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► **To cite this version:**

Federico Andrea Sabbatoli, Houssein El Dirani, Laurène Youssef, Linda Gianini, Luca Zatti, et al.. Nonlinear coupling of linearly uncoupled resonators through a Mach–Zehnder interferometer. *Applied Physics Letters*, 2022, 121 (20), pp.201101. 10.1063/5.0109392 . hal-03997031

HAL Id: hal-03997031

<https://hal.science/hal-03997031>

Submitted on 20 Feb 2023

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Nonlinear coupling of linearly uncoupled resonators through a Mach-Zehnder interferometer

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(Dated: 24 September 2022)

Optical nonlinear processes in linearly uncoupled resonators are being actively studied as a convenient way to engineer and control the generation of non-classical light. In these structures, one can take advantage of the independent combs of resonances of two linearly uncoupled ring resonators for field enhancement, with the phase-matching condition being significantly relaxed compared to a single resonator. However, previous implementations of this approach have shown a limited operational bandwidth along with a significant reduction of the generation efficiency. Here we experimentally demonstrate that a Mach-Zehnder interferometer can be used to effectively linearly uncouple two resonators and, at the same time, allows for their efficient nonlinear coupling. We demonstrate that this structure can lead to an unprecedented control over the rings' interaction and can operate over more than 160 nm, covering the S-, C- and L- telecom bands. In addition, we show that the photon pair generation efficiency is increased by a factor of four with respect to previous implementations.

Parametric processes are at the basis of the generation of quantum states of light. In particular, spontaneous four-wave mixing (SFWM) is arguably one of the most exploited process to generate photon pairs in integrated devices, either in the low- or high-gain regimes¹⁻⁷. Indeed, since SFWM is a third-order nonlinear process, it is widely compatible with almost all material platforms, including silicon-on-insulator and silicon nitride^{8,9}. Among integrated sources, ring resonators are been widely used, for they are compatible with open-fab industrial production, they can provide high generation rates thanks to large field enhancements^{10,11}, and they are naturally characterized by a resonance comb, which can be exploited to guarantee energy conservation in resonant SFWM¹². Yet, ring resonators can exhibit shortcomings in controlling nonlinear optical processes. For instance, in the case of intense pump fields, self- and cross-phase modulation can lead to a

dependence of the resonances **frequencies** on the pumping powers, limiting SFWM efficiency¹⁰. Furthermore, in dual-pump SFWM a comb with several evenly-spaced resonances can be associated with unwanted parasitic processes, reducing the quality of the generated quantum state¹³.

A possible solution to these issues comes from harnessing the design complexity offered by the silicon integrated photonic platform. Efficient and controllable nonlinear interactions have been demonstrated in photonic molecules and crystals, in which the position of the resonances involved in the nonlinear process are engineered by controlling the linear coupling of two or more resonators¹⁴⁻¹⁶. Such a strategy has also been proposed to selectively suppress parasitic nonlinear processes in the generation of squeezed light¹⁷.

An alternative approach involves the nonlinear interaction of modes belonging to resonators that are linearly uncoupled. For instance, Menotti et al. considered two racetrack resonators placed side-by-side to form a directional coupler (DC), the length of which was chosen so that the two resonators were linearly uncoupled and thus exhibited two independent combs of resonances¹⁸. But since the DC is shared by the modes of both racetracks, one can still achieve nonlinear coupling between modes belonging to different resonators. In this structure, dispersion engineering is not critical to achieve the resonant configuration enabling the selectively enhancement of the desired nonlinear interaction, and SFWM is more tolerant of deviations from the phase-matching condition. Finally, in the presence of active tuning elements, one can reconfigure the relative position of the two resonance combs on-demand, either to compensate for fabrication imperfections, or to operate at different pump power levels regardless non-

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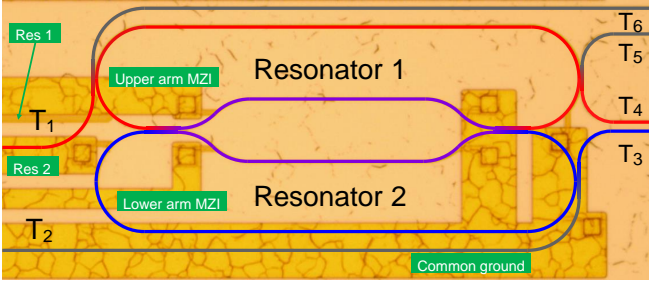


