will find SUC that enforces the GSUC.

The HOIBC reads

$$(I + b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{E}_t = (a_0 I + a_1 \mathcal{L}_D - a_2 \mathcal{L}_R) \vec{J}$$

► HOIBC with $a_1 = a_2 = b_1 = b_2 = 0$ (Leontovich)

$$\Re(a_0) \geq 0$$

► HOIBC with $b_1 = b_2 = 0$

- $\Re(a_0) \geq 0$
- $\Re(a_1) \leq 0$
- $\Re(a_2) \leq 0$

► HOIBC

- $-\Re(\bar{a}_0 z) \leq 0$
- $-\Re(b_1 \bar{a}_1) \leq 0$
- $\Re(b_1 a_0 \bar{a}_1 \bar{a}_2) \leq 0$
- $\Re(\bar{a}_1 z) \leq 0$
- $-\Re(b_2 \bar{a}_2) \leq 0$
- $\Re(b_2 a_0 \bar{a}_1 \bar{a}_2) \leq 0$
- $\Re(\bar{a}_2 z) \leq 0$
- $\Re(a_2) \leq 0$
- $\Re(a_1) \leq 0$

Where $z = |a_1| |a_2|^2 - b_1 a_0 \bar{a}_1 |a_2|^2 - b_2 a_0 \bar{a}_2 |a_2|^2$

► Other SUC exist

Find $x = (a_0, a_1, a_2, b_1, b_2) \in \mathbb{C}^5$ that minimises

$$F(x) = \frac{1}{2} \|Ax - b\|_2^2 \quad \forall x \in \underbrace{\mathbb{C}}_{SUC}$$

Where

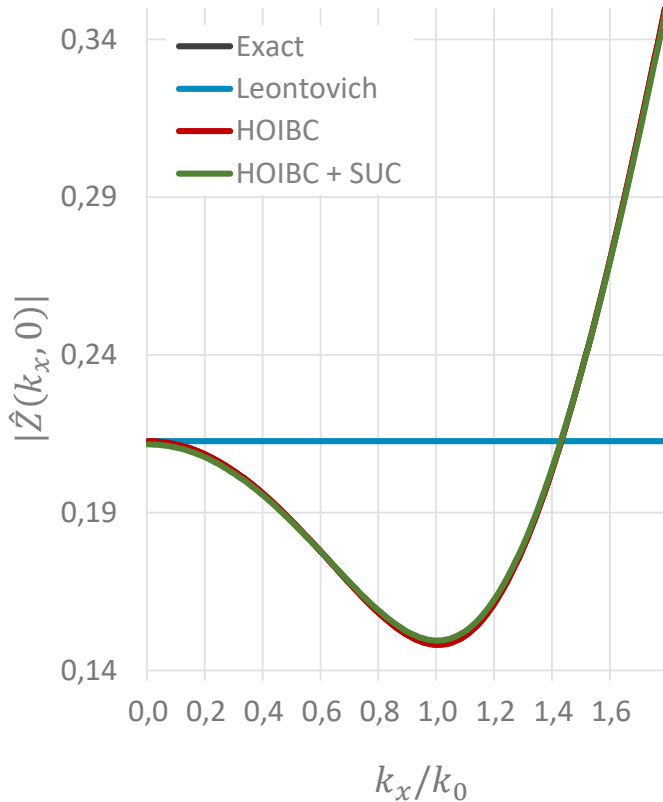
$$\begin{aligned} b_i &= \hat{Z}(\vec{k}_i)_{11} \\ b_{i+1} &= \hat{Z}(\vec{k}_i)_{12} \\ b_{i+2} &= \hat{Z}(\vec{k}_i)_{21} \\ b_{i+3} &= \hat{Z}(\vec{k}_i)_{22} \end{aligned}$$

$$\begin{aligned} A_{i,:} &= [1 \quad L_D(\vec{k}_i)_{11} \quad -L_R(\vec{k}_i)_{11} \quad -(\hat{Z}(\vec{k}_i)L_D(\vec{k}_i))_{11} \quad (\hat{Z}(\vec{k}_i)L_R(\vec{k}_i))_{11}] \\ A_{i+1,:} &= [0 \quad L_D(\vec{k}_i)_{12} \quad -L_R(\vec{k}_i)_{12} \quad -(\hat{Z}(\vec{k}_i)L_D(\vec{k}_i))_{12} \quad (\hat{Z}(\vec{k}_i)L_R(\vec{k}_i))_{12}] \\ A_{i+2,:} &= [0 \quad L_D(\vec{k}_i)_{21} \quad -L_R(\vec{k}_i)_{21} \quad -(\hat{Z}(\vec{k}_i)L_D(\vec{k}_i))_{21} \quad (\hat{Z}(\vec{k}_i)L_R(\vec{k}_i))_{21}] \\ A_{i+3,:} &= [1 \quad L_D(\vec{k}_i)_{22} \quad -L_R(\vec{k}_i)_{22} \quad -(\hat{Z}(\vec{k}_i)L_D(\vec{k}_i))_{22} \quad (\hat{Z}(\vec{k}_i)L_R(\vec{k}_i))_{22}] \end{aligned}$$

