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A mathematical collection found in the “House of the *āšipus*”. The art of metrology in Achaemenid Uruk

Christine Proust^{1 2}

Abstract – Most of the mathematical texts from Late Babylonian Uruk were found in the “House of the *āšipus*”, altogether with other scholarly texts, including astronomical, astrological, medical, lexical texts, rituals, prayers and others. This chapter provides a close examination of the content of a set of mathematical tablets discovered in a room of the Achaemenid level of this house. These tablets deal mainly with diverse methods to evaluate surfaces. One of the more striking aspects of these methods is the way in which they confront ancient and new metrological systems. Indeed, the metrological systems for lengths, surface and capacity differ in many respects in Old-Babylonian and Late-Babylonian sources. Mathematical texts from the Achaemenid period echo a cross-fertilization between various metrological cultures, the ones resulting from a long transmission of mathematical knowledge from generations to generations during several centuries, the others developed in Mesopotamia in late periods for administrative or juridical purposes. This contribution analyses how different mathematical and metrological cultures were combined by scholars linked to the milieu of the *āšipus* of Achaemenid Uruk. The conclusion discusses the reasons of the interest shown by these scholars for the art of evaluating surfaces.

The known Late Babylonian mathematical corpus represents more than one hundred tablets, which come mainly from Babylon and Uruk, as well as, to a lesser extent, Kiš, Sippar and Nippur. These tablets contain a variety of texts, including metrological and numerical tables, coefficient lists, and lists of problems with detailed procedures.³ The Urukean part of this corpus represents about a quarter of it, and was found at two sites in the city: the “House of the *āšipus*” and at or near the *Bīt Rēš* temple. All of the mathematical documents dated to the Achaemenid period and whose exact location of discovery can be identified were found in the “House of the *āšipus*”.

After a broad description of the mathematical texts found in the Achaemenid levels of the “House of the *āšipus*”, this chapter examines in detail a group of five mathematical tablets which were stored together in the same room of the house. It will be shown how this group sheds some light on the place of mathematics in Urukean scholarship. Why were the *āšipus*, kind of exorcists belonging to the high clergy of Uruk, interested in mathematics, and more specifically, in mathematical methods developed in this collection of five tablets, which deal essentially with the problem of the evaluation of surfaces? Why do the mathematical texts of

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³ This corpus was recently extended with the discovery and publication of several Late Babylonian metrological tables from Babylon by Steele (2015).

the “House of the *āšīpus*” use a metrology inherited from a far past, despite the fact that this metrology was obsolete at the time? Why is so much effort deployed to reconcile metrologies which were produced in completely different contexts for different purposes? To address these questions, the multiple links which connect these five texts are analysed. These connections concern the individuals involved in the production of the tablets and quoted in the colophons (see Sect. 1), the metrologies used, and the mathematical methods employed. The analysis of the content of the texts shows how metrological tables (examined in Sect. 2 and 3) were used for solving problems (see Sect. 4). This analysis draws a picture of the network of textual links between the five texts. On this basis, the chapter concludes with an attempt to infer how and why scholars belonging to the milieu of *āšīpus* of Uruk were engaged in mathematics.

1 Sources

1.1 Mathematical tablets found in levels IV and III of the “House of the *āšīpus*”

Eleven mathematical tablets were found in the “House of the *āšīpus*”.⁴ Identifying the exact composition of the Achaemenid mathematical corpus is made difficult by the perturbation of the stratigraphic levels, especially level III. Moreover, some tablets escaped the control of excavators. Fortunately, the date of the tablets found in disturbed archaeological contexts or which were illegally excavated can sometimes be restored according to the content of a colophon.⁵ Tables 1 and 2 offer an attempt to list the mathematical texts which can be either firmly or possibly dated to the Achaemenid period.⁶ I numbered 1 to 5 the texts discussed in this chapter (first column of Table 1).

According to Friberg, who relies on a personal communication with von Weiher, three of the four mathematical tablets (texts 1, 3 and 4, see Table 1) were stored in the same jar.⁷ Another tablet (text 5) seems to have found its way until the Iraqi Museum without being registered by the archaeologists.⁸ However, as shown by Friberg, the content of text 5 presents strong similarities with text 4, and the colophon indicates clearly that the tablet belongs to the same collection as the other mathematical documents found in room 4. It is interesting to note that

