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Chapter 5

Place-value notations in the Ur III period: Marginal numbers in administrative texts

Xiaoli OUYANG and Christine PROUST*

Abstract – The use of sexagesimal place value notation (hereafter SPVN) is one of the most striking features of cuneiform mathematics. The earliest attestations of a systematic use of SPVN in cuneiform sources are found in a small set of mathematical texts dated to the Ur III period (ca. 2112 to 2004 BCE). Besides this mathematical corpus, traces of numbers written in positional notations have been found in some Ur III administrative texts. Just a few dozen tablets, among tens of thousands of known Ur III administrative documents, exhibit such numbers noted in positional notation. Moreover, such numbers noted in positional notation appear almost only as a kind of graffiti in the margins. This chapter focuses on these scanty ‘marginal numbers’. Marginal numbers in Ur III administrative texts testify to diverse practices with positional notations and sexagesimal factors in an administrative context, and, in this way, exemplify different ‘cultures of computation and quantification’ in the Ur III period. Through a close analysis of the positional notations found in Ur III administrative texts, we detect a diversity of graphical systems for what was considered until now as a uniform notion of SPVN. We show that these graphical systems vary according to the operations (multiplication, reciprocal, addition, subtraction), the contexts (administrative or mathematical), and the archaeological sites considered in this study (Umma, Girsu, Puzriš-Dagan, Nippur). Our goal is to show that this diversity of notations reflects different computational methods.

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5.1 Introduction

The use of sexagesimal place value notation (hereafter SPVN) is one of the most striking features of cuneiform mathematics. The earliest attestations of a systematic use of SPVN in cuneiform sources are found in a small set of mathematical texts dated to the Ur III period (ca. 2112 to 2004 BCE – see chronology in Appendix 5.I.3). Besides this mathematical corpus, traces of numbers written in positional notations have been found in some Ur III administrative texts. Just a few dozen tablets, among tens of thousands of known Ur III administrative documents, exhibit such numbers noted in positional notation. Moreover, such numbers noted in positional notation appear almost only as a kind of graffiti in the margins. This chapter focuses on these scanty ‘marginal numbers’.

By ‘marginal numbers,’ we mean numbers noted on the tablet outside the main text, on the four edges and at bottoms of columns on both the obverse and reverse side, or in special boxes inserted in the main text. The phenomenon of marginal numbers has already been observed by several authors, who referred to ‘marginal notes’ (Ellis 1970: 267, see Text 3 below), numbers roughly noted on ‘scratch pads’ (Powell 1976a, see Text 10 below), ‘mathematical notations’ (Garfinkle 2008: 65, see Text 7 below) or ‘scratch calculations’ (Robson 2008: 79). However, no systematic study of marginal numbers exists to date.

Marginal numbers in Ur III administrative texts testify to diverse practices with positional notations and sexagesimal factors in an administrative context, and, in this way, exemplify different ‘cultures of computation and quantification’ in the Ur III period. Through a close analysis of the positional notations found in Ur III administrative texts, we detect a diversity of graphical systems for what was considered until now as a uniform notion of SPVN. We show that these graphical systems vary according to the operations (multiplication, reciprocal, addition, subtraction), the contexts (administrative or mathematical), and the archaeological sites considered in this study (Umma, Girsu, Puzriš-Dagan, Nippur – see map Fig. 5.25). Our goal is to show that this diversity of notations reflects different computational methods. More precisely, we intend to highlight a relationship between the notations of numbers and the operations which acted on or produced these numbers. Thus, for all of the texts examined, the following questions will be raised: Which operations does the text deal with? On what kind of numbers do the operations act?

This chapter offers, in Sect. 5.1, a broad overview of the previous studies devoted to SPVN in the Ur III period, and discusses some of the most important issues dealing with the notation of numbers and quantities in Ur III documentation. Sect. 5.2 analyses in detail a set of ten administrative texts from the Ur III period exhibiting positional notations in the margins (on the edges of the tablet or in boxes inside the text). The list of all the tablets known to us containing positional notations in their margins can be found in Appendix 5.III, and a complete edition of those, among the texts discussed, which were not edited before are available in Appendix 5.IV. Sect. 5.3 attempts to shed light on the various practices of computation as evidenced by the diverse uses of positional notations in Ur III documentation, and suggest hypothesis related to the use of a material device for computation.

5.1.1 In Search of the Origins of SPVN

The description of sexagesimal place value notation as the main numerical system used in mathematical texts was popularized by the pioneers of the history of cuneiform mathematics, François Thureau-Dangin and Otto Neugebauer. Neugebauer claimed assuredly that SPVN

‘appears in the mathematical texts of the Old-Babylonian period,’ while Thureau-Dangin stated more cautiously that it appears ‘at least as soon as the first dynasty of Babylon, and probably at an earlier date’.¹ Subsequently, the date of the appearance of SPVN has become a subject of debate among historians of cuneiform mathematics.

Explicit traces of the use of SPVN in Ur III texts could have been recognized as early as the beginning of the twentieth century if historians had paid more attention to the publication of the reciprocal table Ist L 7375 by Louis Delaporte in 1911 and the administrative text YBC 1793 by Clarence Keiser in 1919.² Delaporte had no doubt about the date and the content of the Ist L 7375 tablet, as indicated by the title of his 1911 article, *Document mathématique de l'époque des rois d'Our*, and by the fact that he published the tablet again in 1912 along with many Ur III texts from Girsu (modern Tello).³ Moreover, Delaporte interpreted the content of the table quite well, which was not easy given the poor understanding of place value notation in his time.⁴ Neugebauer himself, somewhat at odds with his claim that SPVN was an Old Babylonian invention, considered this tablet as dating from the reign of Šulgi, the second ruler of the Ur III dynasty.⁵ However, despite the available evidence, only from 1976 and the publication of ‘*The antecedents of Old Babylonian place notation...*’ by Marvin Powell, did it become indisputable for most historians that the use of SPVN preceded the Old Babylonian development of mathematics.⁶ Strangely enough, Powell did not take into account the reciprocal table in Ist L 7375, the existence of which he seemed to ignore, but based his argumentation on the administrative tablet YBC 1793. In his landmark 1976 paper, Powell strongly associated the invention of SPVN with the social and political context of the Ur III state and administration, as he linked ‘the invention of place notation to the creation of the Ur III empire’ (Powell 1976a: 422). However, in the same paper, Powell opened the door to the hypothesis of a much earlier date for the use of place value notation and confirmed this hypothesis in a paper which appeared later in the same year.⁷ Analysing a small group of tablets dated to the Old Akkadian period (ca. 2340-2200 BCE – see chronology in Appendix 5.I.3), containing ‘field texts,’ that is, calculations dealing with the surface of fields, Powell advocated that a ‘place notation of some type’ was used as early as in the Old Akkadian (or ‘Sargonic’) period.⁸ Whiting, and, subsequently, Foster and Robson, published additional field texts from the Old Akkadian period and supported Powell’s conclusions.⁹ Friberg, on the contrary, argued that the sources do not allow us to claim that SPVN was used before the Ur III period (Friberg 2005). However, Friberg

1 Neugebauer (1951: 15), Thureau-Dangin (1932: 50) ‘dès au moins le temps de la première dynastie Babylonienne et probablement à une date plus ancienne encore’.

2 Ist L 7375 is an Ur III mathematical tablet kept at the Archaeological Museum in Istanbul, Delaporte (1911); YBC 1793 (text 10 in the present study) is an Ur III administrative tablet kept at Yale, Keiser (1919). Another mathematical text from Ur III Girsu, AO 2728 (= AOT 304 = RTC 413), was published by Thureau-Dangin in 1903 (Thureau-Dangin 1903: 149), but was recognized as a mathematical text only in 1987 by Friberg (1987-1990: 451). See more on this text in Sect. 5.1.4 and edition in Appendix 5.IV.

3 Delaporte (1912).

4 Delaporte (1911: 132-133).

5 Neugebauer (1935: 10).

6 Powell (1976a: 417). While this point is now widely shared among historians, some doubts persist. For Damerow and Englund, the supposed invention of SPVN as early as the Ur III period is ‘based primarily on the alleged dating to this period of several numerical tables,’ suggesting that for them this dating is far from proven (Nissen et al. 1993: 142).

7 ‘I have rather cautiously suggested that Sumero-Akkadian scribes of the Sargonic period were using a mental construct analogous to Old Babylonian place notation. Having reflected on the matter at length, I see that I have been overly hesitant: place notation of some type, if not the classical type that appears in Old Babylonian mathematical texts, must have been in use by the Sargonic period’ (Powell 1976b: 13).

8 Powell (1976b).

9 Powell (1976b); Whiting (1984); Foster and Robson (2004).

seems to have moved closer to Powell's position recently.¹⁰ Proust analysed the arithmetical properties of numbers used in some field texts dated to the Old Akkadian period, and argued in favour of the hypothesis that a computing device based on sexagesimal factors and a positional system may have been used in the Old Akkadian period.¹¹

The important point which emerges from this discussion is the fact that SPVN cannot be taken as a uniform notion or a ready-to-use tool, but that the different components of SPVN, namely, S for 'sexagesimal,' PV for 'place value,' and N for 'notation,' must be considered separately. To some extent, there is no 'date for the invention of SPVN' because the different components of SPVN have different histories: the history of practices with sixtieths (cutting units into sixty parts), the speculations on the possible use of a physical device based on a positional principle, and the history of the way in which objects used for computing, such as tokens, were represented in texts with archaic or cuneiform signs. Powell emphasized some of these differentiations¹² and did not use the expression 'sexagesimal place value notation' to describe practices in the Old Akkadian period, but instead 'a mental construct analogous to Old Babylonian place notation' or 'place notation of some type, if not the classical type that appears in the Old Babylonian mathematical texts' (see notes above). The present chapter intends to contribute to this complex history by introducing even more differentiations. In particular, it will show that different forms of positional notations were shaped in the Ur III period, along with the stabilized form of SPVN attested in mathematical texts.

The date of the appearance of SPVN has been a subject of debate, but the function of positional systems seems to have been a consensual issue. Indeed, it is widely accepted that SPVN fulfilled the same function as earlier numerical systems. Powell claims that:

Some Sumero-Akkadian scribe accustomed to working with very large and very small numbers invented the place notation system to replace the older and more cumbersome system of standard notation and, with this act, created the facile instrument upon which the success of Old Babylonian mathematics is predicated. (Powell 1976a: 422)

We find again in Nissen et al. (1993: 142) the idea that SPVN was invented to simplify existing systems, notably for representing large and small numbers, and that this 'new system' tended to replace the 'old system'.

However, it is clear that in Old Babylonian mathematical texts, SPVN did not replace earlier numerical systems, but these different systems coexisted with different functions (see Proust 2008a). The present study sheds new light on different functions of numerical systems in Ur III administrative texts. It will be argued that positional numbers were not created to represent quantities in a more convenient way, as supposed by Powell, but to facilitate certain operations.

5.1.2 System S

10 After studying the table of surfaces CUNES 50-08-01, dated from the Early Dynastic period (mid-third millennium), Friberg (2007: 426) concluded that the 'way of counting with small sexagesimal fractions could easily have led to the invention of sexagesimal place value notation already in the middle of the third millennium!'

11 Proust (2008b: Chap. 1). Powell (1976a: 421) had already suggested that SPVN in the Ur III period may have resulted from the use of a kind of 'scratch pad'.

12 'Conceptions about the nature and origin of Babylonian place notation have always been closely linked or inextricably entangled with notions about the sexagesimal system of counting. The two phenomena are not, however, identical' (Powell 1976a: 418).

More information about the different numerical systems attested in Ur III texts is necessary at this point, because some of our key arguments rely on the distinction between positional and non-positional notations.

The numerical system used for noting cardinal numbers (numbers of animals, workdays, years, and so on) can be described as additive and sexagesimal. It is additive (non-positional) in the sense that a number is composed of signs repeated as many times as necessary, and that there is a special sign for each order of magnitude. It is sexagesimal in the sense that the factors defining the value of each sign from the sign of a lower value are alternatively ten and six (see diagram *a* in Fig. 5.26 of Appendix 5.II.1). The use of such an additive sexagesimal system is not peculiar to the Ur III period, as this system has early roots in the history of writing, and remained in use in the Old Babylonian period and onward.¹³ This system was theorized as such in the Old Babylonian period by the scribes themselves, as shown by a lexical tablet which exhibits the complete system (CBS 11319+).¹⁴

The use of the term ‘sexagesimal’ by historians to characterise such a system, whose base is actually not sixty, but an alternation of factors ten and six, is somewhat inappropriate, even if this term reflects the general numerical structure quite well. For the sake of simplicity and consistency with current literature, we also use this terminology. Otherwise, the additive sexagesimal system used in Mesopotamia since the invention of cuneiform writing directly derives from a pre-cuneiform system for counting objects called ‘system S’ (‘sexagesimal system’) by historians of archaic writing.¹⁵ Thus, by extension, we use ‘system S’ for the cuneiform counterpart of the archaic system (see more on system S in Appendix 5.II).

System S on the Side Edges and at the Bottom of Columns

Let us consider administrative texts (mathematical texts will be discussed below, in Sect. 5.1.4). In the main text of administrative tablets, the situation is simple: there is no SPVN attested except in one case, the completely atypical tablet YBC 1793 (see Text 10 below). In these texts, the numerical values are employed for counting items (animals or workdays) or measurement units, and they are followed by the name of the elements counted. These numerical values are noted in system S.¹⁶

In the margins, we generally find a different situation: numbers appear not only separated from the main text, but also isolated, that is, without specification of the elements counted. It is the case, for example, in tablet MAH 16605 from Puzriš-Dagan (Sauren 1974: No.104), a small square tablet recording the receipt of animals. The details of the deliveries are noted on the obverse (transliteration from CDLI):

13 For more information on the early history of numerical and metrological systems and the long process of their decipherment, see for example Powell (1971); Powell (1987-1990); Friberg (1978); Englund (1987); Nissen et al. (1993).

14 Tablet CBS 11319+ was published by Sjöberg (1993); the numerical section is re-published with corrections and analysed in Proust (2009: §3.2.6).

15 See Englund (1988: 185) and Nissen et al. (1993: 28, 140) for the cuneiform counterpart of archaic signs.

16 Another system is also used for the evaluation of large surfaces (system G). See examples in Text 9 below.

Obverse	Transliteration	Translation
1.	2(diš) udu niga	2 barley-fed sheep
2.	3(diš) udu	3 sheep
3.	2(diš) gukkal geš-du ₃	2 fat-tailed breeding sheep
4.	1(diš) gukkal geš-du ₃ babbar	1 white fat-tailed breeding sheep
5.	3(diš) maš ₂ -gal	3 billy goats



Fig. 5.1 MAH 16605 (Courtesy of Jean-Luc Chappaz, Curator of the *Musées d'Art et d'Histoire of Geneva*. Photo CDLI)

On the left edge appears the isolated number $\llcorner \Upsilon$, which corresponds to 11, the total number of delivered animals. Here, nothing helps to decide whether this number is noted in system S or in SPVN. Thus, more information on the function and meaning of this marginal number is necessary to make a judgement. On another tablet from the same archive (MAH 16573, Sauren 1974: No.105) we find a similar delivery of four animals: the first line of the obverse of the main text reads '4(diš) sila₄', that is, '4 lambs'. The number 4 ($\Upsilon \Upsilon$) is followed by the specification of the item counted, lambs, and thus is noted in system S. The corresponding number 4 ($\Upsilon \Upsilon$) (without specification) is noted on the edge with the same paleography as in the main text (two rows of two wedges), which suggests that system S is also applied on the edge. We come back to this important graphical feature in the next section (Sect. 5.1.3).

In the same archive, some individual receipts for cattle repeat the number of animals received on the edge with specification, which confirms that system S is used on the edges. For example, the left edge of receipt MAH 16323 indicates a total of 4 oxen and 5 sheep (4 gu₄ 5 udu).

On tablets MAH 16605 and MAH 16573 (and others from the same archive), the numbers noted on the edges have the same function, which is to provide the total number of animals delivered.

Thus, it is highly probable that they belong to the same numerical system, and we can infer that the isolated number \llcorner on the edge of the tablet MAH 16605 is noted in system S.

Tablet AO 5674 (Genouillac 1922) is a large multi-column tablet from Ur III Umma, and records numbers of workdays. Sub-totals of the items in each column are noted in the margin (outside the main text) at the bottom of the columns and appear as isolated numbers without specification (see Fig. 5.2).



Fig. 5.2 Marginal Numbers on the Lower Edge of Tablet AO 5674 (*Courtesy of the Musée du Louvre, photo C. Proust*)

At the bottom of column ii, the number 8 (first sign starting from the left), noted with two rows of four wedges, appears. At the bottom of column iv (fourth sign on the edge), the number 2 (geš'u) appears. Here, there is no ambiguity because of the use of the sign 'GEŠ'U' is specific to system S. Thus, we can conclude that all of the marginal numbers on this tablet are noted in system S. A similar example can be found on Umma tablet AO 5676 (see Fig. 5.3).



Fig. 5.3 Marginal Numbers on Lower Edge of Tablet AO 5676 (*Courtesy of the Musée du Louvre, photo C. Proust*)

Likewise, tablet AO 6038 (Genouillac 1922) is a large multi-column tablet from Ur III Umma and records numbers of workdays with sub-totals of the workdays at the bottoms of the columns. Here again, these numbers appear in the margin as isolated numbers (see Fig. 5.4).¹⁷

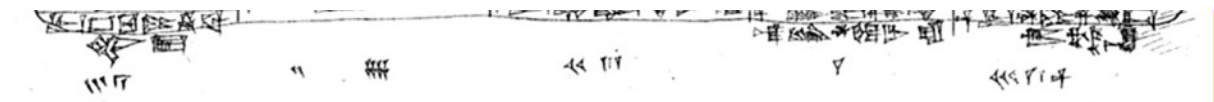


Fig. 5.4 Marginal Numbers on Lower Edge of Tablet AO 6038 (*Copy Genouillac 1922*)

¹⁷ We have provided a copy instead of a photo of the marginal numbers, because the numbers appear faint in a photo and would be hard for a non-specialist to recognize.

At the bottom of column v the number $32 \frac{1}{2}$ ($\llcorner \Upsilon \lrcorner$) appears. The use of a fraction shows clearly that the notation is not positional, and that here again, the marginal numbers belong to system S.

Many other similar examples of numbers written in system S on the edges of administrative tablets denoting totals or sub-totals can be found in Ur III sources.

System S in Boxes

The boxes inserted in texts to house marginal numbers are recognizable because they exhibit no ruling, and contain only isolated numbers (numbers or measurement values without specification of the goods quantified). If the box is intended to house a sub-total and the sub-total turns out to be zero, it is even possible for the box to be empty (see example of MAH 19472 in Fig. 5.5).




		
<p>Erlenmeyer 152, obv. col. ii: box with SPVN¹⁸</p>	<p>MAH 19472, obv. col. ii: box with measurement value (capacity) MAH 19472, rev. col. v: empty box¹⁹</p>	<p>MSR 1, rev. col. v: box with number noted in system S²⁰</p>

Fig. 5.5 Marginal Numbers in Boxes

Similar examples of numbers written in system S in boxes or on the edges of administrative tablets to denote totals and subtotals are omnipresent in Ur III sources. They deserve further study, but such an ambitious enterprise goes beyond the scope of the present article. We focus on positional notations, which are much less common.

With these considerations on the different kinds of marginal numbers, positional or not, in mind, we have specific criteria to define the *corpus* under study in this chapter more precisely.

5.1.3 Paleography

18 Courtesy of Robert Englund, CDLI photo ATU project.

19 Courtesy of Jean-Luc Chappaz, curator of the *Musées d'Art et d'Histoire* of Geneva, photos C. Proust.

20 Courtesy of Benjamin Foster, Yale Babylonian Collection.

Numbers noted in system S or in SPVN are accumulations of graphemes such as ‘diš’, ‘u’, or ‘aš’ (see Table 5.1). The way in which the scribes arranged the graphemes may vary, and a general trend to arrange the signs in rows of three elements maximum (see second column of Table 5.1) was perceived by historians (Oelsner 2001). The paleography using rows of three elements has been considered as innovative, and other arrangements as archaic. This rationalization may reflect a cognitive constraint: the human mind is able to grasp up to three items at first glance without counting. Aware of this phenomenon, scribes may have intentionally developed quick, easy to read notations, perhaps in connection with the disposition of counting tokens. These graphical variations were tentatively, but quite unsuccessfully used by historians for dating texts. Oelsner (2001)) has shown that the paleography of numbers reflects not only change over time, but also differences between genres of texts. He noted that the supposed ‘archaic’ paleography of numbers was used until the late Old Babylonian period in administrative texts, but almost disappeared from mathematical texts in the early Old Babylonian period. In our Ur III marginal numbers we shall discover a subtle use of graphical nuances to contrast different kinds of numbers in the same text. Thus, we avoid opposing the ‘archaic’ and the ‘innovative’ paleography, and we prefer to contrast non-normalized and normalized paleography, as illustrated in Table 5.1.

	Non-normalized paleography	Normalized paleography
diš-numbers		
4		
7		
8		
9		
u-numbers		
40		
50		
aš-numbers		
4		
7		
8		
9		

Table 5.1 Paleographies of Numbers

We have stressed that in some cases it is difficult to decide if the notations are positional or not. However, in the examples discussed above (Figs. 5.1-5.4), the context indicates that the numerical system is non-positional. It is worth mentioning that in all these cases, and in similar cases that we know of, the notation is non-normalized. We will see that, in contrast, positional notations in the margins are normalized. As far as marginal numbers are concerned, the paleography can be used as a criterion to distinguish positional from non-positional notation. We will return to this important point.

5.1.4 Ur III Mathematical Texts

In the Ur III period, ‘mathematical texts’ contain most of the known examples of the use of SPVN. What exactly is a mathematical text in the Ur III period? The list of texts falling under this category varies greatly according to different authors.²¹ The difficulties lie in the identification of the date and the genre: distinguishing Ur III texts from Old Babylonian texts, and administrative texts from mathematical texts. However, a minimal consensus can be reached on a small set of texts dated to the Ur III period and identified as mathematical. The reciprocal table Ist L 7375 published by Delaporte can be safely dated to the Ur III period, thus other very similar tables can also reasonably be considered as dating from the same period. The set of Ur III reciprocal tables includes at least two tables from Nippur (Ist Ni 374 and HS 201) and five tables from Girsu (Ist L 7375, Ist L 9005, Ist L 9006, Ist L 9007, Ist L 9008).²² All of them exhibit the following features:

- They are of type M(2,2).²³
- The entries include all the numbers between 2 and 59 (or more).
- When the numbers are irregular (7, 11, 13, 14, and so on), ‘no reciprocal’ (Sumerian *igi nu*) is noted.
- Moreover, all of these texts adopt the non-normalized paleography.

The mathematical character of these tables is based on the facts that they bear neither a date nor a personal name, that they provide a systematic list of data following an arithmetical logic, and that they use SPVN in the main text. The reciprocal tables are considered as school texts, but the context of scribal schools in the Ur III period is poorly documented.

Beside the reciprocal tables, another document can be considered with certainty as an Ur III mathematical text: the tablet AO 2728 (=AOT 304 = RTC 413).²⁴ According to Thureau-Dangin, the tablet comes from regular excavations led by Ernest de Sarzec in Girsu between 1894 and 1900. Thureau-Dangin dated the tablet to the end of the reign of King Šulgi or to the reign of his successors, that is, the very end of the third millenium (Thureau-Dangin 1903: vii-viii).²⁵ The text is considered to be a mathematical ‘school exercise’ because of its round shape,

21 The Ur III mathematical texts are not the same if we compare for example the list obtained by entering the criteria ‘mathematical’ and ‘Ur III’ in CDLI, the list provided in Robson (2008: 306, Table B7) or Robson (1999, 169-71), and the examples quoted in Friberg (2009: §4). Even the two lists by Robson differ from each other as she explains in (2008: 349, note 9).

22 Ist Ni 374 is published in Proust (2007) and HS 201 in Oelsner (2001); except Ist L 7375, the Girsu tablets are unpublished and were kindly mentioned to C. Proust by B. Lafont. Other tables of unknown provenience were tentatively considered to date from the Ur III period, even if they do not share these features, see Robson (2003-2004: 356-360); Proust (2008b: Chap. 2); Friberg (2009: Sect. 4.2.1-4.25).

23 M(2,2) means a multicolumn tablet, where the text is divided into two columns on the reverse and two columns on the obverse. Old Babylonian reciprocal tables are single column tablets (type S).

24 Thureau-Dangin (1903: 149 No. 413); this tablet was quoted by Friberg (1987-90: 541), and studied by Robson (1999: 66) and Proust (2007: 212); collation Lafont (1985: No. 304). See photo, copy, translation and transliteration in Appendix 5.III.

25 However, according to the archives kept at the Musée du Louvre’s *Antiquités Orientales* library, the tablet was bought in 1898 from the dealers Morel and Géjou.

its rough script, its formulation as a problem with a solution, and the lack of a date and proper name. The text of the obverse provides a length, a width and a height (probably the dimensions of a pile of bricks), and asks for the corresponding volume in standard units of volume and in brick-volume.²⁶ The answer is given, with interesting mistakes (see Robson 1999: 66). The numbers written in SPVN on the reverse correspond to the calculation of the volume. What is of interest for the purpose of this chapter is that this tablet exhibits a clear distinction between numbers used in the statement of the problem, namely metrological notations, and the SPVN numbers used on the reverse for calculations. The reverse is a rough surface with erasures, which seems to have served as a ‘scratch pad’ for calculations with SPVN, probably in connection with the use of a device such as an abacus or tokens.