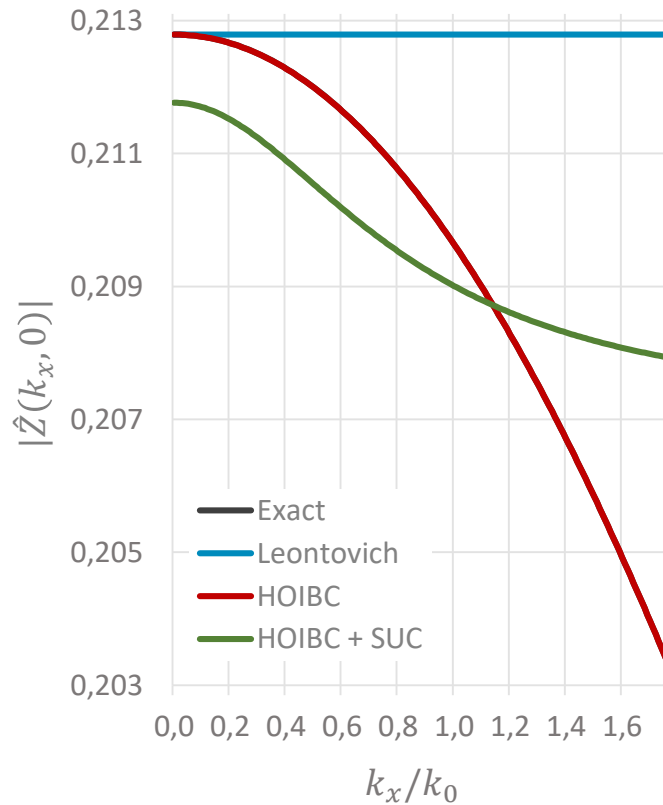
► Gradient based method (SLSQP)

$$\epsilon_r = 1 - i, \mu_r = 1, d = 5 \text{ cm}, f = 200 \text{ MHz}$$

TM polarisation



TE polarisation



L_2^2 relative error	Leontovich	HOIBC	HOIBC + SUC
TM	1.09E+00	4.33E-10	3.16E-05
TE	4.50E-04	3.55E-11	8.24E-05

One layered lossless materials ($\Im(\epsilon_r) = \Im(\mu_r) = 0$) are such that there exists $s_\infty \in \mathbb{R}$ where $\hat{Z}(k_0 s_\infty, 0) = \infty$

E.g., $\exists n \in \mathbb{N}$

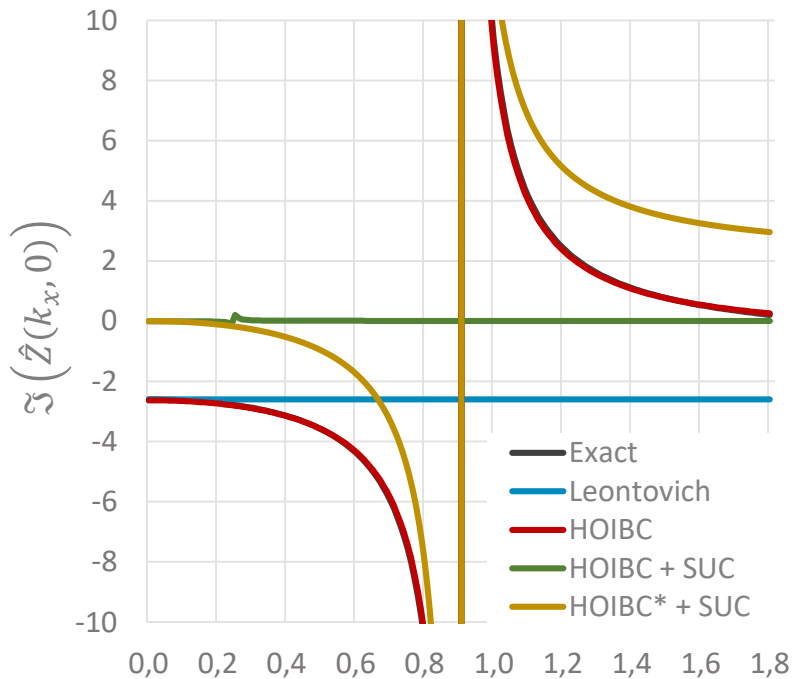
$$s_\infty = \sqrt{\epsilon_r \mu_r - \left(\frac{\pi/2 + n\pi}{k_0 d} \right)^2}$$

It is mandatory to force $b_1 = b_2 = (k_0 s_\infty)^{-2}$ and remove them from the optimisation

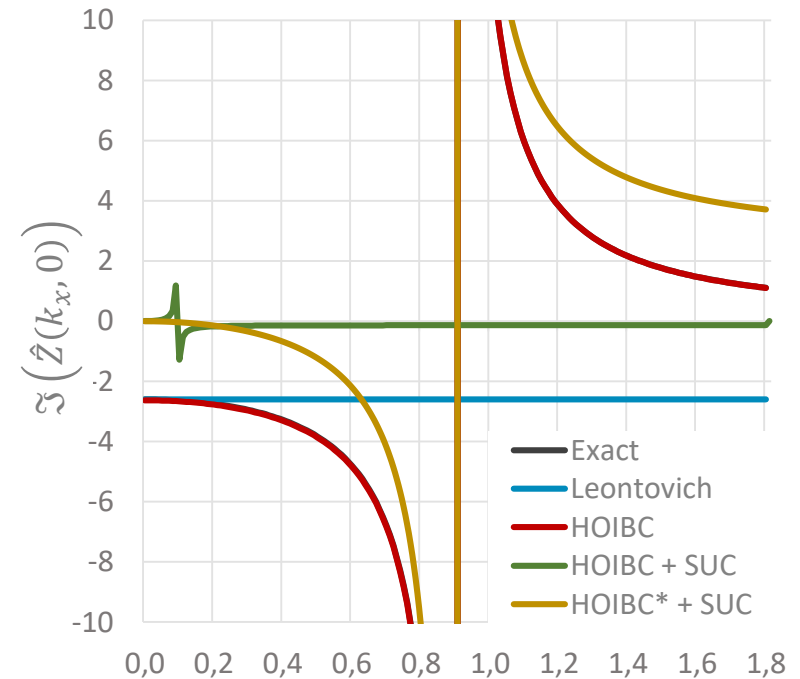
P. Soudais, ICEAA Conference, Verona, 2017

