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**Three dimensional higher-order raypath separation in a  
shallow-water waveguide**

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1       **Abstract:** Separating raypaths in a multipath shallow-water environ-  
2       ment is a challenge problem due to the interferences between them and  
3       colored noise existing in ocean environment, especially for two raypaths  
4       arrive close to each other. Thus, in this paper, a three dimensional (3D)  
5       higher-order raypath separation in an array to array configuration is  
6       proposed. Performance tests using simulation data in a multipath en-  
7       vironment, real data obtained in an ultrasonic waveguide and ocean  
8       shallow-water data, respectively, illustrate that the proposed algorithm  
9       achieves a higher resolution and a stronger robustness comparing to the  
10      existing algorithms.

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## 11 **1. Introduction**

12 Acoustic rays are in multi-path propagation in a shallow-water waveguide due to the re-  
13 flection and (or) the refraction by the surface of the ocean and the continental shelf. The  
14 multiple raypaths cover more parts of the ocean and provide more information than a single  
15 emitted signal. However, they also produce interferences among them. Thus, separating  
16 the raypaths is necessary in many important applications, such as passive sonar, ocean  
17 acoustic tomography<sup>1</sup>, etc. As a classical separation algorithm, the Multiple Signal Clas-  
18 sification (MUSIC) algorithm<sup>2</sup> was proposed to obtain a high-resolution separation of the  
19 sources in a point to array configuration, which is composed of a point source and a verti-  
20 cal receiver array. The MUSIC algorithm improves separation resolution mainly exploiting  
21 the orthogonality between the signal subspace and the noise subspace. However, it fails  
22 when the signals are fully correlated or coherent. Jiang et al.<sup>3</sup> proposed a smoothing active  
23 wideband MUSIC (Smoothing-MUSICAL) algorithm, which is an extension of the MUSIC  
24 algorithm to the case of separating fully correlated or coherent wideband signals by us-  
25 ing the spatial-frequency smoothing. ~~In addition, comparing~~ to the MUSIC algorithm, the  
26 smoothing-MUSICAL algorithm improves the separation resolution through adding the ar-  
27 rival time of each raypath to the signal model as a new discrimination parameter. In addition  
28 to second order moment-based algorithm, applying higher-order statistics to DOA estimation  
29 has been focused on by the researchers and a group of algorithms have been developed<sup>4-10</sup>.  
30 These algorithms exhibit superior estimation performance due to the properties of aperture  
31 extension and noise suppression of higher-order cumulants. Recently, a wideband raypath

32 separation algorithm based on the fourth-order cumulant<sup>11</sup> is proposed in a point to array  
33 configuration in a shallow-water waveguide. Higher resolution and more robust separation  
34 is also obtained while at a cost of computation time. ~~Sequentially, Jiang et al. propose a~~  
35 fast algorithm using low-rank matrix approximation<sup>12</sup> to reduce the computation cost of the  
36 4-smoothing-MUSICAL algorithm. ~~On the other hand, Roux et al<sup>18</sup> develop an array to ar-~~  
37 ray configuration, which is composed of a source vertical array and a receiver vertical array.  
38 A double-beamforming algorithm is introduced ~~experimentally in the configuration and a~~  
39 ~~high-resolution tomography inversion<sup>13</sup> is finally obtained through using its separation re-~~  
40 ~~sults. Moreover, Touz e et al<sup>14</sup> present the double-capon algorithm and the double-MUSICAL~~  
41 algorithm under the assumption of white Gaussian noise. To further improve both the resolu-  
42 tion and the robustness to the colored noise, we present a 3D higher-order raypath separation  
43 algorithm in an array to array configuration ~~in this paper.~~ Its performance improvement is  
44 achieved due to inheriting the merits of using both the higher-order cumulants and the array  
45 to array configuration.

46 The rest of the paper is organized as follows. In Section 2, we give an elaborate  
47 description of the 3D higher-order raypath separation algorithm. In Section 3, we test the  
48 performance of the proposed algorithm using simulation data in a multi-path environment,  
49 real data obtained in an ultrasonic tank and ocean data, respectively. The paper is concluded  
50 in Section 4.

## 51 **2. The 3D higher-order raypath separation algorithm**

### 52 *2.1 Signal model*

53 Assume the raypaths propagate in a double vertical array configuration, which is composed  
 54 of an emission array ( $N$  sources) and a receiver array ( $M$  sensors). The  $P$  ( $P$  is assumed  
 55 known<sup>15,16</sup>) raypaths at the frequency  $\nu$  produced by the  $n^{th}$  source and received on the  $m^{th}$   
 56 receiver is noted as  $x_{m,n,\nu}$  and modeled as follows.

$$x_{m,n,\nu} = s_\nu \sum_{p=1}^P a_p e^{j\Phi_p} + b_{m,n,\nu} \quad (1)$$

57 where  $s_\nu$  is the source spectrum,  $\Phi_p = -j2\pi\nu(T_p + (m - m_0)\tau_p^e + (n - n_0)\tau_p^r)$ ,  $\tau_p^e = d\sin(\theta_p^e)/c$   
 58 and  $\tau_p^r = d\sin(\theta_p^r)/c$ .  $T_p$  notes the arrival time of the  $p^{th}$  raypath.  $m_0$  ( $n_0$ ) refers to the  
 59 reference source (receiver).  $\tau_p^e$  ( $\tau_p^r$ ) is the time delay for the  $p^{th}$  raypath propagates between  
 60 two adjacent sources (receivers).  $d$  is the interval between two adjacent sources (receivers).  
 61  $\theta_p^e$  ( $\theta_p^r$ ) is the direction of emission (reception) of the  $p^{th}$  raypath.  $a_p$  notes the amplitude of  
 62 the  $p^{th}$  raypath.  $b_{m,n,\nu}$  refers to the additive noise.