We do not provide here a detailed analysis of the calculations and interpretations of mistakes, for which the reader is invited to refer to Robson (1999: 66) and Proust (2007: 212). The point to be underlined, to better understand the following text analysis, is the way in which metrological notations of the obverse correspond to the numbers written in SPVN on the reverse. For the sake of simplicity and avoidance of a detailed analysis of mistakes, let us consider the data related to the width. The third line of the obverse of the tablet gives the following information: ‘2 *kuš* 5 *šusi* is the width’ (2 *kuš*₃ 5 *šu-si* dagal). The analysis of the calculation shows that 2 *kuš* 5 *šusi* corresponds to 10:50, the last number noted on the reverse. This kind of correspondence is common in Old Babylonian school texts, and we can suppose that some of the basic tools used for calculation in the Old Babylonian period were known to the Ur III learned milieu.

Among these basic tools are the metrological tables, which provide a complete and coherent system of correspondences between measurement values (of capacities, weights, surfaces and lengths) and numbers written in SPVN. In particular, in the Old Babylonian metrological table for lengths, we read the following entries:

...
 5 *šusi* 50
 ...
 2 *kuš* 10
 ...

Thus, the measurement value 2 *kuš* 5 *šusi* corresponds to the number 10:50 in SPVN in Old Babylonian metrological tables. Exactly the same correspondence occurs both in Old Babylonian metrological tables and in the Ur III mathematical text AO 2728. This observation suggests that a correspondence similar to that attested in Old Babylonian sources was used in the Ur III period.

The reverse of AO 2728 shows that the calculation of the volume was executed by multiplying together the numbers corresponding to the length, the height, and the width in SPVN. Indeed, if we multiply together the (incorrect) numbers in SPVN given on the reverse of the tablet (6:31:50, 3, and 10:50), we obtain a number which corresponds quite well to the (incorrect) volume given in obv. 4 of the tablet, as well as, after multiplication by the coefficient of bricks 7:12, the (incorrect) brick-volume given in obv. 5.

In conclusion, the numbers written in SPVN on the reverse:

²⁶ See Chap. 2 by Heimpel, Chap. 6 by Middeke-Conlin and Chap. 4 by Proust in this volume, for more information on the notion of ‘brick-volume’.

- Correspond to the measurement values noted on the obverse according to the metrological tables similar to those attested in Old Babylonian documentation, and
- Were used for the calculation of volumes.

Old Babylonian metrological tables give, for example, the following correspondences:²⁷

Metrological system	Measurement units	SPVN	Relationship between measurement units (factors in decimal notation)	Relationship between metrological systems
Length	1 <i>šusi</i>	10		
	1 <i>kuš</i>	5	1 <i>kuš</i> = 30 <i>šusi</i>	
	1 <i>ninda</i>	1	1 <i>ninda</i> = 12 <i>kuš</i>	
Surface volume	1 <i>gin</i>	1		1 <i>sar</i> is the measure of a square with 1 <i>ninda</i> side
	1 <i>sar</i>	1	1 <i>sar</i> = 60 <i>gin</i>	
	1(<i>iku</i>) <i>gan</i>	1:40	1(<i>iku</i>) <i>gan</i> = 100 <i>sar</i>	
Capacity	1 <i>še</i>	20		1 <i>gur</i> = 1 <i>gin</i> -volume ²⁸
	1 <i>gin</i>	1	1 <i>gin</i> = 180 <i>še</i>	
	1 <i>silā</i>	1	1 <i>silā</i> = 60 <i>gin</i>	
	1 <i>gur</i>	5	1 <i>gur</i> = 300 <i>silā</i>	
Weight	1 <i>še</i>	20		For each material, relationships between volumes and weights are provided by lists of coefficients
	1 <i>gin</i>	1	1 <i>gin</i> = 180 <i>še</i>	
	1 <i>mana</i>	1	1 <i>mana</i> = 60 <i>gin</i>	
	1 <i>gu</i>	1	1 <i>gu</i> = 60 <i>mana</i>	

Table 5.2 Correspondences between Some Measurement Values and SPVN according to OB Metrological Tables

This global set of correspondences connects all the different metrological systems with each other and, inside a given system, the factors defining the measurement units. We shall see that, in the Ur III administrative texts examined in Sect. 5.2, the same system of correspondences is used to connect the numbers written in SPVN to the measurement values.

5.1.5 Positional Notations in Margins: Sources

As highlighted above (Sect. 5.1.1), for a long time, tablet YBC 1793 (Text 10) was considered by historians to be the only text testifying the use of SPVN in administrative context. Although unique and completely atypical, in the historiography this text represents the archetype of SPVN

²⁷ See the complete set of metrological tables in Proust (2009: Chap.9).

²⁸ 1 *gin*-volume is a volume with a 1/60 *sar* base and a 1 *kuš* height. 1/60 *sar* corresponds to 1 (in floating SPVN) and 1 *kuš* corresponds to 5, thus 1 *gin*-volume corresponds to 5 and, as a result, 1 *gur* also corresponds to 5.

in Ur III administrative texts. When he studied YBC 1793 in detail, Powell quoted other texts containing positional notations: Nik. 2 402, CBS 11661, RTC 408 (= AO 27307), RTC 413 (= AO 2728) and YBC 4179 (Powell 1976a: 435, note 6). In all of these examples, positional notations appear as graffiti in margins (on the edges or in special boxes); however, this coincidence did not attract Powell's attention. Robert Englund found other examples, such as YBC 16487 and Erlenmeyer 152. Since starting our research on marginal numbers within the framework of the SAW project, we have identified several additional sources.²⁹

To our best knowledge and according to our analysis, the complete list of administrative tablets having positional notations is given in Appendix 5.III. This list of tablets is arranged by place of origin and, for a given place, by date. We labelled the tablets we discuss in detail in the present chapter with numbers from 1 to 10 (see first column of table in Appendix 5.III). Other tablets in the list are not discussed in detail here because they are either too damaged to provide adequate information, or will be studied elsewhere.

Note that the sole example we know of a text containing marginal numbers earlier than the Ur III period is an Early Dynastic text (mid-third millennium) which contains graffiti similar to SPVN, but very probably they are not positional (Ist L 9236, from Girsu, published in Genouillac 1921).

5.2 Discussion of Individual Texts

5.2.1 About the Evidence

In this part, we shall focus on ten texts, whose marginal notations are preserved on the tablets and make sense for us. These texts can therefore be presented as examples. Before we delve into the details, some background knowledge about the evidence is in order.

The Ur III dynasty has produced more documents than any other period throughout the history of ancient Mesopotamia. Almost all the extant documents from this period are written in Sumerian, whose linguistic affiliation remains disputed. The latest estimate places the number of known Ur III texts at around 90 000, and the total number of documents surviving from this period may exceed 120 000 (Molina 2008: esp. 20). Two online projects, the Database of Neo-Sumerian Texts (BDTNS) and the Cuneiform Digital Library Initiative (CDLI),³⁰ have made systematic efforts to digitalize the Ur III tablets and provide open access to them. More than 95 per cent of the records originate from five provinces or sites located in the southern part of the kingdom: Umma, Girsu, Puzriš-Dagan (modern Drehem), Ur and Nippur. The site of Umma leads all the others in the volume of documentation with a total of 27 944 tablets, and Girsu follows with 24 460 tablets.³¹ The texts to be discussed thus come from the three major sites of Umma, Girsu and Puzriš-Dagan.

29 Personal communications in the context of CDLI technical meetings. Probably, more examples can be found in different collections, for example at Yale, as noted by Powell (1976a: 435, note 6): 'When I discussed YBC 1793 with W. W. Hallo, curator of the Yale Babylonian Collection, he told me that he remembered seeing sexagesimal notations rather frequently in balanced accounts from the Ur III period and referred me to YBC 4179, published by Ellis [1970]'.

30 See respectively, <http://bdts.filol.csic.es/> and <http://cdli.ucla.edu/>.

31 Molina (2008: 52-53). The statistics concerning Umma and Girsu take into account a small number of texts, 260 and 126 respectively, with an uncertain provenience. Moreover, texts from two sites, Girsu and the much smaller Lagaš, are both counted as Girsu texts because they were sister sites located in the same administrative province we call Girsu or Girsu-Lagaš.

The fundamental problem with the Ur III sources lies in that the vast majority of them lack archaeological context because they came to light through looting, in particular those tablets from Umma, Girsu and Puzriš-Dagan.³² As a result, the provenience of a tablet has to be determined based on other internal (genre, type of calendar attested and prosopographical studies) and external evidence (information about its acquisition). Consequently, we often have to take the provenience of a tablet with a grain of salt.

The vast majority of the Ur III texts fall into the category of administrative documents, in the sense that they were prepared by various state or local administrative officials in an institutional context in order to track the receipt, expenditure and transfer of a wide range of commodities. The texts from Umma and Girsu form respectively two provincial archives, while the records from Puzriš-Dagan constitute a state archive of the central government.

A special kind of administrative record, the so-called balanced account, appears most often in the evidence to be studied (Texts 1-4 and 7). The usual hallmark of such accounts is the phrase, ‘balanced account’ (nig₂-kas₇-ak), toward the end. This Sumerian phrase is frequently attested in conjunction with personal names, professional titles, products, obligations, or organizations, which specify to whom the balanced account belongs or what it concerns or both. People attested as the responsible party for all the transactions in a balanced account included the governor (Text 4), his administrators at various levels (Texts 1, 3 and 9) and professionals such as merchants (Texts 2 and 7).³³ Table 5.3 below illustrates the standard structure of a balanced account.

Section 1
(S)

Remainder carried over from the preceding periods (si-i₃-tum = la₂-NI)
 +
 Product₁ (ku₃-bi x₁)
 +
 ...
 +
 Product_n (ku₃-bi x_n)
 =====
 Total receipts **S**(sag-nig₂-gur₁₁-ra-kam)

Section 2
(Z)

Expenditures (ša₃-bi-ta)
 Product₁ (ku₃-bi x₁)
 +
 ...
 +
 Product_n (ku₃-bi x_n)
 +
 Expenditures carried over from preceding periods (diri)
 =====
 Total expenditures / disbursements **Z**(zi-ga-am₃)

32 For a brief overview of this issue and further references, see Ouyang (2013: 28-29).

33 Although text 7 does not contain the phrase that we translate as ‘balanced account’ (nig₂-kas₇-ak), it adopts the general structure of such an account and features, as discussed below, two other key terms (sag-nig₂-gur₁₁-ra-kam and ša₃-bi-ta) characteristic of a balanced account.

Section 3
(Difference between S and Z)

If $S > Z$, then the difference S-Z is the deficit (la₂-NI)
If $S < Z$, then the difference Z-S is the surplus (diri)

Section 4
(Subscript)

Subscript: balanced account (nig₂-kas₇-ak), followed by personal names, titles, products, obligations, or organizations.

Table 5.3 Standard Structure of a Balanced Account

A balanced account consists of three major sections. The first section, called ‘it is the head of goods’ (sag-nig₂-gur₁₁-ra-kam, abbreviated as S), enumerates the products either received by or made available to an individual or organization affiliated with the administration. Attested products range from comestibles to manufactured goods, raw materials, labour output of work crews calculated in workdays,³⁴ silver and real estate. A number of translations have been proposed for the Sumerian expression (sag-nig₂-gur₁₁-ra-kam), without general acceptance (Ouyang 2013: 38-40). In this chapter, we prefer to translate it as ‘receipts’.

The next section, enclosed within the Sumerian phrase (ša₃-bi-ta ... zi-ga-àm, abbreviated as Z) meaning ‘... is expended out of (the head of goods)’, summarizes the expenditures that the same individual or organization made on behalf of the administration. Likewise, different translations have been attempted without broad acceptance (*ibid.*). Products in this section may include the same as (Texts 3 and 7) or differ from (Texts 1, 2 and 4) those in the previous S section. When the products in the two sections happened to be the same, we translate the Sumerian phrase as ‘disbursements’; otherwise, we translate it as ‘expenditures’.

Finally comes the third section that compares the previous two sections, S and Z, and calculates the difference between the two. This step proves straightforward when the same products appear in both S and Z. If the products in S exceed those disbursed in Z ($S > Z$), the third section designates the difference as the ‘deficit’ (la₂-NI) of the responsible party, which means that he did not disburse all the products placed at his disposal and incurred liabilities toward the government during the current accounting period. If S ends up less than the disbursed Z ($S < Z$), then the third section notes the difference as the ‘surplus’ (diri) of the responsible party. Such a surplus means that his disbursements exceeded his receipts during the current accounting period and is carried forward into the next accounting cycle.

The calculation of the balance (la₂-NI or diri) requires an additional step when the products in the received S section differ from those expended in the Z section. In this case a standard product was introduced to convert the value of the different products in both sections into a standard value, before the comparison of the total value of S with that of Z. Silver appears most often as the standard product (Texts 1, 2 and 4), but barley, wool as well as other goods occur occasionally (see Englund 2012: 435-436). The accounts in which silver served as a standard in order to convert non-silver products are also known as silver accounts. Silver accounts come mostly from the site of Umma and, often, but not always, are associated with merchants, who often received staple goods from the Umma government but supplied it with a far greater variety of products.³⁵

34 Such as the two texts AO 5674 (AS 3 xii) and AO 5676 (ŠS 2) mentioned in Sect. 5.1.

35 See Ouyang (2013: Chap. 5) for a systematic study of the Umma merchants.

Our evidence attests to more occurrences of deficit (Texts 1, 2 and 4) than surplus (Text 3). Both kinds of results carried over to the next accounting period. The content of a surplus simply entered the Z expended section of the next balanced account of the same responsible party. The treatment of a deficit, however, appears different because its content entered the S received section of another account under the new rubric ‘reminder’ (si-i₃-tum – see Text 2).

Next we shall examine the texts one by one and analyse the use of marginal notations in each case. The dates of the texts are provided using the abbreviated year-month-day format (see Appendix 5.I.3).

Remark on terminology of quantities and values

The quantities of goods (for example bitumen in Text 2) can be expressed in different ways:

- Quantity in-kind, that is, actual quantities (e.g. in Text 2 a capacity of 4 *barig* 1 *ban* 9 *silā* of bitumen)
- Value expressed as a quantity of another reference good (the most often, value in-silver, that is, value expressed as a weight of silver, e.g. in Text 2, a weight of 2 5/6 *gin* of silver); barley is also often used as a reference good.

The value is connected to the quantity by a rate. In the majority of cases this rate is the quantity of a good whose value is one unit of the reference good (the most often, 1 *gin* of silver).

5.2.2 Umma

Text 1. Nik. 2 402 (date AS 4)

This tablet³⁶ presents a silver account concerning Lugal-hegal, whose identity remains elusive in the Umma *corpus* because of the popularity of the name.³⁷ He may have served as an administrator in the Umma government. The beginning of the account mentions 13 1/3 *gin* of silver that he received during the current accounting cycle presumably from the Umma government. Following are the two products, lard and a product whose name is broken, which Lugal-hegal provided for the government. The value of both products was converted into silver, in order to calculate the difference between his silver receipt and the total value of the goods that he supplied. The text recognizes, from the viewpoint of the government, a deficit of 1/3 *gin* 6 1/2 *še* of silver on the part of Lugal-hegal. Such a deficit meant that Lugal-hegal had yet to provide goods worth that much silver for the government or to repay that amount of silver in the future. Table 5.4 below summarizes the data in this account:

36 It has thus far been published only in a hand-copy, so we provide a full edition in Appendix 5.IV.

37 In fact, the name Lugal-hegal is attested in all the major corpora of the Ur III period. Despite that, the genre of this text (i.e. the silver account – see discussion above) provides the critical evidence for deducing the provenience of the tablet. In the Umma text, somebody named Lugal-hegal appears identified as a son of Šeš-kala in VAT 7042 (dated to AS 8; OrSP 47 382 53), as a son of Ur-Sin in Erm 14994 (dated to IS 2; Santag 6 340), and as a pig farmer in HMA 9-02824 (dated to AS 4; UCP 9-2-1 100); see Ouyang (2013: p. 84, note 178; p. 145, note 687); Snell (1982: 101).

	Loc.	Quantity in-kind	Loc.	Value in-silver	Loc.	Marginal number
Receipts	o.1			13 1/3 <i>gin</i>		
Expenditures						
Lard	o. 3	1 barig 3 ban 6 1/2 sila 2 <i>gin</i>	o. 4	5 1/2 <i>gin</i> 23 1/2 <i>še</i>		
Unknown product	o. 5	11 <i>mana</i> [...]	o. 6	7 1/3 <i>gin</i>		
Total			r. 1	12 5/6 <i>gin</i> 23 1/2 <i>še</i>	o. 7	12:50 /space/ 23:30
Deficit (o. 1 minus r. 1)			r. 3	1/3 <i>gin</i> 6 1/2 <i>še</i>		

Table 5.4 Summary of Data in Nik. 2 402

A marginal number appears at the bottom of the obverse and consists of two parts, 12:50 and 23:30 (see Fig. 5.6).



Fig. 5.6 Nik. 2 402, End of the Obverse (Photo courtesy of Pushkin State Museum of Fine Arts)

A blank space separates the two parts, and the first part is written slightly above the second part. This marginal number corresponds to the total of the expenditures, 12 5/6 *gin* 23 1/2 *še*, in the following way: the number of *gin* (12 5/6) is transformed into the positional number 12:50 and the number of *še* (23 1/2) into 23:30. As we can see, the entire amount of 12 5/6 *gin* 23 1/2 *še* was not transformed into SPVN as a whole,³⁸ but rather, the number of *gin* and the number of *še* were transformed separately into two positional sexagesimal numbers independent of each other.³⁹ In order to distinguish the two results from different transformations, we label the

38 The transformation of the entire amount into SPVN would yield 12:57:50, according to the correspondence attested in Old Babylonian metrological tables.

39 Powell (1976a: 435, note 6) first noted this kind of strange notation, found on the tablet BM 19027: ‘There is an instructive example of mixed notation (sexagesimal + standard metrological notation) written on the edge of a tablet dated to the last year of Shulgi [King 1898, pl. 30 No. 19027]’. See also Brunke (2011) and Ouyang

notation of this marginal number as ‘partial-SPVN’. Indeed, although this notation appears positional, the factors between digits do not always equal sixty, or in other words, the notation is only partially sexagesimal. The arithmetical structure of the marginal number that we transcribe as ‘12:50 /space/ 23:30’ can be analysed as follows in Table 5.5:

	gin		1/60 gin		še		1/60 še
Factor		60 ←		3 ←		60 ←	
Digit	12		50	/space/	23		30

Table 5.5 Partial-SPVN (partially sexagesimal place value notation) of Weight

The two parts of the marginal number, 12:50 and 23:30, which appear to be separated by a blank space, are not completely independent since they represent the same measurement value, and are connected by the factor 3. We shall, after discussing more examples, revisit this kind of partial-SPVN in detail in Sect. 5.3.

Why are the total expenditures noted both in metrological notation in the main text and as partial-SPVN in the margin? To answer this question, we need to examine the possible mathematical operations involved in this text, which may include the following:

- Multiply by the reciprocal of a rate in-kind (the quantity of barley per unit of weight of silver) to calculate the value in-silver of each expenditure,
- Add the value in-silver of both expenditures to reach their total, and
- Subtract the total value in-silver of the expenditures from the advance to determine the deficit.

On which kind of numbers were these operations performed? Was the marginal number involved in these operations? If yes, then how and why?

The first operations to calculate the value in-silver of the two expenditures have left no trace but the results on the tablet. We have to assume that it had been performed elsewhere, outside the scope of this tablet and only the final results were copied on the tablet.⁴⁰ The second operation to find out the total value in-silver of the expenditures was performed by adding $5 \frac{1}{2}$ gin 23 $\frac{1}{2}$ še and $7 \frac{1}{3}$ gin, a simple operation that could be done mentally and probably did not require the use of any calculation tool. However, we cannot exclude that the sum may have been performed using numbers in partial-SPVN in the intermediate steps. In this case, the possible *modus operandi* can be reconstructed as follows (Table 5.6a):

$5 \frac{1}{2}$ gin	23 $\frac{1}{2}$ še	Two quantities to be added
$7 \frac{1}{3}$ gin		

(2016). Note that the notation of both numbers, 12:50 and 23:30, cannot be confused with system S (see Appendix 5.II.2).

40 The quantity in-kind 1 *barig* 3 *ban* 6 $\frac{1}{2}$ *sila* 2 *gin* (SPVN 1:36:32) of lard was equivalent to $5 \frac{1}{2}$ gin 23 $\frac{1}{2}$ še (SPVN 5:37:50) of silver. Both numbers and their quotient are irregular, thus the rate is irregular, and the calculation would involve approximation (see similar situation in Text 4). From the relation of $1:36:32 \times 3:30 = 5:37:52$, and the fact that 5:37:52 is very close to 5:37:50, we deduce that the rate in-silver was approximately 3:30 (10 $\frac{1}{2}$ še per *sila*). The rate in-kind used by the scribes in this evaluation was probably 17 (17 *sila* per *gin*) because 3:30 is an excellent approximation of the reciprocal of 17. By contrast, it is easy to see that the rate in silver of the copper used here is 1 $\frac{1}{2}$ gin (of cooper) per *gin* of silver. The value in-silver is obtained by dividing 11 by the rate (1:30 in SPVN), that is multiplying 11 by 40, the reciprocal of 1:30, which gives 7:20 in SPVN, or $7 \frac{1}{3}$ gin.

5:30	23:30	Step 1: Split the first quantity into two parts according to different metrological units, then transform it into partial-SPVN; Transform the second quantity into partial-SPVN
7:20		

12:50	23:30	Step 2: Total
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12 5/6 <i>gin</i>	23 1/2 <i>še</i>	Step 3: Transformation into standard metrological notation
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Table 5.6a Operations in Text 1

The sum was noted both in partial-SPVN by the marginal number and in metrological notation in rev. 1 of the main text. The deficit was obtained by subtracting the total value in-silver of the expenditures, 12 5/6 *gin* 23 1/2 *še*, from the total advance of silver, 13 1/3 *gin*. This subtraction appears more complex than the addition above because some elements, including the fractions, had to be carried over during the operation. The operation may have been facilitated by transforming the metrological notations into partial-SPVN. The possible *modus operandi* can be reconstructed as follows:

13 1/3 <i>gin</i>		Quantity from which to subtract (minuend)
12 5/6 <i>gin</i>	23 1/2 <i>še</i>	Quantity to be subtracted (subtrahend)

13:20		Step 1: Transform the minuend into partial-SPVN
12:50	23:30	Transform the subtrahend into partial-SPVN (by reproducing step 2 of the addition above)

13: <u>10</u>	30	Step 2: 10 in minuend carried backward and transformed into 30 (underlined digits)
12:50	23:30	

20	6:30	Step 3: Difference
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1/3 <i>gin</i>	6 1/2 <i>še</i>	Step 4: Transformation into standard metrological notation
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Table 5.6b Operations in Text 1

The result, 1/3 *gin* 6 1/2 *še*, appears on the tablet in rev. 3. According to our reconstruction, the marginal number corresponds both to the sum of the two quantities in the addition and to the quantity to be subtracted in the subsequent subtraction. Therefore, it seems to be linked to the execution of both addition and subtraction. After examining more evidence, we shall offer tentative diagrams in Sect. 5.3 to better illustrate the calculation process.

The paleography of the marginal number seems to be normalized (see the notation of 50). The copy of the tablet indicates traces of erasure in the line where the marginal number is located. The erasures may suggest that this line served as some kind of ‘scratch pad’ during the compilation of this account.

Text 2. YBC 16487 (date AS 5)

This tablet⁴¹ records a balanced account concerning Pada, a merchant active in the Umma province.⁴² The beginning specifies the balance of his receipts (i.e. his unfulfilled obligation toward the Umma government) carried over from the preceding accounting cycle.⁴³ The next part lists his expenditures on behalf of the government, which included an outgoing delivery of silver and other goods such as bitumen, alkaline plants and so on. The value of the goods that he provided was converted into silver, in order to calculate the difference between his receipts and the total value in-silver of his expenditures. At the end of the current period, the merchant incurred a deficit of $2/3$ mana 7 $1/2$ gin 17 še.⁴⁴ Such a deficit meant that Pada had yet to provide goods worth that much silver for the government or to repay it the same amount of silver.