$$\epsilon_r = 4, \mu_r = 1, d = 3.5 \text{ mm}, f = 12 \text{ GHz}$$

TM polarisation



TE polarisation



k_x/k_0

k_x/k_0

L_2^2 relative error

Leontovich

HOIBC

HOIBC + SUC

HOIBC* + SUC

TM

9.95E-01

6.41E-04

1.00E+00

9.81E-03

TE

9.98E-01

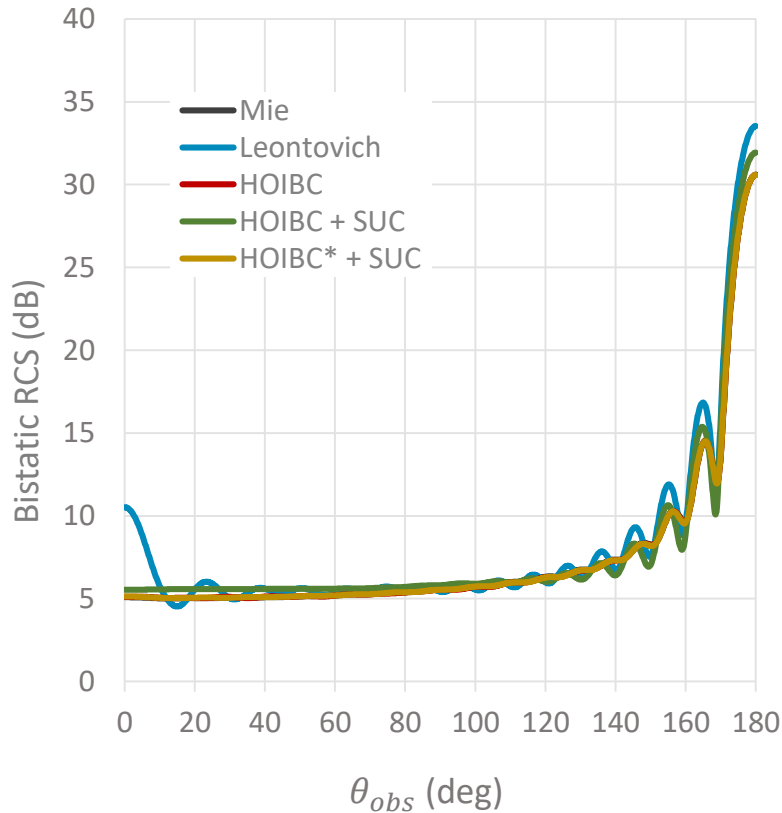
7.46E-06

1.00E+00

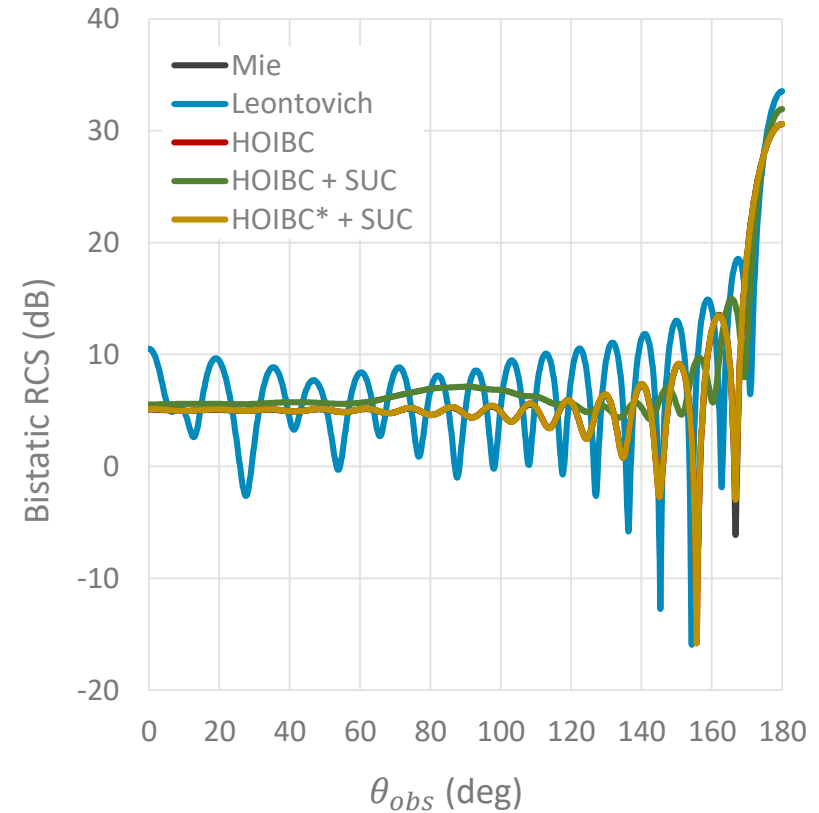
6.01E-03

$$\epsilon_r = 4, \mu_r = 1, d = 3.5 \text{ mm}, f = 12 \text{ GHz}, \theta_{inc} = 0^\circ$$