## 63 2.2 Data model

64 Based on the signal model built in the above section, each signal received in the double-array  
 65 configuration is a data cube for all  $N$  sources,  $M$  sensors and  $F$  frequencies of the wide-band  
 66 signals considered in the algorithm. The data model is built through concatenating the data  
 67 cube's elements into a long vector<sup>19</sup>. First, the elements corresponding to the source  $n^{th}$  at  
 68 the frequency  $\nu$  on all the  $M$  elements of the receiver array is concatenated into a vector as  
 69 follows (+ notes transpose.) :

$$\mathbf{x}_{n,\nu} = [x_{1,n,\nu} \dots x_{M,n,\nu}]^+ \quad (2)$$

70 The elements corresponding to both the  $N$  source and the  $M$  sensors are further  
 71 concatenated into the following vector.

$$\mathbf{x}_\nu = [\mathbf{x}_{1,\nu} \dots \mathbf{x}_{n,\nu}]^+ \quad (3)$$

72 Finally, considering all the  $F$  frequencies, the long vector  $\mathbf{X}$  is obtained as follows.

$$\mathbf{X} = [\mathbf{x}_{\nu_1} \dots \mathbf{x}_{\nu_F}]^+ \quad (4)$$

73 Similarly, the steering vector  $\mathbf{d}(\theta_p^e, \theta_p^r, \mathbf{T}_p)$  and the additive noise  $\mathbf{b}$  can be also concate-  
 74 nated into long vectors. Thus, the received signal in the frequency domain can be rewritten  
 75 in the following matrix form.

$$\mathbf{X} = \sum_{p=1}^P a_p \mathbf{d}(\theta_p^e, \theta_p^r, \mathbf{T}_p) + \mathbf{b} = \mathbf{D}(\theta_p^e, \theta_p^r, \mathbf{T}_p) + \mathbf{b} \quad (5)$$

76 where

$$\begin{aligned} 77 \quad \mathbf{X} &= [\mathbf{x}_{\nu_1}, \mathbf{x}_{\nu_2}, \dots, \mathbf{x}_{\nu_F}]^+, \mathbf{x}_\nu = [\mathbf{x}_{1,\nu}, \mathbf{x}_{2,\nu}, \dots, \mathbf{x}_{N,\nu}]^+ \quad (\nu = \nu_1, \nu_2, \dots, \nu_F), \text{ and } \mathbf{x}_{n,\nu} = \\ 78 \quad &[x_{1,n,\nu}, x_{2,n,\nu}, \dots, x_{M,n,\nu}]^+ \quad (n = 1, 2, \dots, N). \quad \theta_p^e = [\theta_1^e, \theta_2^e, \dots, \theta_P^e]^+, \theta_p^r = [\theta_1^r, \theta_2^r, \dots, \theta_P^r]^+, \\ 79 \quad \mathbf{T}_p &= [T_1, T_2, \dots, T_P]^+. \end{aligned}$$

$$80 \quad \mathbf{d}(\theta_p^e, \theta_p^r, \mathbf{T}_p) = [\mathbf{d}_{\nu_1}(\theta_p^e, \theta_p^r, \mathbf{T}_p), \mathbf{d}_{\nu_2}(\theta_p^e, \theta_p^r, \mathbf{T}_p), \dots, \mathbf{d}_{\nu_F}(\theta_p^e, \theta_p^r, \mathbf{T}_p)]^+, \text{ with } \mathbf{d}_{\nu_i}(\theta_p^e, \theta_p^r, \mathbf{T}_p) =$$

$$81 \quad [s_{\nu_i} e^{-j2\pi\nu_i(T_p + (1-n_0)\tau(\theta_p^e))} \mathbf{d}(\theta_p^r)_{\nu_i}, \dots, s_{\nu_i} e^{-j2\pi\nu_i(T_p + (N-n_0)\tau(\theta_p^e))} \mathbf{d}(\theta_p^r)_{\nu_i}]^+ \quad (i = 1, 2, \dots, F) \text{ and}$$

$$82 \quad \mathbf{d}(\theta_p^r)_{\nu_i} = [e^{-j2\pi\nu_i(1-m_0)\tau(\theta_p^r)}, \dots, e^{-j2\pi\nu_i(M-m_0)\tau(\theta_p^r)}]^+.$$

$$83 \quad \mathbf{b} = [\mathbf{b}_{\nu_1}, \mathbf{b}_{\nu_2}, \dots, \mathbf{b}_{\nu_F}]^+, \text{ with } \mathbf{b}_\nu = [\mathbf{b}_{1,\nu}, \mathbf{b}_{2,\nu}, \dots, \mathbf{b}_{N,\nu}]^+ \quad (\nu = \nu_1, \nu_2, \dots, \nu_F), \text{ and}$$

$$84 \quad \mathbf{b}_{n,\nu} = [b_{1,n,\nu}, b_{2,n,\nu}, \dots, b_{M,n,\nu}]^+ \quad (n = 1, 2, \dots, N)$$



85 *2.3 Principle of the Algorithm*

86 Using the data model built above, the trispectrum matrix of the received signal can be  
 87 theoretically computed according to the following equation.

$$\begin{aligned}
 \mathbf{C} &= E\{(\mathbf{X} \otimes \mathbf{X}^*)(\mathbf{X} \otimes \mathbf{X}^*)^H\} \\
 &- E\{(\mathbf{X} \otimes \mathbf{X}^*)\}E\{(\mathbf{X} \otimes \mathbf{X}^*)^H\} \\
 &- E\{(\mathbf{X}\mathbf{X}^H)\} \otimes E\{(\mathbf{X}\mathbf{X}^H)^*\},
 \end{aligned} \tag{6}$$