	Loc.	Quantity in-kind	Loc.	Value in-silver	Loc.	Marginal number
Receipts						
Previous balance			o. 1	1 $1/2$ mana 6 $1/3$ gin 27 še		
Expenditures						
Delivery			o. 4	$1/2$ mana 8 gin		
Bitumen	o. 6	4 barig 1 ban 9 sila	o. 7	2 $5/6$ gin		
Alkaline plants	o. 8	[...]+1 barig 5 ban 6 sila	o. 9	[...]		
Rest broken						
Total			r. 1'	$2/3$ mana 8 $5/6$ gin 10 še	r. 6'	48:40 ^{sic?} /space/ 10 (48:50 /space/ 10 expected)
Deficit (o. 1 minus r. 1')			r. 3'	$2/3$ mana 7 $1/2$ gin 17 še		

Table 5.7 Summary of Data in YBC 16487 (AS 5)

A marginal number appears at the end of the reverse, bordering the edge and consists of two parts, 48:50 (which seems to be noted as 48:40 – see discussion in appendix) and 10, separated by a blank space (see Fig. 5.7). This marginal number corresponds to the total expenditures, $2/3$ mana 8 $5/6$ gin 10 še, in the same way as in Text 1. In this case the numbers of *mana* ($2/3$) and *gin* (8 $5/6$) are transformed into the sexagesimal number 48:50 (corrected value) and the number of *še* (10) into 10.⁴⁵ The partial-SPVN attested in the margins of Text 1 was likewise adopted

41 It has thus far been published in a hand-copy by Snell (1982: No. 5) and in transliteration with a study by Englund (1992: 85-86). Scattered references to it appear in other studies concerning the Umma merchants. See, for example, Snell (1982: 25-26) cited as AS5PdB; Ouyang (2013: 125, note 452) cited as Ledgers pl. 8 5. We provide a full edition of this tablet in Appendix 5.IV.

42 Pada appeared to be one of the three best-documented merchants in Umma. For his documentation, see Ouyang (2013: 220-2); for the most recent discussion of his business activities, see *ibid.*, Chap. 5.

43 Englund (1992: 85) has pointed out that this carried-over balance is recorded as the deficit (la₂-ia₃) of Pada in the balanced account Ashm. 1924-667 (AS 5 xi) compiled earlier in the same year.

44 As Snell (1982: 107) and Englund (1992: 85) have noted, this deficit appears almost the same as (only 10 še less than) the carried-over balance ($2/3$ mana 7 $1/2$ gin 27 še) in a balanced account of the next year, PUL Ex 662 (AS 6 xi). Images of that tablet show that between the two signs denoting the number 20 in '27 še', the first U sign appears smaller and fainter than the second U sign, but does not look like an erasure.

45 Englund (1992: 96, note 23) has noted the correspondence between the marginal number and the total of the expenditures: '48.40.10 appears to be a sexagesimal reconstruction of the total 48:50',10 expressed in shekel. The final '10' may have represented the 10 grains of the total, graphically separated from the notation itself since not a consequent part of a position value notation. A parallel situation seems to be attested in the text *Nik. 2 402'*.

in the marginal number here. The transformation of the entire amount into SPVN based on the Old Babylonian metrological tables would yield 48:43:20.



Fig. 5.7 YBC 16487, Bottom of Reverse (*Photo courtesy of Yale Babylonian Collection*)

The compilation of this account may have involved the same arithmetical operations as in Text 1:

- Multiply by the reciprocal of a rate in-kind to calculate the value in-silver of each expenditure,⁴⁶
- Add the value in-silver of both expenditures to reach a total, and
- Subtract the total value in-silver of the expenditures from the advance to determine the deficit.

The second operation, to find out the total value in-silver of the expenditures cannot be reconstructed, as the text preserves only two expenditures and the rest have been lost. But we may hypothesize that, just as in Text 1, the same kind of operations were carried out on numbers written in partial-SPVN to reach the sum of the value in-silver of the expenditures.

The deficit was obtained by subtracting the total value in-silver of the expenditures, $2/3 \text{ mana } 8 \frac{5}{6} \text{ gin } 10 \text{ še}$, from the receipt $1 \frac{1}{2} \text{ mana } 6 \frac{1}{3} \text{ gin } 27 \text{ še}$. This subtraction appears complex because it involves three metrological units and certain elements, including the fractions, have to be carried over during the operation. The calculation may have been facilitated by transforming the metrological notations into partial-SPVN. The possible *modus operandi* can be reconstructed as follows (Table 5.8):

$1 \frac{1}{2} \text{ mana } 6 \frac{1}{3} \text{ gin}$	27 še	Quantity from which to subtract (minuend)
$2/3 \text{ mana } 8 \frac{5}{6} \text{ gin}$	10 še	Quantity to be subtracted (subtrahend)
1:36:20	27	Step 1: Transform the minuend into partial-SPVN
48:50	10	Transform the subtrahend into partial-SPVN (first part noted as 48:40 ^{sic} on tablet, but correctly on the calculation device)

⁴⁶ The quantity in-kind, 4 *barig* 1 *ban* 9 *sila* (SPVN 4:19) of bitumen, was equivalent to $2 \frac{5}{6} \text{ gin}$ (SPVN 2:50) of silver. Both numbers and their quotient are irregular. The rate in-kind hovers around 1:30, and the rate in-silver around 40 (reciprocal of 1:30). We cannot say anything about the other expenditure as its value in-kind is partially lost.

<u>96</u> :20	27	Step 2: 1 in the minuend carried backward and transformed into 60 in following position (underlined digits)
48:50	10	
<u>95</u> :80	27	Step 3: 1 in the minuend carried backward and transformed into 60 in following position (underlined digits)
48:50	10	
47:30	17	Step 4: Difference
$2/3$ mana 7 $1/2$ gin	17 še	Step 5: Transformation into standard metrological notation

Table 5.8 Operations in Text 2

The result of $2/3$ mana 7 $1/2$ gin appears on the tablet in rev. 3’.

The fact that the marginal number does not correspond to the correct total of the value in-silver of the expenditures noted in the main text suggests that the latter was not transformed directly from the former. Rather, it is likely that the correct total expressed in metrological notation was copied from a calculation device. When the same sum was carried out on the device and the marginal number recorded as an intermediate result for use in the subsequent subtraction, the scribe made a mistake by writing down 48:40 /space/ 10 instead of the expected 48:50 /space/ 10. When he carried out the subtraction, the scribe, perhaps by checking against the correct sum in the main text, somehow recognized this mistake, corrected it and prevented it from propagating into the final result of the difference.

The paleography of the marginal number is again normalized, while that of the numbers in metrological notations of the main text is not. Moreover, the script of the marginal number appears smaller than that of the main text.

Texts 1 and 2 share several features. Both texts contain a marginal number with the same format and meaning: each number adopts normalized paleography and partial-SPVN with a blank space separating the two parts, and corresponds to the total value in-silver of the expenditures in the main text. Both texts involve the same set of operations that include the multiplication by the reciprocal of a rate in-kind, addition and subtraction. Therefore, it seems reasonable to suppose that the marginal numbers in both texts serves as some sort of shorthand to record an intermediate result, which equals the sum of an addition to be used as the subtrahend in the next step.

The location of the marginal number relative to its correspondent in metrological notation in the main text appears different in the two cases: the marginal number precedes its correspondent in Text 1 while it *follows* its correspondent in Text 2. Thus, the location of marginal numbers does not seem to be dependent on their correspondent in the main text.

Text 3. YBC 4179 (date AS 6)

This tablet⁴⁷ presents a multi-year balanced account of the Umma official Lu-Ninšubur. It lists the quantities of barley that he received and disbursed for the brewing of beer (še kaš) over the

⁴⁷ It has been published in a hand-copy and studied by Ellis (1970). A full edition of it appears in Appendix 5.IV.

course of twelve years from Š 43 to AS 6. Lu-Ninšubur received (60+48) *gur 4 barig 5 ban* of barley in total. His annual receipt consisted mostly of the barley designated for the preparation of beer for the ‘chariot’s crescent’ (u₄-sakar_x(SAR) ^{giš}gigir-ra) (*ibid.*: 266), which may refer to some lunar cultic activity. The barley received for this purpose amounted to 8 *gur 1 ban 8 sila* every year, with an additional 3 *barig 1 ban 6 1/2 sila* for the intercalary month in the four years Š 47, AS 2, AS 4, and AS 6, and 3 *barig 2 ban 1 1/2 sila* for the intercalary month in the year Š 44. The barley received by Lu-Ninšubur also included deliveries (mu-DU), which add up to 8 *gur 4 barig 4 ban 6 1/2 sila* for the twelve years. His disbursements during the same period reached (60+54) *gur 4 barig 5 ban 1 1/2 sila* and exceeded his receipts by 5 *gur 1 1/2 sila*, the amount which appears in rev. ii 3.⁴⁸ This positive balance (*diri*) means that Lu-Ninšubur surpassed his obligation toward the Umma government in these twelve years.

Receipts					
Year	Loc.	Quantity in-kind			Marginal number
		Delivery of barley for beer related to cultic activity (u ₄ -sakar _x ^{giš} gigir-ra)	Delivery of barley for beer (še kaš mu-DU)		
		In 12 months (iti-12-kam)	In additional month (iti diri)		
Š 43	o. i 1	8 <i>gur 1 ban 8 sila</i>			
Š 44	o. i 5-7	8 <i>gur 1 ban 8 sila</i>	3 <i>barig 2 ban 1 1/2 sila</i>		
Š 45	o. i 11	8 <i>gur 1 ban 8 sila</i>			
Š 46	o. i 15	8 <i>gur 1 ban 8 sila</i>			
Š 47	o. i 19	8 <i>gur 1 ban 8 sila</i>	3 <i>barig 1 ban 6 1/2 sila</i>		
Š 48	o. i 24	8 <i>gur 1 ban 8 sila</i>			
AS 1	o. ii 2	8 <i>gur 1 ban 8 sila</i>			
AS 2	o. ii 6-8	8 <i>gur 1 ban 8 sila</i>	3 <i>barig 1 ban 6 1/2 sila</i>		
AS 3	o. ii 13	8 <i>gur 1 ban 8 sila</i>			
AS 4	o. ii 17-18	8 <i>gur 1 ban 8 sila</i>	3 <i>barig 1 ban 6 1/2 sila</i>		
AS 5	o. ii 22-24	8 <i>gur 1 ban 8 sila</i>		3 <i>barig 3 ban</i>	
AS 6	o. ii 26- r. i 3	8 <i>gur 1 ban 8 sila</i>	3 <i>barig 1 ban 6 1/2 sila</i>	4 <i>barig 3 ban</i>	
Š 43- AS 4	r. i 5			7 <i>gur 1 barig 4 ban 6 1/2 sila</i>	7(aš) 1:46:30 (upper edge)
Subtotal		(96 <i>gur 3 barig 3 ban 6 sila</i>)	(3 <i>gur 1 barig 2 ban 1/2 sila</i>)	(8 <i>gur 4 barig 4 ban 6 1/2 sila</i>)	
Total	r. i 7	60+48 <i>gur 4 barig 5 ban</i>			
Disbursements					
Year	Loc.	Quantity in-kind			
Š 43	r. i 9	8 <i>gur</i>			
Š 44	r. i 10	8 <i>gur</i>			
Š 45	r. i 11	9 <i>gur 1 ban 5 sila</i>			

48 Actually, the amount appears with a mistake: a sign 5(*diš*) is inserted, but it cannot appear at this place according to the syntax of the measures of capacity. Perhaps the sign 5(*diš*) is an older sign that was badly erased.

Š 46	r. i 13	9 gur 1 ban 5 sila	
Š 47	r. i 15	9 gur 1 ban 5 sila	
Š 48- AS 4	r. i 17	39 gur 1 barig 6 1/2 sila	
AS 3	r. i 20	9 gur 3 barig	
AS 5-6	r. i 22	22 gur	
Total	r. ii 1	60+53 gur 4 barig 5 ban 1 1/2 sila	
Surplu s (diri: r. ii 1 minus r. i 7)	r. ii 3	5 gur 1 1/2 <<5>> sila (5 gur 1 1/2 sila expected)	5(aš) /space/ 1:30 (middle of rev. ii)

Table 5.9 Summary of Data in YBC 4179 (AS 6)

Two marginal numbers appear respectively on the upper edge of the tablet and in a special box in the middle of column ii on the reverse. The first marginal number, 7(aš) 1:46:30, corresponds to the sub-total of the barley deliveries during the ten-year period from Š 43 to AS 4,⁴⁹ 7 gur 1 barig 4 ban 6 1/2 sila, in the following way: the number of *gur* (7) is expressed with the same AŠ notation as in the main text, but with normalized paleography (three rows of AŠ instead of two rows in the main text), and the numbers of *barig*, *ban* and *sila* are transformed into the sexagesimal number 1:46:30 (see Fig. 5.8).



Fig. 5.8 YBC 4179, Upper Edge (Photo courtesy of Yale Babylonian Collection)

The partial-SPVN adopted in the marginal numbers of Texts 1, 2 and later in Text 4, was also adopted here, but in a slightly different way. Indeed, an additional new grapheme, AŠ, appears in this marginal number in order to denote the number of *gur*. Here a new phenomenon is manifest: the notation is not purely positional, as the shape of the grapheme AŠ provides information on the orders of magnitude. The expression ‘partial-SPVN’, therefore, refers not only to the fact that the notation is partially sexagesimal (presence of the factor 5, see Table 5.10), but also to the fact that the notation is partially positional (presence of the graphemes AŠ).

The arithmetical structure of the marginal number that we transcribe as ‘7(aš) 1:46:30’ can be analysed as follows in Table 5.10:

⁴⁹ The correspondence between metrological notations in the main text and marginal numbers is well analyzed by Ellis (1970: 267).

	<i>gur</i>		<i>barig</i>		<i>ban-sila</i>		$1/60 \text{ sila}$
Factor		5 ←		60 ←		60 ←	
Digit	7(aš)		1		46		30

Table 5.10 Partial-SPVN of Capacity

The second marginal number, 5(aš) /space/ 1:30, features partial-SPVN too and corresponds to the expected quantity of the surplus, $5 \text{ gur } 1 \frac{1}{2} \text{ sila}$, in a similar way (see Fig. 5.9). The transformation of these two quantities in-kind, $7 \text{ gur } 1 \text{ barig } 4 \text{ ban } 6 \frac{1}{2} \text{ sila}$ and $5 \text{ gur } 1 \frac{1}{2} \text{ sila}$, into SPVN would give 36:46:30 and 25:1:30 respectively based on the Old Babylonian metrological tables.



Fig 5.9 YBC 4179, Middle of rev. ii (Photo courtesy of Yale Babylonian Collection)

The compilation of this account is likely to involve the following operations:

- Addition of the annual receipts to reach their total,
- Addition of the annual disbursement to obtain their total, and
- Subtraction of the total of the disbursements from the total of the receipts to determine the difference.

The operation of addition appears more difficult than that in Texts 1, 2, as each addition would involve a dozen or more items. The correspondence of the first marginal number to the subtotal of the mu-DU deliveries during the ten-year period (Š 43-AS 4) may well imply that the addition had been performed using partial-SPVN numbers.

The surplus, $5 \text{ gur } 1 \frac{1}{2} \text{ sila}$, was obtained by subtracting the total quantity in-kind of the receipts, $(60+48) \text{ gur } 4 \text{ barig } 5 \text{ ban}$, from the total quantity of the disbursements, $(60+53) \text{ gur}$

4 *barig* 5 *ban* 1 1/2 *sila*. This subtraction looks simple as it involves neither elements to be carried over nor fractions. Nevertheless, the fact that the difference is noted correctly in the second marginal number but incorrectly in the metrological notation in the main text leads to the possibility that the subtraction had first been performed in partial-SPVN and subsequently the result was transformed into metrological notation. In other words, the scribe may have made a mistake in the latter step. The possible *modus operandi* can be reconstructed as follows (Table 5.11):

60+53 <i>gur</i>	4 <i>barig</i> 5 <i>ban</i> 1 1/2 <i>sila</i>	Quantity from which to subtract (minuend)
60+48 <i>gur</i>	4 <i>barig</i> 5 <i>ban</i>	Quantity to be subtracted (subtrahend)
1:53	4:51:30	Step 1: Transform into partial-SPVN
1:48	4:50	
5	1:30	Step 2: Difference
5 <i>gur</i>	1 1/2 <i>sila</i>	Step 3: Transform into standard metrological notation

Table 5.11 Operations in Text 3

The result, 5 *gur* 1 1/2 *sila*, is noted by mistake as 5 *gur* 1 1/2 <<5>> *sila*₃ in rev. ii 3 of the tablet.

The paleography of both marginal numbers is normalized, while that of the numbers in metrological notations in the main text is not. We observe traces of erasure in the box of the middle of rev. ii, where the second marginal number appears. Such erasures may indicate the use of this space as a scratch pad.

Text 4. YBC 16607 (date ŠS 5)

This tablet⁵⁰ records a balanced account on the silver that the governor of Umma received during a five-year period from ŠS 1 to ŠS 5. According to this account, the governor's total receipts of silver, 6 5/6 *mana* 7 1/3 *gin* 15 *še*, came from the sale of three batches of barley and the repaid arrears from the overseer of a mill house. His expenditures included 14 2/3 *gin* for the purchase of metals and 4 1/2 *mana* for an undisclosed purpose. Since the governor received more silver than he expended, the text registers a deficit of 2 *mana* 13 *gin* on his part, for which he remained accountable to the Umma government.

	Loc.	Quantity in-kind	Rate in-kind	Loc.	Quantity in-silver	Marginal number
Receipts						
Sale of barley in ŠS 1	o. 1	60+10 <i>gur</i>	1 <i>gur</i> 5 <i>ban</i>	o. 3	2 1/2 <i>mana</i> 7 <i>gin</i> 9 1/2 <i>še</i>	
Sale of barley in ŠS 1	o. 2	60+50 <i>gur</i>	1 <i>gur</i> 4 <i>ban</i>			
Sale of barley in ŠS 2	o. 5	60+10 <i>gur</i>		o. 6	1 <i>mana</i> 10 <i>gin</i>	

⁵⁰ It has been published in a hand-copy by Snell (1982: No. 13) and mentions of it appear in Ouyang (2013). A full edition is provided in Appendix 5.III.

Sale of barley in ŠS 3	o. 8	60x2+14 gur		o. 9	2 1/2 mana 7 1/2 gin	
Repaid arrears				r. 1	1/2 mana 2 5/6 gin 5 še	
Total				r. 4	6 5/6 mana 7 1/3! gin 15 še*	6:57:20 /space/ 15 (Lower edge)

Expenditures

Silver to buy metals				r. 6	14 2/3 gin	
Silver				r. 7	4 1/2 mana	
(Total)					(4 1/2 mana 14 2/3 gin)	

Deficit

(r. 4 minus r. 7)

				r. 9	2 mana 13 gin**	
--	--	--	--	------	-----------------	--

Table 5.12 Summary of Data in YBC 16607 (ŠS 5)

Notes: *Trailing part rounded up from 14 1/2 še.

**Rounded up from 2 mana 12 2/3 gin 15 še.

A marginal number appears on the lower edge and consists of two parts, 6:57:20 and 15, separated by a blank space in between (see Fig. 5.10). This marginal number corresponds to the total expenditures, 6 5/6 mana 7 1/3 gin 15 še, in the same way as in Texts 1 and 2: the numbers of *mana* (6 5/6) and *gin* (7 1/3) are transformed into the sexagesimal number 6:57:20 and the number of *še* (15) into 15. This marginal number adopts the same partial-SPVN as in those of the first two texts. The transformation of the entire amount into SPVN based on the Old Babylonian metrological tables would yield 6:57:25.



Fig. 5.10 YBC 16607, Lower Edge (Photo courtesy of Yale Babylonian Collection)

The compilation of this account may have involved the same arithmetical operations as used in Texts 1 and 2, but with an extra addition to determine the total value in-silver of the receipts:

- Multiplication by the reciprocal of a rate in-kind to calculate the value in-silver of each receipt,
- Addition of the value in-silver of all the receipts to reach their total,
- Addition of the value in-silver of both expenditures to obtain their total, and
- Subtract the total value in-silver of the expenditures from the receipts to determine the deficit.

For the first time in our evidence, this text provides rates in-kind for goods, two batches of barley available in the year ŠS 1, enabling us to calculate the amount of silver obtained from their sale.⁵¹ A modern calculation, using SPVN, is given in the last column of Table 5.13. Do the values given by the cuneiform text provide some evidence on the ancient method of calculation?

The first observation is that, for batches 1 and 2, the quantities and rates correspond to non-regular numbers, 5:50, 9:10 and 5:40 (see Table 5.13). It is not certain that the ancient scribes were aware of this property. However, we must highlight that the modern distinction between regular and non-regular numbers reflect an essential distinction which emerges from the known Ur III reciprocal tables: for the former, these tables provide the reciprocal, but for the latter, they provide the mention ‘no reciprocal’ (*igi nu* – see Sect. 5.1.4). Did the authors of our administrative Text 4 know the reciprocal tables? The known reciprocal tables come from Nippur and Girsu, not from Umma, but a positive answer is plausible. We come back to this question below.

	Quantity in-kind (SPVN)	Rate in-kind (SPVN)	Value in-silver according to modern calculation (SPVN)	Value in-silver according to tablet (SPVN)	Modern calculation
Barley (batch 1)	60+10 <i>gur</i> (5:50)	1 <i>gur</i> 5 <i>ban</i> (5:50)	1 <i>mana</i> (1)	1 <i>mana</i> (1)	(5:50/5:50 = 1)
Barley (batch 2)	60+50 <i>gur</i> (9:10)	1 <i>gur</i> 4 <i>ban</i> (5:40)	ca. 1 1/2 <i>mana</i> 7 <i>gin</i> 10 <i>še</i> (ca. 1:37:3:21)	1 1/2 <i>mana</i> 7 <i>gin</i> 9 1/2 <i>še</i> (1:37:3:10)	(9:10/5:40 ≈ 1:37:3:21:36)

Table 5.13 Value in-silver in YBC 16487

Anyway, for batch 1, the quantity (60+10 *gur* of barley) and the rate (1 *gur* 5 *ban* of barley per *gin* of silver) differ by a factor of sixty, thus the value in-silver is evident (60 *gin*, that is, 1 *mana*).

For batch 2, the quantity and the rate in-kind both correspond to irregular SPVN numbers (9:10 = 50×11 and 5:40 = 20×17) with no common factor, which means that the division of the quantity by the rate cannot be expressed with an exact value. Only an approximation of the value in-silver can be found. A modern calculation shows that the exact value in-silver is between 1 1/2 *mana* 7 *gin* 10 *še* and 1 1/2 *mana* 7 *gin* 10 1/2 *še*. The value in-silver provided in the text is 1 1/2 *mana* 7 *gin* 9 1/2 *še*, which is an excellent approximation of the exact value (error less than 1‰). The method used by the ancient calculators to get such impressive

51 Texts 4, 5 and 10 provide the rates explicitly. This situation is rare, as noted by Englund (2012: 440): ‘Texts such as *MVN* 11, 101, with multiple instances of explicit equivalency values in the form of 1(aš) 4(barig) 4(ban₂) 6 sila₃ mungur 3(aš) gur-ta / ku₃-bi 2/3 (gin₂) la₂ 3(diš) še (obverse 19–reverse 1) ... are very rare’.

precision is not clear. We can only infer that they used sophisticated mathematical tools, such as the reciprocal tables, developed in the scholarly milieu.

The total amount of the governor’s receipts can be found by adding the four values in-silver using partial-SPVN. The possible *modus operandi* can be as follows in Table 5.14a:

2 1/2 <i>mana</i> 7 <i>gin</i>	9 1/2 <i>še</i>	Four quantities to be added
1 <i>mana</i> 10 <i>gin</i>		
2 1/2 <i>mana</i> 7 1/2 <i>gin</i>		
1/2 <i>mana</i> 2 5/6 <i>gin</i>	5 <i>še</i>	
2:37	9:30	Step 1: Split the first quantity into two parts according to different metrological units, then transform into partial-SPVN; Transform the three other quantities into partial-SPVN
1:10		
2:37:30		
32:50	5	
6:57:20	14:30	Step 2: Total
6:57:20	15	Step 3: Round-up approximation
6 5/6 <i>mana</i> 7 1/3 <i>gin</i>	15 <i>še</i>	Step 4: Transformation into standard metrological notation

Table 5.14a Operations in Text 4

The sum of the addition is noted both in partial-SPVN in the marginal number and in metrological notation in rev. 4 of the main text. The deficit is obtained by subtracting the total expenditures, 4 1/2 *mana* 14 2/3 *gin*, from the total receipts, 6 5/6 *mana* 7 1/3 *gin* 15 *še*. The calculation may have been facilitated by transforming these two amounts into partial-SPVN. The possible *modus operandi* can be as follows in Table 5.14a (Table 5.14b):

6 5/6 <i>mana</i> 7 1/3 <i>gin</i>	15 <i>še</i>	Quantity from which to subtract (minuend)
4 1/2 <i>mana</i> 14 2/3 <i>gin</i>		Quantity to be subtracted (subtrahend)
6:57:20	15	Step 1: Transform both quantities into partial-SPVN
4:44:40		
6:56:80	15	Step 2: 1 <i>gin</i> carried backward and transformed into 60 sixtieths of <i>gin</i> (underlined digits)
4:44:40		
2:12:40	15	Step 3: Difference
2 <i>mana</i> 12 2/3 <i>gin</i>	15 <i>še</i>	Step 4: Transform into standard metrological notation

Table 5.14b Operations in Text 4

The calculated difference in the partial-SPVN may have been rounded up to 2:13, which then corresponds to 2 *mana* 13 *gin* as attested in rev. 9 of the tablet.

The paleography of the marginal number is likewise normalized, while that of the numbers in metrological notations of the main text is not. The first four signs of the marginal number look fuzzy and seem to have been written over erasures. In addition, the script of the marginal numbers appears smaller than that of the main text. All these observations may indicate the use of the lower edge, where the marginal number appears, as a scratch pad. As the marginal number appears on the lower edge of the tablet and thus in the middle (instead of the end) of the list of items to be added, such a position confirms that the spatial location of the marginal number may not depend on the main text, as previously observed in Texts 1 and 2.

