

Measurement of Individual Color Space using Luminous Vector Field

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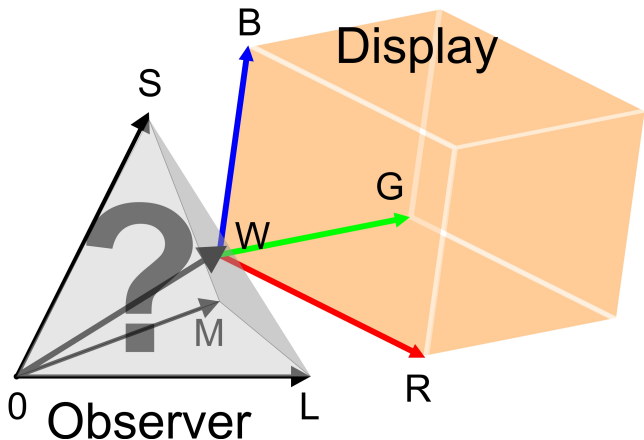
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Geometry of color vision

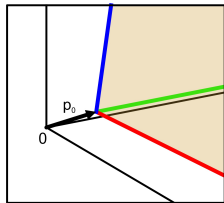
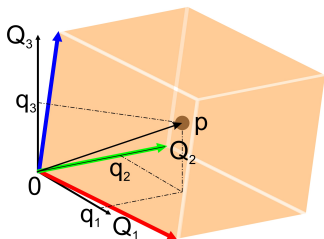
An observer watching a computer screen



- Display's color space is a cube defined by RGB.
- Observer's color space is defined by LMS.
- Origin of observer's color space is not necessarily the origin of display's color space.

Display color space

An orthogonal space where each point is a light the display can produce

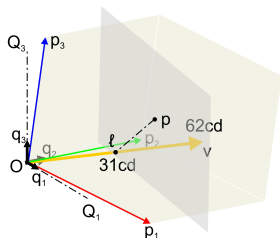


- We set up an orthogonal coordinate system $Q_1 Q_2 Q_3$ with $Q_2 = G$.
- Light spectra produced by the display is a point p .
- A point p has coordinates (q_1, q_2, q_3) , i.e. $p(\lambda) = q_1 Q_1(\lambda) + q_2 Q_2(\lambda) + q_3 Q_3(\lambda)$.
- In Q -space the integral $683.002 \int_{\lambda} a(\lambda)b(\lambda)d\lambda = a \cdot b$ is equal to the scalar product of corresponding vectors a and b .
- Q -space is Euclidean space where orthogonality and norm (radiometric) are well defined

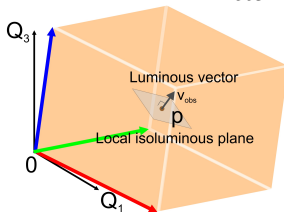
Achromatic measurement of display's light

Any point p of display's color space, is attached to a luminous vector v_{obs}

Colorimetric observer - $V(\lambda)$



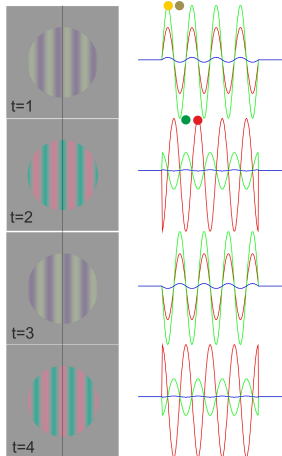
Real observer - v_{obs}



- Spectral efficiency function of the eye $V(\lambda)$ is the vector v in Q-space of norm $\|v\| = \sqrt{v \cdot v} = 62cd$ for our display.
- Luminance of p is the affix of the intersection ℓ between the plane orthogonal to v passing through p and vector v .
- Another vector v_{obs} represents observer's luminous vector defined by a direction and a magnitude.
- The local portion of plane orthogonal to the vector v_{obs} at point p is the local iso-luminous surface.

Heterochromatic photometry

Minimum Motion stimulus for measuring equi-luminosity



Minimum motion stimulus produces a motion when the two colors (Red and Green) composing the heterochromatic frames are not of the same luminosity for the observer.