88 where  $*$  notes conjugate,  $H$  notes conjugate transpose and  $\otimes$  notes kronecker product,  
 89 respectively. However, in a real ocean experiment, it is costly to obtain several realizations  
 90 for computing the expectation of the stochastic process. Moreover, the raypaths are reflected  
 91 or refracted by the emitted signal thus they are fully correlated or coherent, which will lead  
 92 to a rank deficiency of the trispectrum matrix. To resolve the problems, we use a three  
 93 dimensional smoothing technique<sup>14</sup> to generate several realizations based on the received  
 94 signal. The spatial-frequency smoothing algorithm has two major advantages : (1) it is easy  
 95 to implement; (2) it first combines the spatial and frequency smoothing to provide more  
 96 useful information for estimation. To completely separate all the raypaths, it is necessary to  
 97 select the number of sub-antennas greater than the number of raypaths<sup>3</sup>. The smoothing is  
 98 performed in the dimensions of the emitted array, the receiver array and the frequency bins,  
 99 respectively. That is, the emitted (receiver) array is divided into  $K_e$  ( $K_r$ ) subarrays for a  
 100 length of  $N_e^s = N - K_e + 1$  ( $N_r^s = M - K_r + 1$ ). Similarly the  $K_f$  frequency subbands are  
 101 respectively composed of  $N_f^s = F - K_f + 1$  frequency bins. Through these operation, we

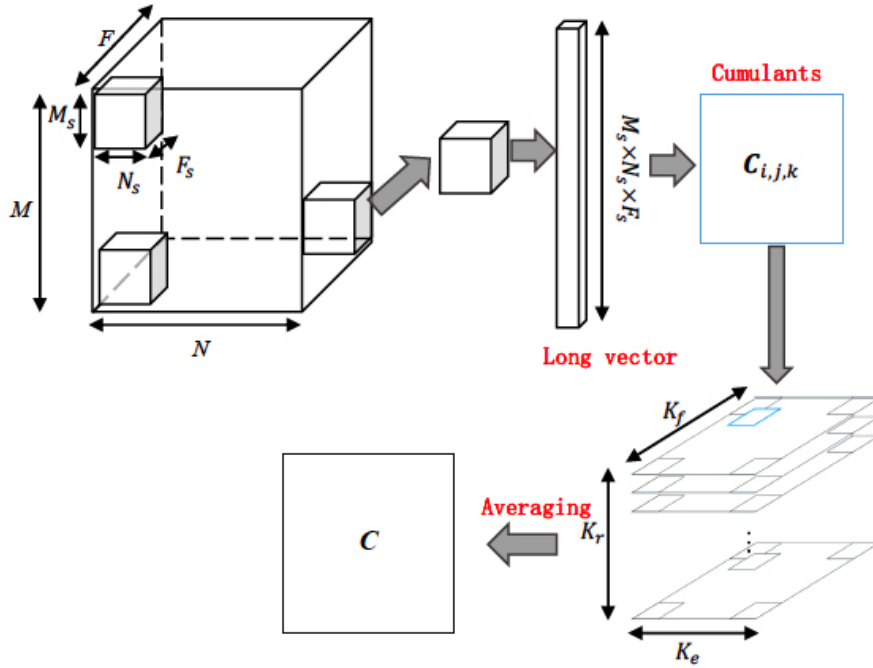


Fig. 1. Schematic diagram of the smoothing technique for the cumulants. The  $C$  and  $C_{i,j,k}$  are computed based on the long vector through the Eq. (7). The size of the matrix  $C$  is equal to  $(N_e^s \times N_r^s \times N_f^s)^2$ .

102 generate  $K_e \times K_r \times K_f$  small data cubes, whose dimensions are  $N_e^s \times N_r^s \times N_f^s$ . The  $X_{i,j,k}$  is  
 103 obtained through connecting the elements of all the three dimensions into a long vector. To  
 104 be easily understood, the schematic diagram of the smoothing technique for the cumulants  
 105 is provided in Fig. 1.

106 The expectation of the trispectrum matrix of the received signal is finally estimated  
 107 through an average of these smaller trispectrum matrices computed through the small data  
 108 cubes, which is specifically described in Eq. 7.

$$\begin{aligned} \widehat{\mathbf{C}} &= \frac{1}{K_e K_r K_f} \sum_{k=1}^{K_f} \sum_{j=1}^{K_e} \sum_{i=1}^{K_r} \mathbf{C}_{i,j,k} = \frac{1}{K_e K_r K_f} \sum_{k=1}^{K_f} \sum_{j=1}^{K_e} \sum_{i=1}^{K_r} E\{(\mathbf{X}_{i,j,k} \otimes \mathbf{X}_{i,j,k}^*)(\mathbf{X}_{i,j,k} \otimes \mathbf{X}_{i,j,k}^*)^H\} \\ &\quad - E\{(\mathbf{X}_{i,j,k} \otimes \mathbf{X}_{i,j,k}^*)\} E\{(\mathbf{X}_{i,j,k} \otimes \mathbf{X}_{i,j,k}^*)^H\} - E\{(\mathbf{X}_{i,j,k} \mathbf{X}_{i,j,k}^H)\} \otimes E\{(\mathbf{X}_{i,j,k} \mathbf{X}_{i,j,k}^H)^*\}, \end{aligned} \quad (7)$$

109 Then, we apply the eigenvalue decomposition to  $\widehat{\mathbf{C}}$  as follows.

$$\widehat{\mathbf{C}} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H = \sum_{i=1}^{(N_e^s \times N_r^s \times N_f^s)^2} \lambda_i \mathbf{u}_i \mathbf{u}_i^H = \sum_{i=1}^{P^2} \lambda_i \mathbf{u}_i \mathbf{u}_i^H + \sum_{i=P^2+1}^{(N_e^s \times N_r^s \times N_f^s)^2} \lambda_i \mathbf{u}_i \mathbf{u}_i^H \quad (8)$$