FIG. 1. Optical image of the device, which is composed of two resonators side-by-side forming a Mach-Zehnder interferometer. The optical waveguides of the device, which are partially covered by the metal circuits, have been highlighted with coloured lines: red and blue traces have been used for the optical modes of Resonator 1 and Resonator 2, respectively, while the purple line highlights the section shared by both modes and where the nonlinear interaction takes place.

linear effects, such as self- and cross-phase modulation. Stimulated and spontaneous four-wave mixing have been experimentally demonstrated for this kind of structure in both silicon nitride¹⁹ and silicon¹³ devices.

The use of a DC results in quite compact systems, but the design and fabrication of such structures can be challenging, for the DC splitting ratio critically depends on its length and the distances between the two waveguides. Longer DCs with more distant waveguides are less sensitive to fabrication imperfections, but they come with an increasing of the resonator size and a reduction of its finesse. Finally, the DC response can be significantly frequency dependent, with the desired behaviour achieved in a limited bandwidth, typically a few tens of nm at telecom wavelengths. A possible solution would be to adjust the DC optical response, employing asymmetrical tuning elements that lead to a differential variation of the mode effective index in the two side-by-side waveguides²⁰. However, in silicon devices this approach can require up to 50 mW of electrical power to switch from a zero-coupling to a full-coupling condition.

In this work we demonstrate that nonlinear coupling of linearly uncoupled resonators can also be achieved by replacing the DC with a Mach-Zehnder interferometer (MZI). This solution can provide significant improvements in terms of linear isolation of the two resonators, operation bandwidth, and SFWM efficiency. The device is shown in Fig. 1. It is composed of an integrated MZI, the input and output ports of which are connected to form two resonators that are point coupled to three bus waveguides for light I/O coupling.

The linear coupling between the two resonators depends on the interference resulting at the output of the MZI, which acts as an effective coupler. When scattering losses are negligible, the MZI cross-coupling transmission is given by

$$\kappa_{\text{eff}}^2 = 2\kappa^2(1 - \kappa^2)[1 + \cos(\Delta\phi)], \quad (1)$$

where κ^2 is the cross-coupling transmission of the two DCs that are part of the MZI; we assume they are identical, and $\Delta\phi = (2\pi n_{\text{eff}}/\lambda_0)\Delta L$ is the phase difference of the two arms,

with ΔL the MZI path difference, n_{eff} the mode effective index, and λ_0 the wavelength in vacuum. When $\Delta\phi = (2m + 1)\pi$, with m integer, light entering from the upper (lower) MZI input port is entirely redirected to the upper (lower) output port, so that the two rings are perfectly linearly uncoupled. Notably, it follows from (1) that, at a given wavelength λ_0 , two resonators can always be made linearly uncoupled regardless of the DC splitting ratio, provided that the condition $\Delta\phi = (2m + 1)\pi$ is satisfied. However, if one is interested in the nonlinear coupling of the two resonators, it is convenient working with $\kappa^2 \approx 0.5$ to maximize the nonlinear field overlap of the modes of the two resonators in the MZI arms.

The structure is potentially reconfigurable by using a phase shifter (e.g. an electrical heater) in one of the arms to control the effective coupling of the two resonators. In addition, even in the absence of any active tuning element, the design is resilient to fabrication imperfections and can have a wide operation bandwidth. In particular, if one works at the shortest useful path difference $\Delta L_{\text{min}} = \lambda_0/(2n_{\text{eff}})$, which corresponds to $\Delta\phi = \pi$ and guarantees the MZI largest operation bandwidth. For this value it can be shown that

$$\kappa_{\text{eff}}^2 \approx \left(\frac{\pi n_g \Delta\lambda}{n_{\text{eff}} \lambda_0} \right)^2 \kappa^2 (1 - \kappa^2), \quad (2)$$

where we assumed linear dispersion, with n_g being the group index. It follows from (2) that in a single-mode silicon waveguide, with $\lambda_0 = 1550$ nm, $n_{\text{eff}} = 2.4$, and $n_g = 4$, one can expect a maximum MZI cross transmission of only about 1% over a bandwidth of some 150 nm, where we assume $\kappa^2 \approx 0.5$.