TM polarisation



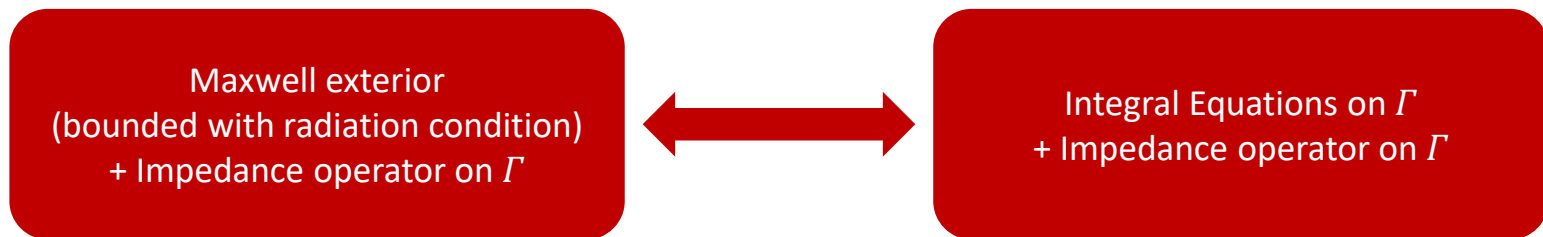
TE polarisation



L_∞ error (dB)	Leontovich	HOIBC	HOIBC + SUC	HOIBC* + SUC
TM	5.43	0.109	2.34	0.148
TE	24.4	4.77	21.2	4.79

Integral Equations with HOIBC

From **J.-C. Nédélec**, *Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems*. Springer Science & Business Media, 2001



The standard EI are the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE).

Unknowns are

$$\vec{J} = \vec{n} \times \vec{\mathcal{H}}, \vec{M} = \vec{n} \times \vec{E} \quad \in H_{div}(\Gamma)$$

► EFIE

$$\begin{aligned}
 & \int_{\Gamma} \overline{E_t^{inc}}(\vec{x}) \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) (\vec{x}) \\
 & = ik_0 \int_{\Gamma} \left(\int_{\Gamma} G(\vec{x} - \vec{y}) \vec{J}(\vec{y}) ds(\vec{y}) \right) \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) \\
 & - \frac{i}{k_0} \int_{\Gamma} \left(\int_{\Gamma} G(\vec{x} - \vec{y}) \vec{\nabla} \cdot \vec{J}(\vec{y}) ds(\vec{y}) \right) \vec{\nabla} \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) \\
 & - \int_{\Gamma} \left(\int_{\Gamma} \vec{\nabla} G(\vec{x} - \vec{y}) \times \vec{M}(\vec{y}) ds(\vec{y}) \right) \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) + \frac{1}{2} \int_{\Gamma} \overline{E_t}(\vec{x}) \cdot \vec{\phi}(\vec{x}) ds(\vec{x})
 \end{aligned}$$

► MFIE

$$\begin{aligned}
 & \int_{\Gamma} \overline{\mathcal{H}_t^{inc}}(\vec{x}) \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) (\vec{x}) \\
 & = -ik_0 \int_{\Gamma} \left(\int_{\Gamma} G(\vec{x} - \vec{y}) \vec{M}(\vec{y}) ds(\vec{y}) \right) \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) \\
 & + \frac{i}{k_0} \int_{\Gamma} \left(\int_{\Gamma} G(\vec{x} - \vec{y}) \vec{\nabla} \cdot \vec{M}(\vec{y}) ds(\vec{y}) \right) \vec{\nabla} \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) \\
 & - \int_{\Gamma} \left(\int_{\Gamma} \vec{\nabla} G(\vec{x} - \vec{y}) \times \vec{J}(\vec{y}) ds(\vec{y}) \right) \cdot \vec{\phi}(\vec{x}) ds(\vec{x}) + \frac{1}{2} \int_{\Gamma} \overline{\mathcal{H}_t}(\vec{x}) \cdot \vec{\phi}(\vec{x}) ds(\vec{x})
 \end{aligned}$$

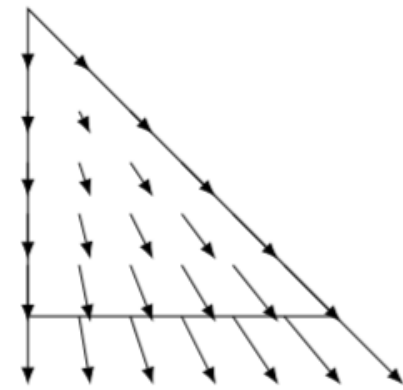
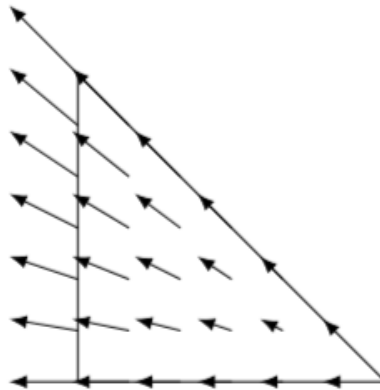
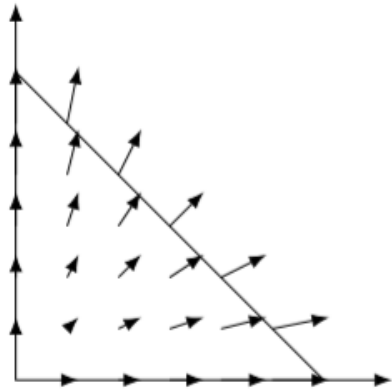
$$\frac{1}{2} \int_{\Gamma} \vec{E}_t \cdot \vec{\phi} = \frac{1}{2} \int_{\Gamma} \left((a_0 + a_1 \mathcal{L}_D - a_2 \mathcal{L}_R) \vec{J} - (b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{E}_t \right) \cdot \vec{\phi}$$