110 where  $\mathbf{U}$  is composed of  $(N_e^s \times N_r^s \times N_f^s)^2$  eigenvectors,  $\mathbf{\Lambda}$  is a diagonal matrix containing the  
 111  $(N_e^s \times N_r^s \times N_f^s)^2$  eigenvalues and  $\mathbf{u}_i$  denotes the  $i^{th}$  eigenvector. Arranging all the eigenvalues  
 112 from largest to smallest  $\lambda_1 > \lambda_2 > \dots > \lambda_{(N_e^s \times N_r^s \times N_f^s)^2}$ , the  $(N_e^s \times N_r^s \times N_f^s)^2 - P^2$  eigenvectors  
 113 corresponding to the smallest eigenvalues project the noise subspace  $\mathbf{U}_n \mathbf{U}_n^H$  where

$$\mathbf{U}_n = \left[ \mathbf{u}_{P^2+1}, \mathbf{u}_{P^2+2}, \dots, \mathbf{u}_{(N_e^s \times N_r^s \times N_f^s)^2} \right] \quad (9)$$

114 Finally, the estimator  $P_{Double\ 4-s-MUSICAL}$  in the double-array configuration can be  
 115 constructed as follows.

$$P_{Double\ 4-s-MUSICAL} = \frac{1}{\mathbf{d}_4^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{d}_4} \quad (10)$$

116 with the steering vector  $\mathbf{d}_4 = \mathbf{d}(\theta_p^r, \theta_p^e, T_p) \otimes (\mathbf{d}(\theta_p^r, \theta_p^e, T_p))^*$ .

### 117 3. Performance test

118 In this section, the performance of the proposed algorithm is tested using simulation data in  
 119 a multi-path environment, small-scale data obtained in an ultrasonic tank and ocean data<sup>18</sup>,  
 120 respectively. The test parameters used in these experiments are shown in Table 1 and Table  
 121 2.  $z_{s_1} \sim z_{s_M}$  notes the depths in which the first source to the  $M^{th}$  source is fixed under the  
 122 water while  $z_{r_1} \sim z_{r_N}$  refers to the ranges which the first receiver to the  $N^{th}$  receiver cover.  
 123  $D$  is the distance between the reference source and the reference receiver.  $c$  indicates the  
 124 acoustic velocity in which the raypaths propagate under the water. The central frequency of  
 125 the emitted signal is  $\nu_c$  and its bandwidth is noted as  $\nu_w$ .  $\nu_n$  notes the number of frequencies  
 126 used in each test choosing the beginning frequency  $\nu_b$  to the last frequency  $\nu_e$ .  $N_s$  refers to  
 127 the number of samples used in each test in the time domain.

128 Figures 2 and 3 show the separation results of the proposed algorithm while the con-  
 129 trast ones of the smoothing-MUSICAL algorithm and the 4-smoothing-MUSICAL algorithm  
 130 are also provided. Specifically, figure 2 (a), (c) and (e) illustrate the separation results of  
 131 a set of simulation data under the assumption that the waveguide provides nearly perfect  
 132 boundary conditions at the water-bottom interface. In the case, five raypaths propagate  
 133 between the double array configuration composed of four emitters and four receivers. The  
 134 signal to noise ratio is equal to 2dB. The SNR is defined as the power ratio of signal to  
 135 noise in the signal band. These powers are estimated in the time domain by the sum of  
 136 squares of samples (signal or noise). The reference source (receiver) is located at 50m under  
 137 the water. The smoothing-MUSICAL algorithm provides a two-dimensional (2D) separation

138 in a plan of the arrival time and the direction of arrival (DOA) in figure 2 (a) while the  
 139 other two algorithms give a 3D separation with the additional parameter: the direction of  
 140 emission (DOE) in figure 2 (c) and figure 2 (e), respectively. In figure 2 (a), (c) and (e),  
 141 each spot corresponds to a raypath and the black crosses mark the theoretical positions. It  
 142 is obviously that the smoothing-MUSICAL algorithm fails in finding the raypath with the  
 143 DOA around at  $-5^\circ$  in figure 2 (a) and the raypath with the DOA around at  $0^\circ$  is deficient  
 144 in the results of the double-MUSICAL algorithm in figure 2 (c). In contrast, the proposed  
 145 algorithm successfully separates all the five raypaths without any artifact in figure 2 (e),  
 146 where the deficient raypaths in figure 2 (a) and figure 2 (c) are highlighted by the red circles.

147 Figure 2(b), (d) and (f) illustrate the performance contrast of the three algorithms  
 148 using a set of real data obtained at a small-scale ultrasonic tank. The small-scale experiment  
 149 reproduces the actual physical phenomena occurring in the real ocean environment in a  
 150 smaller scale inside the laboratory based on the operation multiplying the frequency of the  
 151 signals by a factor and dividing the spatial distances by the same factor. Thus, the small-scale  
 152 experiment is a reduced cost and a totally controlled experiment. Specifically, in this tank,  
 153 a steel bar acts as the bottom, for which the boundary conditions are nearly perfect at the  
 154 water-bottom interface, and a 1.10m-long, 5.4-cm-deep acoustic waveguide is constructed<sup>17</sup>.  
 155 Two coplanar 64-element vertical line arrays (VLA) are placed and the central frequency of  
 156 the transducer is 1 or 3 MHz with a 50% frequency bandwidth. The transducer dimensions  
 157  $0.75mm \times 12mm$  are used to make the linear arrays omni-directional in the plane defined  
 158 by the source-receiver arrays and the beams collimated in the plane perpendicular to the

159 waveguide axis. Figure 2(b), (d) and (f) show a group of separation results with a set of real  
160 data obtained in the small-scale experiment. There are seven predictable raypaths in the  
161 example, whose theoretical positions are indicated by the black crosses. Figures 2(b) and  
162 2(d) show the separation results using the smoothing-MUSICAL algorithm and the double-  
163 MUSICAL algorithm, respectively. It can be clearly seen that both of the two algorithms  
164 are deficient in detecting one raypath with the DOA around at  $5^\circ$ . Comparing to them, the  
165 proposed 3D higher order algorithm correctly find all the seven raypaths in figure 2(f), where  
166 the red circle highlights the deficient raypath in figures 2(b) and 2(d).

