

Supplementary Information: Randomly stacked open cylindrical shells as functional mechanical energy absorber

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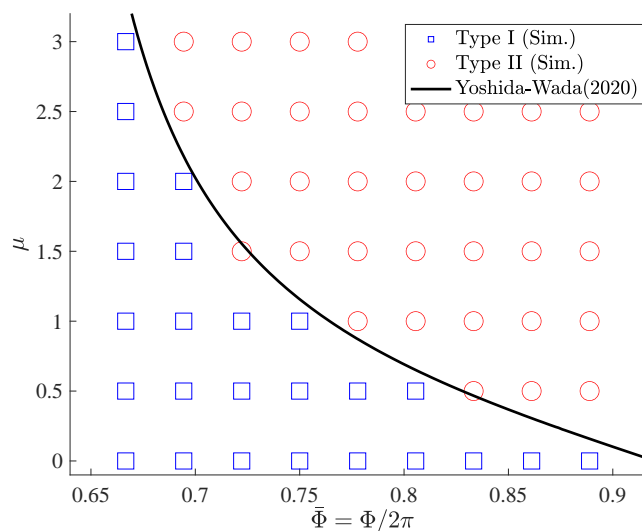
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Supplementary Note 1. VALIDATION OF THE 2D SUPER-HELIX MODEL COUPLED TO SO-BOGUS AGAINST THE SHELL-CYLINDER PROBLEM

The simulation code used throughout our paper consists of the 2D Super-Helix rod model [2] coupled to the non-smooth frictional contact solver of Daviet *et al.* [3] implemented in the so-bogus library. In the main text of our paper, we study, as an elementary micro-structural event, the problem of snap-fitting between two open cylindrical shells, where one shell is compressed against the other one. To validate our code beforehand, we use a slightly different scenario consisting of the snap-fit of a cylindrical shell onto a rigid cylinder of same radius, referring ourselves to the fully-validated theoretical framework of Yoshida and Wada [1]. Note that the latter work is more general, as it considers the case where the radius R of the shell can be different from that of the cylinder, R_c , with $\alpha \equiv R_c/R$ the ratio between the two radii. Using the condition of mechanical equilibrium and the Amontons-Coulomb law with a friction coefficient μ , the analytical formula for the phase boundary between Type I and II is derived as [1, Supplemental, eq. S18]

$$\frac{K(\phi)}{K_{\perp}(\phi)} + \frac{\tan \varphi_0(\phi, \alpha) - \mu}{1 + \mu \tan \varphi_0(\phi, \alpha)} = 0, \quad (1)$$



Supplementary Figure 1. **The phase diagram of Type I and II snap-fits for the shell-cylinder problem, computed with our simulator.** Empty squares and circles correspond to Type I and II snap-fits, respectively. The solid black curve represents the theoretical curve, which has been formulated and validated experimentally by Yoshida and Wada [1].

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where $K(\phi)$, $K(\phi)$ and $\varphi_0(\phi, \alpha)$ are known functions:

$$K(\phi) \equiv \left[\frac{\phi}{2} - \cos \phi \left(\frac{3}{2} \sin \phi - \phi \cos \phi \right) \right]^{-1} \quad (2)$$

$$K_{\perp}(\phi) \equiv \left[\frac{1}{2} - \cos \phi \left(-1 + \phi \sin \phi + \frac{3}{2} \cos \phi \right) \right]^{-1} \quad (3)$$

$$\varphi_0(\phi, \alpha) \equiv \sin^{-1}(\alpha^{-1} \sin \phi). \quad (4)$$

Here, we have introduced the half of Φ as $\phi \equiv \Phi/2$ (note that Yoshida and Wada [1] use Φ as our half angle). To validate our numerical model, we run simulations for different Φ and μ against this shell-cylinder problem, while the radii ratio is fixed throughout as $\alpha = R_c/R = 1$. In this case, equation (1) predicts a single-valued master curve on the Φ - μ plane.

We summarise our simulation results superposed on the analytical curve of Yoshida and Wada [1] in Fig. 1. Our simulation results are in excellent agreement with equation (1). Given that equation (1) is validated experimentally in Yoshida and Wada [1], we can also conclude that the 2D Super-Helix model coupled to So-Bogus will be able to predict physical experimental results.

We stress here that this validation is important because it enriches previous validations made on our numerical model [4]. Indeed, in the latter reference, this code has been validated against the ‘‘pinning test’’ [5], where an elastic strip is vertically compressed against a rigid flat substrate. However, the validation protocol based on the pinning test is limited to the case when the strip only contacts the substrate punctually, which limits the validation range of μ to $0 \leq \mu \lesssim 0.33$. In other words, with this former test, the simulation model is not validated for large μ . In contrast, the snap-fit-based protocol in Yoshida and Wada [1] enables us to test the cases of large friction coefficients. Hence, we can conclude that the predictability of our numerical simulator applies to a large range of friction coefficients.