This is the case of our device, which is fabricated starting from a silicon-on-insulator (SOITEC 200-mm SOITEC Photonics-SOI™ substrate) silica-cladding waveguide having cross-section 600×220 nm². The width of the waveguides is chosen to maximize the nonlinear interaction in this specific structure and results from a trade-off between having small nonlinear effective area and achieving low propagation losses. The two resonators have lengths $\mathcal{L}_1 = 639$ μm and $\mathcal{L}_2 = 632$ μm , and the DCs have a 50:50 splitting ratio. The two interferometer arms have lengths $\mathcal{L}_{A1} = 194$ μm and $\mathcal{L}_{A2} = 194.25$ μm to guarantee the linear isolation of the resonators near 1550 nm and to maximize the operation bandwidth, as discussed above. The fabricated device has four thermal phase shifters composed of TiN resistive circuits, which are driven by applying a voltage through conductive AlCu circuits. Two of them are located in the region of Resonator 1 and 2 far from the MZI to adjust the position of the two resonant combs independently, and two are located over the MZI arms to control the linear coupling of the two resonators.

First, we studied the tunability of the coupling between the resonators through the analysis of the optical response between the ports T_1 and T_6 (see Fig. 1) by varying the voltage applied to the thermal phase shifter placed on one arm of the MZI. The optical spectra are shown in Fig. 2. When no thermal power is applied (panel A), one can recognise the presence of two sets of resonances: a series of prominent dips associated to the resonance comb of Resonator 1, and series of dips with very small visibility associated to the resonance comb of Resonator 2. This last set of resonances is due to a

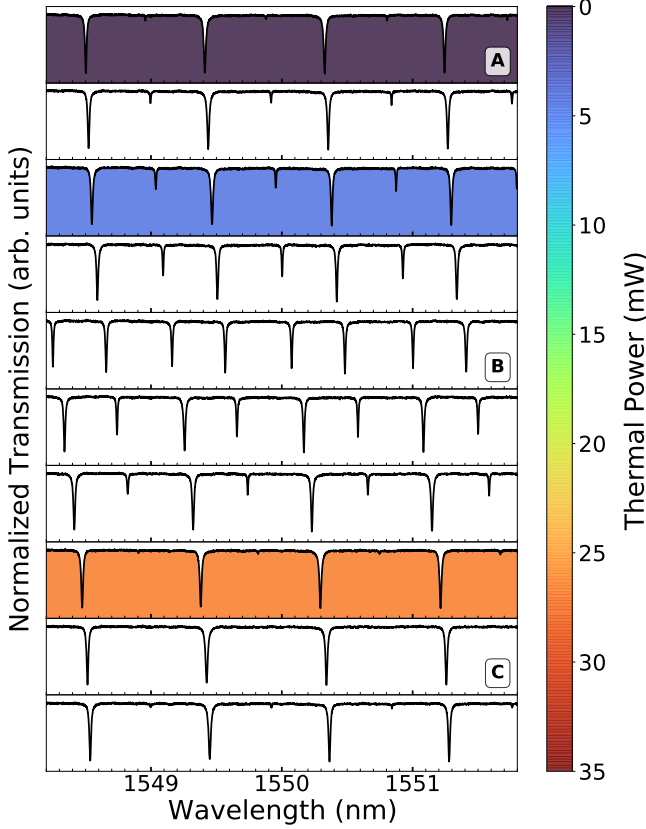


FIG. 2. Plots of the spectra obtained by coupling the laser to port T_1 and collecting the light from port T_6 . Each panel is associated with a different value of the voltage applied to the heater on top of one arm of the Mach-Zehnder (resistance around 1.25 k Ω). About 15 mW of electrical power are sufficient to switch from a configuration with no coupling (curves A and C) to the configuration of maximally-coupled resonators (curve B).

residual coupling between the two resonators. However, by applying an external voltage to the phase shifter (few tens of milliwatts of thermal power) one can adjust the coupling between the resonators so that the device can reach the condition of either maximum (panel B) or minimum (panel C) coupling.