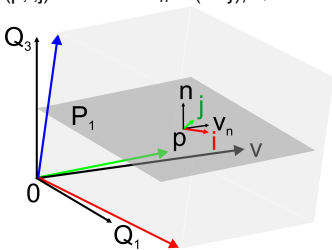
- Anstis, S., & Cavanagh, P. (1983). A minimum motion technique for judging equiluminance. *Color vision : Physiology and psychophysics*/Ed. by JD Mollon, LT Sharpe. p.156-166.
- Moreland, J. D. (1982). Spectral sensitivity measured by motion photometry. *Documenta Ophthalmologica Proceedings Series*.
- Cavanagh, P., MacLeod, D. I., & Anstis, S. M. (1987). Equiluminance : spatial and temporal factors and the contribution of blue-sensitive cones. *JOSA A*, 4(8), 1428-1438.
- Webster, M. A., & Mollon, J. D. (1997). Motion minima for different directions in color space. *Vision Research*, 37(11), 1479-1498.

Measurement of luminous vector

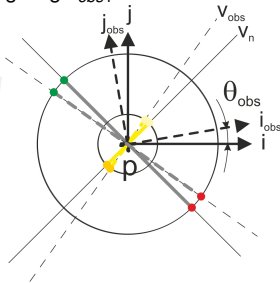
Two successive sessions of minimum motion around p define v_{obs}

Choosing P_1

Define local coordinate system (p, i, j) such that $v_n = (i + j) / \sqrt{2}$.

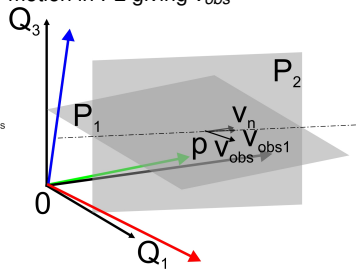


Rotate the coordinate system around p until minimum motion is obtained giving v_{obs1} in P_1



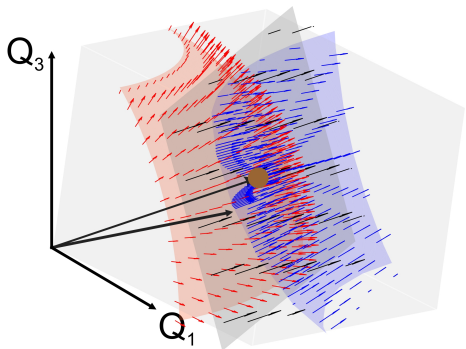
Define P_2 orthogonal to P_1 and collinear to v_{obs1}

Measure the direction of minimum motion in P_2 giving v_{obs}



Hypotheses on iso-luminous surfaces

Iso-luminous surfaces passing through a point can be plane, ellipsoid or hyperboloid



We limit iso-luminous surfaces to be quadratic of constant curvature. To any point x of the surface, one associate a vector v orthogonal to the surface at that point. The set of couples (x,v) is the vector field.

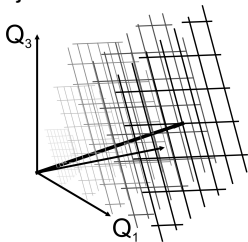
- If the surface is plane (grey), orthogonal vector field is constant (colorimetric situation $V(\lambda)$).
- If the surface is ellipsoidal (red), orthogonal vector field is divergent.
- If the surface is hyperboloidal (blue), orthogonal vector field is convergent.

Projective surfaces

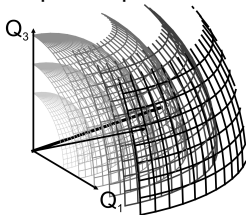
Iso-luminous surfaces are supposed proportional toward (visual) zero

Vector field is completely defined for the whole display's color space using proportionality

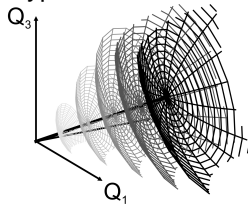
Projective stack of Planes



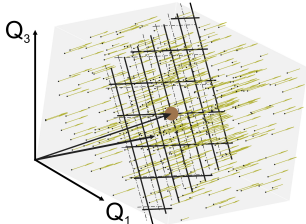
Projective stack of Ellipses/Spheres



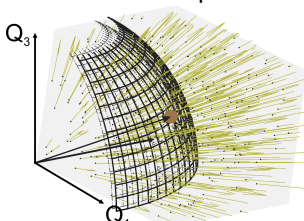
Projective stack of Hyperboloids



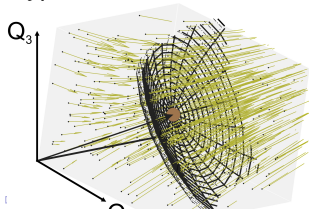
Proportional vector field
Iso-luminous Planes



Divergent vector field
Iso-luminous Ellipsoids



Convergent vector field
Iso-luminous
Hyperboloids

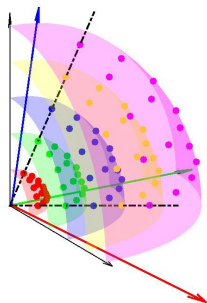


Placement of points for vector field measurement

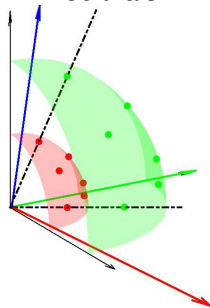
15×5 points for authors, 6×2 for naïve observers

The two authors did 15 points on 5 different spheres, whereas 20 observers did 6 points on 2 different spheres.