⁴ Clancier (2009: 404) mentions also that eleven mathematical tablets were found in this house. However, on the one hand, W22656/2 (SpTU 4, 178) is not mathematical (H. Hunger, personal communication April 2017, indicates that this text contains “some kind of recipe, maybe magical.”). On the other hand, W 22260a (SpTU 1, 101), which was identified as a table of fractions (Rechentabelle (u. a. Namen von Bruchzahlen angehend)) in UVB 26-27: 97, is considered here as mathematical. Moreover, the attributions of tablets to the scholarly archives of the ‘House of the *āšīpus*’ are not exactly the same here and in Clancier 2009. Clancier supposes that W 23021 and W 22715/2 (SpTU 4, 176 and 177) are to be attributed to the scholarly archives of the family Šangî-Ninurta, but I am not sure of this attribution because the archaeological context is unclear (filling of level IV and level III), and these tablets do not have a colophon. Some of the mathematical texts were published along with the other tablets found in the ‘House of the *āšīpus*’ in the SpTU volumes (SpTU 1 = Hunger 1976; SpTU 4 = von Weiher 1993; SpTU 5 = von Weiher 1998; see for more details the introduction to this volume), and complementary editions focused on mathematical texts were published by Jöran Friberg, Hermann Hunger and Farouk Al-Rawi (BagM 21 = Friberg, Hunger, and Al-Rawi 1990; BagM 28 = Friberg 1997 – see list of abbreviations).

⁵ This was the methodology developed by Clancier (2009) who restored the lost information by studying the colophons.

⁶ This list is based on the catalogues compiled by Hunger and von Weiher (UVB 26-27: 79-86 and UVB 29-30: 96-111), updated by Clancier (2009: 386-400) and Robson (2008a: 231-236).

⁷ “The latter tablet [W 23 291] was found in a clay jar together with two other mathematically important Late-Babylonian clay tablets, the large metrological table text W 23 273, and the large many-place table of reciprocals W 23 283.” (Friberg 1997: 252). “Three of them (W 23273, W 23283, and W 23291) were even found together, stored with other tablets in a clay jar (von Weiher, *personal communication*)” (Friberg 1999: 139).

⁸ According to Friberg (1997: 252), this tablet was “confiscated”.

the two fragments W 23283 and W 22905, which belong to the same tablet (text 3, see Table 1), were found in different places, room 4 for the former, and a filling rubble of level III or IV for the latter. As we see, the four tablets found in room 4 (texts 1–4, see Table 1) represent a coherent collection according to archaeological criterion, and text 5 is to be adjoined to this core corpus due to its strong affinities with text 4. Friberg underlined the importance, for the ancient actors themselves, of the “mathematical treasure-trove” of texts 1, 3 and 4.⁹ Along the same line, Eleanor Robson considered that texts 1–5 “comprise a coherent corpus”, because the colophon of the tablets clearly connect them the ones with the others (Robson 2008b: 230). I come back on this important aspect in Sect. 1.2.

Text	Excavation number	Publication number	Tablet type ¹⁰	Locus ¹¹	Campaign	Content
1	W 23273	SpTU 4, 172	M(5,4)	Room 4, jar	30 th	Metrological tables for lengths and surfaces, list of gods, shadow scheme
2	W 23281	SpTU 4, 173	M(2,2)	Room 4	30 th	Metrological equivalencies for measurements of length, surface, time, distance; reciprocal table for large numbers
3	W 23283 + W 22905	SpTU 4, 174	M(2,2)	Room 4, jar + filling rubble of level III or IV	29 th + 30 th	Reciprocal table for large numbers with first digit 1, 2 and 3
4	W 23291	SpTU 4, 175	M(2,2)	Room 4, jar	30 th	22 problems on surfaces
5	W 23291x (Museum no IM 75985)	BagM 21	M(2,2)	“confiscated”	Illegal excavation?	23 problems on surfaces


Table 1: the mathematical tablets dated to the Achaemenid period (square Ue XVIII 1, level IV, room 4)

Some other mathematical tablets were found in a less clear archaeological context. Two were found in the filling rubble of level IV, and four others probably in level III where material from different periods were mixed due to later perturbations (see Table 2). The archaeological information is insufficient to determine if these tablets date to the Achaemenid or the Hellenistic phase of the life of the “House of the *ašīpus*”. Unfortunately, none of the tablets listed in Table 2 exhibits a colophon: the three numerical tablets (W 22715-2, W 23021, W 23016) seem to be school exercises because of their shape (lentils and type S) and bear no colophon; the other tablets are fragments and the end of the text they contain is lost.¹²

⁹ “The fact that three so interesting specimens [W 23273, W 23283, W 23291 = texts 1, 3, 4 in Table 1] were found together, a veritable mathematical treasure-trove, suggests that they were considered to be important already in antiquity” (Friberg 1997: 252).

¹⁰ According to conventions used in Assyriology, a tablet of type M(*m,n*) is a multi-column tablet with *m* columns on the obverse and *n* columns on the reverse; a tablet of type S is a single column tablet.

