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We report the first measurements of dynamic transport properties of an acoustic pulsed wave propagating in a strongly disordered 2D medium by means of time-dependent coherent backscattering. The coherent backscattering peaks are recorded using an array of transducers. Both transport speed and diffusion coefficient have been determined accurately by comparison to a real-space generalization of coherent backscattering in optics. [S0031-9007(97)04374-3]

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In the past, ultrasonic transducer arrays have been used successfully to study propagation of acoustic waves in multiple scattering media. Especially, using time reversal mirrors, we have shown for the first time the robustness of time reversal of acoustic waves, even in strongly disordered media [1–3].

In this Letter, we show that ultrasonic arrays are also highly efficient to measure dynamic transport properties of a multiply scattered acoustic wave by recording the time-dependent coherent backscattering effect.

When studying wave propagation in random media over distances larger than one extinction mean free path ($\ell_{\text{ext}}$), the average transport of energy is described by the diffusion equation,

$$D \Delta - \frac{\partial}{\partial t} \langle I(r,r',t,t') \rangle = \delta(r-r')\delta(t-t'),$$

(1)

where $I(r,r',t,t')$ denotes the intensity Green function. $D$ is the diffusion coefficient, which is related to another dynamic quantity, the transport speed $V_E$, and a stationary one, the transport mean free path $\ell_{tr}$, through [4]

$$D = \frac{1}{d} V_E \ell_{tr},$$

(2)

d representing spatial dimension. Equation (1) is only valid in an ensemble-averaged sense, and phase information seems to be lost. However, interference can survive even in strongly disordered media as proven by the “coherent backscattering effect.” In the past, numerous experiments, mainly in optics [5–7], but seldom in acoustics [8], have confirmed the existence of interference in multiple scattering.

What we present in this Letter are new experimental results taking advantage of ultrasonic arrays. First, unlike optical detectors, ultrasonic transducers can directly record time-dependent fluctuations in both amplitude and phase of a wide-band wave rather than just its intensity. Moreover, the wavelengths and sample scales are relatively large; there is no Brownian motion of the scatterers, so we have access to each particular realization of disorder. Finally, ultrasonic experiments take place in real space rather than in $k$ space; i.e., instead of using plane waves, our sources and detectors have a wide angular spectrum.

The experiment consists in sending an ultrasonic pulse into the scattering sample immersed in water and in recording the $N$ backscattered signals measured by an $N$-elements array (Fig. 1). Since we measure the backscattered amplitude $A(x,t)$, we can deduce either the stationary intensity $I(x) = \int A^2(x,t) dt$ or the time-dependent intensity $I(x,t) = \int_0^T A^2(x,\tau) d\tau$, where $T$ is chosen smaller than the initial wave form duration. Both measurements have been carried out in optics, although these are more difficult [9].

As we show, our dynamic method allows us to determine both transport speed $V_E$ and diffusion coefficient $D$ of the ultrasonic waves, and to monitor the time evolution of the diffusive “halo” inside the sample. The transport velocity $V_E$ is different from both phase and group velocity; it is related to the energy flow [4]; $\ell_{tr}$ can be determined from the stationary intensity profile, and $D$ from dynamic intensity measurements, so $V_E$ can be assessed through relation (2).

![FIG. 1. Experimental setup. (I) A transducer element $S$ sends a pulse into the sample. (II) The backscattered signals are recorded on the whole array. Then the array is translated along the $x$ axis for configuration averaging.](image-url)
FIG. 2. Time-integrated coherent backscattering. Normalized intensity $2\langle I(\theta) \rangle / \max_I \langle I(\theta) \rangle$ as a function of scattering angle $\theta$. The theoretical prediction of Eq. (5) is plotted as a smooth dashed line.

The scattering sample we studied consists of 2400 steel rods ($C_L = 5.9$ mm/µs, $C_T = 3.2$ mm/µs, $\rho = 7.85$, radius 0.4 mm) randomly distributed in the $(x, z)$ plane (Fig. 1). It is 160 mm long and 80 mm large. As a source and detector, we used a linear array of 128 transducers with central frequency 3.5 MHz (which corresponds to an average wavelength of 0.43 mm in water), pitch 0.417 mm, and height 12 mm. The experiment was carried out in a water tank.

The recording sequence can be described as follows: One of the transducer elements transmits a pulsed wave ($\approx 4$ µs, i.e., 15 cycles) into the sample. The backscattered signals are recorded on the whole array as a function of time. Then the array is translated along the $x$ axis by 1 mm, i.e., the distance over which the recorded signals are found to be uncorrelated, and the sequence is repeated; the ensemble average is finally made over 70 realizations. After the transmitting element has been changed, the entire experiment is carried out again, which enables us to average out the differences in the sensitivity of the transducers.

A fit of the time-integrated intensity to theory (Fig. 2) gives $\ell_c = 4$ mm. This theory will be discussed below. Figure 3 describes the time-resolved cone at four different times. We stress that the corrections obtained by deconvoluting the experimental results by the directivity patterns of the receivers are found to be negligible.