$75 \times 2 \times 30$ trials \sim 5 hours on three consecutive days.



$12 \times 2 \times 30$ trials \sim 1 hour



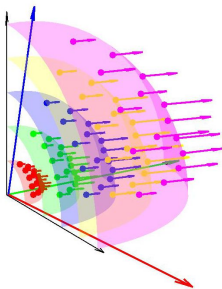
Points are chosen such that they lie on a sphere of constant radiometric norm of radius $r \in [1, 2, 3, 4, 5]$ and are related by a projective line from zero.

We measure the length of the vectors as the minimal contrast needed to detect the homochromatic frame, using a contrast adjustment procedure.

Results

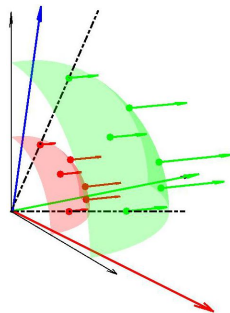
Measured vector field, observer's parameters estimate

N = 75



Vectors are slightly convergent

N = 12



We got matrices $X = \begin{bmatrix} X_{11} & \dots & X_{1N} \\ X_{21} & \dots & X_{2N} \\ X_{31} & \dots & X_{3N} \end{bmatrix}$ and $V = \begin{bmatrix} V_{11} & \dots & V_{1N} \\ V_{21} & \dots & V_{2N} \\ V_{31} & \dots & V_{3N} \end{bmatrix}$ from which we

estimate A and a_0 such that $\tilde{V} = 2AX + a_0$ and $mse = \|\tilde{V} - V\|$ is minimum.

Eigenvalues of A is the signature of the surface.

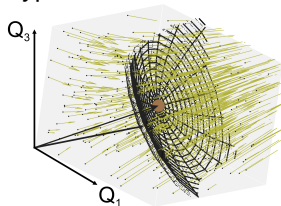
Results

Analysis on A and mse, corresponding surface

TABLE – Analysis of A and mse

OBS	Eigenvalues of $A * 1E3$			mse
2010	-0.134	-0.0844	4.84	5.17e-06
2210	-0.0985	-0.0667	3.91	4.38e-07
100	-0.0605	-0.0237	2.28	9.74e-08
101	-0.0953	-0.0647	3.15	9.94e-08
102	-0.144	0.025	3.39	6.93e-07
103	-0.414	-0.152	5.64	1.14e-06
104	-0.0879	-0.00827	7.1	1.19e-06
105	-0.0684	-0.015	3.62	8.86e-07
106	-0.105	-0.0533	4.5	1.2e-06
107	-0.189	-0.0865	10.6	1.36e-06
108	-0.188	-0.136	8.24	1.09e-06
109	-0.196	-0.0382	6.41	1.22e-06
⋮				⋮

Because one or two eigenvalues are negatives, iso-luminous surface is hyperboloid

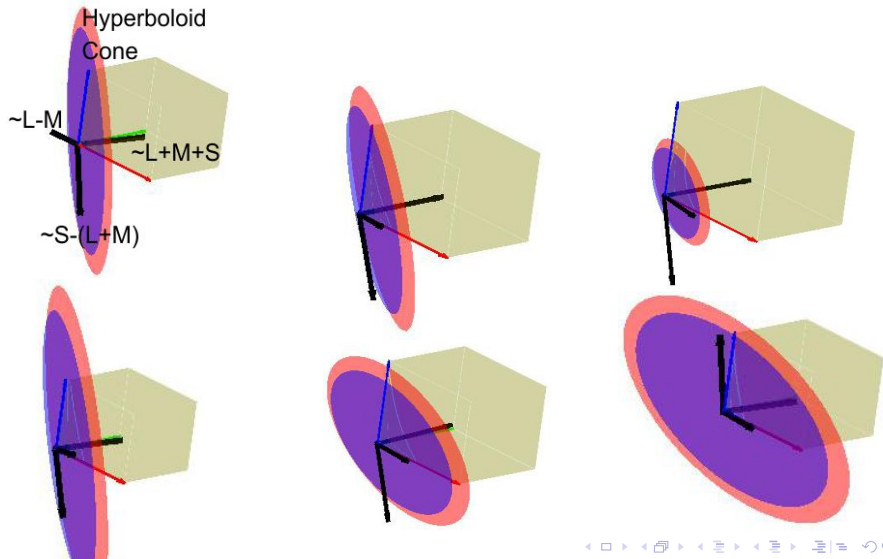


We also check for the two authors that the length $\|v\|$ along each projective line follows a Weber law.