$$\begin{aligned} \frac{1}{2} \int_{\Gamma} \vec{\mathcal{H}}_t \cdot \vec{\phi} &= \frac{1}{2} \int_{\Gamma} \vec{J} \cdot (\vec{n} \times \vec{\phi}) \\ &= \frac{1}{2a_0} \int_{\Gamma} \left((I + b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{E}_t - (a_1 \mathcal{L}_D - a_2 \mathcal{L}_R) \vec{J} \right) \cdot (\vec{n} \times \vec{\phi}) \\ &= \frac{1}{2a_0} \int_{\Gamma} \left(-(I - b_1 \mathcal{L}_R + b_2 \mathcal{L}_D) \vec{M} - (-a_1 \mathcal{L}_R + a_2 \mathcal{L}_D) (\vec{\mathcal{H}}_t) \right) \cdot \vec{\phi} \end{aligned}$$

- ▶ HOIBC adds $\vec{\mathcal{H}}_t = -\vec{n} \times \vec{J}$, $\vec{E}_t = -\vec{n} \times \vec{M} \notin H_{div}(\Gamma)$
- ▶ Need to define L_D and L_R

FEM method: Let Γ_h be a triangular meshing of Γ

Raviart-Thomas function : let $(\vec{\phi}_i)_i$ be the basis of $H_{div}(\Gamma_h)$



$$\vec{J} \Rightarrow \sum_{j=1}^N I_j \vec{\phi}_j, \vec{M} \Rightarrow \sum_{j=1}^N m_j \vec{\phi}_j$$

By definition

$$\int_{\Gamma_h} \mathcal{L}_D(\vec{\phi}_i) \cdot \vec{\phi}_j = - \int_{\Gamma_h} \vec{\nabla} \cdot (\vec{\phi}_i) \vec{\nabla} \cdot (\vec{\phi}_j) := L_{Dij}$$

$$\int_{\Gamma_h} \mathcal{L}_R(\vec{n} \times \vec{\phi}_i) \cdot (\vec{n} \times \vec{\phi}_j) = - \int_{\Gamma_h} \mathcal{L}_D(\vec{\phi}_i) \cdot \vec{\phi}_j$$

Problems arise with

$$\int_{\Gamma_h} \underbrace{\mathcal{L}_R}_{rot}(\vec{\phi}_i) \cdot \vec{\phi}_j = - \int_{\Gamma_h} \mathcal{L}_D(\vec{n} \times \vec{\phi}_i) \cdot (\vec{n} \times \vec{\phi}_j)$$

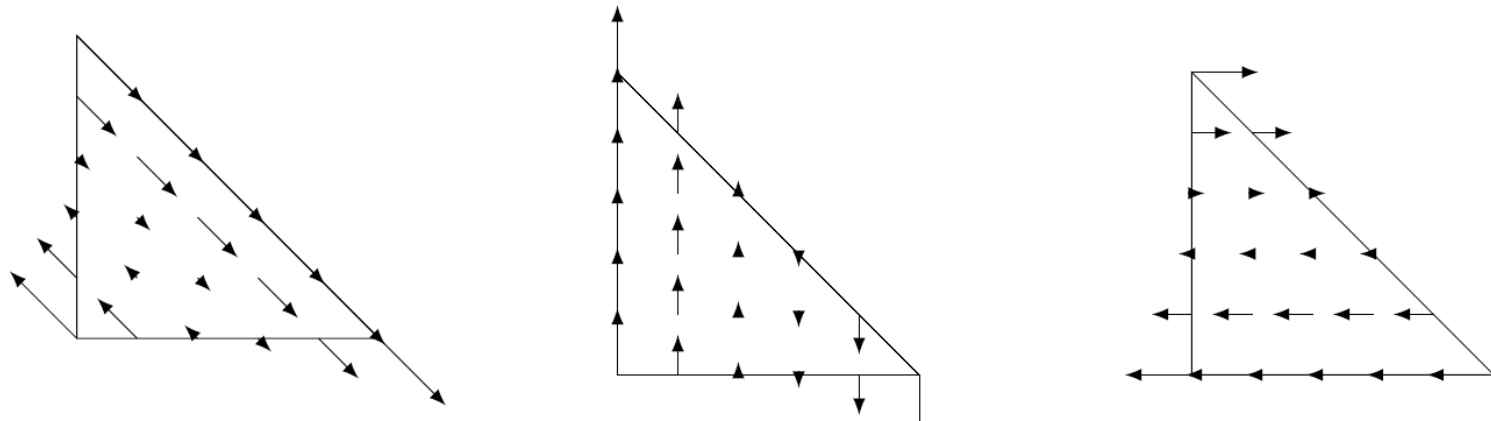
- ▶ B. Stupfel, *Implementation of High-Order Impedance Boundary Conditions in Some Integral Equation Formulations*. IEEE Transactions on Antennas and Propagation. (2015).