Figure 3 shows that the “cone” narrows with time, and a linear regression $\log(k\Delta \theta) = f(\log(t))$, where $k$ denotes the wave number and $\Delta \theta$ the angular width of the cone (Fig. 4), provides a slope of 0.49. This slope is consistent with the expected $1/\sqrt{D t}$ dependence of the intensity profile on the basis of common diffusion theory [10,11]. Extrapolation to the origin gives $D = 2.5$ mm$^2$/µs. Knowing both the transport mean free path and the diffusion coefficient, we deduce the transport velocity $V_E = 1.25$ mm/µs, not as much as the velocity of sound in water, 1.5 mm/µs.

We now sketch the diffusion theory used to explain our experiments. This theory is a generalization of the plane wave approach, common in optics, to the real space measurements in acoustics. In the far field approximation, i.e., $kR \gg 1$, the 2D free space Green function at a distance $R$ from the source is given by $G(R) = -[(1 + i)/4\sqrt{\pi kR}] e^{i k R}$. In the layer defined by $a < z < \ell_{ext} + a$ (Fig. 5), the ensemble averaged Green function can be approximated by $G(x, z) = \left[-(1 + i)/4\sqrt{\pi k} \right] \left[e^{ik\sqrt{a^2 + x^2}}/(a^2 + x^2)^{1/4}\right] e^{-z/2\ell_{ext}}$, with

FIG. 3. Normalized intensity $2\langle I(\theta) \rangle / \max_I \langle I(\theta) \rangle$ as a function of scattering angle $\theta$, at four different times. The theoretical predictions of Eq. (4) are plotted as a smooth dashed line.
$\mu = \cos \theta$. The incoherent background is obtained by

$$|\Psi_3(t)|^2 = \int \int \int dr_1 dr_2 dr_3 dr_4 G(r_1) G^*(r_3)$$

$$\times L(r_1, r_2, r_3, r_4) G(r_2) G^*(r_4).$$  \hspace{1cm} (3)

In the ladder approximation, one has $L(r_1, r_2, r_3, r_4) = F(r_1, r_2) \delta(r_1 - r_3) \delta(r_2 - r_4)$. For a slab geometry with boundary conditions $F = 0$ at $z = -z_0$ and $z = L + z_0$ ($z_0 = l_v \times \pi / 4$ in 2D), $F$ can be expressed as an infinite series of diffusive eigenmodes [12].

For the coherent peak $|\Psi_c(t)|^2$, the calculation is the same except that $L$ is replaced by $C(r_1, r_2, r_3, r_4) = F(r_1, r_2) \delta(r_1 - r_3) \delta(r_2 - r_3)$. The expression differs from the incoherent background by phase factors only, which are unity at backscattering. Furthermore, if we consider the diffusive domain defined by $t > \tau_{th} = B^2 / \pi^2 D$, where $B = L + 2z_0$, then only the slowest diffusive mode contributes to the peak.

Finally, within the approximation $a^2 << 4Dt$, we find for the time-resolved intensity

$$I(x, t) = 1 + \frac{1}{b} \int_{-\pi/2}^{\pi/2} g(\theta) \exp \left( - \frac{Dt}{a^2} (kx \cos \theta)^2 \cos^4 \theta \right) d\theta,$$  \hspace{1cm} (4)

where $g(\theta) = \cos^2 \theta (\pi/4 + \cos \theta)^2$ and $b = \int_{-\pi/2}^{\pi/2} g(\theta) d\theta$.

From Eq. (4) the FWHM of the dynamic intensity profile $I(x, t)$ is $\Delta x = 1.12(a / \sqrt{Dt})$, thus the cone narrows as the inverse square root of time, as we found experimentally. If the approximation $a^2 << 4Dt$ does not hold, Eq. (4) is no longer valid, and $\Delta x$ turns out to $\lambda / 2\pi$.

Integrating $|\Psi_c(t)|^2$ and $|\Psi_3(t)|^2$ over time, we obtain for the stationary intensity profile

$$I_{st}(x) = 1 + \frac{1}{\pi^2/4 + 4/3} \int_{-\pi/2}^{\pi/2} \frac{\mu^2}{(1 + Q\mu)^2} d\theta \exp \left( - \frac{\mu^2}{(1 + Q\mu)^2} \right),$$  \hspace{1cm} (5)

where $Q = (\ell_v / a) \mu^3 kx$.

These results have been used to fit our experimental results to determine $\ell_v$, $D$, and $V_E$. This provides an accurate method to measure dynamic transport properties of ultrasonic waves in strongly multiple scattering media.

In conclusion, we have presented an efficient method to determine stationary as well as dynamic transport properties using the coherent backscattering effect for a pulsed acoustic wave in a 2D random medium. In particular, we have emphasized that the use of linear arrays of transducers provides an efficient way to record time-dependent coherent backscattering.