With the MZI set to achieve the highest possible linear isolation between the two resonators, we performed a linear characterization of the device by injecting light from a CW tunable laser (Santec TSL-710) into the ports T_1 and T_2 of the sample, and we detected the optical response at the ports T_6 and T_3 , respectively, with a power meter. The two spectra are plotted in Fig. 3, in which two sets of resonances (having different FSRs and finesse) can be observed. In order to evaluate the linear coupling between the two resonators, we injected the light into port T_1 and measure the cross transmission in at port T_3 . The resulting curve is plotted with the blue dashed line in Fig. 3. We performed the characterization over the full bandwidth of the tunable laser source (from 1480 nm to 1640 nm, covering the C- and L-band and part of the S-band). We observe on resonance transmission as low as 50 dB due to the low effective coupling provided by the red MZI coupler, which

leads to an isolation of more than 30 dB over the whole range. This shows a significant improvement with respect to previous results obtained with DCs (see Fig. 2(c) in Ref.¹³), for which such isolation was achieved over a bandwidth of only a few tens of nanometers for a device realized with the same material platform to operate around the same central wavelength.

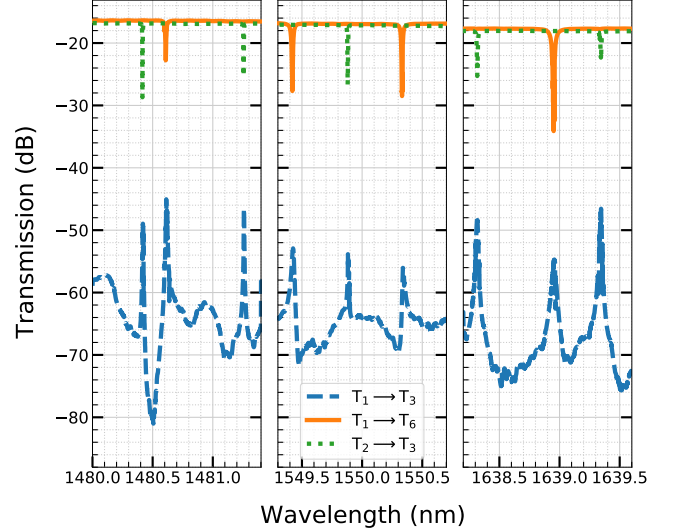


FIG. 3. High-resolution spectra of the linear characterization of the device in three selected ranges of the S-, C- and L- bands. The solid orange and dotted green lines correspond to the optical response in the configurations $T_1 \rightarrow T_6$ and $T_2 \rightarrow T_3$, respectively. The dashed blue line corresponds to the cross transmission of the device from ports T_1 to port T_3 . High linear isolation of the modes is achieved in the entire spectrum (data not shown) from 1480 nm to 1640 nm.

We now turn to the nonlinear characterization of our devices by performing a SFWM experiment. First, we adjusted the position of the resonances of Resonator 2, by means of the corresponding heater, such that two resonances (signal and idler) of Resonator 2 are equally spaced in frequency with respect to a resonance (pump) of Resonator 1. In this condition, due to the nonlinear interaction occurring in the MZI, triply resonant degenerate SFWM can take place, by pumping the device with a CW laser through port T_1 tuned at the pump resonant frequency and detecting the generated signal and idler photons in port T_3 . Optical filters have been used to filter out the residual pump laser, and two superconductive single-photon detectors are used to measure the coincidence rate. This is plotted in Fig. 4 as a function of the optical pump power. The data are obtained from the measured coincidence rate by taking into accounts the linear losses of the set-up and the detection efficiency of the detectors. As expected, the generation rate grows quadratically with the pump power.