¹¹ The data on the *locus* in Tables 1 and 2 come from the catalogue compiled by Hunger in UVB 26–27 and by von Weiher in UVB 29–30. These data or some of them are also provided in Robson (2008a: 337–338), Clancier (2009: part I), and Friberg, Hunger, and Al-Rawi (1990).

Fincke and Ossendrijver (2016: 185, 196) observed that SpTU 4, 176 and SpTU 5, 316 are similar to BM 46550, probably to be dated to the Achaemenid period and possibly originated from Borsippa. However, tablet W 23021 (SpTu 4, 176) exhibits a paleographic difference with the other Achaemenid texts: the shape of the digit 9 () is different from those used in other tablets found in level IV. This feature may point to a later date, which is not impossible since the filling rubble of level IV contained tablets dated to different periods. For example, the astronomical tablet SpTU 4, 168 (W 22925), also found in a filling rubble of level IV, belongs to the so-called “Ekur-zakir Library” (see John Steele’s chapter in this volume).

Excavation number	Publication number	Tablet type	Locus	Campaign	Content
W 23021	SpTU 4, 176	Lentil	Filling rubble of level IV	30 th	List of reciprocal pairs, reduction by factorisation
W 23016	SpTU 5, 316	Lentil	Filling rubble of level IV	30 th	Powers of 5 (geometrical progression, reason 5, from 1.2.12.28.48 to 1)
W 22715-2	SpTU 4, 177	S	Level III, next a water system	29 th	Multiplication table: $\times 45$
W 22260a	SpTU 1, 101	Fragment	Level not specified	27 th	Metrological tables for <i>še</i> and <i>gin</i> , with conversions into decimal notations ; table of conversion of fractions into SPVN, multiplication table by 100
W 22309a+b	SpTU 1, 102	Fragment	Northern half of the trench Ue XVIII 1	27 th	List of metrological equivalencies for lengths; metrological table for capacities
W 22661-3a+b	SpTU 5, 317	Fragment	Level III, next a water system	29 th	Mathematical? Traces of numbers. Vertical alignments and lines which evoke an abacus

Table 2: mathematical tablets dated possibly to the Achaemenid period (square Ue XVIII 1 or 2, filling rubble of level IV, level III or unknown)

The mathematical tablets from the “House of the *āšīpu*” have already been studied in several publications: Friberg published most of the Achaemenid mathematical texts (with Hunger and Al-Rawi for text 5), and Eleanor Robson has provided a general presentation of mathematics in Achaemenid and Seleucid Uruk.¹³ A broad description of Late Babylonian metrology was included by Otto Neugebauer and Abraham Sachs in their discussion of the Hellenistic mathematical text VAT 7848.¹⁴ A complete overview of the metrologies developed in late periods was offered by Marvin Powell in his landmark paper on Mesopotamian metrology “Masse und Gewichte”, and his pioneer study of “Late Babylonian Surface Mensuration”.¹⁵ Friberg elucidated the various metrological systems used in texts 1-5 discussed here.¹⁶ Important improvements on the history of the Late-Babylonian metrologies, were made recently by Heather Baker in her studies of land surveying and legal documents. Baker provides new evidence of the use, in administrative archives, of measurement units attested in mathematical texts.¹⁷

This chapter focuses on aspects that were not specifically examined in previous publications, namely the interconnections between the five mathematical tablets found in room 4. This small collection of tablets offers a rare opportunity to analyze a set of metrological tables and mathematical problems which were apparently kept together as a group in antiquity.

¹³ Robson (2008b) and (2008a: 227–240; 240–260).

¹⁴ Neugebauer and Sachs (1945: 143–145).

¹⁵ Powell (1984) and (1987-1990: 482-4).

¹⁶ Friberg, Hunger, and Al-Rawi (1990); Friberg (1993) and (1997); Friberg and Al-Rawi (2017: Sect. 3).

¹⁷ Baker (2004) and (2011).

1.2 The colophons

The five mathematical tablets found in room 4 have similar format and type (type M, that is, multi-column tablets). Among them, four bear a readable colophon (texts 1, 2, 3, 5), and the last one (text 4) seems to have also contained a colophon, but the end of the text is damaged. The colophons are presented in Table 3.