Supplementary Note 2. HISTOGRAM FOR Δ_{\max} AND η AGAINST DIFFERENT $(\bar{\Phi}, \mu)$

In the main text, we characterise our 30-shell experiment by defining the two descriptors Δ_{\max} and η . Thanks to the computational efficiency of the 2D Super-Helix model coupled to So-Bogus, we could obtain sufficient statistics for Δ_{\max} and η across a range of 6×4 values for $(\bar{\Phi}, \mu)$, by using 100 different initial conditions per tuple $(\bar{\Phi}, \mu)$. All the data, gathered in fig. 2 (Δ_{\max} and η), feature Gaussian distributions with small standard or median absolute deviations.

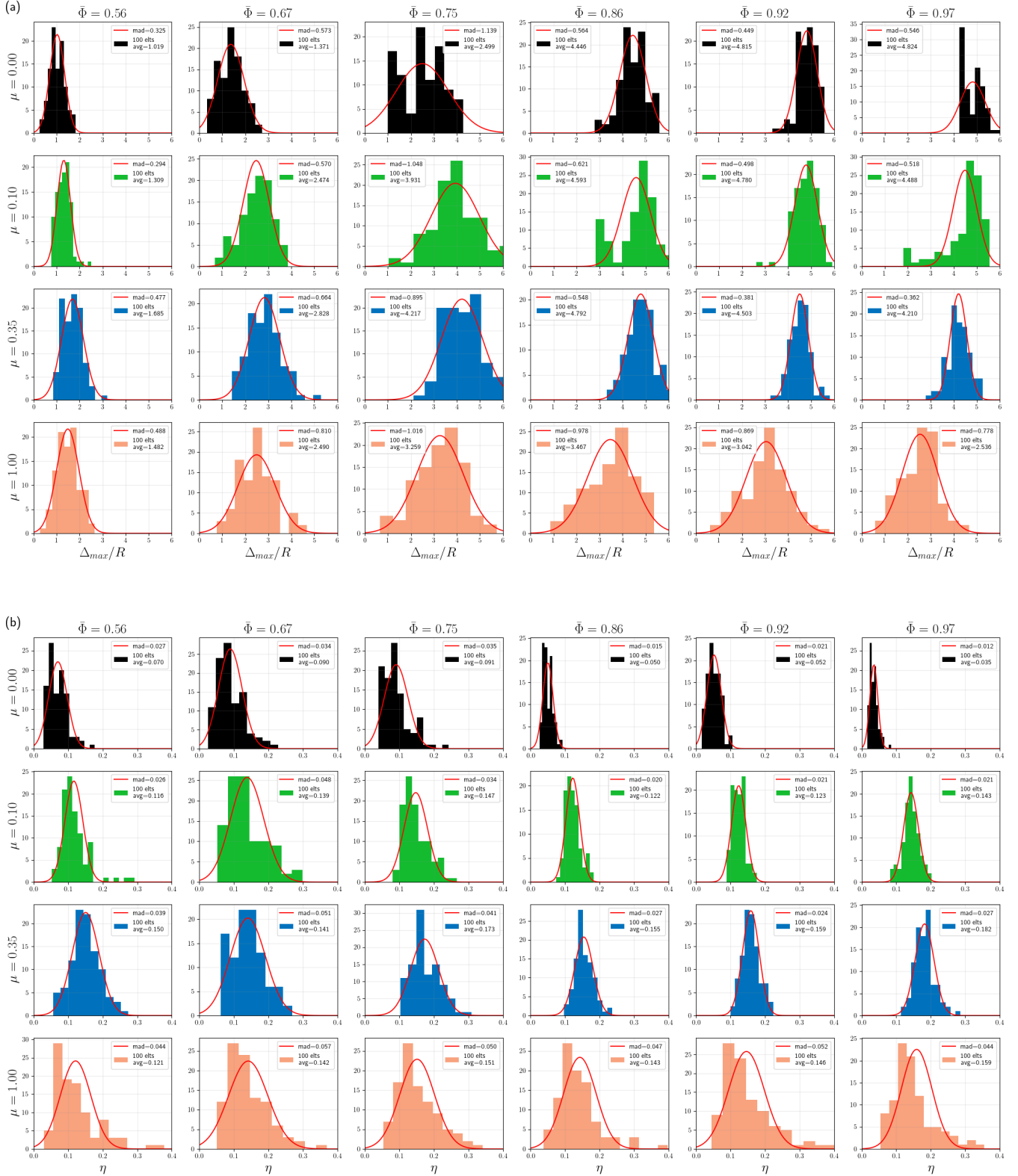
Supplementary Note 3. IMPACT OF μ ON Δ_{\max}

As observed in fig.5(c) in the main text, the indentation Δ_{\max} required to reach the prescribed force F_{\max} significantly depends on the friction coefficient in the type II regime (for which shell-shell snap-fit do not occur during compression). This effect can be interpreted in the context of almost closed shells ($\bar{\Phi} \gtrsim 0.85$): in this case, the shells are geometrically close to rings, and as such, present nearly no entanglement; the system deforms through elastic bending and frictional interactions between the shells. Increasing the friction coefficient μ thus naturally stiffens the system, as the Coulomb frictional threshold – proportional to μ – must be overcome to get tangential motion, which leads to an increase of compression forces, and therefore decreases the maximal indentation Δ_{\max} .