Text 5. E 15550 (date IS 2)

This tablet⁵² calculates the volumes of earth relating to the construction or maintenance of different segments of a dike. The main body of the text includes twelve entries, each of which specifies the length in the unit *ninda* of one section of the dike and the earthwork calculated in the volume unit *sar* for this section. All the entries adopt the format, ‘(one section) x *ninda* long at the rate of y *sar* per (*ninda*), its earthwork z *sar*’ (x *ninda* gid₂ / y *sar*-ta / sahar-bi z), except for the entry in obv. 3, which we translate as ‘(one part) 6 *ninda* long without (work to be done)’ (6(diš) *ninda* gid₂ nu-tuku). Following the individual entries appears the total volume, (60+39) 5/6 *sar*, of the earthwork for the dike.

Loc.	Length of section	Work rate (volume per <i>ninda</i>)	Volume of earth to be dug (length × rate)	Marginal number	Possible modus operandi
o. 2	1 1/2 <i>ninda</i>	1 <i>sar</i>	1 1/2 <i>sar</i>		Mentally
o. 3	6 <i>ninda</i>		‘nothing to do’ (nu-tuku)		
o. 4-5	3 <i>ninda</i>	2/3 <i>sar</i>	2 <i>sar</i>		Mentally
o. 6-7	20 <i>ninda</i>	1/2 <i>sar</i>	10 <i>sar</i>		Mentally
o. 8-9	10 <i>ninda</i>	1 <i>sar</i>	10 <i>sar</i>		Mentally
o. 10-11	60+37 <i>ninda</i>	1/2 <i>sar</i>	48 1/2 <i>sar</i>	48:30 (left edge)	1:37×30 = 48:30
o. 12-13	3 <i>ninda</i>	1 <i>sar</i>	3 <i>sar</i>		Mentally
o. 14-15	20 <i>ninda</i>	1/3 <i>sar</i>	6 2/3 <i>sar</i>		Mentally
o. 16-17	15 <i>ninda</i>	1/2 <i>sar</i>	7 1/2 <i>sar</i>		Mentally
o. 18-19	[6] <i>ninda</i>	1 <i>sar</i>	6 <i>sar</i>		Mentally
r. 1-2	16 <i>ninda</i>	1/2 <i>sar</i>	8 <i>sar</i>		Mentally
r. 3-4	14 <i>ninda</i>	1/3 <i>sar</i>	4 2/3 <i>sar</i>		Mentally
r. 5	Total		60+39 5/6 <i>sar</i>		

Table 5.15 Summary of Data in E 15550

The marginal number, 48:30, appears on the left edge of the tablet and corresponds to 48 1/2 *sar*, which appears in the main text and specifies the volume of earth to be dug for the longest section of the dike.

52 An edition of the text appears in Ouyang and Brookman (2012: Sect. 3.4).

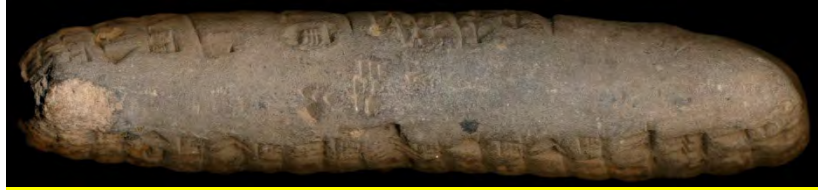


Fig. 5.11 E 15550, Left Edge (*Courtesy Peabody Essex Museum*)

The notation is clearly sexagesimal and positional, and cannot be interpreted as belonging to system S (see Appendix 5.II.2). Moreover, the transformation of the volume into SPVN, according to the correspondence attested in Old Babylonian metrological tables, would yield the same result. Thus, this marginal number is written in SPVN and not in the partially sexagesimal or partially positional notations exhibited in Texts 1-4.

The mathematical operations involved in the main text include the following:

- Multiply the length of each section of the dike by the work rate to calculate the volume of earth dug for each, and
- Add the volumes for the twelve sections dug to reach the total volume for the entire dike.

The multiplications seem simple enough for mental calculation except for the longest section, the length of which exceeds 60 *ninda*. The correspondence between SPVN and the result of the length of the longest section multiplied by the work rate indicates that the operation had probably been performed in SPVN (see last column of Table 5.15 above), and the result was subsequently transformed into metrological notation in the main text.

Summary

Among the five texts from Umma discussed above, the marginal numbers appear most often in a special type of administrative record called balanced accounts (Texts 1-4) and are used in the operations of addition and subtraction. Each of these texts bears one marginal number, except Text 3, which contains two. In Texts 1, 2 and 4 the marginal number consists of two parts, features partial-SPVN, and corresponds to a total value or quantity in-silver noted in metrological notation in the main text. This total in turn serves as a minuend (quantity from which to subtract) as in Text 4, or subtrahend (quantity to be subtracted) as in Texts 1 and 2, in the operation of subtraction, the final step to calculate the balance in a balanced account. In Text 3, each of the two marginal numbers consists of two parts and is written in the partial-SPVN as well, but in this case the partial-SPVN features the AŠ grapheme not attested in the marginal numbers in Texts 1, 2 and 4. The marginal number on the reverse of Text 3 corresponds to the difference of a subtraction concerning the quantities of barley, the final result of this balanced account.

In Texts 1-4, there seems to be no pattern for the location of a marginal number on a tablet, as it may appear near the bottom of the obverse or reverse, on the upper or lower edge, or in the middle of a column. Nor can we detect a link between the location of marginal number and its correspondent in metrological notation in the main text. What we do observe is that the position where some marginal numbers appear seems to have been scratched, and that some marginal numbers were written over erasures. This may hint at the temporary nature of the marginal numbers in the compilation of a balanced account.

Unlike Texts 1-4, Text 5 contains a list of different sections of a dike and the associated earthwork. The marginal number on the left edge proves to be SPVN, representing the result of a multiplication.

We have at this point identified several different ways of dealing with positional notations. In the margins of Texts 1-2 and 4, the notation is positional and partially sexagesimal. In the margins of Text 3, the notation is partially sexagesimal and partially positional. In Text 5, the notation is fully sexagesimal and positional, and its correspondence with the measurement values follows metrological tables attested in Old Babylonian sources. For simplicity, we decided to use the term ‘partial-SPVN’ for notations that are either partially sexagesimal, or partially positional, or both (Texts 1-4), and ‘SPVN’ for those sexagesimal place value notations reflecting the system of correspondence attested in metrological tables known from Old Babylonian sources (Text 5).

5.2.3 Puzriš-Dagan

Text 6. PTS 473 (date Š 48 vii)

Turning to the evidence from another site, Puzriš-Dagan, we now continue with Text 6. This tablet⁵³ records a list of livestock as booty (nam-ra-ak) from conquered territories, including two batches from Kimaš and Harši, two northeastern vassal kingdoms of the Ur III dynasty. One batch was conveyed by Bubu and the other by Šu-Enlil. The name of Susa appears on the reverse.

Batch conveyed by	Loc.	Cows	Loc.	Sheep and goats	Marginal number
Bubu	o. 1	2×60 + 45 cows			
	o. 2		o. 2	[x] + 8×60 + 34 sheep	
	o. 3		o. 3	[x] + 7×60 + 21 sheep	
	o. 4		o. 4	2×600 + 2×60 + 34 f. goats	
	o. 5		o. 5	3×600 + 27 f. goats	
Total	o. 6	2×60 + 45 cows	o. 7	2×3600 + 8×60 + 56 sheep (and f. goats)	2 /space/ 8:56 (middle of rev.)
Šu-Enlil		no details			
Total	o. 8-9	60 + 6 cows	o. 10	5×600 sheep and f. goats	

Note: ‘f’ stands for female.

Table 5.16 Summary of Data in PTS 473 (Š 48 vii)

A marginal number, 2 /space and break/ 8:56, appears in a box inserted in the middle of the reverse and corresponds to the total number of sheep and goats conveyed by Bubu, 2(šar₂) 8(geš₂) 5(u) 6(diš). Therefore, the same sexagesimal number is noted in system S in the main text on the obverse and in SPVN in a box on the reverse.

53 It has thus far been published only in a hand-copy by Sigrist (1990: No. 60), so we provide a full edition in Appendix 5.IV.



Fig. 5.12 PTS 473, Middle of Reverse (*courtesy of Princeton Theological Seminary*)

The only calculation in this text seems to be the addition of four flocks of sheep and goats in order to obtain their total. As Table 5.16 above shows, the addition does not involve any fractions, but deals with quite large and complex quantities. One can wonder if performing an addition of this type required the aid of a calculating device of some sort, or was done without such assistance, for example mentally. In the first hypothesis, the marginal number may have been a copy of the sum and served as a mnemonic before the sum was transformed into a number in system S in the main text (obv. 7). In the second hypothesis, if the addition was done mentally and the sum directly noted in system S, the marginal number may have been noted later in order to facilitate an operation in the next step that goes beyond the scope of this text. We hypothesize that, in this case, the next operation might have involved the calculation of the value in-silver of the sheep and goats by multiplying respectively their numbers by their rates in-kind. We find evidence of this operation in other administrative records from Ur III Umma.⁵⁴ At the light of subsequent texts, we will argue in favor of the second hypothesis.

Note that the paleography of the marginal number is normalized, while that of the numbers in system S of the main text is not. The space in the marginal number may indicate the absence of a sub-digit, *gešu*, in SPVN. The location of the box containing the marginal number (middle of the reverse) is not correlated to the location of its correspondent representation in system S in the main text (middle of the obverse). We see no traces of erasure in the box.

Text 7. NBC 6641(date ŠS 8 ix)

This text⁵⁵ registers seventy *mana* of silver distributed to probably eight individuals and four groups of people, three of which came from Sippar, Ur and possibly from Šuruppak respectively, for the purchase of gold at the rate of ten *gin* of silver per *gin* of gold (nig₂-sam_x(SA₁₀)-ma ku₃-sig₁₇ 10-ta-še₃). The main body of the text consists of individual entries that start with a personal name or the name of a group followed by the amount of silver distributed. We can identify all these individuals and groups as merchants thanks to rev. 18: ‘Copy of the sealed receipt of the merchants’ (gaba-ri kišib dam-gar₃-e-ne).

	Loc.	Quantity in-silver	Marginal number
Receipts	o. 1	1 <i>gu</i> 10 <i>mana</i>	
Disbursements	o. 4	9 1/2 <i>mana</i> 2 2/3 <i>gin</i> 11 <i>še</i>	
	o. 6	9 1/2 <i>mana</i> 2 2/3 <i>gin</i> 11 <i>še</i>	
	o. 8	9 1/2 <i>mana</i> 2 2/3 <i>gin</i> 11 <i>še</i>	

⁵⁴ As cited by Englund (2012: 441), the quantity of animals, including bandicoot rats (peš₂-giš-gi), two kinds of birds (amar-sag niga maš₂ and amar-sag u₂-ga maš₂) and fish, and their respective value in-silver are recorded on the Umma tablet MAH 19353 (Š 39). Additional evidence can be found in Ouyang (2013: 291-2, Table 5.3.M-1), which lists quantities of fish, donkeys, cows, sheep and goats alongside their value in-silver.

⁵⁵ Both Garfinkle and Paoletti (Garfinkle 2008; Paoletti 2008; Paoletti 2012: 216-223, 448-9) have edited and analysed this text in detail. Additional discussion of it appears in Ouyang (2011).

	o. 10	$7 \frac{5}{6}$ mana $7 \frac{1}{4}$ gin 3* še	
	o. 12	7 mana $9 \frac{1}{2}$ gin $8 \frac{1}{2}$ še $\frac{1}{2}$ še	
	o. 14	$6 \frac{1}{3}$ mana $1 \frac{5}{6}$ gin	
Subtotal of obverse	Lower edge	(50 mana $6 \frac{5}{6}$ gin)	50 /space/ 6:50
	r. 16	$4 \frac{2}{3}$ mana $6 \frac{1}{3}$ gin $5 \frac{1}{2}$ še	
	r. 18	3 mana $10 \frac{5}{6}$ gin 15 še	
	r. 20	3 mana $10 \frac{5}{6}$ gin 15 še	
	r. 22	$2 \frac{2}{3}$ mana $2 \frac{5}{6}$ gin $18 \frac{1}{2}$ še	
	r. 24	$2 \frac{1}{3}$ mana $3 \frac{1}{6}$ gin 3 še	
	r. 27	[x] mana [x] gin 3 še	
	r. 29	[x] mana [x] gin $22 \frac{1}{2}$ še	
Subtotal of reverse	Left edge	(20 mana $12 \frac{1}{6}$ gin $22 \frac{1}{2}$ še)	20 /space/ 12:17:30
Total	r. 32	1 gu 10 mana (1 gu 10 mana 19 gin $22 \frac{1}{2}$ še expected)	

Table 5.17 Summary of Data in NBC 6641

*Note: Published as $2 \frac{1}{2}$ še by both Garfinkle (2008: 66) and Paoletti (2012: 448). Our collation of the tablet reveals that the presumed MAŠ sign representing $\frac{1}{2}$ turns out to be a DIŠ sign with a tiny crack perpendicular to it. This crack conveys the misleading impression of a horizontal stroke, but the absence of the triangular head at the beginning of the crack makes it unlikely to be so.

Two marginal numbers appear on the lower and left edge of this tablet. Garfinkel (2008: 65) observes the ‘mathematical notations on the edges’, but does not comment on them. The marginal number on the lower edge, 50 /space/ 6:50, corresponds to the measurement value 50 mana $6 \frac{5}{6}$ gin, the subtotal of the silver distributed in the entries on the obverse of the tablet, which is not explicitly provided by the text but is the result of the addition of items listed obv. 1-14 (see Fig. 5.13). This correspondence reflects those attested in Old Babylonian metrological tables.



Fig. 5.13 NBC 6641, Lower Edge (Photo courtesy of Yale Babylonian Collection)

The second marginal number, 20 /space/ 12:17:30 is likewise a number written in SPVN and corresponds to the measurement value 20 *mana* 12 1/4 *gin* 7 1/2 *še* (see Fig. 5.14). This amount probably equals the subtotal of the silver distributed in the entries of the reverse side, despite the fact that some data have been lost in several damaged entries. Neither subtotal is noted in metrological notation on the tablet.



Fig. 5.14 NBC 6641, Left Edge (Photo courtesy of Yale Babylonian Collection)

Possible mathematical operations in this text may include:

- Addition, respectively, of the individual amounts of silver listed on the obverse and reverse to obtain two subtotals,
- Addition of these two subtotals to reach a total, and
- Multiplication of the total by the reciprocal of the rate in-silver (10) to calculate the corresponding quantity of gold.

As the amounts to be added on the obverse feature fractions with different denominators and three metrological units, the addition was likely to be performed in the partial-SPVN system attested in Texts 1, 2 and 4 from Umma. The same process would produce the subtotal of the entries on the reverse. The addition of the two subtotals corresponds to 1 *gu* 10 *mana* 19 *gin* 22 1/2 *še* and exceeds the total noted in rev. 32 of the text by 19 *gin* 22 1/2 *še*. The total of 1 *gu* 10 *mana* recorded on the tablet may be obtained by rounding down the exact value of 1 *gu* 10 *mana* 19 *gin* 22 1/2 *še* and indicates a rough approximation by truncature.

The correspondence between marginal numbers and the measurement values in the main text of the reverse reveals an interesting phenomenon. The sum (1 *gu* 10 *mana* 19 *gin* 22 1/2 *še*) of the quantities listed on the reverse appears twice: the exact counterpart in SPVN (20:12:17:30) appears on the left edge, and only a rough approximation by truncature (1 *gu* 10 *mana*) appears in the main text, rev. 32. We perceive here two different forms of the total: the total transformed into SPVN reflects a step in the flow of calculation, while the truncated total provides a rough idea of the actual total quantity.

The marginal numbers in this text may be linked to the execution of the addition of the quantities. However, the fact that the corresponding measurement value of the total does not appear on the obverse, but appears only truncated on the reverse, orients us toward another explanation. The calculation seems more linked to the application of the rate in-silver, 10, in order to facilitate the calculation of the total amount of gold that the distributed silver could be exchanged for. The latter hypothesis finds further evidence in two other Puzriš-Dagan texts

dated to the same month and year as our Text 7. The individual attested in rev. 19 of our text, Ilum-bāni, delivered 19 *gin* 16 $\frac{1}{2}$ *še* of gold in FLP 972 (cited as MVN 3 290 in Garfinkle 2008: 68). This quantity proves to be the same as that calculated by dividing the amount of silver that he received, 3 *mana* 10 $\frac{5}{6}$ *gin* 15 *še*, by the rate in-silver, 10. Another individual attested in obv. 11, Ilī-andulli, provided $\frac{2}{3}$ *mana* 7 $\frac{2}{3}$ *gin* of gold in NBC 6501 (Paoletti 2012: 447). This quantity approximates the result calculated by dividing the amount of silver distributed to him, 7 $\frac{5}{6}$ *mana* 7 $\frac{1}{4}$ *gin* 2 $\frac{1}{2}$ *še*, by the same rate in-silver.⁵⁶

As usual, the paleography of the marginal numbers differs from that of the numbers of metrological notation in the main text: the former is normalized, while the latter is not. Both marginal numbers are written in a smaller script than that of the main text. The conspicuous space in the first marginal number, 50 /space/ 6:50, may indicate the absence of the sub-digit of units in 50 and that of the sub-digit of tens in 6. The blank space in the second marginal number, 20 /space/ 12:17:30, may similarly mark the absence of the sub-digit of units in 20. In both cases, the blank space may reflect the state of a calculation device, for example the absence of a token or tokens in certain positions on an abacus. In contrast, the blank space in the partial-SPVN marginal numbers in Texts 1-4 seems to reflect the split of a number into two parts, the factor between which is not sixty. These observations underline a key aspect of the calculation: the numbers are not shaped in the same manner according to the goal or method of the calculations, and the function of the blank space is not the same in the different cases.

Text 8. YBC 13418 (no date)

This tablet⁵⁷ presents a record of gold deliveries and consists of nine entries, each of which starts with a certain amount of ‘red’ gold (*ku₃-sig₁₇ huš-a*) and ends with a personal name. None of the individuals carries any further identification other than his name, but four of them – Lu-Zabalam, Nūr-Šamaš, Ilum-bāni and Lu-Ašgi – reappear in Text 7 above, where they are identified as merchants. Two totals appear at the end of the text: one tallies the total amount of gold from the preceding nine entries, and the other specifies the corresponding value in-silver of that much gold.

	Loc.	Quantity in-kind (weight of gold)	Value in-silver (weight of silver)	Loc.	Marginal number
Delivery of gold	o. 1	1 $\frac{1}{3}$ <i>mana</i>			
	o. 3	1 <i>mana</i> 6 $\frac{2}{3}$ <i>gin</i>			
	o. 5	$\frac{5}{6}$ <i>mana</i> 3 $\frac{1}{3}$ <i>gin</i>			
	o. 7	$\frac{1}{3}$ <i>mana</i>			
	o. 8	$\frac{1}{3}$ <i>mana</i> 6 $\frac{2}{3}$ <i>gin</i>			
	o. 10	13 $\frac{1}{3}$ <i>gin</i>			
	o. 11	$\frac{1}{2}$ <i>mana</i> 3 $\frac{1}{6}$ <i>gin</i> 8 <i>še</i>			
	r. 2	2 $\frac{5}{6}$ <i>mana</i> 6 <i>še</i>			
	r. 4	$\frac{5}{6}$ <i>mana</i> 6 <i>gin</i>			

⁵⁶ The amount of silver corresponds to SPVN 7:57:16. Divide it by 10, that is, multiply it by 6 (the reciprocal of 10), we would get the result 47:43:36. The scribe might have rounded it down to 47:40 and then transformed it into $\frac{2}{3}$ *mana* 7 $\frac{2}{3}$ *gin*.

⁵⁷ It has been published in transliteration by Sigrist and Ozaki (2009: No. 20) and studied by Ouyang (2011). A full edition appears in Appendix 5.IV.

Total	r. 6-7	5 <i>mana</i> 11 <i>gin</i> 14 <i>še</i> (8 1/2 <i>mana</i> 9 1/6 <i>gin</i> 14 <i>še</i> expected)	1 <i>gu</i> 28 <i>mana</i> minus [?] 10 <i>gin</i>	Upper edge	1(aš) 27:50 xxx
				Bottom of rev.	20 /space/ [45 [?]]

Table 5.18 Summary of Data in YBC 13418 (no date)

Marginal numbers appear in two places on the reverse. After the last line of the reverse, the number 20 /space/ 45[?], is noted toward to the right edge. We cannot establish any relationship between this marginal number and any quantity noted in the main text. This number may be connected with the value in-silver of some of the gold amounts.

On the upper edge, a number, 1(aš) 27:50, is noted. It corresponds to the total value in-silver attested in the main text, 1 *gu* 28 *mana* minus[?] 10 *gin*, in the following way: the number of *gu* (1) is represented by the same AŠ notation, but the numbers of *mana* and *gin* are transformed into the sexagesimal number 27:50. This marginal number not only testifies to the partial-SPVN (notation partially sexagesimal) adopted in the marginal numbers of Texts 1, 2 and 4, but also features the AŠ grapheme (notation partially positional) attested in the case of Text 3. The number 1(aš) 27:50 is followed by some unclear figures (twice 40[?]), which may indicate the erasure of previous calculations.



Fig. 5.15 YBC 13418, Bottom of Reverse and Right Edge (*Photo courtesy of Yale Babylonian Collection*)



Fig. 5.16 YBC 13418, Upper Edge (*Photo courtesy of Yale Babylonian Collection*)

Possible mathematical operations in this text may include the following:

- Addition of the individual amounts of ‘red’ gold delivered to obtain a total, and
- Multiplication of the total by the reciprocal of a rate in-kind to find out the total value in-silver.

Or, in the reverse order:

- Multiplication of each amount of gold by the reciprocal of a rate in-kind to get the correspondent value in-silver, and
- Addition of the value in-silver of the individual amounts of gold to obtain the total value in-silver.

In light of the potential order of computation in Text 7, the calculation in this text may take the first order, that is, addition followed by multiplication. The subtractive notation that seems to have been adopted for the total (1 *gu* 28 *mana* minus⁷ 10 *gin*) may be another clue that the total was reached directly from adding the amount in metrological notation, and not from calculation with SPVN (see similar phenomenon in Text 10).

The addition was possibly performed using partial-SPVN as is also the case in Texts 1, 2, 4 and 7, or using other methods as suggested by the use of subtractive notation in the total in the main text.

Since the text does not spell out the rate in-kind of the gold delivered, we can only assume that the expected total in SPVN, 8:39:14:40, would be multiplied by the reciprocal of this rate in order to calculate the value in-silver of the gold. The product of this multiplication is noted as 1 *gu* 28 *mana* minus 10 *gin* in metrological notation in the main text and as 1(aš) 27:50 in partial-SPVN in the marginal number.⁵⁸

Both marginal numbers are noted on a scratched surface, and the one on the upper edge uses normalized paleography. The paleography of the other marginal number on the bottom of the reverse seems not normalized.

Summary

The marginal numbers of Texts 6-8 from Puzriš-Dagan, the site of the state archives, are mainly written in SPVN. The partial-SPVN characteristic of Texts 1-5 from the provincial archives of Umma is attested only in Text 8 where it corresponds to the total value in-silver of the amounts of gold delivered by various individuals. However, for each of the three Puzriš-Dagan texts, partial-SPVN may have been adopted in order to perform additions outside the tablet. In Text 6, the number in SPVN corresponds to the total number of sheep and goats. Was this number in SPVN the result of an addition or the input of further calculations, namely those to determine the value in-silver of the animals? In Text 7, the numbers in SPVN correspond to the sub-totals of quantities of silver distributed to various individuals. Again, this situation raises the same kind of question. Were these numbers in SPVN the results of additions, or rather both the output of previous calculations and the input for further calculations to compute the value in-silver of the gold? For Texts 6 and 7, related texts make the second hypothesis credible. The unclear marginal number at the bottom of the reverse of Text 8 (20 / space / 45³) may be linked in the same way with the calculation of the value in-silver of the gold distributed. In all the three texts from Puzriš-Dagan, marginal numbers are noted in normalized paleography except the one on

58 The product of 1:27:50 and 5:54:42:8 approximates to 8:39:14:40. Thus the rate in-silver may fall between 5:54 and 5:55 (i.e. the reciprocal of the rate in-kind may fall between 10:7 and 10:9).

the bottom of the reverse of Text 8. Moreover, on the tablet recording Text 8, the surface of the clay around the marginal numbers exhibits irregularities and erasures which evoke a scratch pad.

5.2.4 Girsu

Text 9. AO 27307 (= AOT c-61 = RTC 408) (date lost)

This text⁵⁹ features an interesting example dealing with seeding rates per unit of surface, and exhibits a kind of scratch pad on the reverse, where numbers apparently in SPVN may be associated to calculations using rates. The tablet contains two columns on each side, and the lower part is lost. Four sections, which we label A, B, C and D, are completely or partially preserved. Each section is devoted to one field, itself composed of plots. For each plot, the surface and a rate that gives a quantity of seeds (measured in units of capacity) per unit of surface are provided. Each section is concluded by the total quantity of seeds for the field concerned. The second column of the reverse ends with a large space containing erasures and numbers written in SPVN.