Text	Excavation number	Content	Colophon ¹⁸
1	W 23273	Metrological tables	<p>al-til ... [ki]-i pi-i tup-pi gaba-ri ^rE^{ki} ¹Ri-mut An-šar₂-gal [a ša₂^{ld}]utu-sum-na a^{lu2}sanga-^dmaš [x x] ^rx¹ u ib-ri</p> <p>Finished ... According to a tablet, original of Babylon, Rīmūt-Ani, [son of] Šamaš-iddin, descendant of Šangi-Ninurta, [wrote] and checked it.</p>
2	W 23281	Metrological equivalencies; reciprocal table	<p>al-^rtil¹ ki-i ka tup-pi-[meš? X x]-meš sar-sar-ma igi-^rkar₂¹</p> <p>Finished According to tablet(s?) [...] written and checked.</p>
3	W 23283 +W 22905	Reciprocal table	<p>^rnu-al¹-til ki-i ka BE¹(libir)-š_{u2}¹Ri-^rmut¹-[^dA-nu] ^rdumu¹ ša₂^{ld}gis-šir-id-di-nu dumu^{lu2}sanga-[^dmaš] a!¹-na lit-mu-di-šu in-sar-ma ^rib₂-ri?¹</p> <p>Not finished. According to its original, Rīmūt-Ani, son of Šamaš-iddin, descendant of Šangi-[Ninurta]. For his learning, he wrote and checked it.</p>
4	W 23291	22 problems on surfaces	Colophon (?) damaged
5	W 23291x =IM 75985	23 problems on surfaces	<p>še-numun u gi-meš al-til gaba-ri ^{gis}da libir-ra-bi-gim ab-sar ba-an-e₃ tup-pi^{ld}utu-sum-na dumu-a-ni ša ¹na-di-nu dumu lu₂ e₂-maš-^dnin-urta lu₂-^rmaš¹-maš lu₂ qaq-qar^da-nu^{ki}-u₂</p> <p>še-numun and gi, finished. Copy of a wooden board, identical to its original, written and checked. Tablet of Šamaš-iddin, son of Nādinu, descendant of Šangi-Ninurta āšipu from Uruk.</p>

Table 3: colophons of Achaemenid mathematical tablets found room 4 (reading Hunger)

¹⁸Transliteration and translation by Hunger (personal communications April 2005 and March 2016).

The colophons provide the following information:

- The four readable colophons mention the fact that the tablet was copied from an original. The nature of the original is specified in three instances: a clay tablet (*tuppu*) in two cases (texts 1 and 2), and a wooden board (^{gis}da) in one case (text 5).
- The tablet was “checked” (*barû*) in three cases (texts 1, 2 and 3); the three texts “checked” are the metrological and numerical tables.
- In the three colophons where proper names appear (texts 1, 3 and 5), Šamaš-Iddin, descendant of Šangî-Ninurta, is involved. In texts 1 and 3, Šamaš-Iddin appears as the father of the writer of the tablet, Rīmūt-Anu, and in text 5 he appears as the owner of the tablet.
- The profession of Šamaš-Iddin appears in text 5: “*āšipu* (lu₂-maš-maš) from Uruk”.

The names which appear in the colophons are that of Šamaš-Iddin and his son Rīmūt-Anu, who represented one of the most prominent family of *āšipus* of Uruk at the time. They seem to have shown some interest in mathematics.¹⁹ However, the exact way in which they were involved in mathematics remains to be elucidated. Was Šamaš-Iddin (or his son) an expert in mathematics? It is possible, but not certain. The only information we have is that Šamaš-Iddin owned one of the two mathematical tablets containing a collection of problems (copied from a wooden board), and that his son Rīmūt-Anu copied two mathematical tablets containing metrological and numerical tables. Moreover, as pointed out by Friberg in the passage quoted above, the very fact that some of these tablets were found stored in the same jar suggests that Šamaš-Iddin and his son Rīmūt-Anu attached a particular importance to the preservation or constitution of a mathematical collection.

The colophons do not inform us who composed the mathematical content of texts 1–5, or who compiled them. The fact that at least four of the five texts were copied from “older” sources may reflect that the collection of mathematical tablets found in room 4 was composed prior Šamaš-Iddin, and, in this case, that Šamaš-Iddin only inherited a mathematical tradition. Why were the members of Šamaš-Iddin’s family, or their ancestors, or the community they belonged to, interested in the kind of mathematical traditions conveyed by the mathematical texts found in room 4? In this chapter, it is argued that a tentative answer can be drawn by examining the interconnections between the texts.

2 Metrological systems

A simple glance at the contents of the mathematical texts found in room 4 suffices to see that they deal mainly with metrology and surfaces. Among the tablets found in room 4, one contains a set of metrological tables (text 1), one contains a table of conversions of measurement units (text 2), one contains a list of pairs of reciprocals (text 3), and two contain lists of problems concerning surfaces (texts 4–5). The metrology of lengths and surfaces appears to be an essential concern of the milieu Šamaš-iddin and his son Rīmūt-Ani belonged to.