Observer's color space seen a computer screen

Three axis, two orthogonal to L+M+S and a metric surface

Origin of observer's metric is always negative

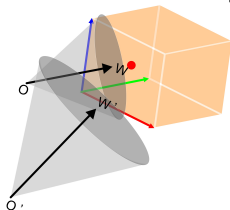


About the method

Advantages

- Few measurements for the whole color space. Origin and metric.
- Minimum motion is a simple task.
- Method precise enough to show, hyperbolic metric and observer's variability.

Two different observers seen the same point



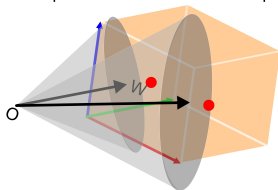
Further works

- Dependencies between the two successive minimum motion ?
- Interpretation of second positive eigenvalue ?
- Observer's color space is defined only for the display's primaries
- Check the reliability of the observer parameters for different points in display color space

Conclusion

- Assuming iso-luminous surfaces are proportional and quadratic, we measured those surfaces for an observer using two minimum motion in two different planes.
- Estimated surfaces for several observers indicate for all a hyperbolic geometry of iso-luminous surface.
- We can use several displays for constructing the whole observer function.
- What about adaptation? Adaptation may explain why we found spectral cone with large aperture.

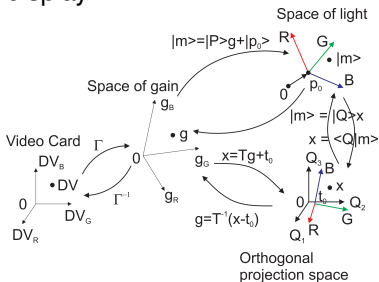
Adaptation around two different points



Supplementary Method

Display calibration

Model of light produces by the display



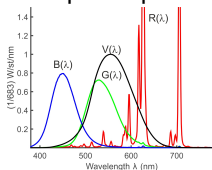
$$\Gamma: DV \in [0, 1]^3 \rightarrow g \in [0, 1]^3$$

$$DV \mapsto g = \Gamma(DV) = \begin{bmatrix} \Gamma_R(DV_R) \\ \Gamma_G(DV_G) \\ \Gamma_B(DV_B) \end{bmatrix}$$

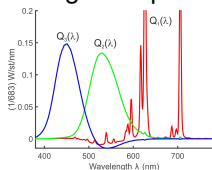
$$\Gamma_c(DV_c) = \{a_{c,i} + b_{c,i}x^{c,i}, x_{c,i-1} \leq x \leq x_{c,i}\}_{i=1..N}$$

$$c \in \{R, G, B\}, a_{c,1} = 0, b_{c,N} = 1, x_{c,0} = 0, x_{c,N} = 1$$

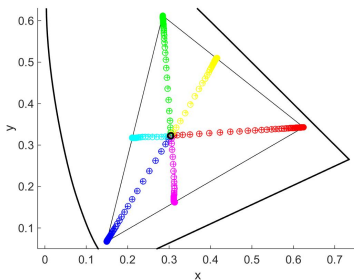
Phosphor spectra



Orthogonal spectra



Model evaluation

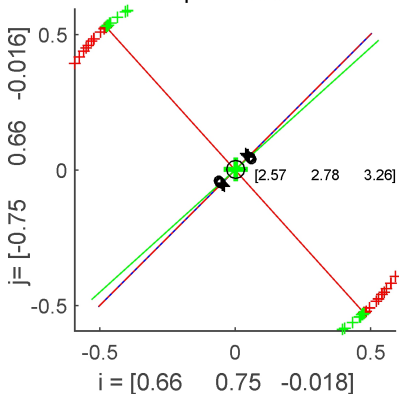


Adaptive procedure

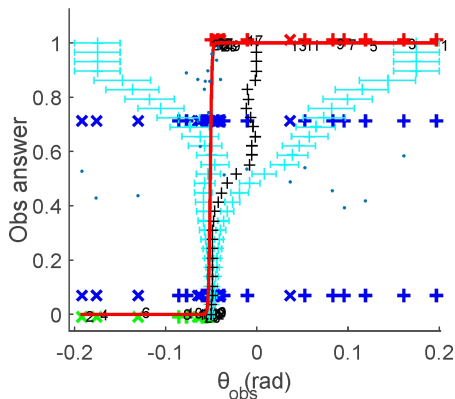
Two intervals that join along trials

After thirty trials PSE is estimated which corresponds to a rotation angle from the initial v .