Let Π be the projection $\Pi(\vec{\phi}_i) = \sum_{j=1}^N P_{ji} \vec{p}_j$

$$\begin{aligned} \int_{\Gamma_h} \Pi(\vec{\phi}_i) \cdot \mathcal{L}_R(\Pi(\vec{\phi}_j)) &= \sum_{l=1}^N \sum_{m=1}^N P_{li} P_{mj} \int_{\Gamma_h} (\vec{p}_l \cdot \mathcal{L}_R(\vec{p}_m)) \\ &= - \sum_{l=1}^N \sum_{m=1}^N P_{li} P_{mj} \int_{\Gamma_h} (\vec{\phi}_l \cdot \mathcal{L}_D(\vec{\phi}_m)) \\ &= -(P^t L_D P)_{ij} \end{aligned}$$

- ▶ We define L_R as $-P^t L_D P$
 - L_{Rij} is an approximate of $\int_{\Gamma_h} (\vec{\phi}_i) \cdot \mathcal{L}_R(\vec{\phi}_j)$
- ▶ Which projection ???

- ▶ A. Bendali *et al*, *A boundary-element solution of the Leontovich problem*, IEEE Transactions on Antennas and Propagation, vol. 47, no. 10, pp. 1597–1605, 1999
- ▶ They are written $(\vec{q}_i)_i$



- ▶ Let $\vec{W} = \sum_{j=1}^N w_j \vec{q}_j$
 - $\vec{U} := \sum_{j=1}^N \int_{\Gamma_h} (\vec{W} \cdot \vec{\phi}_j) \vec{\phi}_j = \sum_{j=1}^N u_j \vec{\phi}_j$
 - $\vec{V} := \sum_{j=1}^N \int_{\Gamma_h} (\vec{W} \cdot \vec{p}_j) \vec{p}_j = \sum_{j=1}^N v_j \vec{p}_j$
- ▶ Let \underline{w} the components of \vec{W} in the $(\vec{q}_j)_j$ basis
 - $\underline{u} = G^{\phi q} \underline{w}$
 - $\underline{v} = G^{pq} \underline{w}$
- ▶ The projection is $P = G^{pq} (G^{\phi q})^{-1}$

$$G_{ij}^{pq} = \int_{\Gamma_h} \vec{p}_i \cdot \vec{q}_j$$

We still need to approximate

$$\int_{\Gamma_h} (b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{E}_t \cdot \vec{\phi}_i$$

Since by definition, $\vec{E}_t = -\vec{n} \times \vec{M}$

$$\begin{aligned} \int_{\Gamma_h} (b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{E}_t \cdot \vec{\phi}_i &= \sum_{j=1}^N m_j \int_{\Gamma_h} (b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{p}_j \cdot \vec{\phi}_i \\ &\simeq \sum_{j=1}^N \sum_{l=1}^N m_j P_{lj} \int_{\Gamma_h} (b_1 \mathcal{L}_D - b_2 \mathcal{L}_R) \vec{\phi}_l \cdot \vec{\phi}_i \\ &\simeq \left((b_1 L_D - b_2 L_R) P \underline{m} \right)_i \end{aligned}$$

► Same goes for

$$\begin{aligned}
 & \int_{\Gamma_h} (a_1 \mathcal{L}_D - a_2 \mathcal{L}_R) (\vec{n} \times \overline{\mathcal{H}}_t) \cdot (\vec{n} \times \overline{\phi}_i) \\
 &= - \int_{\Gamma_h} (a_1 \mathcal{L}_R - a_2 \mathcal{L}_D) \overline{\mathcal{H}}_t \cdot \overline{\phi}_i \\
 &\simeq - \sum_{j=1}^N I_j \int_{\Gamma_h} (a_1 \mathcal{L}_R - a_2 \mathcal{L}_D) \overline{p}_j \cdot \overline{\phi}_i \\
 &\simeq - \sum_{j=1}^N \sum_{l=1}^N I_j P_{lj} \int_{\Gamma_h} (a_1 \mathcal{L}_R - a_2 \mathcal{L}_D) \overline{\phi}_l \cdot \overline{\phi}_i \\
 &\simeq - \left((a_1 L_R - a_2 L_D) P \underline{I} \right)_i
 \end{aligned}$$

Let $M = G\phi\phi$

The IBC0 gives

$$\left(\begin{bmatrix} A & B \\ B & -A \end{bmatrix} + \frac{1}{2} \begin{bmatrix} a_0 M & 0 \\ 0 & -M \\ 0 & a_0 \end{bmatrix} \right) \begin{bmatrix} I \\ m \end{bmatrix} = \begin{bmatrix} b_E \\ b_H \end{bmatrix}$$