A brief review of the main features of the metrological systems used in the Achaemenid and previous periods is needed at this point. What I mean by a “metrological system” (sometimes abbreviated as “metrology”) is a set of measurement units linked by fixed factors. I use the

¹⁹ By contrast, the name of Anu-Ikšur, the brother of Rīmūt-Anu, does not appear in colophons of mathematical texts. This interest was already underlined by Robson (2008a: 227-260) and (2008b), and Friberg (1997: 252).

convenient representation of a metrological system by means of a “factor diagram” introduced by Friberg.²⁰

The metrological systems were described by the ancient scribes themselves in specific texts. Different kinds of linguistic devices were used for such descriptions. The most widely used was the list, that is, enumerations which make clear the factors between measurement units. For example, in the Old Babylonian tablet HS 241 (see Figure 1), the enumeration ... / 8 *šu-si* / 9 *šu-si* / 1/3 *kuš* / ... makes it clear that 10 *šu-si* is equal to 1/3 *kuš*, that is, 1 *kuš* is equal to 30 *šu-si*. These lists are omnipresent in sources produced in Old Babylonian scribal schools.

1 <i>šu-si</i>	
2 <i>šu-si</i>	
3 <i>šu-si</i>	
4 <i>šu-si</i>	
5 <i>šu-si</i>	
6 <i>šu-si</i>	
7 <i>šu-si</i>	
8 <i>šu-si</i>	
9 <i>šu-si</i>	
1/3 <i>kuš</i> ₃	
1/2 <i>kuš</i> ₃	
2/3 <i>kuš</i> ₃	
5/6 <i>kuš</i> ₃	
1 <i>kuš</i> ₃	
1 1/3 <i>kuš</i> ₃	
[1 1/2 <i>kuš</i> ₃]	

Figure 1: Translation and copy of tablet HS 249, rev. col. v, extract of metrological list for lengths, school tablet from Nippur, Old Babylonian period (Jena University, Proust 2008: pl. 3)

By contrast, in certain Late Babylonian texts, for example in two tablets from the “House of the *āšipus*” (text 2 (W 23281) and W 22309), another layout is adopted: the equivalencies between the measurement values are explicitly stated. For example, the relation “1 *kuš* is equal to 30 *šu-si*” just mentioned is expressed in text 2 (W 23281, obv. i, line 3) as an explicit equivalency (for translation conventions adopted in this chapter, see Appendix 4):

30 *šu-si* (are) a *kuš*
 (3(u) *šu-si* *kuš*₃)

In the Old Babylonian period, each measurement value was associated with a number written in sexagesimal place value notation (see definition below) for the purposes of computation. This correspondence was established by means of metrological tables, which were probably memorized by scribes during their elementary education. To what extent was this system of correspondence adopted or modified in the Achaemenid period in Uruk? The mathematical tablets found in room 4 shed interesting light on this question.

To underline the specificities of Late Babylonian approaches of metrology, and to understand how they contrast with the Old Babylonian legacy, a quick overview of the main features of the

²⁰ Friberg (1978). The measurement units of a given system are enumerated in the order they appear in texts (from the smallest on the right to the largest on the left), and they are linked with an arrow indicating the ratio between each unit and its multiple or submultiple (see the examples in Table 4).

Old Babylonian metrology is useful. These features are clearly illustrated by the tablet HS 241, an Old Babylonian metrological table for lengths from Nippur (see Figure 2).

1(diš) šu-si	[10]	
2(diš) šu-si	20	
3(diš) šu-si	30	
4(diš) šu-si	40	
5(diš) šu-si	50	
6(diš) šu-si	1	
7(diš) šu-si	1.10	
8(diš) šu-si	1.20	
9(diš) šu-si	1.30	
1/3 kuš₃	1.40	
[1/2] kuš₃	2.30	
[2/3] kuš₃	3.20	
[1(diš)] kuš₃	5	

Figure 2: Translation and copy of tablet HS 241, metrological table for lengths, school tablet from Nippur, Old-Babylonian period (Jena University, Proust 2008: pl. 20)

Many aspects of the Old-Babylonian metrological tables, namely their layout, notations, order of entries, order of the sub-columns, metrological factors, correspondence between measurement values and sexagesimal place value numbers, as well as the variability of the texts among the hundreds known sources, contrast with those found in Achaemenid metrological tables.

The layout – Unlike the metrological lists, which are simple enumerations of measurement values displayed into a one dimensional column (example HS 249, Figure 1), the Old Babylonian metrological tables are displayed into two sub-columns (example HS 241, Figure 2). In the left sub-column, measurement values are enumerated, and in the right sub-column, the sexagesimal place value numbers corresponding to the measurement values are provided.

The sexagesimal place value notation²¹ does not include any mark for indicating the order of magnitude of the numbers. For example, in HS 241 (see Figure 2), the number corresponding to the measurement value 6 *šu-si* is 1, and the number corresponding to the measurement value 1 *kuš*, which is sixty times greater, is also 1. As a consequence, identifying the measurement value corresponding to a given number in SPVN requires an approximate evaluation of the order of magnitude of the expected result.