OBS 00 C3 S1
Slope = 0.90269



OBS 00 C3 S1
Thres = -0.051098 ± 0.002578



Supplementary Method

Estimation of observer's data A and a_0

From matrices $X = \begin{bmatrix} x_{11} & \dots & x_{1N} \\ x_{21} & \dots & x_{2N} \\ x_{31} & \dots & x_{3N} \end{bmatrix}$ and $V = \begin{bmatrix} v_{11} & \dots & v_{1N} \\ v_{21} & \dots & v_{2N} \\ v_{31} & \dots & v_{3N} \end{bmatrix}$ we defined new

matrices $\bar{X} = \begin{bmatrix} x_{11} & \dots & x_{1N} \\ x_{21} & \dots & x_{2N} \\ x_{31} & \dots & x_{3N} \\ 1 & \dots & 1 \end{bmatrix}$ and $\bar{V} = \begin{bmatrix} v_{11} & \dots & v_{1N} \\ v_{21} & \dots & v_{2N} \\ v_{31} & \dots & v_{3N} \\ 1 & \dots & 1 \end{bmatrix}$.

The model is $\tilde{V} = 2BX + a_0$ which with new variables writes $\tilde{V} = M\bar{X}$.

with $M = \begin{bmatrix} 2B & a_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \overline{VX}^t (\overline{XX}^t)^{-1}$. Resulting matrix B is not symmetric.

Another symmetric matrix $A = (B + B^t)/2$ produce the same surface but not the same vector field.

Supplementary Method

Vector field from surface

General equation of a quadratic surface in \mathbb{R}^3 is given for $x = (x_1, x_2, x_3)$ by :

$$f(x_1, x_2, x_3) = ax_1^2 + bx_2^2 + cx_3^2 + 2dx_2x_3 + 2ex_1x_3 + 2fx_1x_2 + gx_1 + hx_2 + ix_3 + j = 0$$

which writes in matrix-vector form :

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \underbrace{\begin{bmatrix} a & f & e \\ f & b & d \\ e & d & c \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \underbrace{\begin{bmatrix} g \\ h \\ i \end{bmatrix}}_{a_0} + j = 0 \Leftrightarrow x^t Ax + x^t a_0 + j = 0$$

Coordinates of orthogonal vector v to any point of the surface is given by the gradient of $f(x_1, x_2, x_3)$ which writes :

$$v = \nabla f(x_1, x_2, x_3) = \begin{bmatrix} \frac{\partial f(x_1, x_2, x_3)}{\partial x_1} \\ \frac{\partial f(x_1, x_2, x_3)}{\partial x_2} \\ \frac{\partial f(x_1, x_2, x_3)}{\partial x_3} \end{bmatrix} = 2 \begin{bmatrix} ax_1 + fx_2 + ex_3 \\ fx_1 + bx_2 + dx_3 \\ ex_1 + dx_2 + cx_3 \end{bmatrix} + \begin{bmatrix} g \\ h \\ i \end{bmatrix} = 2Ax + a_0$$

Thus, from the affine model of the orthogonal vector field, $v = 2Ax + a_0$ surface is completely defined by $x^t Ax + x^t a_0 = -j$.

Supplementary Method

Surfaces from A and a_0 .

The set of surfaces that follow the metric A (symmetric by $A \rightarrow (A + A^t)/2$) and shift of the origin a_0 are given by :

$$x^t A x + x^t a_0 = k^2$$

which can be written :

$$x^t A x + x^t a_0 = k^2 \Leftrightarrow \left(x - \frac{1}{2} A^{-1} a_0\right)^t A \left(x - \frac{1}{2} A^{-1} a_0\right) = k^2 + \frac{1}{4} a_0 A^{-1} a_0$$

Posing $x_0 = \frac{1}{2} A^{-1} a_0$, $A = U S U^t$, $\rho^2 = k^2 + \frac{1}{4} a_0 A^{-1} a_0$ we have :

$$(x - x_0)^t H (x - x_0) = 1$$

with $H = U^t \frac{\sqrt{|S|}}{\rho} \mathcal{J} \frac{\sqrt{|S|}}{\rho} U$ with $\mathcal{J} = \frac{1}{2} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is the surface metric and x_0

the origin of the visual space. With variable $z = \frac{\sqrt{|S|}}{\rho} U(x - x_0)$, surface writes $z^t \mathcal{J} z = 1$ is the canonical hyperboloid.