The expected generation rate can be calculated following an asymptotic field²¹ approach in the backward Heisenberg picture²² framework. Assuming a CW pump at ω_p the generation rate is

$$R_{\text{pair}} = \frac{16v_g^4 \gamma^2 P_p^2 \omega_S \omega_I}{\omega_p^4 (\omega_S Q_I + \omega_I Q_S)} \frac{Q_p^4 Q_S^2 Q_I^2}{Q_{C,p}^2 Q_{C,s}^2 Q_{C,I}} \left(\frac{\mathcal{L}_{\text{MZI}}}{\mathcal{L}_1 \mathcal{L}_2} \right)^2, \quad (3)$$

where γ is the usual nonlinear parameters, P_P is the pump power in the bus waveguide, \mathcal{L}_{MZI} is the MZI length (where we assume $\mathcal{L}_{\text{MZI}} = \mathcal{L}_{A1} \simeq \mathcal{L}_{A2}$), Q_m and $Q_{C,m}$ are the loaded and the coupling quality factors, respectively, with $m = S, P$, and I referring to the signal, pump, and idler resonances. It is worth noting that the generation rate predicted by (3) is four times larger than that expected for a systems of two resonators linearly isolated using a DC, assuming the same optical length of the resonators \mathcal{L}_1 and \mathcal{L}_2 , the same interaction length ($\mathcal{L}_{\text{MZI}} \approx \mathcal{L}_{\text{DC}}$), and the same quality factors Q_m and $Q_{C,m}$ ¹⁸. This improvement is associated to the fact that, unlike when the linear uncoupling is achieved via a DC^{13,18}, **in the MZI arms the intensities of pump, signal, and idler fields are constant, while in the DC case evanescent coupling between the waveguides leads to oscillating intensities in the coupling region where the nonlinear interaction takes place (see Fig. 1 in Ref.¹⁸).** Eventually, this results in a larger nonlinear overlap integral for the MZI case. The curve resulting from (3), when considering the nominal parameters of our structure and the resonant quality factor obtained from the linear characterization²³, is reported in Fig. 4, which shows a very good agreement with the experimental results. **The measured generation rate is lower than theoretical value when the pump power exceeds 1 mW. This is likely due to two photon absorption²⁴, which has not been included in our model.**

In conclusion, we demonstrate that efficient SFWM can take place in two resonators that are linearly uncoupled using an MZI. We show that this approach is more efficient than previous implementations based on DCs, with significantly broader isolation bandwidth that can exceed 150 nm, enabling its use also for experiments and devices where the involved modes are significantly spaced in frequency. This approach is versatile, as it allows one to precisely control the isolation between the resonators and reconfigure the device to compensate for fabrication imperfections by means of thermal heater requiring only few tens of milliwatts. This approach can also be generalized to enhance other classical and quantum nonlinear processes, including those based on second-order nonlinear interactions.

ACKNOWLEDGMENTS

This work has been supported by Ministero dell’Istruzione, dell’Università e della Ricerca [MIUR grant Dipartimenti di Eccellenza 2018-2022 (F11I18000680001)]. J.E.S. acknowledges support from the Natural Sciences and Engineering Research Council of Canada. The device has been designed using the open source Nazca designTM framework.

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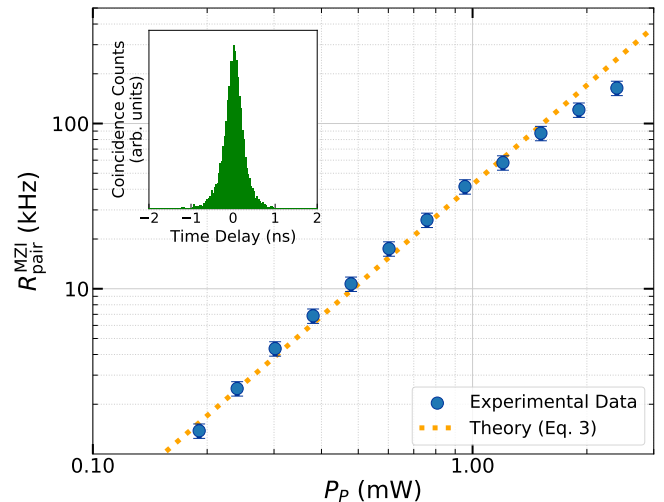


FIG. 4. Power scaling of the SFWM coincidences. A time-tagging unit is used to correlate the arrival times of the idler and signal photons generated through SFWM. In this way, a histogram of the coincidences is obtained (as shown in the inset), and the rate of the events are obtained by integrating the peak over a sufficiently wide window (4 ns) and subtracting the histogram background.

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