167 Finally, the performance of the double-4-smoothing algorithm is tested using ocean  
168 data. The experiment has been performed in July 2005 north of Elba Island, Italy<sup>18</sup>. It uses  
169 a similar experimental setup to the small-scale ultrasonic experiment, although at a much  
170 larger scale. Two equally spaced vertical linear arrays are implemented in 120 m water and  
171 the distance between them is 4.071 km. The source array (SA) is composed of 29 transducers  
172 covering 78 m and the receiver array (RA) has 32 hydrophones spanning 62m. The central  
173 frequency of the transducers is 3.2 kHz with 1 kHz bandwidth. Figure 3 shows the contrast  
174 of separation results using a set of ocean data, which has been acquired in a down-refraction  
175 profile. There are three expected rays, a surface-reflected ray and two refracted rays. Each  
176 refracted ray has a turning point near the depth of maximum sound-speed variability. Due  
177 to the sound speed variation described in the Ref.<sup>18</sup>, the three raypaths arrived at close  
178 time. Figure 3(a) shows that the separation result of the smoothing-MUSICAL algorithm  
179 in the plan of the arrival time and the DOA. It detects just a mixed spot for two of the

180 three raypaths due to their close arrival times and DOAs. Figure 3(b) shows that the  
 181 separation result of the double-MUSICAL algorithm in the plan of the emitted angle and  
 182 the DOA, which has a raypath deficiency with the DOA around at  $10^\circ$ . However, the  
 183 proposed algorithm gives an accurate separation for the three raypaths shown in the plan of  
 184 the emitted angle and the DOA in Figure 3 (c) because of the different emitted angles and  
 185 a strong robustness of the proposed algorithm to the fluctuant ocean. Figure 3(d) displays  
 186 the three raypaths in Figure 3(a)-(c), which propagate between the centers of source and  
 187 receive arrays.

188 Moreover, we test the performance of the double-4-smoothing-MUSICAL algorithm  
 189 using the simulation data at different SNRs in a red noise inference environment and compare  
 190 its separation performance in terms of root-mean-square errors (RMSE) with that of the  
 191 double-MUSICAL algorithm. In these Monte Carlo experiments, we define the average  
 192 RMSE of the 3D separation result  $\hat{R}_p$  for the  $p^{th}$  raypath as follows.

$$RMSE_{R_p} = \sqrt{\frac{1}{K_p} \sum_{k=1}^{K_p} \|\hat{R}_p - R_p\|^2} \quad (11)$$

193 where  $K_p$  is the number of the trials, which is equal to 20 for these Monte Carlo experiments.  
 194 That is, the simulations are performed 20 times at each different SNR. 400 simulations in  
 195 total are performed in red noise environments with their SNRs ranging from -20 dB to 20  
 196 dB. The configuration parameters used in the simulations are consisted with those shown in  
 197 Table I and Table II. For those raypaths which can be separated by both the two algorithms,  
 198 the RMSEs are computed at each dimension of the 3D separation result. Specifically, figures

	$M$	$N$	$z_{s_1} \sim z_{s_M}(m)$	$z_{r_1} \sim z_{r_N}(m)$	$d$ (m)
Simulation	4	4	47.5 ~ 52.5	47.5 ~ 52.5	2.5
Small-scale experiment	4	5	$26.375 \times 10^{-3} \sim 28.625 \times 10^{-3}$	$25.625 \times 10^{-3} \sim 28.625 \times 10^{-3}$	$0.75 \times 10^{-3}$
Ocean data	4	4	93.098 ~ 101.456	94 ~ 100	2

Table 1. The configuration parameters of simulation, small-scale experiment and the at-sea experiment.

199 4 (a), (b) and (c) show the curves of the average RMSEs for the DOE, the DOA and the time  
 200 of arrival (TOA) produced by the two algorithms, respectively. In figure 4 (a), the double-4-  
 201 MUSICAL algorithm generally produces a smaller bias in the low SNR range (from -20dB to  
 202 0dB) and a relatively stable performance in moderate-to-high SNR. For the average RMSEs  
 203 shown in the Figures 4(b) and 4(c), the double-4-MUSICAL algorithm generally shows a  
 204 better and stable performance. Based on the performance contrasts with the simulation  
 205 data, small-scale data and ocean data, a conclusion is drawn that the double-4-smoothing-  
 206 MUSICAL algorithm enables to achieve an aperture extension, a stronger robustness to the  
 207 fluctuant environment and a resolution improvement for the extension of cumulants-based  
 208 algorithm to the double-array configuration.

#### 209 4. Conclusion

210 In this paper, we extended a higher-order algorithm to a double-array configuration. Higher-  
 211 resolution and more robust separation for close arrivals even in a fluctuant ocean environment