Section B, which is totally preserved, allows the determination of the unit of surface used in the expression of the seeding rate. Thureau-Dangin showed that the seeding rates give the quantity of seeds for each unit of 1 *bur* (1(*bur*₃) GAN₂, ca. 6.5 ha)⁶⁰. Thus, for example, the rate 1(aš) 1(*barig*)-ta means ‘1 *gur* 1 *barig* (of seeds) per (*bur* of surface)’.

	Loc.	Surface of a plot	Rate (capacity of seed per <i>bur</i> of surface)	Total quantity of seeds for the field
A (field for seeding)				
Plot 1	o. i 1	x + 3 <i>buru</i> 6 <i>bur</i> [1 <i>eše</i>] 3 <i>iku gan</i>	1 <i>gur</i> 2 <i>barig</i> 3 <i>ban</i>	
Plot 2	o. i 2	x + 4 <i>bur</i> 1 <i>iku gan</i>	1 <i>gur</i> 4 <i>barig</i>	
Plot 3	o. i 3	2 <i>buru</i> 8 <i>bur</i> 1 <i>eše</i> 2 <i>iku gan</i>	1 <i>gur</i> 1 <i>barig</i>	
Total field A	o. i 4			4×600 + 7×60 + 41 <i>gur</i> 2 <i>barig</i> 2 <i>ban</i> 5 <i>sila</i>
B (field for <i>bala</i>)				
Plot 4	o. i 6	2 <i>šar</i> 7 <i>bur</i> 2 <i>eše gan</i>	1 <i>gur</i> 2 <i>barig</i> 3 <i>ban</i>	
Plot 5	o. i 7	1 <i>šar</i> 2 <i>buru</i> 4 <i>bur</i> 2 <i>eše</i> 1 1/4 <i>iku gan</i>	1 <i>gur</i> 4 <i>barig</i>	
Plot 6	o. i 8	4 <i>buru</i> 1 <i>bur</i>	1 <i>gur</i> 1 <i>barig</i>	
Plot 7	o. i 9	4 <i>bur</i> 2 <i>eše</i> 5 <i>iku gan</i>	1 <i>gur</i>	

59 After its partial publication and interpretation by Thureau-Dangin (1897: 26-27; pl. 30 No. 79) at the end of the nineteenth century, this text has received little attention. It was mentioned by Powell (1976a: 435 note 6) who notes: ‘Two undated texts from this period which indicate the use of place notation in making calculations are Thureau-Dangin [1903, nos. 408 and 413]’. Lafont (1985) catalogued it under No. 61. We found neither the reference of the text nor its study elsewhere. The transliteration, translation (in Appendix 5.IV) and analysis offered here complete Thureau-Dangin’s publication.

60 For this reason, Thureau-Dangin (1897: 26) argued that ‘thus, it appears to be certain that it was considered as a unit’ (Il paraît donc certain que c’était considéré comme unité). The sign is now transliterated as 1(*bur*₃).

Total field B	o. i 10			$6 \times 60 + 38$ gur 5 ban 5/6 sila
Total fields A+B+...	o. i 12			$(x + 38)$ gur 3 barig [?] 1 ban 3 sila
C (field for seeding)				
Plot 8	o. ii 3	$x + 4 \frac{1}{2}$ iku gan + x	[1 gur 2 barig 3 ban]	
Plot 9	o. ii 4	3 šar 8 bur 2 eše 1 $\frac{1}{4}$ iku gan	1 gur 4 barig	
Plot 10	o. ii 5	3 šar 4 buru 2 bur 2 eše 3 $\frac{1}{2}$ $\frac{1}{4}$ iku gan	1 gur 1 barig	
Plot 11	o. ii 6	1 bur 2 eše gan	1 gur	
Total field C	o. ii 7			$3 \times 600 + 8 \times 60 + 38$ gur 3 barig 3 ban 4 5/6 sila 5 gin
D				
Plot 12	o. ii 9	2 šar 4 buru 4 bur 1 eše 2 iku gan	1 gur 2 barig 3 ban	
Plot 13	o. ii 10	2 buru 2 bur [+x gan ...]	[1 gur 4 barig]	
	[...]			

Table 5.19 Summary of Data in AOT c-61

Calculations

The calculations involved in this text yield the quantity of seeds for each plot (multiplication) and the total quantity of seeds for each field (addition). The quantity of seeds is obtained by multiplying the surface of the fields by the seeding rate (capacity per *bur*). For each plot of field B, Thureau-Dangin (1897: 26) calculated this product by:

- Converting the surface into the number of *bur*,
- Converting the rate into the number of *gur* per *bur*, and
- Multiplying the number of *bur* by number of *gur* per *bur*, an operation which gives the quantities of seeds expressed in number of *gur* (GOUR in Thureau-Dangin 1897); this is then transformed into standard capacity notations, which provide the quantity of seeds. The total calculated is exactly the total found in the text.

	127 $\frac{2}{3}$ GAN à 1 $\frac{2}{3}$ $\frac{3}{30}$ (GOUR de semence)
	84 $\frac{2}{3}$ $\frac{1}{18}$ $\frac{1}{72}$ GAN à 1 $\frac{4}{3}$ (GOUR de semence)
	41 GAN à 1 $\frac{1}{3}$ (GOUR de semence)
	4 $\frac{2}{3}$ $\frac{5}{18}$ GAN à 1 (GOUR de semence)
Ce qui fait : 398 $\frac{5}{30}$ GOUR $\frac{5}{3}$ QA de grains.	

Les calculs s'établissent ainsi:

$$\begin{array}{rcl}
 127 \frac{2}{3} & \times 1 \frac{2}{3} \frac{3}{30} & = 191 \frac{8}{3} \frac{3}{30} \text{ GOUR} \\
 84 \frac{2}{3} \frac{1}{18} \frac{1}{72} & \times 1 \frac{4}{3} & = 152 \frac{8}{3} \frac{3}{30} \text{ GOUR } 7 \frac{1}{3} \text{ QA} \\
 41 & \times 1 \frac{1}{3} & = 49 \frac{1}{3} \text{ GOUR} \\
 4 \frac{2}{3} \frac{5}{18} & \times 1 & = 4 \frac{4}{3} \frac{4}{30} \text{ GOUR } 3 \frac{1}{3} \text{ QA} \\
 \hline
 \text{TOTAL :} & & 398 \frac{5}{30} \text{ GOUR } \frac{5}{3} \text{ QA}
 \end{array}$$

Il parait donc bien certain que \leftarrow était considéré comme unité.

Fig. 5.17 Thureau-Dangin's Calculations (1897: 26)

This reasoning confirms that the rates provided in the text are capacities per *bur*. However, Thureau-Dangin's multiplications act on fractions. His calculations involve conversions into *bur* and *gur* which are not attested in cuneiform sources, and demand the ability to calculate with abstract fractions (multiplying and adding a large repertoire of fractions and integers). Such methods are alien to Ur III and Old Babylonian practices of calculation. Thus, the ancient process of calculation was certainly different. The fact that numbers in SPVN appear in col. ii of the reverse on a kind of 'scratch pad' suggests that the multiplications were performed in SPVN. If we assume that Ur III scribes involved in administration had access to metrological tables similar to those used in the Old Babylonian period as suggested in Sect. 5.1.4, the reconstructed calculations, given in Table 5.20 below, should include the following steps (we illustrate these steps with the example of plot 4, obv. i 6):

- The surface is transformed into SPVN using the metrological table of surfaces (col. I of Table 5.20) – (example: 2 *šar* 7 *bur* 2 *eše gan* is transformed into SPVN 1:3:50)
- The seeding rate is transformed into SPVN using the metrological table of capacities (col. II) – (example: 1 *gur* 2 *barig* 3 *ban* is transformed into SPVN 7:30)
- These two numbers (in SPVN) are multiplied together (col. III) – (example: 1:3:50 × 7:30 = 7:58:45)
- In the metrological table of surfaces, 1 *bur* corresponds to 30. Thus, the product of surface and the rate is to be divided by 30, that is, multiplied by 2 (col. IV) – (example: 7:58:45 × 2 = 15:57:30).
- The resulting number is transformed into a capacity value using the metrological table of capacities (col. V) – (example: 15:57:30 is transformed into (3 × 60 + 11) *gur* 2 *barig* 3 *ban*).
- These capacities correspond to lines indicated in col. IV of Table 5.20, but are not explicitly provided on the tablet, whereonly the total for each field is registered (in bold in Table 5.20).⁶¹

61 Explanations on the reconstruction. For section A, the text is too damaged to be completely reconstructed. For section B, the calculations can be completely reconstructed, as the text is perfectly preserved. In section C, the first entry (obv. ii 3) containing the surface and the seeding rate for plot 8 is partially destroyed. However, the

		I	II	III = I×II	IV = III×2	V (transformation of IV)
	Loc.	Surface	Rate	Surface × rate	Surface × rate × 2	Quantity of seeds (metrological notations)
A						
Plot 1	o. i 1	x + 18:15	7:30	x + 2:16:52:30	x + 4:33:45	x + 54 gur 3 barig 4 ban 5 sila
Plot 2	o. i 2	x + 2:01:40	9	x + 18:15	x + 36:30	x + 7 gur 1 barig 3 ban
Plot 3	o. i 3	14:13:20	6	1:25:20	2:50:40	24 gur 4 ban
Calculated total						x + 1×60 + 26 gur 5 ban 5 sila
Attested total	o. i 4					4×600 + 7×60 + 41 gur 2 barig 2 ban 5 sila
B						
Plot 4	o. i 6	1:03:50	7:30	7:58:45	15:57:30	3×60 + 11 gur 2 barig 3 ban
Plot 5	o. i 7	42:22:05	9	6:21:18:45	12:42:37:30	2×60 + 32 gur 2 barig 3 ban 7 1/2 sila
Plot 6	o. i 8	20:30	6	2:03	4:06	49 gur 1 barig
Plot 7	o. i 9	2:28:20	5	12:21:40	24:43:20	4 gur 4 barig 4 ban 3 1/3 sila
Calculated total					33:10:50:50	6×60 + 38 gur 5 ban 5/6 sila
Attested total	o. i 10					6×60 + 38 gur 5 ban 5/6 sila
C						
Plot 8	o. ii 3	9:29:57:29:40	7:30	1:11:14:41:12:30	2:22:29:22:25	(2×600 + 8×60 + 29 gur 4 barig 2 ban 2 1/3 sila 5 gin)
Plot 9	o. ii 4	1:34:22:05	9	14:9:18:45	28:18:37:30	5×60 + 39 gur 3 barig 3 ban 7 1/2 sila
Plot 10	o. ii 5	1:51:26:15	6	11:8:37:30	22:17:15	4×60 + 27 gur 2 barig 1 ban 5 sila
Plot 11	o. ii 6	1:20	5	6:40	13:20	1 gur 3 barig 2 ban
Calculated total						3×600 + 8×60 + 38 gur 3 barig 3 ban 4 5/6 sila 5 gin
Attested total	o. ii 7					3×600 + 8×60 + 38 gur 3 barig 3 ban 4 5/6 sila 5 gin

Table 5.20 Reconstructed Calculations and Transformations in AOT c-61 for Sections A, B and C
(in bold the notations attested on the tablet)

seeding rate is probably the same as in the first entry of section B for plot 4 (1 gur 2 barig 3 ban, SPVN 7:30), and the missing surface can be reconstructed. Indeed, we know the total quantity of seed for the four plots of field C (obv. ii 7) and we can calculate the quantity of seed for plots 9, 10 and 11 thanks to the information given in obv. ii 4-6. We obtained the quantity of seed for plot 8 by subtracting the seed for the other plots from the total, and finally, we obtained the surface of plot 8. In SPVN, the surface of plot 8 must be 9.29.57.29.40 (col. I of Table 5.20), which corresponds to 1 šaru 8 šar 5 buru 9 bur 2 eše 4 1/4 iku gan 24 sar 2/3 gin. Thus, line obv. ii 3 for plot 8 can be restored as follows: [1(šar'u) 8(šar₂) 5(bur'u) 9(bur₃) 2(eše₃) 4 1/4 (iku) GAN₂ [24 sar 2/3 gin₂ 1(aš) 2(barig) 3(ban₂)-ta]. In this case, 4 1/2 (iku) GAN₂ must be read '4 1/4 (iku) GAN₂'. For section D, the text is too damaged to be reconstructed. Another possibility is that this method, namely the multiplication by 2 and the use of a metrological table based on the correspondence of 1(bur₃) GAN₂ to 30 as in the Old Babylonian period, was replaced by a unique step, using a non-standardized table of capacity based on the correspondence of 1(bur₃) GAN₂ to 1 (instead of 30). In Old Babylonian B period, a unique set of standardized metrological tables served for all the calculations. A set of coefficients allowed the use of these tables. Here, this coefficient is 2.

Note the extravagant precision of the quantities of seeds involved: for field C, the total quantity of seeds is $(3 \times 600 + 8 \times 60 + 38)$ gur 3 barig 3 ban 4 5/6 sila 5 gin, which is approximately equivalent to 695 614.92 litres with a precision of 2 cl. This detail implies that the results come from purely theoretical calculations.

The last column of the reverse contains a large space with traces of numbers and erasures. Close observation of these graffiti leaves no doubt that they are SPVN: normalized paleography, no trace of graphemes other than DIŠ and U. These numbers may have been employed during the calculation process, similar to the calculations reconstructed in Table 5.20. However, we are not able to draw a clear connection between the numbers in SPVN displayed in columns I-IV of Table 5.20 and marginal numbers preserved on the tablet. In fact, we have little chance of finding such correspondence because a significant part of the text is lost.⁶²



Fig. 5.18 AO 27307, rev. col ii (Photo by C. Proust, courtesy of the Musée du Louvre)

Anyway, even if we found no clear connections between the preserved part of the text and marginal numbers, there is little doubt that these numbers are traces of the calculations of the capacities of grain to be seeded, that is, traces of multiplications.

62 Perhaps, the trace [...]2:[...]:40 (second line of marginal numbers) may correspond to the number 12:21:40 possibly involved in the calculation of the seeds for plot 7 of field B (obv. i 9). The number 45 (last line of marginal numbers – see rev. col. ii 8' of the transliteration, Appendix 5.IV) may correspond to one of the numbers ending with the digits 45 in column III of Table 5.20 (seeds for plot 4 or 5 of field B, or for plot 9 of field C).

Summary

In this text from Girsu, marginal numbers appear to be numbers written in SPVN, but their relationship with the preserved main texts is not clear. Numbers in SPVN appear on a kind of scratch pad occupying a large portion of col. ii of the reverse, and seem to be linked to the calculation of the quantity of seeds for each plot, that is, multiplications.

5.2.5 Unknown provenience

Text 10. YBC 1793 (date AS 5 ix)

Well known to historians of cuneiform mathematics, this tablet has been studied in many publications.⁶³ Our ongoing discussion on marginal numbers will shed further light on the understanding of the text and show that it belongs to the corpus examined here.

The tablet bears writing only on the obverse, and the reverse side is blank. The text starts in column i with four amounts of silver in SPVN followed by their subtotal in metrological notation, $1 \frac{1}{2}$ *mana* $3 \frac{1}{2}$ *gin* minus 7 *še*, and labeled as ‘various deliveries’ (mu-ku_x(DU) didli). Next appears 7 *mana* 19 *gin* of silver designated as ‘a delivery for the lustration of the king’ (mu-ku_x(DU) a-tu₅-a lugal). These two amounts add up to the total of $8 \frac{5}{6}$ *mana* $2 \frac{1}{2}$ *gin* minus 7 *še*. Column ii begins with another amount of silver, $8 \frac{5}{6}$ *mana* $4 \frac{1}{2}$ *gin* minus $x+2$ *še*, of unclear meaning.

	Loc.	SPVN	Loc.	Value in-silver	Loc.	Marginal number
Various deliveries						
	o. i 1	14:54		($14 \frac{5}{6}$ <i>gin</i> 12 <i>še</i>)		
	o. i 2	29:56:50		($29 \frac{5}{6}$ <i>gin</i> 20 $\frac{1}{2}$ <i>še</i>)		
	o. i 3	17:43:40		($17 \frac{2}{3}$ <i>gin</i> 11 <i>še</i>)		
	o. i 4	30:53:20		($30 \frac{5}{6}$ <i>gin</i> 10 <i>še</i>)		
Total		(1:33:27:50)	o. i 6	$1 \frac{1}{2}$ <i>mana</i> $3 \frac{1}{2}$ <i>gin</i> minus 7 <i>še</i> ($1 \frac{1}{2}$ <i>mana</i> $3 \frac{1}{2}$ <i>gin</i> minus $6 \frac{1}{2}$ <i>še</i> expected)		
Lustration of king					o. ii blank	2:[54] [45] [28] 17 2:28 27
Total			o. i 8	7 <i>mana</i> 19 <i>gin</i>		(7:19)
Grand total			o. i 10	$8 \frac{5}{6}$ <i>mana</i> $2 \frac{1}{2}$ <i>gin</i> minus 7 <i>še</i>		

63 It has been published in a hand-copy by Keiser (1919: No. 293) and studied by Powell (1976a: 420-2), Friberg (2005: 8-10) and Robson (2008: 78-9). Additional discussion appears at the end of Sect. 1 of this chapter and a full edition is provided in Appendix 5.IV.

Unspecified			o. ii 1	$8 \frac{5}{6}$ <i>mana</i> $4 \frac{1}{2}$ <i>gin</i> minus? $x+2$ <i>še</i>		
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Table 5.21 Summary of Data in YBC 1793 (AS 5 ix)

Six marginal numbers are noted in the blank space of col. ii (see Figure 19). Each of them is written in SPVN,⁶⁴ and their total, 7:19, corresponds to the amount of silver in metrological notation, 7 *mana* 19 *gin*, in obv. i 8. Here, the sum does not appear in SPVN as a marginal number but only in metrological notation in the main text. Meanwhile, the items to be added are not provided in the main text but only as marginal numbers.



Fig. 5.19 YBC 1793, Lower Section of obv. ii (Photo C. Proust, courtesy of Yale Babylonian Collection)

Four numbers written in SPVN are listed at the beginning of this tablet (o. i 1-4). This is the only example, in our evidence, in which numbers written in SPVN appear as part of the main text as indicated by the rulings (see Fig. 5.20). As in the case of Text 7 above, the total of these four numbers does not appear in SPVN but only in metrological notation as $1 \frac{1}{2}$ *mana* $3 \frac{1}{2}$ *gin* minus 7 *še* in obv. i 6 of the main text.

⁶⁴ In theory, the six numbers could also be partial-SPVN numbers, as we cannot distinguish the two systems when a number has no digits correspondent to the weight unit *še*. But in view of the parallelism with the four SPVN numbers at the start of the tablet, these six numbers are more likely to be SPVN numbers.



Fig. 5.20 YBC 1793, Beginning of obv. i (*Photo C. Proust, courtesy of Yale Babylonian Collection*)

Thanks to the presence of these two sets of numbers written in SPVN, the tablet has long been considered as the archetype of SPVN in the Ur III period in the historiography of cuneiform mathematics. But what kind of operations could the numbers written in SPVN hint at?

As we see, we have on the one hand numbers in SPVN, but not their total in SPVN, and on the other hand the corresponding total in metrological notation, but not any individual metrological notation correspondent to each number in SPVN. Previous studies of this text assume that the total (in metrological notation) was reached by adding the numbers in SPVN. However, one detail suggests another hypothesis.⁶⁵ A subtractive notation, 'minus 7 še' (la₂ 7(diš) še), is used to express the total amount of silver in obv. i 6. If the four numbers in SPVN had been added directly, the total would have equalled 1:33:27:50 (see Table 5.21), which corresponds to the exact value of 1 1/2 *mana* 3 1/3 *gin* 23 1/2 *še* in metrological notation. It is difficult to explain how this exact value was rounded to 1 1/2 *mana* 3 1/2 *gin* minus 7 *še*. It seems, therefore, more probable that the total in obv. i 6 was obtained by an addition performed directly on the individual metrological notations (correspondent to the four numbers in obv. i 1-4 but not recorded on the tablet), without the intermediacy of SPVN. In other words, the four numbers in SPVN in obv. i 1-4 would not have been used for the addition to give the total in obv. i 6. We can maybe draw the same conclusion about the relationship between the six numbers in SPVN on the reverse of the tablet and their total in metrological notation in obv. i 8. Since the tablet registers the two total values in-silver of the two deliveries designated respectively as 'various deliveries' and 'lustration of the king', the two sets of numbers in SPVN may reflect the conversion of the values in-kind of the goods into values in-silver, and represent, then, the results of the multiplication of the values in-kind by the rates. A possible pattern of calculation will be presented in Sect. 5.3.

In our previous reconstruction of the operations involved in Texts 1, 2 and 4, we propose multiplications of quantities in-kind by the reciprocals of rates in-kind in order to calculate the

65 The following hypothesis relies on Middeke-Conlin (2015).

value in-silver of individual receipts or expenditures. This step seems to have left no trace on the tablet other than the results denoted in metrological notations in the main text. The numbers in SPVN in Text 10, both at the beginning and in the margin, might represent the results in SPVN before their transformation into metrological notation.

Summary

This tablet of unknown provenience shares certain features typical of some of the tablets from Umma, Puzriš-Dagan and Girsu, discussed above. For example, the marginal numbers in col. ii are in the normalized paleography and seem to have been written on a scratched surface (Powell 1976a: 421). Their location on the tablet does not seem to relate to that of the correspondent of their total. Moreover, the additions in this text may have been performed by using partial-SPVN, even though no number written in partial-SPVN appears on the tablet.

Meanwhile, the tablet exhibits a number of peculiarities rarely attested on the tablets discussed above. To begin with, for the first time in our evidence, SPVN appears in the main text (obv. i 1-4) instead of in the margins and uses both normalized and non-normalized paleography even in the same notation.⁶⁶ The subtractive notation, ‘minus 7 še’, in the total in obv. i 6 does not appear anywhere else except perhaps in the total value in-silver in Text 8.⁶⁷ This subtractive notation offers the critical clue indicating that the total may have been reached by means other than the use of SPVN (such as partial-SPVN). Last but not least, (Powell 1976a: 421) speculated that the *entire* tablet, rather than just the blank space in o. ii, served as a kind of ancient ‘scratch pad’ because ‘[i]t has a form similar to a school text, being rather thick and having flat edges. The writing surface [i.e. the obverse] is extremely flat, and the back side, which was not used, is convex. The writing surface shows clear traces of having been previously used. The appearance of the tablet suggests that it was moistened and smoothed off after use’. But in fact, only the box containing the marginal numbers in col. ii exhibits traces of use of the clay as a scratch pad (erasures, traces of fingers and rough signs). One of the most surprising aspects of the tablet has to do with its shape, with well-made sharp angles that we never observe on other Ur III tablets. Therefore, it is hard to imagine that such an atypical tablet can illustrate the usual practices with SPVN during the Ur III period.

5.3 Numbers and Operations

Texts 1-10 studied above come from at least three different Ur III sites, and the observation of numbers noted in margins demonstrates a variety of ancient practices with regard to positional notations. In this part, we discuss the relationship between the notations of these marginal numbers and the arithmetical operations carried out with them. We wonder if in this respect a local culture could be detected.

5.3.1 Common Features: Material Aspects of the Margins

All the positional notations, except in Text 10, appear in the margins, edges or boxes, as isolated numbers, and with normalized paleography. By contrast, numbers in the main text are metrological notations or numbers in system S or system G, with non-normalized paleography and they are always associated with specific measurement units and quantified goods (except again in Text 10). Moreover, margins containing positional notations often exhibit traces of

66 Such as the two 4s in obv. i 1. For normalized paleography, see the second 4 in line obv. i 1 7 in obv. i 3, and the first 40 in obv. i 3; for non-normalized paleography, see the first 4 in obv. i 1 and the second 40 in obv. i 3.

67 As 1 *gu* 28 *mana* minus² 10 *gin* in rev. 7.

digits and erasures, and marginal numbers appear as small, rough and faintly marked signs. As observed by Powell (1976a: 421), marginal numbers often seem to have been noted on dried and then re-moistened clay. The marginal numbers seem to have been written and erased several times throughout the process of writing on the tablet and possibly after drying. The location of the area containing marginal notations on the tablets is quite independent from the main text, and seems to have been prepared before the process of writing. These material aspects suggest that the margins (boxes or edges) may have served as scratch pads, and that marginal numbers may have been traces of temporary notations. It is striking to observe that the material aspect of the margins containing non-positional notations (system S or metrological notations) is different, and, as far as we can judge according to the sources we know, that these kinds of margins do not present such features of scratch pads (see Sect. 5.1.2 and Fig. 5.5). These observations strongly indicate that positional notations in administrative texts were traces of ephemeral notes taken during a process of calculation. Note that, insofar as we understand them, these ephemeral notes do not reflect the calculations themselves, but only numbers which operations acted on or produced, that is, the inputs or outputs of the calculations.

The detailed steps of the calculations remain to be reconstructed: which operations acted on which kinds of numbers and in which order? How were the processes of calculation articulated with the process of writing on the tablet? Some possible answers emerge through our observation of the diversity of the positional notations.