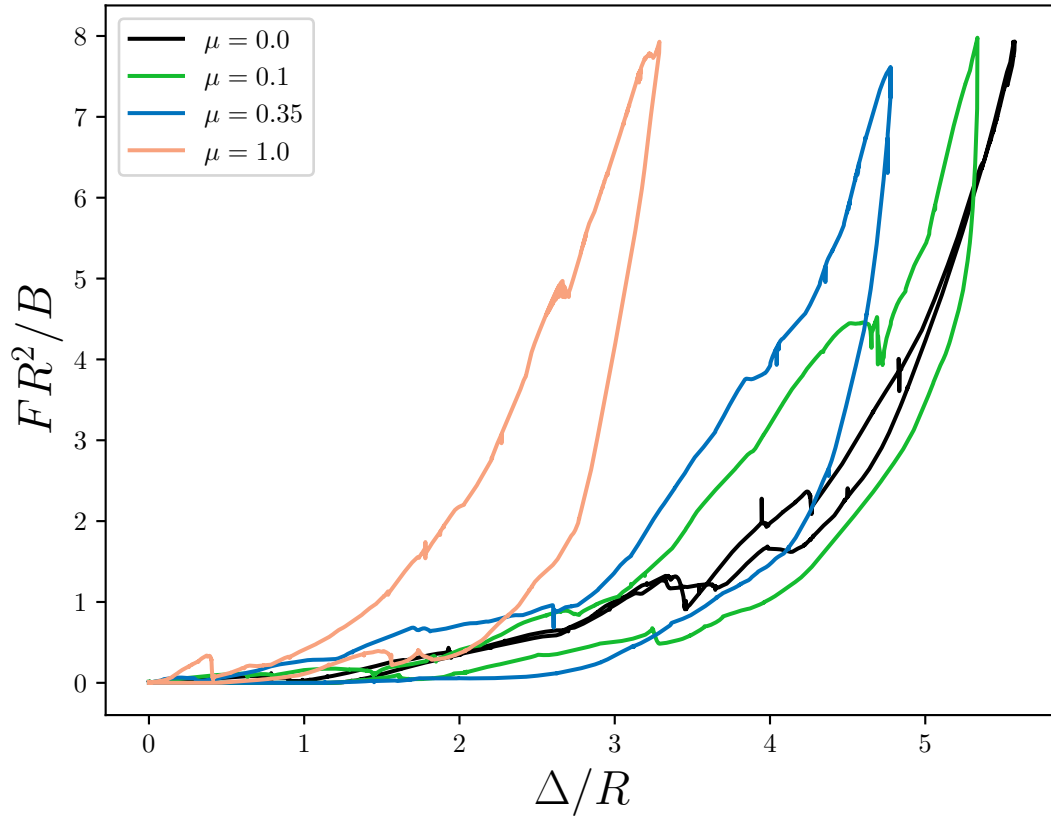
The increase of the compression forces with the friction coefficient is shown in fig. 3, where we have plotted the – simulated – force compression-decompression cycle for a system of 30 $\bar{\Phi} \approx 0.97$ shells with different friction coefficient (keeping the same initial configuration). Naturally, we also observe the decrease of Δ_{\max} as μ increases.

Supplementary Note 4. LIST OF EXPERIMENTAL AND SIMULATION MOVIES

- **Supplemental Movie 1:** The experimental movie and simulation results with $\mu = 0.35$ for $\bar{\Phi} = 0.75$ (Type I).
- **Supplemental Movie 2:** The experimental movie and simulation results with $\mu = 0.35$ for $\bar{\Phi} = 0.97$ (Type II).
- **Supplemental Movie 3:** The simulation movie with $\mu = 0.35$ for different $\bar{\Phi}$: ($\bar{\Phi} = 0.56, 0.75, 0.97$).
- **Supplemental Movie 4:** The simulation movie with $\bar{\Phi} = 0.86$ for different μ ($\mu = 0.0, 0.35, 1.0$).



Supplementary Figure 2. **Histogram of (a) Δ_{\max} and (b) η for 100 different initial conditions.** Solid lines are Gaussian distribution functions with the average (avg) and median absolute deviation (mad) calculated from the statistics of each $(\bar{\Phi}, \mu)$ simulation data.



Supplementary Figure 3. **Force-Displacement curves for a given initial condition and varying friction coefficient μ .** Black, green, blue, yellow curves are the force-displacement curves for $\mu = 0.0, 0.1, 0.35, 1.0$, respectively.

SUPPLEMENTARY REFERENCES

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