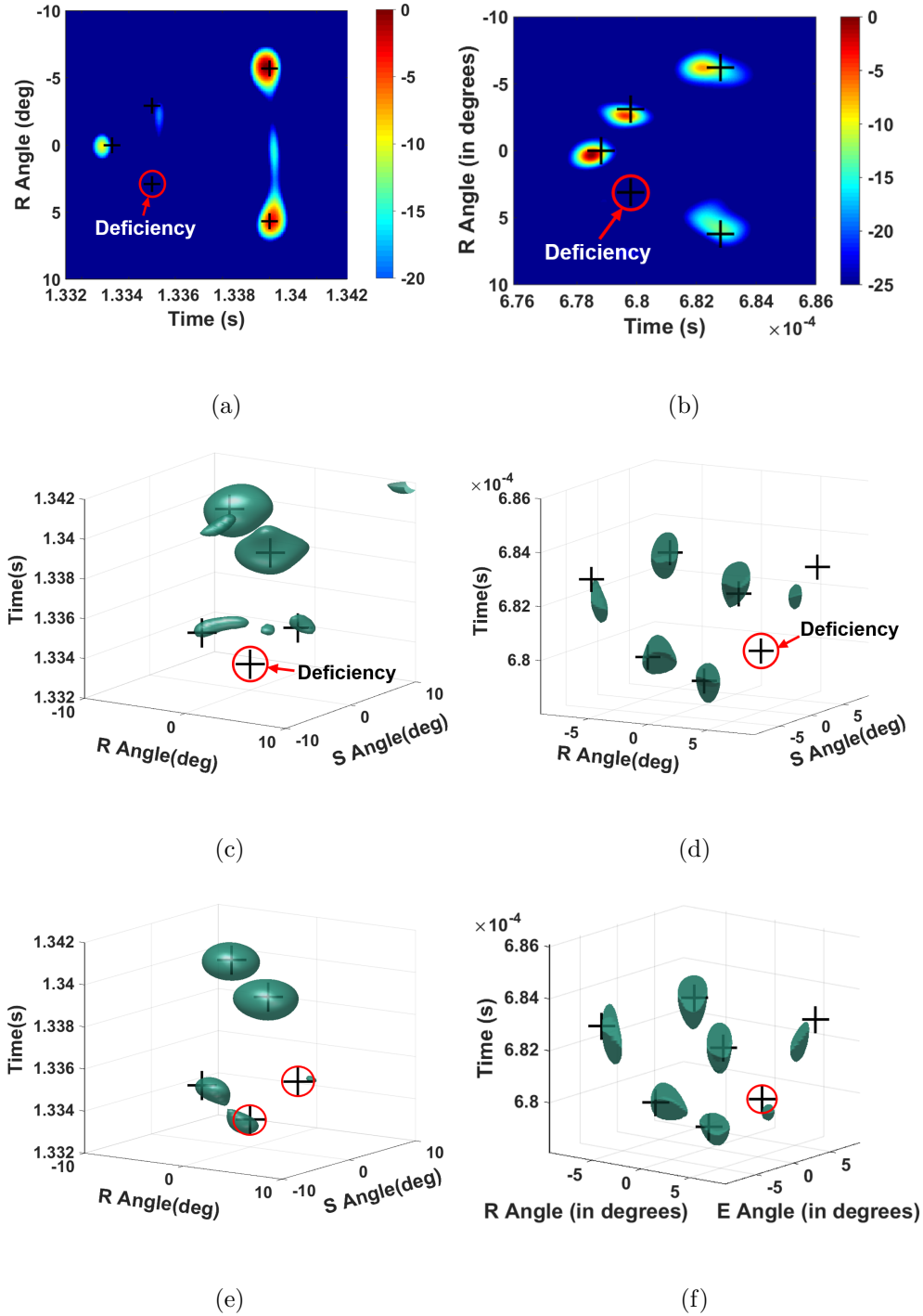


Fig. 2. Separation results comparison of different algorithms using simulation datas ((a), (c) and (e)) or using real data obtained in an ultrasonic tank ((b), (d) and (f)). The black crosses denote the theoretical values. The red circles highlight the raypaths used to differentiate the separation ability of the algorithms. (a) and (b) show the results of the smoothing-MUSICAL algorithm; (c) and (d) show the results of the double-MUSICAL algorithm; (e) and (f) show the results of the double-4-MUSICAL algorithm (The iso-surfaces are located at max-value/3.)