5.3.2 Differences: Shaping Numbers for Operations

The discussion of individual texts (Sect. 5.2) has shown that different kinds of positional notations are adopted in the margins. In the balanced accounts from Umma (Texts 1-4), where one total needs to be subtracted from another, a composite system is used. This system is neither fully positional nor fully sexagesimal, and we characterized it as ‘partial-SPVN’. To some extent, the marginal numbers in Text 8, which feature an aš-notation, may be seen as partial-SPVN as well. Tables 5.22 and 5.23 summarize the different forms of partial-SPVN attested in our texts.

	gu	60 ←	mana	60 ←	gin	60 ←	1/60 <i>gin</i>	3 ←	še	60 ←	1/60 <i>še</i>
Text 1					12		50	/space/	23		30
Text 2					48		50	/space/	10		
Text 4			6		57		20	/space/	15		
Text 8	1(aš)		27		50						

Table 5.22 Partial-SPVN for Silver

		gur	5 ←	bariga	60 ←	ban-sila	60 ←	1/60 <i>sila</i>
Text 3		7(aš)		1		46		30
Text 3		5(aš)		/space/		1		30

Table 5.23 Partial-SPVN for Grain

In Texts 1, 2 and 4 from Umma, the marginal numbers correspond to weights of silver. The numbers are split into two parts separated by a space. The first part is a sexagesimal place value representation of the quantity in *mana* and *gin*, and the second part is a sexagesimal place value

representation of the quantity in *še*. However, as a whole, the marginal numbers are not fully sexagesimal, as shown in Table 5.22 by the factor 3 between the digits representing *še* and the digits representing sixtieths of *gin*. In Text 8, from Puzriš-Dagan, the digit representing *gu* is written in the *aš*-notation, while the digits representing *mana* are written in the standard *diš*-notation. Here, the marginal number is not fully positional, as different signs represent different orders of magnitude. Tables 5.22 and 5.23 above further suggest that a number in partial-SPVN (e.g. 12:50 // 23:30 in Text 1) is a single number composed of two parts. However, such a number in partial-SPVN was perhaps conceived by ancient scribes as a pair made up of two different numbers (e.g. 12:50 and 23:30), as denoted by the space separating the two parts, and by the fact that these two parts are not always exactly aligned (12:50 is slightly above 23:30 in Text 1).

In Text 3 from Umma, the marginal numbers correspond to capacities of grain. Here again, the marginal numbers are not fully positional, as different signs represent different orders of magnitude (*aš* for digits representing *gur*, and standard notation for digits representing *bariga*, *ban* and *sila*).

Although different, the systems used in Texts 1, 2, 3, 4 and 8 in what we label ‘partial-SPVN’ share the common feature that the numerical notations reproduce the structure of metrological notations, with the same factors and the same use of *aš* to represent *gur* and *gu*. The writing system used in partial-SPVN includes marks to identify the order of magnitude of each digit, such as a space between digits which represent *gin* and digits which represent *še*, or the *aš*-number for digits which represent *gu* or *gur*. In short, unlike SPVN, partial-SPVN is not a floating notation. Essentially, the use of partial-SPVN allowed the scribes to avoid fractions of measurement units, which are replaced by integer numbers of sexagesimal sub-units. This representation facilitates the addition and subtraction of measurement values which include fractions, as argued in the discussion of texts (Sect. 5.2.2).

In Text 5, from Umma, the marginal number, written in SPVN, is probably linked to the calculation of the volume of earth and obtained by a multiplication. In Text 9, from Girsu, even if their relationship with the main text is not clear, the marginal numbers are clearly written in SPVN probably linked to the calculation of the quantities of seeds and obtained by multiplication. In Texts 6 and 7, from Puzriš-Dagan, the marginal numbers are written in SPVN, but it is not clear if these notations are linked to additions or the applications of a rate (a reciprocal followed by a multiplication). However, related texts seem to indicate that the marginal numbers in our Puzriš-Dagan texts are the result of (or were prepared for) the application of a rate (see summary of Sect. 5.2.3).

5.3.3 Patterns of Calculations

From these observations we can attempt to reconstruct the processes of calculation. These processes certainly followed different steps on different *media*, and some of the steps appeared external to the tablet as we have no trace of the details of the operations. The elements we have to hand are the following.

- The main texts, containing metrological notations or system S, both of which represent quantities, and
- margins or scratch pads, containing marginal numbers which correspond more or less to the quantities noted in the main text and adopt a format compatible with the operations to be performed.

But we have no trace of the details of the operations themselves, which were executed elsewhere, probably with the aid of a calculating device.

We can thus consider marginal numbers as an intermediate step necessary to transform quantities noted in the main text into inputs for a device where calculations were executed or to transform outputs from the device into quantities noted in the main text. The following diagrams are tentative representations of the possible patterns of calculation. These patterns are just hypotheses, which will certainly be improved or amended in further studies, perhaps after the examination of more material.

Insert here Fig.5.21a		Insert here Fig.5.21b		Insert here Fig.5.
Main text	Transformation	Margin	Transfer	Calculation device
Quantities	➔	Marginal numbers	➔	Inputs
Quantities	➔	Marginal numbers	➔	Outputs

Fig. 5.21 Main Text, Margin and Possible Calculation Device (with example of Text 9)

If this scheme is likely, we may further posit the ephemeral character of marginal numbers: intermediate notations are noted in margins during their transfer from text to device or from device to text, and then erased and replaced by new data used in the following calculation. Note that, at some points in the process, the transfer may not have been necessary, and the scratch pad may have been left empty. Moreover, data may have been erased from the scratch pad when the tablet was finished. This may explain why we often find empty boxes with erasures in administrative texts (for example, MLC 1980, Snell 1982: No.2, pl. III, and many others published as copies in Snell 1982 and elsewhere). The normalized format of marginal numbers would represent the concrete numbers used for calculation with a device, and perhaps the shapes and disposition of tokens.

The pattern of calculation in Text 1 would be illustrated by Fig. 5.22. In this pattern, the marginal numbers are shaped as partial-SPVN because the operations which act on the numbers are additions and subtractions. The same pattern may have applied to the subtractions, and, to some extent, the additions, in the silver balanced accounts Texts 2 and 4.⁶⁸

68 In Figs. 22-24, continuous lines represent elements attested in our sources, discontinuous lines represent elements partially attested in our sources, and dotted line represent elements absent from our sources.

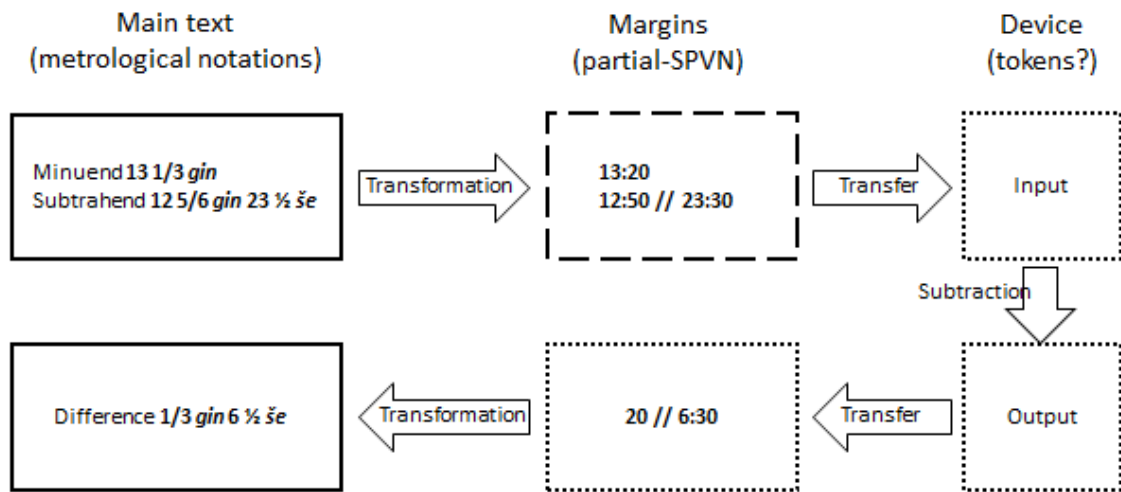


Fig. 5.22 Possible Pattern of Calculation for Subtraction in Text 1

The pattern of calculation in Text 5 would be represented by Fig. 5.23. In this pattern, the marginal numbers are shaped as SPVN on which the multiplications act.

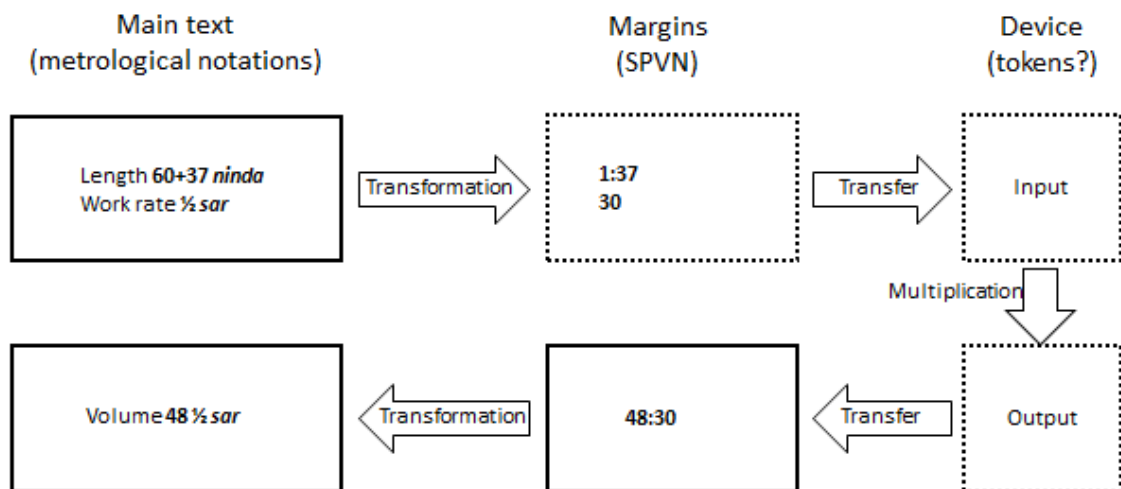


Fig. 5.23 Possible Pattern of Calculation for Text 5

In Text 7, the calculation may have involved a nexus of tablets.

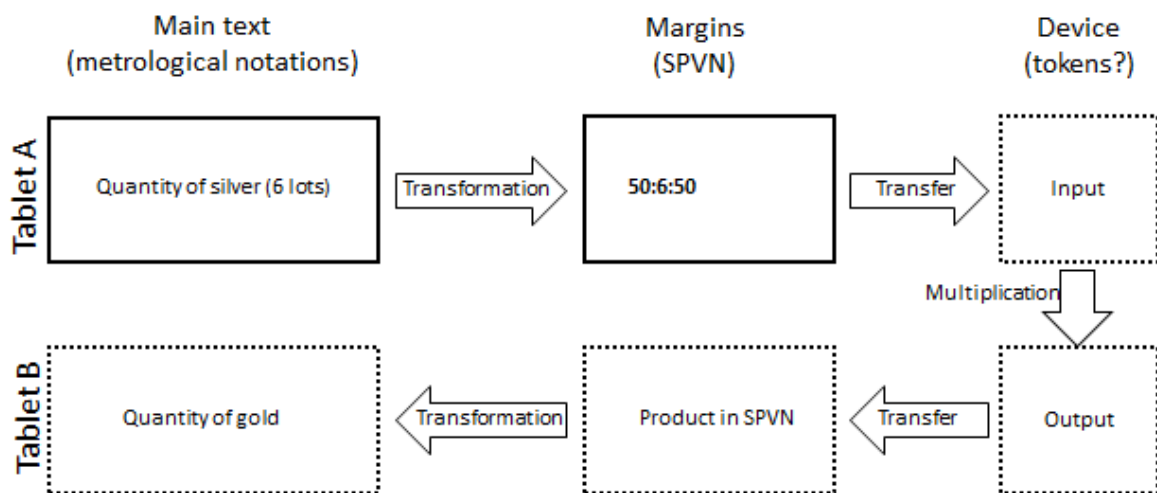


Fig. 5.24 Pattern of Calculation for Text 7

To conclude, marginal numbers may evidence the practice of transforming quantities into numbers to be transferred to a calculation device and vice-versa. The shape of the marginal numbers would represent the shape of the numbers used in calculations, and would depend on the nature of the operation: partial-SPVN would be adopted for subtractions and probably for additions, and SPVN for multiplication and reciprocals (application of a rate). In both cases, the paleography is normalized, which may imply the use of tokens and their disposition in rows of three elements. The use of temporary notes for the transformation of quantities into inputs or outputs for the calculation device seems not to have been systematic. Perhaps, this kind of notes was necessary in certain cases, but not in others, depending on the complexity of the notations and the skill of the scribes who wrote the tablets. The fact that the location of marginal numbers seems not to be related to the location of corresponding metrological notations in the main text may reflect different relationships between temporality and spatiality during the writing process. In the main text, the signs and lines are noted down one after the other, namely, from left to right and top to bottom. The order of the lines reflects the order in which they were written. In the margins, numbers or sets of numbers are noted down, then erased, and new ones noted down in their place, in the same way as we write on a blackboard. Each step erases the previous one, the spatiality vanishes, and only the last step is visible.

5.4 Appendix I: Notations and Chronology

(A map of Southern Mesopotamia in Ur III period is provided at the end of the book, Annex B, map 1)

5.4.1 Transliterations and translations

- For the transliterations and translations of numbers and measurement values, we follow the conventions provided in Annex A1 at the end of this volume. However, given the peculiarity of the sources studied in this chapter, we add the following adjustment: when

a semi-positional number uses horizontal wedges, we specify ‘aš’ in the transliteration, as in the example of 5(aš) 1:30 in Text 3.

- In commentaries, we use capital letters to refer to individual cuneiform signs isolated from their context, e.g. AŠ versus aš, such as in our discussion of how the number of *gur* is written in the marginal numbers of Text 3.
- The symbol ‘/’ is used to indicate layout features (e. g. /space/).
- The notation <<x>> means that the sign ‘x’ is not expected.
- In the tables summarizing the data from the texts (Sect. 5.2), we use parentheses to mark quantities and numbers in our calculation that are not attested in the texts.

5.4.2 Location and orientation

The location of a certain line of a text on a tablet is specified in the following format: ‘side, column, line’. For example, ‘obv. ii 3’ means ‘obverse, column ii, line 3’; ‘rev. i 2’ means ‘reverse, column i, line 2’.

Upper and lower edges are defined in relation to the obverse. As tablets are read by rotation around the lower edge, notations on the upper edge are a continuation of the text of the reverse.

5.4.3 Chronology

General chronology, following the so-called middle chronology, after CDLI⁶⁹

Period	Approximate dates
Early Dynastic I-II	ca. 2900-2700 BCE
Early Dynastic IIIa	ca. 2700-2500 BCE
Early Dynastic IIIb	ca. 2500-2340 BCE
Old Akkadian (or Sargonic)	ca. 2340-2200 BCE
Lagash II	ca. 2200-2100 BCE
Ur III (or Third Dynasty of Ur)	ca. 2100-2000 BCE
Old Babylonian	ca. 2000-1600 BCE
Old Assyrian	ca. 1950-1850 BCE

Ur III chronology

Ur III administrative texts are almost always dated, and historians refer to the dates provided by tablets using the year-month-day format in the following way: abbreviated name of the ruler, year of his reign, month in roman numerals (i, ii, ..., xii and ‘diri’ which means ‘additional’ for intercalary months) and day.

The Ur III dynasty includes five rulers (the dates follow the middle chronology):

Ruler	Abbreviation
Ur-Namma (2112-2095)	UN
Šulgi (2094-2047)	Š
Amar-Suen (2046-2038)	AS
Šu-Suen (2037-2029)	ŠS
Ibbi-Suen (2028-2004)	IS

For example, the date labelled in modern publications as ‘AS 5 ii 17’ represents the seventeenth day of the second month of the fifth year of Amar-Suen’s reign. Of course, the Sumerian system

69 http://cdli.ox.ac.uk/wiki/doku.php?id=adopted_periodisation_in_cdli, accessed October 2015.

for naming years and months in cuneiform texts has nothing to do with this modern representation. Appendix 5.II: System S.

Appendix 5.II: System S

5.5.1 A Broad Definition of System S

System S, a sexagesimal additive system, is attested since the beginning of writing in Mesopotamia, and was used primarily for counting items such as animals, workers, days, years and other items of discrete collections. Diagram *a* of Fig. 5.26 represents the shape of the graphemes of system S used in the Ur III period, as well as the factors which define the value attached to each grapheme.

The numerical system used for expressing most of the measurement values exhibits the same features as system S (see diagram *c*, *c'* and *d* in Fig. 5.26). The only variant is that the number 1 is represented by a horizontal wedge (aš) for counting the highest measurement units of capacity and weight (see diagram *b* of Fig. 5.26).⁷⁰

Small measurement units, namely, all of the units except the largest of each metrological system, were generally counted with numerical values less than sixty, noted with graphemes one (𐎶) and ten (𐎵) repeated as many times as necessary (see diagram *c* in Fig. 5.26). However, in the Ur III period, the notations were quite flexible, and, for example, the units *ninda* and *sar* may have been counted with numbers larger than sixty (see diagram *c'* in Fig 5.26 and examples in Text 5).

Fig. 5.26 shows that the systems represented by diagrams *a*, *b*, *c*, *c'* and *d* share the same graphical repertoire, the same base (alternation of factors ten and six), and the same additive principle. The only variation is the sign for 1, which is a horizontal wedge in *b*, but vertical in *a*, *c*, *c'* and *d*. For the sake of simplicity, we term all of them as ‘system S’.⁷¹

a) System used for counting items (animals, workers, workdays, years, ...)										
	×1		×6		×10		×6		×10	
	0		←		←		←		←	
	←									
šar'u		šar ₂		geš'u		geš ₂		u		diš
36		3 600		600		60		1		1
000								0		
b) System used for counting <i>gur</i> and <i>gu</i> (the largest units of capacity and weight)										

⁷⁰ For counting *gan*, the largest unit of surface, another system, the so-called ‘system G’ was used (see Text 9). The graphical repertoire of signs used in system G exhibits some similarities with system S, but the factors are different (see Annex A1). Two units of capacity (ban and bariga) used different principles, with no graphical separation between numerical value and measurement unit. See more on this in Annex A1.

⁷¹ Note again that the term ‘sexagesimal’ is confusing because in fact system S is not purely sexagesimal, as underlined in Sect. 1.2. In particular, the first factor is ten, thus for low values, the system looks like a decimal numeration.

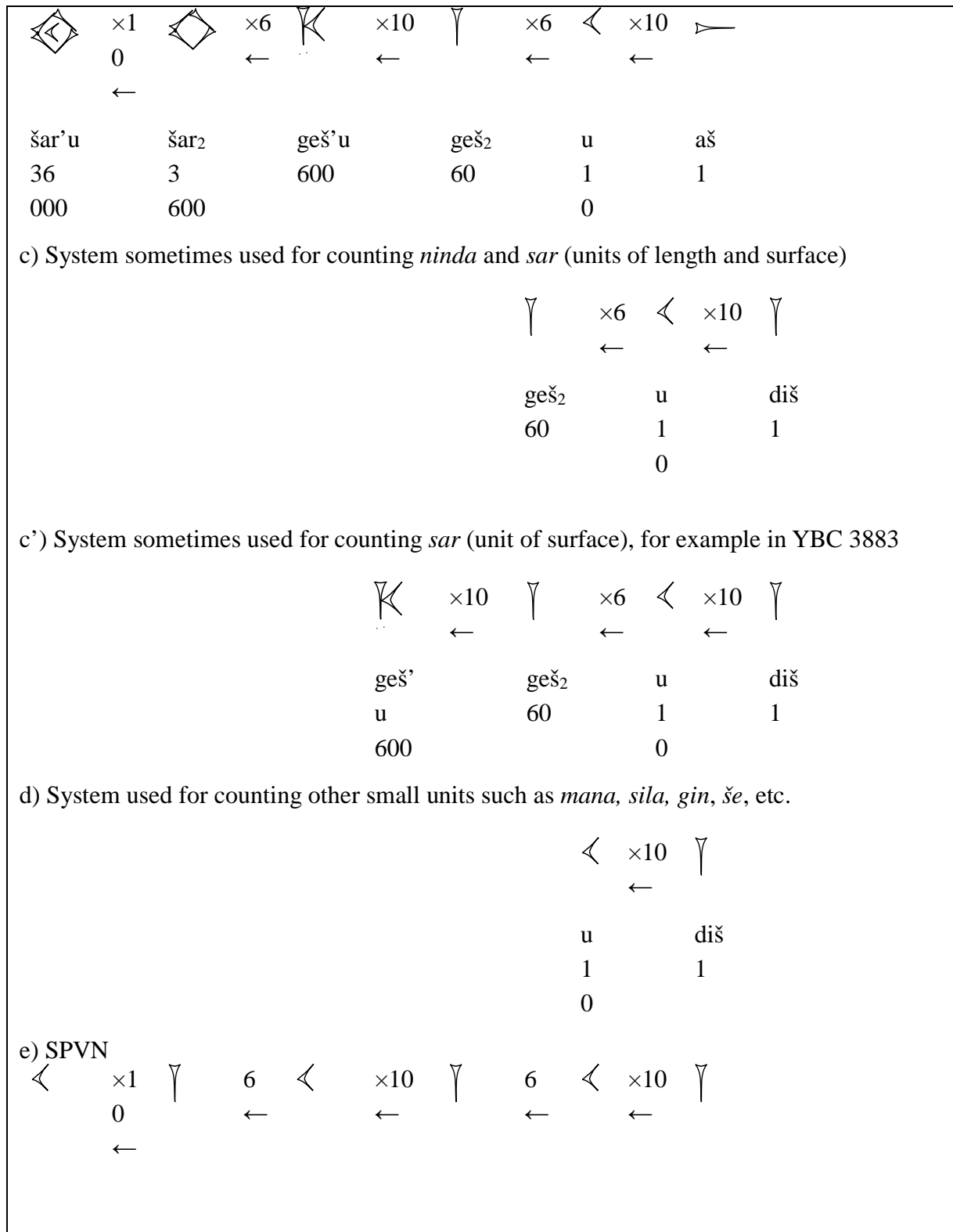



Fig. 5.26 Diagrams Representing various Forms of System S and SPVN





5.5.2 Distinguishing System S from SPVN

In diagrams *a*, *c* and *c'*, the cuneiform graphemes representing 1 and 60 are the same, namely, a vertical wedge (). In early texts, the wedge for sixty is bigger than the wedge for one. However, the sizes of both signs tended to become the same and confusion between one and

sixty became inevitable. Subsequently, in the Old Babylonian period, the scribes differentiated one from sixty in ambiguous cases by specifying ‘sixty’ (šu-ši) when necessary (see Proust 2009: Sect. 5 for details), or by introducing new units (for example the length unit UŠ for sixty *ninda*), or by using fractions of superior units (for example, $1/2$ *gan* for fifty *sar*).

In some instances, when numbers are less than 600 and use vertical wedges as units (see diagrams *a* and *c*), there is no graphical difference between SPVN and system S. However, some criteria can be used to recognize system S:

- The function: system S is used for counting discrete items or measuring units.
- The textual context: in main texts, a number in system S is always followed by the name of the items counted or a measurement unit; moreover, fractions never appear just after a number in SPVN.⁷²The paleography: in the example we analyse in this article, the non-normalized paleography is adopted in system S, while the normalized paleography is adopted for positional systems (SPVN and different forms of partial-SPVN)

Of course, for numbers more than 600, there is no ambiguity. For example, in Text 1, the marginal number  (transliteration 23:30) cannot be interpreted as a number noted in system S, the paleography of which should be  (transliteration 2(*geš*'u) 3(*ges*₂) 3(*u*)). Another example: the positional system adopted for the number  48:30 found in Text 6 cannot be confused with system S .

72 The fact that SPVN always consists of isolated numbers (number without specification) is widely attested in Ur III and Old Babylonian mathematical texts, more details in Proust (2008a). The same rule applies in the texts examined in this chapter. Numbers in system S can appear isolated only in margins (see Sect. 5.1.2).

Appendix 5.III: Catalogue of Ur III Administrative Texts with Positional Notations

Text numbers are the numbers used in the present chapter. Documents without a text number are not studied in detail here.

Typology of tablets

M(*n*,*n'*) means a multi-column tablet, on which the text is divided into *n* columns on the obverse, and *n'* columns on the reverse.

S means single-column tablet.

Date: See chronology in Appendix 5.I.3.

Numerical systems:

SPVN means sexagesimal place value notation.

Partial-SPVN means that the notation is partially sexagesimal or partially positional or both.

MN means marginal number.

∅ means not inscribed.

Text No.	Museum No.	Date	Type of tablet	Type of text	Quantity in relation to MN	Location of MN	Numerical system of MN
Umma							
	MM 396	AS 1 [?]	Frag.	Delivery of precious metals and objects	Weight of silver and gold	Left edge	SPVN [?]
1	Nik. 2 402	AS 4	S	Balanced account for silver in-value	Weight of silver	Obv. bottom	Partial-SPVN
2	YBC 16487	AS 5	S	Balanced account for silver in-value	Weight of silver	Rev. bottom	Partial-SPVN
3	YBC 4179	AS 6	M(2,2)	Balanced account for grain	Capacity of grain	Rev. ii middle section	Partial-SPVN
						Rev. upper Balanced account on aromatics edge	Partial-SPVN
	FMB 39.2	AS 7	S		Weight and capacity of aromatics	Left edge	SPVN [?]
	Nik. 2 403	AS 9	S	Balanced account for silver in-value	Capacity of cash crops or weight of silver	Rev. middle section	Partial-SPVN [?] or SPVN [?]