²¹ The sexagesimal place value notation (SPVN thereafter) was used, mainly in mathematical texts, from the end of the third millennium to the end of the first millennium. As indicated by its name, this notation is sexagesimal (base sixty) and positional (the value of a sign derives from its place in the number). This notation uses 59 digits, written with ones (𐎶) and tens (𐎵) repeated as many times as necessary. According to the place value principle, each sign represents sixty times the same sign occupying the previous place (at the right hand). For example, in 𐎶𐎵 (1.21), the left hand wedge represent sixty times the right hand wedge. Moreover, the place of the unit in the number is not indicated in cuneiform writing, at least as far as mathematical texts are concerned (other features appear in some Seleucid astronomical texts). For example, the number 𐎶 represents at the same time 1, and 60, and 1/60, and any power of 60; the number 𐎶𐎶 represents at the same time 3, and 3×60, and 3/60, and 3 multiplied by any power of 60.

The order of the metrological lists in Old Babylonian texts is fixed. When all of the metrological lists, or several of them, are written on the same tablet, they always appear in the same order, which is: capacity, weight, surface, and length.²² This order reflects the curriculum, that is, the order in which these texts were learnt in scribal schools, at least at Nippur.²³ The order of the metrological tables is the same.

The factors – The metrological systems, that is, the factors defining each measurement unit in relation to the others, are expressed in Old-Babylonian texts as lists, as underlined above. In Table 4 this information is represented with the modern synthetic “factor diagrams”.

Units of capacity gur ←×5– bariga ←×6– ban ₂ ←×10– sila ₃ ←×60– gin ₂
Units of weight gun ₂ ←×60– ma-na ←×60– gin ₂ ←×180– še
Units of surface GAN ₂ ←×100– sar ←×60– gin ₂ ←×180– še
Units of length danna ←×30– UŠ ←×60– ninda ←×12– kuš ₃ ←×30– šu-si

Table 4: factor diagrams describing the metrological systems learnt in Old-Babylonian scribal schools

The correspondences – In Old-Babylonian sources, each measurement value is associated with a sexagesimal place value number following strict rules. The measurement of length of 1 *ninda* is associated with the number 1 (in SPVN) when the measured line is horizontal, and the measurement of length of 1 *kuš* is associated with the number 1 (in SPVN) when the measured line is vertical. All of the other numbers (in SPVN) associated to measurement values result from these two correspondences. For example, as 1 *ninda* corresponds to 1, then 1 ½ *ninda* corresponds to 1.30, 2 *ninda* corresponds to 2, and so on, and 1 *sar* (surface of 1 *ninda*-side square) corresponds to 1. This system of correspondence is represented in ancient texts by tables such as HS 241 (Figure 2); it is represented in modern synthetic form by the diagram in Table 5 below.

²² This order appears for example in tablet HS 249, which contains all of the metrological lists – see Proust 2008: 23).

²³ Proust (2007).

Units of capacity								
gur	←×5-	bariga	←×6-	ban ₂	←×10-	sila ₃	←×60-	gin ₂
5		1		10		1		1
Units of weight								
gun ₂	←×60-	ma-na	←×60-	gin ₂	←×180-	še		
1		1		1		20		
Units of surface								
GAN ₂	←×100-	sar	←×60-	gin ₂	←×180-	še		
1.40		1		1		20		
Units of length (horizontal lines)								
danna	←×30-	UŠ	←×60-	ninda	←×12-	kuš ₃	←×30-	šu-si
30		1		1		5		10
Units of height (vertical lines)								
danna	←×30-	UŠ	←×60-	ninda	←×12-	kuš ₃	←×30-	šu-si
6		12		12		1		2

Table 5: factor diagrams and correspondences with sexagesimal place value numbers

It is important to keep in mind the distinction between the metrological systems (the factors defining the measurement units), and the correspondences (the sexagesimal place value number associated to each measurement value). Indeed, in the Achaemenid mathematical texts found in room 4, the authors played with both features: several different correspondences may be used for the same metrological system; conversely, a measurement unit, say *kuš*, may appear in different systems, while being associated to the same number in SPVN.