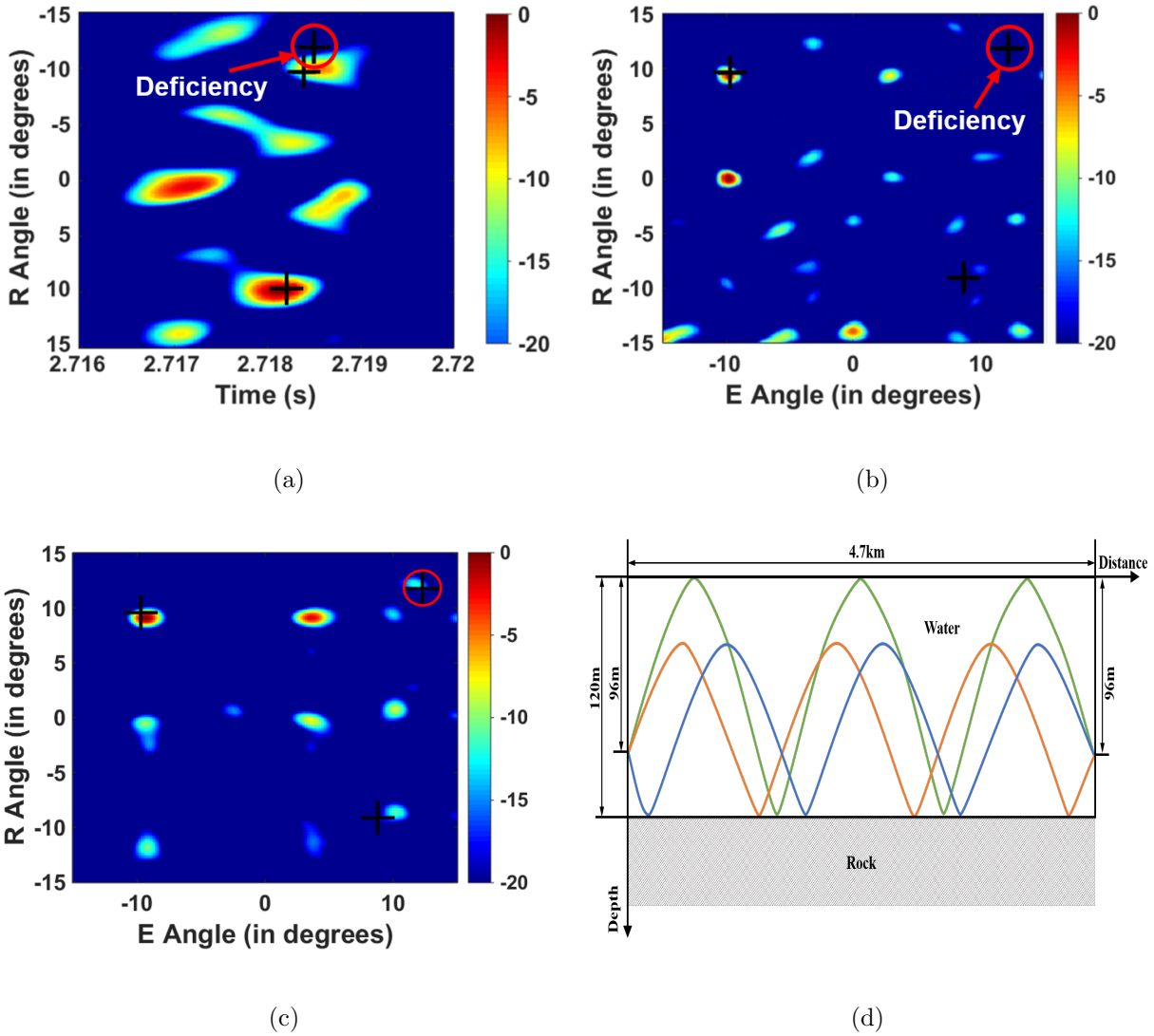
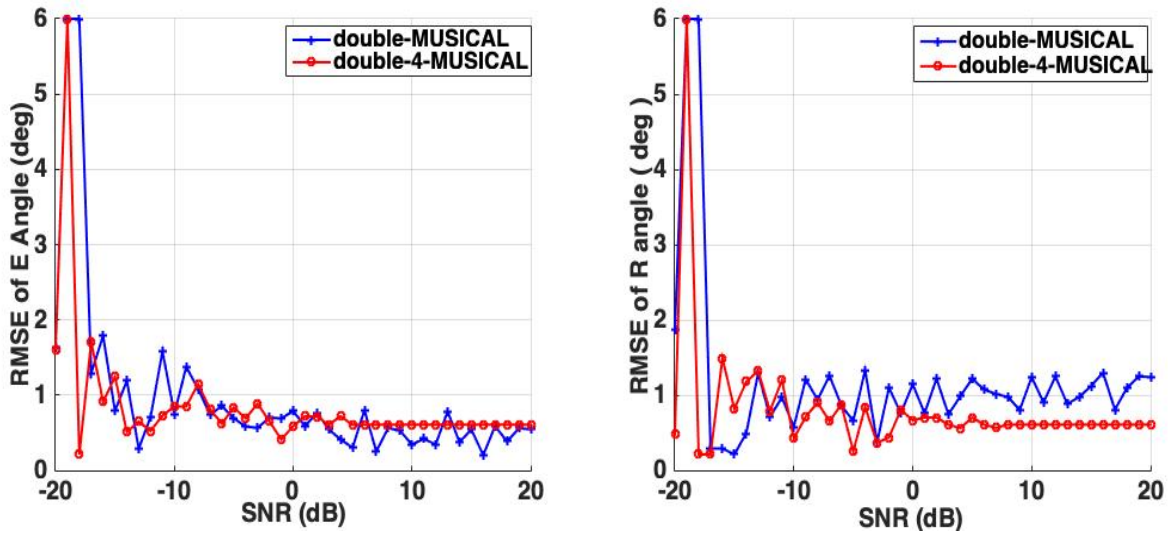
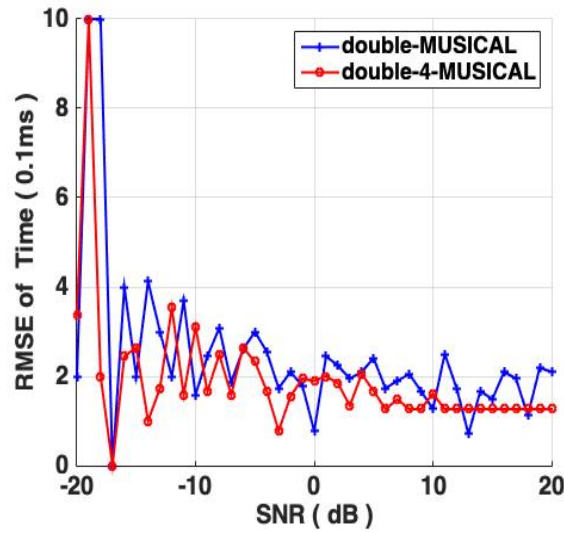


Fig. 3. Separation results comparison of different algorithms using ocean data. The black crosses denote the theoretical values. The red circles highlight the raypaths used to differentiate the separation ability of the algorithms (a) the smoothing-MUSICAL algorithm; (b) the double-MUSICAL algorithm; (c) the double-4-MUSICAL algorithm; (d) Raypaths between the source and the reference sensor for the three rays.



(a)

(b)



(c)

Fig. 4. The RMSE comparison of different algorithms using simulation data interfered by red noise.

(a) the RMSE ( in degree) of the direction of emission (DOE) vs SNR variation ; (b) the RMSE (in degree) of the direction of arrival (DOA) vs SNR variation; (c) the RMSE (in 0.1ms) of the time of arrival (TOA) vs SNR variation.