	Erlenmeyer 152	ŠŠ 2	M(5,5)	Balanced account for workdays	Number of workdays	Obv. iii upper section Lo. edge of obv. iii-v, rev. i-iv	SPVN System S
4	YBC 16607	ŠŠ 5	S	Balanced account for silver	Weight of silver	Lower edge	Partial-SPVN
	YBC 3883	ŠŠ 5	M(2,2) sealed	Agricultural work measured in workdays	Number of workdays	Obv ii upper section	SPVN? (normalized)
5	E 15550	IS 2	S	Earthwork for different sections of a dike	Volume of earth	Left edge	SPVN
Puzriš-Dagan							
6	PTS 473	Š 48 vii	S	Two batches of livestock as booty	Number of animals	Rev. lower section	SPVN
	WCMA 20.1.07	ŠŠ 6 diri	S(1, Ø)	Distribution of barley as rations	Capacity of grain	Obv. lower section	Partial-SPVN
7	NBC 6641	ŠŠ 8 ix	S	Distribution of silver for purchase of gold	Weight of silver	Lower edge Left edge	SPVN SPVN
8	YBC 13418	No date	S	Deliveries of gold	Weight of silver	Upper edge Rev. bottom	Partial-SPVN? Unknown
	YBC 1778	Date damaged	S	Balanced account for sheep and goats	Numbers of animals	Obv. middle section	SPVN? (normalized)
Girsu							
	BM 19027	Š 48	M(4,4)	Grain balanced account	Capacity of grain	Left edge	Partial-SPVN [?] or SPVN [?]
	SM 1909.05.22 4	No date [?]	S	Survey of two fields	Surface of fields	Obv. lower section	SPVN
9	AO 27307	Date lost	M(2,2)	Seed for given surfaces	Capacity of seeds	Rev. ii lower section	SPVN
Nippur							
	CBS 11661	IS 2 xi 30	M(2,2)		Capacity of grain		SPVN? (damaged)
Uncertain							

10	YBC 1793	AS 5 ix	M(2,Ø)	Value in-silver of different deliveries, or deliveries of silver	Weight of silver	Obv. i upper section (main text)	SPVN
						Obv. ii lower section	SPVN

Appendix 5.IV: Texts

5.7.1 Mathematical Text

AO 2728 (=AOT 304 = RTC 413)

Location of marginal number: reverse

Provenience: Girsu

Date: Ur III (no date written on the tablet)

Location of tablet: Musée du Louvre, Paris, France

Reference No.: BDTNS 000853; CDLI P128566

Ed. Thureau-Dangin 1903: No. 413

Photo: <http://www.cdli.ucla.edu/dl/photo/P128566.jpg>

	Transliteration	Translation
Obverse		
1.	6(diš) ninda 4(diš) 1/3 kuš ₃ [gid ₂]	6 <i>ninda</i> 4 1/3 <i>kuš</i> the length,
2.	1/2 ninda sukud	1/2 <i>ninda</i> the height,
3.	2(diš) kuš ₃ 5(diš) šu-si dagal	2 <i>kuš</i> 5 <i>šusi</i> the width.
4.	sahar-bi 3(diš) 1/2 sar 2(diš) 1/2 gin ₂	Its volume 3 1/2 <i>sar</i> 2 1/2 <i>gin</i> ,
5.	sig ₄ -bi 2(u) 5(diš) 1/2 sar	its brick-volume 25 1/2 <i>sar</i> .
Lower edge	30? 6	
Reverse		
1.	6:31:50	6:31:50
2.	3 10:50	3 10:50

Commentary:

Our transliteration follows the works of Thureau-Dangin (1903: No. 413) and Robson (1999: 66), and the collation in the Musée du Louvre on February 6, 2014, by C. Proust.

Lower edge: The numerical signs are visible, but they do not appear in Thureau-Dangin's copy.

5.7.2 Administrative Texts

The following editions of texts rely on previous publications, on transliterations provided by CDLI and BDTNS, on digital photos, and, in most cases, on our own examination of the tablets. Both databases were last accessed in February 2014 unless otherwise stated. Only photos of the tablets (obverse and reverse) without images in CDLI are provided here. Photos of the edges with marginal numbers in Texts 1-10 appear in Sect. 5.2.

Text 1-Nik. 2 402

Location of marginal number: Obverse, bottom

Provenience: Umma

Date: AS 4

Dimensions (cm): ***

Location of tablet: Pushkin State Museum of Fine Arts, Moscow, Russia

Museum No.: Unavailable

Reference No.: BDTNS 004641; CDLI P122085

Copy: Nikol'skij 1915: No. 402



Fig. 5.27 Nik. 2 402 (Photos courtesy of Pushkin State Museum of Fine Arts)

	Transliteration	Translation
Obverse		
1.	1(u) 3(diš) 1/3 gin ₂ ku ₃	13 1/3 <i>gin</i> of silver,
2.	ša ₃ -bi-ta	from its within:
3.	1(barig) 3(ban ₂) 6(diš) 1/2 sila ₃ 2(diš) gin ₂ i ₃ -šah ₂	1 <i>barig</i> 3 <i>ban</i> 6 1/2 <i>sila</i> 2 <i>gin</i> of lard
4.	ku ₃ -bi 5(diš) 1/2 gin ₂ 2(u) 3(diš) 1/2 še	its value in-silver 5 1/2 <i>gin</i> 23 1/2 <i>še</i> ;
5.	1(u) 1(diš) ma-na [uruda [?]]	11 <i>mana</i> of ...
6.	ku ₃ -bi 7(diš) 1/3 gin ₂	its value in-silver 7 1/3 <i>gin</i> ;
7.	12:50 /space/ 23:30	12:50 /space/ 23:30
Reverse		
1.	šu-nigin ₂ 1(u) 2(diš) 5/6 gin ₂ 2(u) 3(diš) 1/2 še	in total: 12 5/6 <i>gin</i> 23 1/2 <i>še</i>
2.	zi-ga-am ₃	expended;
3.	la ₂ -NI 1/3 gin ₂ 6(diš) 1/2 še	deficit 1/3 <i>gin</i> 6 1/2 <i>še</i> .
4.	Blank line	
5.	nig ₂ -kas ₇ -ak lugal-he ₂ -gal ₂	A balanced account of Lugal-hegal.
6.	mu en-mah-gal-an-na en ^d nanna ba-hun	Year when Enmahgal-ana, the en- priestess of Nanna, was installed.

Commentary:

Obv. 5: The name of the product appears lost in BDTNS but is restored as [uruda] in CDLI. We find the rate in-kind of this product to be 1 1/2 *mana* per *gin* of silver, which is close to the rates in-kind of copper calculated by Snell (1982: 150) based on the Ur III evidence.

Obv. 7: The copy of the tablet indicates an erasure of three vertical wedges right above the second sign, 2(diš), of the marginal number. The images of the tablet show only an erasure, probably from the tip of a finger, in the same place, but we cannot tell how many wedges may have been erased.

Text 2-YBC 16487

Location of marginal number: Reverse, bottom

Provenience: Umma

Date: AS 5

Dimensions (cm): 4.9x4.5x2

Location of tablet: Yale Babylonian Collection, New Haven, Connecticut, USA

Reference No.: BDTNS 024111; CDLI P112497

Copy: Snell 1982: No. 5



Fig. 5.28 YBC 16487 (Photos courtesy of Yale Babylonian Collection)

	Transliteration	Translation
Obverse		
1.	1(diš) 1/2 ma-na 6(diš) 1/3 gin ₂ 2(u) 7(diš) še ku ₃ -babbar	1 1/2 <i>mana</i> 6 1/3 <i>gin</i> 27 <i>še</i> of silver,
2.	si-i ₃ -tum	carried-over balance,
3.	ša ₃ -bi-ta	from its within:
4.	1/2(diš) ma-na 8(diš) gin ₂ ku ₃ - babbar	1/2 <i>mana</i> 8 <i>gin</i> of silver
5.	mu-ku _x (DU)	as delivery;
6.	4(barig) 1(ban ₂) 9(diš) sila ₃ esir ₂ e ₂ - a	4 <i>barig</i> 1 <i>ban</i> 9 <i>sila</i> of bitumen
7.	ku ₃ -bi 2(diš) 5/6 gin ₂	its value in-silver 2 5/6 <i>gin</i> ;
8.	[...] 1 ¹ (barig) 5 ¹ (ban ₂) 6(diš) sila ₃ naga si-e ₃ gur	[...]+1 <i>barig</i> 5 <i>ban</i> 6 <i>sila</i> sprouted alkaline plants,
9.	[ku ₃ -bi x] 1 ¹ gin ₂ [x] 1 ¹ še ¹ Rest broken	9. its value in-silver x <i>gin</i> x <i>še</i> ;
Reverse		
	Beginning broken Blank line	
1'.	1 ¹ šu-nigin ₂ 2/3 ¹ ma-na 8(diš) 5/6 gin ₂ 1(u) še ku ₃	in total: 2/3 <i>mana</i> 8 5/6 <i>gin</i> 10 <i>še</i> of silver
2'.	zi-ga-am ₃	expended;
3'.	la ₂ -NI 2/3 ma-na 7(diš) 1/2 gin ₂ 1(u) 7(diš) še ku ₃	deficit 2/3 <i>mana</i> 7 1/2 <i>gin</i> 17 <i>še</i> of silver.
4'.	nig ₂ -kas ₇ -ak pad ₃ -da dam-gar ₃	A balanced account concerning Pada the merchant.
5'.	mu en-unu ₆ -gal ^d inanna ba-hun	Year when En-unugal of Inanna was installed.

Text 3-YBC 4179

Location of marginal numbers: Upper edge and middle section of rev. ii

Provenience: Umma

Date: AS 6

Dimensions (cm): 15.2x8.8x2.5

Location of tablet: Yale Babylonian Collection, New Haven, Connecticut, USA

Reference No.: BDTNS 015950; CDLI P111807

Copy: Ellis 1970: 268-269

**Fig. 5.29** YBC 4179 (Photos courtesy of Yale Babylonian Collection)

	Transliteration	Translation
Obverse		
Col. i		
1.	8(aš) 1(ban ₂) 8(diš) sila ₃ še gur	8 gur 1 ban 8 sila of barley
2.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
3.	Erased line	
4.	mu en ^d nanna	year of Š 43;
5.	8(aš) 1(ban ₂) 8(diš) sila ₃ gur	8 gur 1 ban 8 sila
6.	iti 1(u) 2(diš)-kam	for twelve months,
7.	3(barig) 2(ban ₂) 1(diš) 1/2 sila ₃ iti diri	3 barig 2 ban 1 1/2 sila for the additional month,
8.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
9.	(erasure at beginning) še kaš mu-ku _x (DU)	delivery of barley for beer
10.	mu si-mu-ru-um lu-lu-bum ₂ ^{ki}	for the year of Š 44;
11.	8(aš) 1(ban ₂) 8(diš) sila ₃ gur	8 gur 1 ban 8 sila
12.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent

13.	(erasure at beginning) še kaš mu- ku _x (DU)	delivery of barley for beer
14.	mu ur-bi ₂ -lum ^{ki}	for the year of Š 45;
15.	8(aš) 1(ban ₂) 8(diš) sila ₃ gur	8 gur 1 ban 8 sila
16.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
17.	(erasure at beginning) še kaš mu- ku _x (DU)	delivery of barley for beer
18.	mu ki-maš ^{ki} ba-hul	for the year of Š 46;
19.	[8(aš)] 1(ban ₂) 8(diš) sila ₃ iti 1(u) 2(diš)-kam	8 gur 1 ban 8 sila for twelve months
20.	[3(barig) 1(ban ₂)] [6](diš) 1/2 sila ₃ iti diri	3 barig 1 ban 6 1/2 sila for the additional month
21.	[še kaš] [u ₄]-sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
22.	(erasure at beginning) [sila ₃] še kaš mu-ku _x (DU)	delivery of barley for beer
23.	[mu us ₂]-[sa] ki-maš ^{ki}	for the year of Š 47;
24.	[8(aš) 1(ban ₂) 8(diš) sila ₃] gur	8 gur 1 ban 8 sila for twelve months
25.	[še kaš u ₄ -sakar ^{giš} gigir]-[ra]	barley for beer for the chariot's crescent
26.	(erasure at beginning) še kaš [mu]- [ku _x (DU)]	delivery of barley for beer
Col. ii		
1.	mu ha-ar-ši ^{ki}	for the year of Š 48 ;
2.	8(aš) [1(ban ₂)] [8(diš) sila ₃] [gur]	8 gur 1 ban 8 sila
3.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
4.	(erasure at beginning) še kaš mu- ku _x (DU)	delivery of barley for beer
5.	mu ^d amar- ^d suen lugal	for the year of AS 1 ;
6.	8(aš) 1(ban ₂) 8(diš) sila ₃ gur	8 gur 1 ban 8 sila
7.	iti 1(u) 2(diš)-kam	for twelve months,
8.	3(barig) 1(ban ₂) 6(diš) 1/2(diš) sila ₃ iti diri	3 barig 1 ban 6 1/2 sila for the additional month,
9.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
10.	(erasure at beginning) še kaš mu- ku _x (DU)	delivery of barley for beer
11.	mu ^d amar- ^d suen lugal-e	for the year of AS 2
12.	ur-bi ₂ -lum ^{ki} mu-ku _x (DU)	delivery;
13.	8(aš) 1(ban ₂) 8(diš) sila ₃ gur	8 gur 1 ban 8 sila
14.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
15.	(erasure at beginning) sila ₃ še kaš mu-<ku _x (DU)>	delivery of barley for beer
16.	mu gu-za ^d en-lil ₂ -la ₂	for the year of AS 3 ;
17.	8(aš) 1(ban ₂) 8(diš) sila ₃ iti 1(u) 2(diš)-kam	8 gur 1 ban 8 sila for twelve months
18.	3(barig) 1(ban ₂) 6(diš) 1/2(diš) sila ₃ iti diri	3 barig 1 ban 6 1/2 sila for the additional month,
19.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
20.	(erasure at beginning) še kaš mu- ku _x (DU)	delivery of barley for beer
21.	mu en-mah-[gal] [...]	for the year of AS 4;
22.	8(aš) 1(ban ₂) 8(diš) sila ₃ gur	8 gur 1 ban 8 sila

23.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
24.	3(barig) 3(ban ₂) še kaš mu-ku _x (DU)	3 <i>barig</i> 3 <i>ban</i> , delivery of barley for beer
25.	mu en-unu ₆ -gal	for the year of AS 5 ;
26.	8(aš) 1(ban ₂) 8(diš) sila ₃ [gur]	8 <i>gur</i> 1 <i>ban</i> 8 <i>sila</i>
27.	iti 1(u) 2(diš)- [kam]	for twelve months
Reverse		
Col. i		
1.	3(barig) 1(ban ₂) 6(diš) 1/2(diš) sila ₃ iti [diri]	3 <i>barig</i> 1 <i>ban</i> 6 1/2 <i>sila</i> for the additional month
2.	še kaš u ₄ -sakar ^{giš} gigir-ra	barley for beer for the chariot's crescent
3.	4(barig) 3(ban ₂) še kaš mu-ku _x (DU)	4 <i>barig</i> 3 <i>ban</i> , delivery of barley for beer
4.	mu ša-aš-ru ^{ki} ba-[hul]	for the year of AS 6;
5.	7(aš) 1(barig) 4(ban ₂) 6(diš) 1/2 sila ₃ še kaš mu-ku _x (DU)	7 <i>gur</i> 1 <i>barig</i> 4 <i>ban</i> 6 1/2 <i>sila</i> , delivery of barley for beer
6.	mu en ^d nanna-ta mu en-mah-gal-še ₃	from the year of Š 43 to AS 4;
7.	šu-nigin ₂ 1(geš ₂) 4(u) 8(aš) 4(barig) 5(ban ₂) še gur	in total: (60+48) <i>gur</i> 4 <i>barig</i> 5 <i>ban</i> .
8.	ša ₃ -bi-ta	From its within:
9.	8(aš) gur mu en ^d nanna maš ₂ -e i ₃ -pa ₃	8 <i>gur</i> in the year of Š 43,
10.	8(aš) gur mu si-mu-ru-um lu-lu-bu- um ^{ki}	8 <i>gur</i> in the year of Š 44,
11.	9(aš) 1(ban ₂) 5(diš) sila gur	9 <i>gur</i> 1 <i>ban</i> 5 <i>sila</i>
12.	mu ur-bí-lum ^{ki}	in the year of Š 45,
13.	9(aš) 1(ban ₂) 5(diš) sila ₃ gur	9 <i>gur</i> 1 <i>ban</i> 5 <i>sila</i>
14.	mu ki-maš ^{ki}	in the year of Š 46,
15.	9(aš) 1(ban ₂) 5(diš) sila ₃ gur	9 <i>gur</i> 1 <i>ban</i> 5 <i>sila</i>
16.	mu us ₂ -sa ki-maš ^{ki}	in the year of Š 47,
17.	3(u) 9(aš) 1(barig) 6(diš) 1/2 sila ₃ gur	39 <i>gur</i> 1 <i>barig</i> 6 1/2 <i>sila</i>
18.	mu ha-ar-ši ^{ki} -ta	from the year of Š 48
19.	mu en-mah-gal-še ₃	to the year of AS 4,
20.	9(aš) 3(barig) gur	9 <i>gur</i> 3 <i>barig</i>
21.	mu gu-za ba-dim ₂	in the year of AS 3,
22.	2(u) 2(aš) še gur	22 <i>gur</i> of barley
23.	mu en-unu ₆ -gal u ₃ mu ša-aš-ru ^{ki}	in the years of AS 5 and AS 6,
24.	[kišib ⁷] lu ₂ - ^d nin-šubur	sealed tablet of Lu-Ninšubur,
Col. ii		
1.	šu-nigin ₂ 1(geš ₂) 5(u) 3(aš) 4(barig) 5(ban ₂)1(diš) 1/2 sila ₃ še gur	in total: (60+53) <i>gur</i> 4 <i>barig</i> 5 <i>ban</i> 1 1/2 <i>sila</i> of barley
2.	zi-ga-am ₃	expended.
3.	diri 5(aš) 1(diš) 1/2 <<5(diš)>> sila ₃ še gur 5(aš) /erasure/ 1 :30	Surplus: 5 <i>gur</i> 1 1/2 <i>sila</i> of barley. 5(aš) 1:30
4.	nig ₂ -ka ₉ -ak še kaš u ₄ -sakar ^{giš} gigir-ra	The balanced account concerning the barley for beer for the chariot's crescent
5.	lú- ^d nin-šubur	in the charge of Lu-Ninšubur.
6.	mu 1(u) 2(diš)-kam mu en ^d nanna maš ₂ -e i ₃ -pa ₃ -ta	Twelve years from the year of Š 43
7.	mu ša-aš-ru ^{ki} ba-hul-šè	to AS 6.

Upper
edge

7(aš) 1:46:30

7(aš) 1:46:30

Commentary:

Obv. i 7: The amount for the additional month happens to equal the annual total in obv. i 5 divided by twelve. However, the amount for the additional month in other years (Š 47, AS 2, AS 4, AS 6) turns out to be 5 *сила* less than the annual total divided by twelve (see summary of data in Sect. 5.2).

Obv. i 19: Restoration of [8(aš)] based on parallel amounts attested for other years.

Obv. i 20: Restoration of [3(barig) 1(ban₂)] [6](diš) 1/2 sila₃ based on the numerical relationship between quantities of different years (see summary of data in Sect. 5.2).

Obv. i 24: Restoration of [8(aš) 1(ban₂) 8(diš) sila₃] based on parallel amounts attested for other years.

Obv. ii 2: Restoration of [8(diš) sila₃] based on parallel amounts attested for other years.

Rev. i 5: Although there appears to be no ruling between this line and the following line 6, the latter does not have an indenture at the beginning. So we separate these two lines in our numbering.

Text 4-YBC 16607

Location of marginal number: Lower edge

Provenience: Umma

Date: ŠS 5

Dimensions (cm): 8.8x4.7x2.2

Location of tablet: Yale Babylonian Collection, New Haven, Connecticut, USA

Reference No.: BDTNS 024119; CDLI P112505

Copy: Snell 1982: No. 13

Photo: <http://www.cdli.ucla.edu/dl/photo/P112502.jpg>

	Transliteration	Translation
Obverse		
1.	1(geš ₂) 1(u) še gur 1(aš) 5(ban ₂) gur-ta ¹	60+10 <i>gur</i> of barley (at the rate of one <i>gin</i> of silver) per 1 <i>gur</i> 5 <i>ban</i> ,
2.	1(geš ₂) 5(u) gur 1(aš) 4(ban ₂) gur-ta	60+50 <i>gur</i> (at the rate of one <i>gin</i> of silver) per 1 <i>gur</i> 4 <i>ban</i> ,
3.	ku ₃ -bi 2(diš) 1/2 ma-na 7(diš) gin ₂	their value in-silver 2 1/2 <i>mana</i> 7 <i>gin</i> 9
4.	9(diš) 1/2 še mu ^d šu- ^d suen lugal	1/2 <i>še</i> , year when Šu-Sin (became) king (ŠS 1);
5.	1(geš ₂) 1(u) gur	60+10 <i>gur</i> ,
6.	ku ₃ -bi 1(diš) ma-na 1(u) gin ₂	its value in-silver 1 <i>mana</i> 10 <i>gin</i> ,
7.	mu ma ₂ dara ₃ ab-zu ^d en-ki ba-ab-du ₈	year when the boat 'ibex of the abzu' was caulked for Enki (ŠS 2);
8.	2(geš ₂) 1(u) ʾ4ʾ(aš) gur	60x2+14 <i>gur</i> ,

9.	ku ₃ -bi 2(diš) 1/2 ʾma ¹ -<na> 7(diš)	its value in-silver 2 1/2 <i>mana</i> 7 1/2 <i>gin</i> ,
	1/2 <gin ₂ >	
10.	mu si-ma-num ₂ ^{ki} ba-hul	year when Simanum was destroyed (ŠŠ 3);
11.	Blank line	
12.	še ku ₃ -še ₃ sa ₁₀ -a	barley sold for silver;
Lower edge		
	6:57:20 /space/ 15	6:57:20 /space/ 15
Reverse		
1.	1/2 ma-na 2(diš) 5/6 gin ₂ 5(diš) še ku ₃ -babbar	1/2 <i>mana</i> 2 5/6 <i>gin</i> 5 <i>še</i> of silver,
2.	la ₂ -NI su-ga ugula kikken ₂ sa ₂ -du ₁₁ ensi ₂ -ka	repaid arrears of the overseer of a milling house, as the <i>sadu</i> -offering to the governor;
3.	Blank line	
4.	[6(diš)] ʾ5/6 ¹ ma-na 7(diš) 1/3 ¹ gin ₂ 1(u) 5(diš) še	6 5/6 <i>mana</i> 7 1/3 <i>gin</i> 15 <i>še</i>
5.	ʾki ² [...] -ta	from ...
6.	1(u) 4(diš) 2/3(diš) gin ₂ sám x x x zabar uruda šu-nir-ra x	14 2/3 <i>gin</i> as price ... for bronze and copper for the emblem ...;
7.	4(diš) 1/2 ma-na ku ₃ -babbar	4 1/2 <i>mana</i> of silver,
8.	ʾkišib ¹ ʾensi ₂ ¹ -ka	sealed receipt of the governor;
9.	la ₂ -NI 2(diš) ma-na 1(u) 3(diš) gin ₂ ku ₃ -babbar	deficit: 2 <i>mana</i> 13 <i>gin</i> of silver.
10.	nig ₂ -ka ₉ -ak ku ₃ ensi ₂ -ka	A balanced account on the silver of the governor,
11.	gir ₃ lu ₂ -kal-la	conveyed by Lu-kala.
12.	mu ʾšu- ^d suen lugal-ta	From the year when Šu-Sin became king (ŠŠ 1)
13.	mu us ₂ -sa bad ₃ mar-tu ba-du ₃ -še ₃	to the year after the western wall was built (ŠŠ 5)

Commentary:

Rev. 4: Restoration of [6(diš)] based on the relationship that the quantities in obv. 3, 6, 9 and rev. 1 add up to the total in rev. 4 (see summary of data in Sect. 5.2).

Regarding the fraction before gin₂, both BDTNS and CDLI read it as 1/2, but we expect it to be 1/3 instead. Our collation of the tablet reveals two short vertical strokes below and above the horizontal stroke but to the right of the vertical stroke in the MAŠ sign representing 1/2. These two may indicate a long but half-erased vertical stroke. If so, then we may restore the fraction as 1/3.

Text 5-E 15550

Location of marginal number: left edge

Provenience: Umma

Date: IS 2

Dimensions (cm): 10.6×5.5×2.5

Location of tablet: Peabody Essex Museum, Salem, Massachusetts, USA

Reference No.: BDTNS 191620; CDLI P416398

For photos, transliteration and translation, see Ouyang and Brookman 2012: 3.4.