The variability – No substantial variations are observed among the metrological lists and tables produced in scribal schools. The Old Babylonian metrological documentation exhibits a great stability of the layout, the order of tables, the factors defining the measurement units, and the correspondences between measurement values and sexagesimal place value numbers. The systems expressed throughout metrological tables were used in a consistent way in Old Babylonian mathematical and administrative texts over a large part of Mesopotamia. These systems reflect a relatively high level of standardization, even if some local variants can be observed, for example in Syrian capacity systems.²⁴

The metrology taught in Old Babylonian scribal schools was no longer used in Mesopotamia for economic and administrative purposes after the Kassite period.²⁵ However, mathematical texts show that this metrology remained known and used by scholars until the end of the Hellenistic period. The following sections analyze the modalities and meaning of the persistence of this “traditional” metrology, and the way in which the new metrological systems were articulated with the traditional ones by the Urukian scholars in the Achaemenid period.

²⁴ See synthesis and references in Colonna d’Istria (2015).

²⁵ Powell (1987–1990: 482).

3 The metrological tables: the tradition revisited

3.1 Text 1 (W 23273 = SpTU 4, 172)

According to its colophon, text 1 (W 23273)²⁶ was copied from an older tablet by Rīmūt-Anu, son of Šamaš-Iddin. The text exhibits a complete overview of the traditional metrological systems for lengths and surfaces in the sense that the tables it contains are based on the same measurement units and the same factors as those from Old-Babylonian period. However, important innovations affect the layout, the order of the tables, the order of the signs, and other epigraphical details. This section attempts to elucidate the meaning of these elements of permanency and innovations.

The text contains six metrological tables, and additional sections on diverse topics. The structure of the text was elucidated by Friberg (1993: 400). I numbered the tables and sections A, B, B', C, and so on (see first column of Table 6) differently from Friberg.²⁷ The section on the shadow-length scheme (H in my numbering) was elucidated by John Steele.²⁸ I summarize the content of the tablet in Table 6, with some minor changes from Friberg's interpretation.²⁹ A complete translation is provided in Appendix 1.

#	Place	Content	Metrology
A	Obv. <i>i</i>	Numbers associated to gods	
B	Obv. <i>i-iii</i>	Metrological table for lengths (horizontal lines)	Length (traditional) danna ←30– UŠ ←60– ninda ←12– kuš ₃ ←30– šu-si 30 1 1 5 10
B'	Obv. <i>iii-iv</i>	Idem - Inversion of the order of the signs	Idem
C	Obv. <i>iv-v</i>	Metrological table for heights (vertical lines)	Length (traditional) UŠ ←60– ninda ←12– kuš ₃ ←30– šu-si 12 12 1 2
C'	Obv. <i>v</i>	Idem - Inversion of the order of the signs	Idem
D	Rev. <i>i</i>	“Range table for lengths and surfaces” ³⁰	Relation between units of length and surface: 1 sar = 1 ninda □ 1 ninda
E	Rev. <i>i-iii</i>	Metrological table for surfaces	Surface (traditional) GAN ₂ ←100– sar ←60– gin ₂ ←180– še 1.40 1 1 20
E'	Rev. <i>iv</i>	Metrological table for weights (or surfaces and weights) - Inversion of the order of the signs	Weight or Surface (traditional) + Weight gun ₂ ? ←60– ma-na? ←60– gin ₂ ←180– še 1 1 1 20
F	Rev. <i>iv</i>	Catch line	
G	Rev. <i>iv</i>	“finished” (al-til)	
H	Rev. <i>iv</i>	Shadow-length scheme	
	Rev. <i>iv</i>	Colophon	See Table 3

Table 6: content of text 1 (W 23273 = SpTU 4, 172)

²⁶ See publication information and translation in Appendix 1.

²⁷ Friberg (1993: 400) labelled the sections as follows: for Friberg, my section A is section G (gods); B is L_n (length in ninda), B' is L_n', C is L_c (lengths in cubits), C' is L_c', D is L&A (lengths and areas), E is A (areas), E' is M (mass), F is C (catch line), G is also C, H is L-T a and L-T b (cubits and month names).

²⁸ Steele (2013).

²⁹ The table in section E' is interpreted as a table of weights by Friberg (1993: 400) and Robson (GKAB). However, this table is similar to the previous one, a table for surfaces (E), with inversion of the order of the signs. This would fit with the general structure of the text, even if the traces of sign “ma-[na]” and “gun₂” at the end of the table argue in favor of Friberg's interpretation.

³⁰ Friberg (1993: 401).

The metrological systems are displayed on the tablet by means of tables, according to ancient textual traditions developed more than one millennium before in the Old Babylonian scribal schools. The metrological systems adopted in this text are, broadly speaking, the traditional ones, but some important differences from the Old Babylonian versions are worth mentioning:

- Additional sections are inserted: a list of gods with corresponding numbers (section A), a shadow scheme (section H),³¹ and a notice which explains the links between the units of length and surface (section D).
- The order of the tables is inverted: here the metrological tables for lengths (B-B'-C-C') come before the tables for surfaces (E and perhaps E').
- The sub-columns are inverted in tables B, C, and E: the sexagesimal place value numbers appear in the left sub-column, and the measurement values in the right sub-column.
- The order of the signs is inverted in some of the sections: the unusual order measurement unit + numerical value is adopted in sections B', C' and E'. Example: the first items of tables B and B' are respectively the following:

10	1(diš)	šu-si	(table B)
šu-si	1(diš)	10	(table B')

What is the meaning of these features, namely, the layout of the tables (order of the tables, the sub-columns and the signs) and the composition of the text (insertion of new sections)? What do these new features inform us about the way in which ancient metrologies were understood by scholars of the late period?