	$\nu_c$ (Hz)	$\nu_w$ (Hz)	$\nu_n(\nu_b \sim \nu_e)$ (Hz)	$N_s$	$D(m)$	$c$ (m/s)
Simulation	$1.5 \times 10^3$	$5 \times 10^3$	75 ( $0 \sim 5 \times 10^3$ )	135	2000	1500
Small-scale experiment	$1.2 \times 10^6$	$5 \times 10^6$	150( $0 \sim 5 \times 10^6$ )	132	1	1473
Ocean data	$3.2 \times 10^3$	$6 \times 10^3$	50 ( $0 \sim 6 \times 10^3$ )	60	$4.701 \times 10^3$	1509

Table 2. The test parameters used in the simulations, the small-scale experiment and the at-sea experiment.

is achieved by the higher-order 3D algorithm. In future work, we will study the inversion for ocean acoustic tomography based on the separation results.

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224 **References and links**

225 <sup>1</sup>W.H. Munk, and P. F. Worcester. "Ocean acoustic tomography", *Oceanography*, vol.1, no.  
226 1, pp. 8-10, 1988.

227 <sup>2</sup>R. Schmidt, " Multiple emitter location and signal parameter estimation," *IEEE Transac-*  
228 *tions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, 1986.

229 <sup>3</sup>L. Jiang, P. Roux, and J. I. Mars, " Raypath separation with a high-resolution algorithm  
230 in a shallow-water waveguide," *IEEE Journal of Oceanic Engineering*, vol. 43, no. 1, pp.  
231 119-130, 2018.

232 <sup>4</sup>Y. H. Chen and Y. S. Lin, "Doa estimation by fourth-order cumulants in unknown noise en-  
233 vironments," *IEEE International Conference on Acoustics, Speech, and Signal Processing*,  
234 vol. 4, pp. 296-299, 1993.

235 <sup>5</sup>M. C. Dogan and J. M. Mendel, "Applications of cumulants to array processing. I. Aperture  
236 extension and array calibration," *IEEE Trans. Signal Processing*, vol. 43, no. 5, pp.1200-  
237 1216,1995.

238 <sup>6</sup>M. C. Dougan and J. M. Mendel, "Applications of cumulants to array processing. II. Non-  
239 Gaussian noise suppression," *IEEE Trans. Signal Processing*, vol. 43, no. 7, pp.1663-1676  
240 ,1995.

241 <sup>7</sup>E. Gonen, J. M. Mendel, and M. C. Dogan, "Applications of cumulants to array processing.  
242 iv. direction finding in coherent signals case," *IEEE Trans. Signal Processing*, vol. 45, no.  
243 9, pp. 2265-2276,1997.

- 244 <sup>8</sup>P. Chevalier, A. Ferreol, and L. Albera, "High-resolution direction finding from higher  
245 order statistics:The 2q-music algorithm," *IEEE Trans. Signal Processing*, vol. 54, no. 8,  
246 pp. 2986-2997, 2006.
- 247 <sup>9</sup>G. Birot, L. Albera, and P. Chevalier, "Sequential high-resolution direction finding from  
248 higher order statistics," *IEEE Trans. Signal Processing* vol. 58, no. 8, pp. 4144-4155, 2010.
- 249 <sup>10</sup>P. Pal and P. Vaidyanathan, "Multiple level nested array: An efficient geometry for 2qth  
250 order cumulant based array processing," *IEEE Trans. Signal Processing*, vol. 60, no. 3, pp.  
251 1253-1269, 2012.
- 252 <sup>11</sup>L. Jiang, Y. Hong, P. Roux, J. Wu, and H. Shu, "Active wideband higher-order raypath  
253 separation in multiple environment," *The Journal of the Acoustical Society of America*,  
254 vol. 141, no. 1, pp. EL38-EL44, 2017.
- 255 <sup>12</sup>L. Jiang, W. Song, Z. Zhang, C. Yang, S. Wang, and P. Roux, "Fast raypath separation  
256 based on low-rank matrix approximation in a shallow-water waveguide," *The Journal of*  
257 *the Acoustical Society of America*, vol. 143, no. 1, pp. EL271-EL278, 2017.
- 258 <sup>13</sup>I. Iturbe, P. Roux, B. Nicolas, and J. Mars, "Ocean acoustic tomography using a double-  
259 beamforming algorithm" *IEEE Journal of Oceanic Engineering*, vol. 123, no. 5, pp. 3912-  
260 EL278, 2008.
- 261 <sup>14</sup>G. Touzé, B. Nicolas, J. Mars, P. Roux, B. Oudompheng "Double-Capon and double-  
262 MUSICAL for arrival separation and observable estimation in an acoustic waveguide,"  
263 *Eurasip Journal on Advances in Signal Processing*, vol. 143, no. 1, pp. EL271-EL278, 2017.

264 <sup>15</sup>L. Jiang and J. I. Mars, "Automatic detection of the number of raypaths in a shallow-water  
265 waveguide," *IEEE Journal of Oceanic Engineering*, vol. 39 (4): 713-723, 2014.

266 <sup>16</sup>M. Wax and T. Kailath, Detection of signals by information theoretic criteria, *IEEE Trans.*  
267 *Acoust. Speech Signal Process.*, vol. 33, no. 2, pp. 387-392, 1985.

268 <sup>17</sup>Roux P and Nicolas B, "Inverting for a deterministic surface gravity wave using the  
269 sensitivity-kernel approach", *Journal of the Acoustic Society of America*, vol.135, no. 4,  
270 pp. 1789-1799, 2014.

271 <sup>18</sup>Roux P., Cornuelle B. D., Kuperman W. A, WS Hodgkiss, "The structure of raylike arrivals  
272 in a shallow-water waveguide", *The Journal of the Acoustical Society of America*, vol.124,  
273 no. 6, pp. 3430-3439, 2008.

274 <sup>19</sup>C. Paulus, P. Gounon, and J. I. Mars, "Wideband spectral matrix filtering for multicom-  
275 ponent sensors array," *Signal Process.*, vol. 85, no. 9, pp. 1723-1743, 2005.