Text 6-PTS 473

Location of marginal number: Reverse, middle section

Provenience: Puzriš-Dagan

Date: Š 48 vii

Location of tablet: Princeton Theological Seminary, Princeton, New Jersey, USA

Dimensions (cm): 7.5x4.5x1.5

Reference No.: BDTNS 032993; CDLI P126749

Copy: Sigrist 1990: No. 60

Photo: <http://www.cdli.ucla.edu/dl/photo/P126749.jpg>

	Transliteration	Translation
Obverse		
1.	[2(geš ₂)] 4(u) ṛ5(diš)ṛ ab ₂	2x60+45 cows
2.	[...] [6+]2(geš ₂) [3(u)] 4(diš) udu	[...]+8x60+34 sheep
3.	[...] [5+]2(geš ₂) 2(u) 1(diš) maš ₂	[...]+7x60+21 sheep
4.	2(geš ₂ 'u) 2(geš ₂) ṛ3(u)ṛ 4(diš) ud ₅ gun ₃	2x600+2x60+34 speckled female goats
5.	[2+]1(geš ₂ 'u) ṛ2(u)ṛ 7(diš) ud ₅	3x600+27 female goats,
6.	(blank space) 2(geš ₂) 4(u) 5(diš) ab ₂	(in total) 2x60+45 cows
	2(šar ₂) 8(geš ₂) 5(u) 6(diš) udu	2x3600+8x60+56 sheep
7.	ṛgir ₃ ṛ bu-bu	conveyed by Bubū;
8.	1(geš ₂) 6(diš) ab ₂	60+6 cows
9.	5(geš ₂ 'u) udu maš ₂ hi-a	5x600 sheep and goats of various kinds
10.	gir ₃ šu- ^d en-lil ₂	conveyed by Šu-Enlil;
11.	nam-ṛraṛ-ak	booty
12.	ki-maš ^{ki} ṛhaṛ-ar-š ^{ki}	of Kimaš and Harši
Reverse		
1.	[...] -x ṛkiṛ	[...]
2.	šušin x [...] -x-ra	Susa ...
3.	kišib sa-ṛsṛṛ [...] x	Sealed receipt of ...
4.	šu-ṛ ^d ṛ [...] ṛTI ^ṛ .A [...] di-ku ₅ -ṛxṛ	...
5.	ṛtum ₃ ṛ-dam	to bring.
	2 /space/ 8:56	2:8:56
	1	1
6.	iti [ezem]-ṛ ^d ṛ šul-gi	The month of the 'Festival of Šulgi'.
7.	mu ṛha-arṛ-š ^{ki} u ₃ ki-maš ṛkiṛ ba-hul	The year when Harši and Kimaš were destroyed.

Commentary:

Obv. 1: Restored according to the first part of obv. 6.

Obv. 2: Partially restored based on the numerical relationship of this line with obv. 3-6 (see summary of data in Sect. 5.2).

Obv. 3: Partially restored based on the numerical relationship of this line with obv. 2, 4-6 (see summary of data in Sect. 5.2).

Obv. 4: In contrast to the hand-copy by Sigrist and the reading of BDTNS and CDLI, our collation of the tablet shows no A sign at the end.

Rev. middle section: A DIŠ sign appears in a second line and right below the 8(diš) sign of the marginal number. We do not understand the meaning of this sign.

Text 7-NBC 6641

Location of marginal numbers: Lower edge and left edge

Provenience: Puzriš-Dagan

Date: ŠS 8 ix

Dimensions (cm): 11.1x6.2x2.7

Location of tablet: Yale Babylonian Section, New Haven, Connecticut, USA

Reference No.: BDTNS 077468; CDLI P298493

Copy: Garfinkle 2008

Edition: Garfinkle 2008, Paoletti 2008 and Paoletti 2012: 448-9.



Fig. 5.30 NBC 6641 (Photos courtesy of Yale Babylonian Collection)

	Transliteration	Translation
Obverse		
1.	1(aš) gu ₂ 1(u) ma-na ku ₃ -babbar	1 <i>gu</i> 10 <i>mana</i> of silver
2.	nig ₂ -sa ₁₀ -ma ku ₃ -sig ₁₇ 1(u)-ta-še ₃	as purchase price of gold at the rate of 10:1
3.	sag-nig ₂ -gur ₁₁ -ra-kam ša ₃ -bi-ta	This is the total of the receipts. From its within:
4.	9(diš) 1/2 ma-na 2(diš) 2/3 gin ₂ 1(u)9 1/2 <i>mana</i> 2 2/3 <i>gin</i> 11 še	
5.	kur-bi-la-ak	for Kurbilak;
6.	9(diš) 1/2 ma-na 2(diš) 2/3 gin ₂ 1(u)1/2 <i>mana</i> 2 2/3 <i>gin</i> 11 še	
7.	lu ₂ - ^{daš} ašgi ^{gi4}	for Lu-Ašgi;

8. 9(diš) 1/2 ma-na 2(diš) 2/3 gin₂ 1(u)9 1/2 *mana* 2 2/3 *gin* 11 še
1(diš) še
9. lu₂-zabala₄^{ki} for Lu-Zabala;
10. 7(diš) 5/6 ma-na 7(diš) gin₂ igi-4(diš)-7 5/6 *mana* 7 1/4 *gin* 3 še
gal₂ 3(diš)⁷³ še
11. i₃-li₂-an-dul₃ for Ili-andul;
12. 7(diš) ma-na 9(diš) 1/2 gin₂ 8(diš) 1/2 še7 *mana* 9 1/2 *gin* 9 še
1/2 še
13. ^dsuen-kal for Sin-kal;
14. 6(diš) 1/3 ma-na 1(diš) 5/6 gin₂ 6 1/3 *mana* 1 5/6 *gin*
15. nu-ur₂-^dutu for Nur-Utu;
- Reverse
1. 4(diš) 2/3 ma-na 6(diš) 1/3 gin₂ 5(diš)4 2/3 *mana* 6 1/3 *gin* 5 1/2 še
1/2 še
2. lu₂ zimbir^{ki}-ke₄-ne for people of Zimbir;
3. 3(diš) ma-na 1(u) 5/6 gin₂ 1(u) 5(diš) še3 *mana* 10 5/6 *gin* 15 še
4. dingir-ba-ni for Dingir-bani;
5. 3(diš) ma-na 1(u) 5/6(diš) gin₂ 1(u)3 *mana* 10 5/6 *gin* 15 še
5(diš) še
6. bu₃-su₂-num₂ for Busunum;
7. 2(diš) 2/3 ma-na 2(diš) gin₂ igi-6(diš)-2 2/3 *mana* 2 1/6 *gin* 18 1/2 še
gal₂ 1(u) 8(diš) 1/2 ṛše¹
8. ku₃ lu₂ uri₅^{ki}-ke₄-ne for people of Ur;
9. 2(diš) [1/3 ma]-na 3(diš) gin₂ igi-6(diš)-2 1/3 *mana* 3 1/6 *gin* 3 še
gal₂ 3(diš) še
10. ṛku₃¹ [...] -ru^{ki}-ke₄-ne for [...];
11. [...] -la [...]
12. 2(diš) [1/3 ma-na 3(diš) gin₂ igi-6(diš)]-2 [1/3 *mana* 3 1/6 *gin*] 3 še
gal₂ 3(diš) še
13. [ku₃ lu₂ ...^{ki}]-ke₄-ne for people of [...];
14. [...] 2(u) 2(diš) 1/2(diš) še [...] + 22 1/2 še of silver
15. [...] -um [...]
- Blank line
17. [šu-nigin₂] 1(aš) ṛgu₂¹ 1(u) ma-na ku₃-In total: 1 *gu* 10 *mana* of silver.
babbar
18. gaba-ri kišib dam-gar₃-e-ne Copy of the sealed receipt of the merchants.
19. iti ezem-^dšu-^dsuen The month of the ‘Festival of Šu-Sin’.
20. mu ^dšu-^dsuen lugal uri₅^{ki}-ma-ke₄ ma₂-Year when Šu-Sin, king of Ur, constructed a
gur₈-mah ^den-lil₂ ^dnin-lil₂-ra mu-ne-grand boat for the gods Enlil and Ninlil.
dim₂
- Bottom 5(u) /space/ 6(diš) 5(u) 50 /space/ 6:50
edge
- Left edge 2(u) /space/ 1(u) 2(diš) 1(u) 7(diš) 3(u) 20 /space/ 12:17:30

Commentary:

Obv. 12: Last two signs read as <<1/2 (diš) še>>^{sic} in CDLI, but as 1/2 (diš) še in the two editions above and BDTNS.

73 See note to Table 5.17 in Sect. 2.

Rev. 9: The fraction before ma-na read as $[2/3]$ in BDTNS, Paoletti 2008 and Paoletti 2012: 448-9, but as $[1/3]$ in Garfinkle 2008 and CDLI.

Text 8-YBC 13418

Location of marginal numbers: Upper edge and bottom of reverse

Provenience: Puzriš-Dagan

No date

Dimensions (cm): 9.6x5.5x2.5

Location of tablet: Yale Babylonian Section, New Haven, Connecticut, USA

Reference No.: BDTNS 061651; CDLI P210091

No copy of this tablet is available.



Fig. 5.31 YBC 13418 (Photos courtesy of Yale Babylonian Collection)

	Transliteration	Translation
Obverse		
1.	1(diš) 1/3 ma-na ku ₃ -sig ₁₇ huš-a	1 1/3 <i>mana</i> of 'red' gold
2.	lu ₂ -zabalam ₄ ^{ki}	(from) Lu-Zabalam,
3.	1(diš) ma-na 6(diš) 2/3 gin ₂	1 <i>mana</i> 6 2/3 <i>gin</i>
4.	puzur ₄ -er ₃ -ra	(from) Puzur-Erra,
5.	5/6 ma-na 3(diš) 1/3 gin ₂	5/6 <i>mana</i> 3 1/3 <i>gin</i>
6.	nu-ur ₂ - ^d utu	(from) Nūr-Utu,
7.	1/3 ^{ša} ba-sa ₆ -ga	1/3 <i>mana</i> (from) Basaga,
8.	1/3 ^{ša} 6(diš) 2/3 gin ₂	1/3 <i>mana</i> 6 2/3 <i>gin</i>
9.	dingir-ba-ni	(from) Ilim-bani,
10.	1(u) 3(diš) 1/3 gin ₂ i-re-eb	13 1/3 <i>gin</i> (from) Ireb,
11.	1/2 ma-na 3(diš) gin ₂ igi-6-gal ₂ 8(diš) še	1/2 <i>mana</i> 3 1/6 <i>gin</i> 8 še

Reverse

1.	lu ₂ - ^{daš} aš ₇ -gi ₄	(from) Lu-Ašgi,
2.	2(diš) 5/6 ma-na 6(diš) še	2 5/6 <i>mana</i> 6 <i>še</i>
3.	nu-ur ₂ -eš ₄ -tar ₂	(from) Nūr-Eštar,
4.	5/6 ma-na 6(diš) gin ₂	5/6 <i>mana</i> 6 <i>gin</i>
5.	er ₃ -ra-dan	(from) Erra-dan,
6.	šu-nigin ₂ 5(diš) ma-na 1(u) 1(diš) ┌gin ₂ ┐ ⁷ 1(u) 4(diš) še	in total: 5 <i>mana</i> 11 <i>gin</i> 10 <i>še</i>
7.	ku ₃ -bi 1(aš) gu ₂ 2(u) 8(diš) ma-na ┌la ₂ ┐ 1(u) gin ₂	its value in-silver 1 <i>gu</i> 28 <i>mana</i> minus 10 <i>gin</i> .
8.	2(u) /space/ 4(u) 5(diš)	8. 20 /space/ 45
Upper edge	1(aš) 2(u) 7(diš) 5(u)	1(aš) 27:50

Commentary:

Obv. 7-8: The ša serves as the phonetic complement for the fraction of 1/3 (read as šušana). In the absence of the weight unit ma-na, this ša combined with the fraction implies that 1/3 *mana* is meant (Powell 1971: 133).

Rev. 6: This total turns out to be 3 *mana* 28 1/6 *gin* less than the expected total 8 1/2 *mana* 9 1/6 *gin* 14 *še*, which is calculated by adding all the numbers from the preceding individual entries. Our collation of the tablet reveals that a fraction before the MA sign might have been erased.

Rev. 7: Restoration of ┌la₂┐ based on our collation of the tablet and the correspondence between the metrological notation here and the marginal number on the upper edge.

Rev. 8: The first part of the marginal number, 2(u), appears slightly above the second part, 4(u) 5(diš). For the digit 4(u), the two U signs on the top row appear much fainter and smaller than the two on the bottom.

Text 9- AO 27307 (=AOT c-61 = RTC 408)

Location of marginal number: Reverse, end of col. ii

Provenience: Girsu

Date: Ur III (date probably written on the tablet, but lost)

Location of tablet: Musée du Louvre, Paris, France

Reference No.: BDTNS 000848; CDLI P128561

Edition: Thureau-Dangin 1897: 26-27; pl. 30 No. 79; Thureau-Dangin 1903: No. 408

Photo: <http://www.cdli.ucla.edu/dl/photo/P128561.jpg>, accessed October 2015.

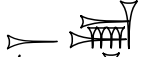


Obverse	Transliteration	Translation
Col. i		
1.	[...] 3(bur'u) 6(bur ₃) [1(eše ₃) 3(iku) GAN ₂ 1(aš) 2(barig) [3(ban ₂)- ta]	[...] 3 <i>buru</i> 6 <i>bur</i> [1 <i>eše</i>] 3 <i>iku gan</i> ; (the rate is) 1 <i>gur</i> 2 <i>barig</i> 3 <i>ban</i> [per (<i>bur</i>)]
2.	1(bur'u) 4(bur ₃) 1(iku) GAN ₂ 1(aš) 4(barig)-ta	1 <i>buru</i> 4 <i>bur</i> 1 <i>iku gan</i> ; (the rate is) 1 <i>gur</i> 4 <i>barig</i> per (<i>bur</i>)

3.	2(bur'u) 8(bur ₃) 1(eše ₃) 2(iku) GAN ₂ 1(aš) 1(barig)-ta	2 buru 8 bur 1 eše 2 iku gan; (the rate is) 1 gur 1 barig per (bur)
4.	[še]-bi 4(geš'u) 7(geš ₂) 4(u) 1(aš) 2(barig) 2(ban ₂ / 5 sila ₃ gur	Its grain 4×600+7×60+41 gur 2 barig 2 ban 5 sila
5.	GAN ₂ uru ₄ -a	The field for seeding
6.	2(šar ₂) 7(bur ₃) 2(eše ₃) GAN ₂ 1(aš) 2(barig) 3(ban)-ta	2 šar 7 bur 2 eše gan; (the rate is) 1 gur 2 barig 3 ban per (bur)
7.	1(šar ₂) 2(bur'u) 4(bur ₃) 2(eše ₃) 1(iku) 1/4 GAN ₂ 1(aš) 4(barig)-ta	1 šar 2 buru 4 bur 2 eše 1 iku 1/4 gan 1 gur 4 barig per (bur)
8.	4(bur'u) 1(bur ₃) GAN ₂ 1(aš) 1(barig)-ta	4 buru 1bur gan; (the rate is) 1 gur 1 barig per (bur)
9.	4(bur ₃) 2(eše ₃) 5(iku) GAN ₂ 1(aš)-ta	4 bur 2 eše 5 iku gan; (the rate is) 1 gur per (bur)
10.	še-bi 6(geš ₂) 3(u) 8(aš) 5(ban ₂) 5/6 / sila ₃ gur	Its grain is 6×60+38 gur 5 ban 5/6 sila
11.	GAN ₂ bala-a	The field of bala
12.	[šu-nigin ₂] 3(u) 8(aš) [3(barig) 1(ban)] ² 3 sila ₃ gur] [...]	[Total] 38 gur 3 barig? 1 ban 3 sila [...]
Col. ii	[...]	[...]
3.	[1(šar'u) 8(šar ₂) 5(bur'u) 9(bur ₃) 2(ese ₃)] 4(iku) 1(ubu) ^{sic} GAN ₂ [24 sar 2/3 gin ₂ 1(aš) 2(barig) 3(ban ₂)-ta]	[1 šaru 8 šar 5 buru 9 bur 2 ešê] 4 iku 1/4 ¹ gan [24 sar 2/3 gin (the rate is) 1 gur 2 barig 3 ban]
4.	3(šar ₂) 8(bur ₃) 2(eše ₃) 1(aš) 1/4 GAN ₂ 1(aš) 4(barig)-ta	3 šar 8 bur 2eše 1 iku 1/4 gan; (the rate is) 1 gur 4 barig per (bur)
5.	3(šar ₂) 4(bur'u) 2(bur ₃) 2(eše ₃) 3(iku) 1(ubu) 1/4 GAN ₂ / 1(aš) 1(barig)-ta	3 šar 4 buru 2 bur 2 eše 3 iku 1/2 1/4 gan; (the rate is) 1 gur 1 barig per (bur)
6.	1(bur ₃) 2(eše ₃) GAN ₂ 1(aš)-ta	1 bur 2 eše gan; (the rate is) 1 gur per (bur)
7.	še-bi 3(geš'u) 8(geš ₂) 3(u) / 8(aš) 3(barig) 3(ban) 4 5/6 sila ₃ 5 gin ₂	Its grain 3×600 + 8×60 + 38 gur 3 barig 3 ban 4 5/6 sila 5 gin
8.	GAN ₂ [uru ₄ -a]	The field for seeding
9.	2(šar ₂) 4(bur'u) 4(bur ₃) 1(eše ₃) 2(iku) [GAN ₂] 1(aš) 2(barig) [3(ban ₂)]-[ta]	2 šar 4 buru 4 bur 1 eše 2 iku gan; (the rate is) 1 gur 2 barig 3 ban [per (bur)]
10.	[2(bur'u) 2 bur ₃] [...]	2 buru 2 bur [gan ...]
Reverse Col. i	[...] [...]	[...] [...]
1'.	šu-nigin ₂ [...]	Total [...]
2'.	šu-nigin ₂ [...]	Total
3'.	še-bi [...]	Its grain
4'.	GAN ₂ bala-a	The field for bala
5'.	šu-nigin ₂	Total
6'.	šu-nigin ₂	Total
7'.	šu-nigin ₂	Total
8'.	šu-nigin ₂	Total

9'	še-bi	Its grain
10'	GAN ₂ bala-[a]	The bala of the field
Col. ii		
1'	[...]	[...]
2'	x x ŠU x [...]	In Girsu? [...]
	/space/	/space/
3'	20 ² 2 ² 4 ² [...]	20 ² 2 ² 4 ² [...]
	5:41:56 ² :55	5:41:56 ² :55
	[3]	[3]
	/space/	/space/
	10 ² 3 ² 1:20	10 ² 3 ² 1:20
	1:20	1:20
	45	45

Commentary:

Note on fractions of iku:

Cuneiform	Transliteration	Translation
	1(iku) GAN ₂	1 <i>gan</i>
	1(ubu) GAN ₂	1/2 <i>gan</i>
	1/4 GAN ₂	1/4 <i>gan</i>

Obv. i 2: Contra Thureau-Dangin's copy, the first sign of the line is visible; it is 1(bur'u).

Obv. ii 3: For explanations on the reconstruction of the damaged part, see Text 9 in Sect. 5.2. '4(iku) 1/2 GAN₂' is noted on the tablet, but, according to the reconstitution, '4(iku) 1/4 GAN₂' is expected.

Rev. ii 2': CDLI and BDTNS restore [ša₃[?]] [gir₂[?]-šu²ki] [x] (in Girsu).

Text 10-YBC 1793

Location of marginal numbers: Obverse, beginning of col. i and lower section of col. ii

Provenience: Unknown

Date: AS 5 ix

Dimensions (cm): 9.3x7x2.2

Location of tablet: Yale Babylonian Section, New Haven, Connecticut, USA

Reference No.: BDTNS 005335; CDLI P142357

Copy: Keiser 1919: No. 293



Fig. 5.32 YBC 1793 (Photos C. Proust, courtesy of Yale Babylonian Collection)

	Transliteration	Translation
Obverse		
Col. i		
1.	14:54	14:54
2.	29:56:50	29:56:50
3.	17:43:40	17:43:40
4.	30:53:20	30:53:20
5.	Blank line	
6.	šu-nigin ₂ 1(diš) 1/2 ma-na 3(diš) 1/2 gin ₂	In total: 1 1/2 <i>mana</i> 3 1/2 <i>gin</i> minus 7
	la ₂ 7(diš) še ku ₃ -a	še of silver,
7.	mu-ku _x (DU) didli	various deliveries;
8.	7(diš) ma-<na> 1(u) 9(diš) gin ₂ ku ₃ -a mu-	7 <i>mana</i> 19 <i>gin</i> of silver, delivery for the
	ku _x (DU) a-tu ₅ -a lugal	lustration of the king;
9.	Blank line	
10.	šu-nigin ₂ 8(diš) 5/6 ma-na 2(diš) 1/2 gin ₂	in (grand-)total: 8 5/6 <i>mana</i> 2 1/2 <i>gin</i>
	la ₂ 7(diš) še ku ₃ -a	minus 7 še of silver,
11.	ša ₃ im UD	within the daily tablets.
12.	Blank line	
13.	[mu]-ku _x (DU) iti ezem-mah	Delivery in the month of 'Grand
		Festival'
14.	┌mu┐ en ^d inanna	during the year when the en-priest of
		Inanna (was installed).
Col. ii		
1.	8(diš) 5/6 ma-na 4(diš) 1/2 gin ₂ la ₂ ┌x┐	8 5/6 <i>mana</i> 4 1/2 <i>gin</i> minus <i>x</i> +2 še
	2(diš) še	
2.	ša ₃ [...] -a	[...]
	2:┌54┐	2:┌54┐
	┌45┐	┌45┐
	┌28┐	┌28┐
	17	17

2:28	2:28
27	27
Reverse	
Not inscribed	

Commentary:

We follow CDLI for the unknown provenience of this tablet. BDTNS identifies it as a tablet from Puzriš-Dagan.

Obv. i 6, 8, 10: Our search in both BDTNS and CDLI (accessed 2013 August) did not find the expression ‘kù-a’ in any other Ur III document. The exact meaning of this phrase remains unclear to us.

Obv. i 11: Read as ‘šà im-babbar’ in CDLI. Our search in both BDTNS and CDLI (accessed 2013 August) found the phrase šà im UD only in this text. We follow Friberg (2005: 9) for the translation here.

Obv. ii 1: Friberg (*ibid.*) restored the amount in this line as the same in obv. i 10, but our collation does not support his restoration. Instead of the 2 1/2 gin₂ as read by him, both the copy and our collation of the tablet show that the amount involved should be 4 1/2 gin₂. As for the amount subtracted, since the two DIŠ signs do not appear under the horizontal stroke of the LA₂ sign (cf. la₂ 7 še in obv. i 6, 10) but immediately after this sign, the amount subtracted may well exceed two še.

Obv. ii 2: Friberg (*ibid.*) restored the lost sign as bala without producing any evidence.

Primary sources

Museum number	Primary publication	CDLI number
AO 2728 = AOT 304 = RTC 413	Thureau-Dangin 1903: 149, No. 413	P128566
AO 27307 = AOT c-61 = RTC 408 (Text 9)	Thureau-Dangin 1897: 26-27; pl. 30 No. 79; Thureau-Dangin 1903: No. 408	P128561
AO 5674	Genouillac 1922	P131745
AO 5676	Genouillac 1922	P131747
AO 6038	Genouillac 1922	P131752
Ashm 1924-0667	Grégoire 1996	P142827
BM 19027	King 1898: pl. 27	P108470
CBS 11661	Myhrman 1910: 56	P105608
E 15550 (Text 5)	Ouyang and Brookman 2012: Sect. 3.4	P416398
Erlenmeyer 152	Englund 2003: No. 1	P109319
Erm 14994	Koslova 2000: No. 340	P212246
FLP 972	Owen 1975: No. 290	P113850
FMB 39.2	Clevenstine, forthcoming	P427641
HMA 9-02824	Lutz 1928: No. 100	P136004
HS 201	Neugebauer 1935; Oelsner 2001	P254568
Ist L 7375	Delaporte 1911	P254700
Ist L 9236	Genouillac 1921	P227599
Ist Ni 374	Proust 2007	P257557
MAH 16323	Sauren 1974: No. 168	P113467
MAH 16573	Sauren 1974: No. 105	P113404

MAH 16605	Sauren 1974: No. 104	P113403
MAH 19353	Sauren 1974: No. 24	P113323
MAH 19472	Sauren 1974: No. 175	P113474
MM 0396	Schneider 1932	P101631
MSR 1	Goetze and Foster 2010: No. 158	P142394
MVN 3 290	Garfinkle 2008: 68	P215676
NBC 6501	Paoletti 2012: 447	P298413
NBC 6641 (Text 7)	Garfinkle 2008	P298493
Nik. 2 402 (Text 1)	Nikol'skij 1915: No. 402	P122085
Nik. 2 403	Nikol'skij 1915: No. 403	P122086
PTS 473 (Text 6)	Sigrist 1990: No. 60	P126749
PUL Ex 662	Chiera 1922	P130373
SM 1909.05.224	Owen 1982: 91	P116105
VAT 07042	Schneider 1930: No. 382	P125272
WCMA 20.1.07	unpublished according to CDLI	P424374
YBC 01778	Keiser 1919: No. 304	P142368
YBC 01793 (Text 10)	Keiser 1919: No. 293	P142357
YBC 03883	Keiser 1919: No. 225	P142289
YBC 04179 (Text 3)	Ellis 1970: 268	P111807
YBC 13418 (Text 8)	Sigrist and Ozaki 2009: No. 20	P210091
YBC 16487 (Text 2)	Snell 1982: No. 5	P112497
YBC 16607 (Text 4)	Snell 1982: No. 13	P112505

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