The HOIBC gives

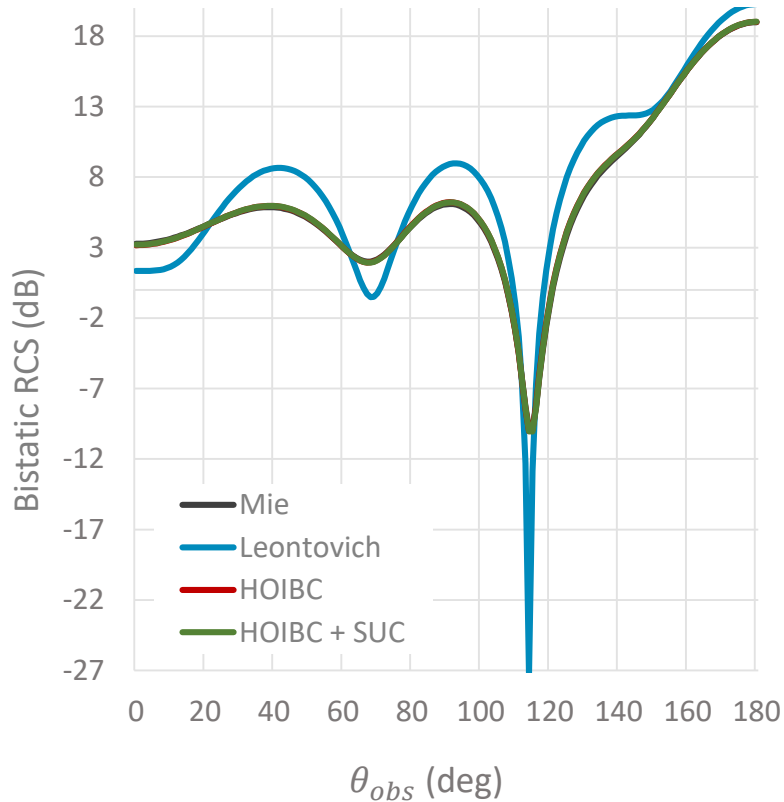
$$\left(\begin{bmatrix} A & B \\ B & -A \end{bmatrix} + \frac{1}{2} \begin{bmatrix} a_0 M + a_1 L_D - a_2 L_R & -(b_1 L_D - b_2 L_R)P \\ -\frac{(a_1 L_R - a_2 L_D)P}{a_0} & -\frac{(M - b_1 L_R + b_2 L_D)}{a_0} \end{bmatrix} \right) \begin{bmatrix} I \\ m \end{bmatrix} = \begin{bmatrix} b_E \\ b_H \end{bmatrix}$$

- ▶ Full (2N x 2N) matrix
- ▶ Sparse (2N x 2N) matrix

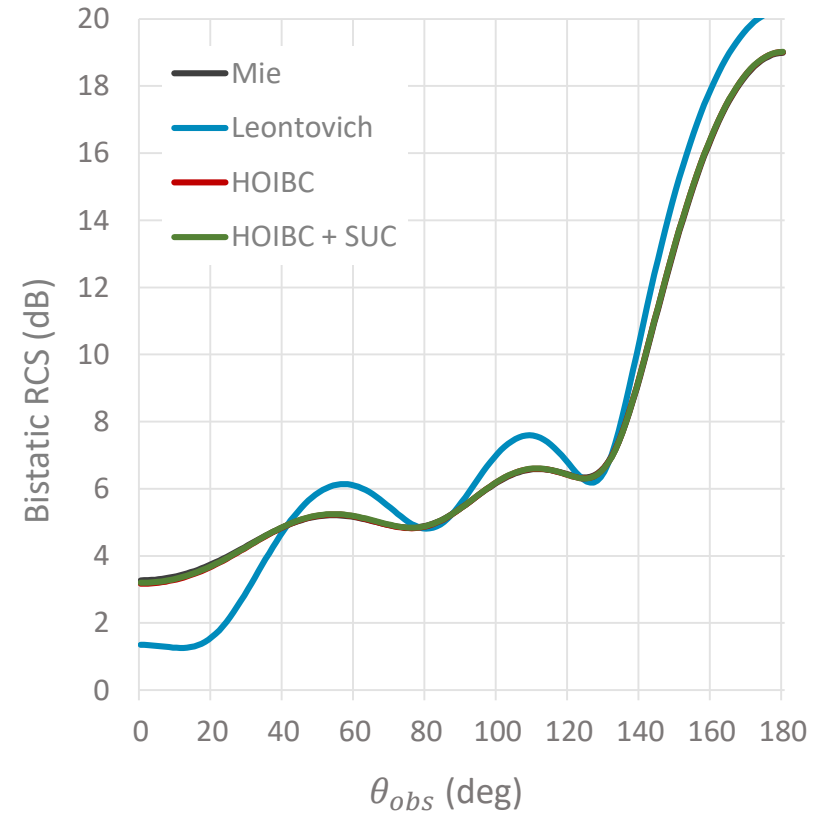
RCS of a sphere (R=1m) with 4240 triangles

 $\varepsilon = 1 - i, \mu = 1, d = 0.05\text{m}, f = 0.2 \text{ GHz (meshed } \lambda_0/18), \theta_{inc} = 0^\circ$

TM polarisation



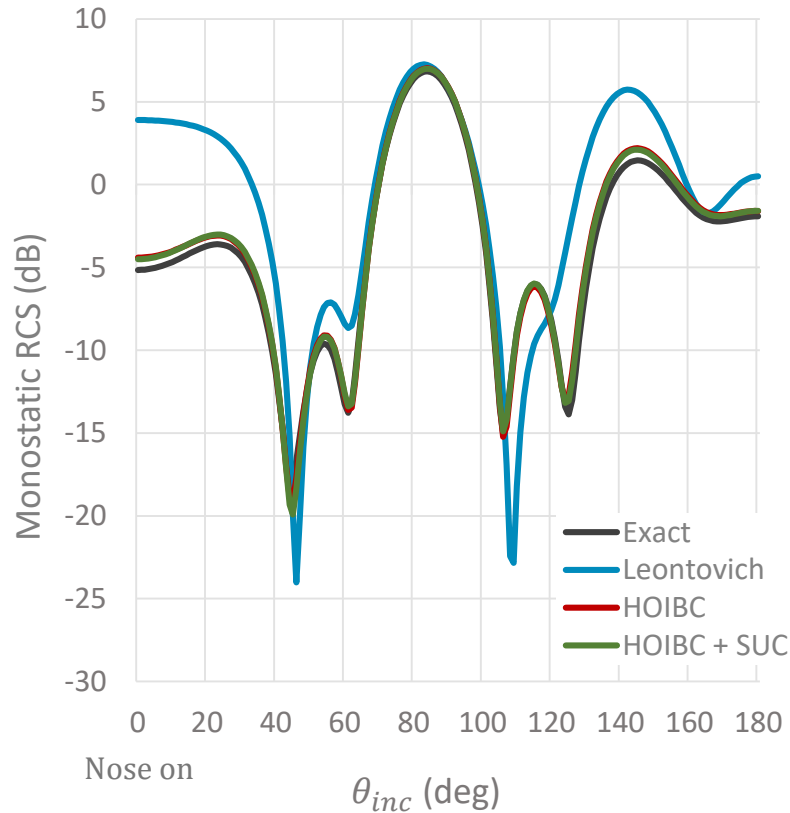
TE polarisation



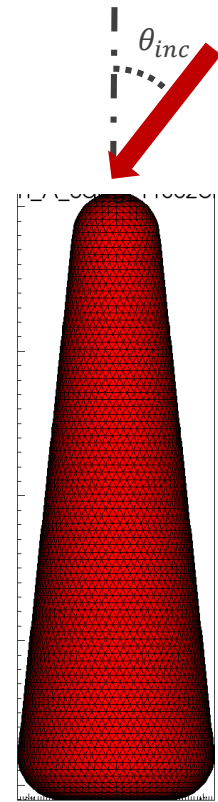
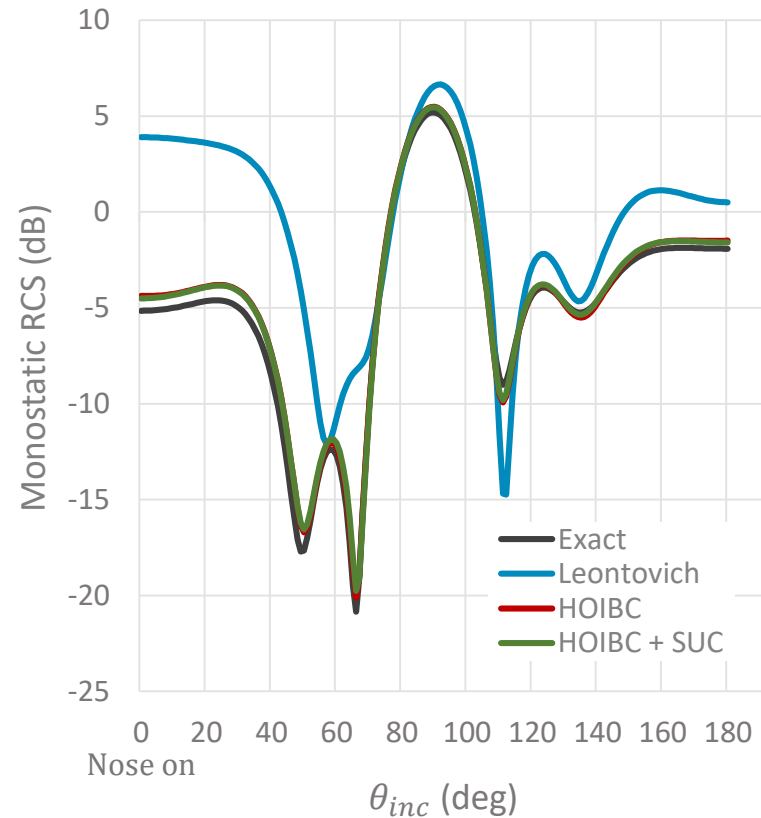
L_∞ error (dB)	Leontovich	HOIBC	HOIBC + SUC
TM	17.5	0.251	0.203
TE	2.25	0.101	0.076

RCS of a cone-sphere ($\alpha = 8^\circ$, $L=2m$) with 11352 triangles
 $\varepsilon = 1 - i$, $\mu = 1$, $d = 5\text{cm}$, $f = 200\text{ MHz}$ (meshed $\lambda_0/55$), $\theta_{obs} = \theta_{inc}$

TM polarisation



TE polarisation



L_∞ error (dB)	Leontovich	HOIBC	HOIBC + SUC
TM	12.5	1.70	2.19
TE	13.8	1.72	1.75

Conclusion

- ▶ HOIBC outperforms Leontovich
- ▶ SUC can be applied
- ▶ Computational time of HOIBC coefficients is negligible
- ▶ Easily interfaced with BEM codes

Outlooks

- ▶ More HOIBC and corresponding SUC



Thanks for your attention