The issue of the order

As for the order of the tables, the puzzling aspect is not the order of the tables in this text—lengths, and then surfaces, which looks quite logical for modern observers—but the order of the tables in the Old Babylonian school texts—capacities, weight, surfaces, and lengths, which is hard to explain for the modern historian, and possibly even for the users in later periods. This order may reflect a historical process whose details and logic are lost. It seems that the Urukian authors or compilers abandoned an arrangement that was no longer understood.

The inversion of the sub-columns in the items of tables B, C, and E may reflect profound changes in writing techniques between the Old-Babylonian period and the Neo- or Late Babylonian period, in particular with the profusion of alternative writing media such as wooden boards, leather and papyrus in the late period.³² The coexistence, in the same text, of different orders of the signs and sub-columns may reflect the variety of writing techniques in use in the scholarly milieu of Achaemenid Uruk.

The inversion of signs in the items of tables B', C', and E' may have another explanation. In the Old Babylonian period, the metrological tables were read from left to right to transform measurement values into SPVN, and from right to left to transform SPVN into measurement

³¹ John Steele (personal communication) notes that: “It could be argued that the shadow scheme is not part of the metrological material because it appears after a catchline and the remark ‘finished’. In fact we seem to have two catchlines, one before and one after the shadow length scheme. Perhaps we should see this tablet as containing a copy of a metrological tablet, complete with the catchline etc, followed by an additional (unrelated?) text and a colophon.”

³² Jens Høyrup suggested that the Late Babylonian metrological tables may result from copies of Aramaic texts written from right to left (personal communication).

values. Perhaps the Late Babylonian practitioners felt it was useful to have two different tables for these two directions of reading.

Anyway, these variants in the order of the elements of the tables do not affect the actual metrological and numerical content of the table: the same measuring units, the same factors, and the same correspondence with sexagesimal place value numbers as in the Old Babylonian period are adopted. The Urukean compilers reformulated in their own way the Old Babylonian metrological tradition.

Section D

However, it seems that some features of the traditional metrological system had to be explained to the Neo or Late Babylonian audience. This may have been the function of section D.

Section D (Reverse, *i*)³³

1. nam-lugal-la nam-tar-ma
2. šu-si-meš *a-na*³⁴ še-meš
3. 1(diš) kuš₃-meš *a-na* [x³⁵] gin₂
4. gi-meš *a-na* [x³⁶] sar
5. 1(u) ninda-meš *a-na* 1(iku) GAN₂
6. 1(diš) UŠ-meš *a-na* bur GAN₂
7. 1(diš) danna-meš *a-na* 1(šar₂) GAN₂
8. TA 6 UŠ *a-na* 1(šar₂) [GAN₂]
9. ta-nam [x]
10. nig₂-a-ra₂-šu₂-nu gi-na
11. nig₂-šid-šu₂-nu ul ih-haš-ši

Translation

1. Kingship, destiny
2. *šu-si* to *še*
3. 1 *kuš* to [x] *gin*
4. *gi* to [x] *sar*
5. 10 *ninda* to 1 *iku GAN*
6. 1 UŠ to *buru GAN*
7. 1 *danna* to 1 *šar GAN*
8. From 6 UŠ to 1 *šar [GAN]*
9. You do [...]
10. Their multiplication (will be) correct
11. Their calculation will not be chopped (?).

At first glance, this section seems to contain the surfaces of squares which sides are respectively 1 *šu-si*, 1 *kuš*, 1 *gi* (= ½ *ninda*), 10 *ninda*, 1 UŠ, 1 *danna*, and perhaps others. The corresponding surfaces are the following:

Side	Surface
1 <i>šu-si</i>	5 <i>še</i>
1 <i>kuš</i>	¼ <i>gin</i>
1 <i>gi</i>	¼ <i>sar</i>
10 <i>ninda</i>	1 <i>iku GAN</i>
1 UŠ	2 <i>bur GAN</i>
1 <i>danna</i>	3 <i>šar'u GAN</i>

³³ Transliteration and translation by Hunger (personal communication, May 2016).

³⁴ Here, the value 5 is expected. The place where 5 is expected is damaged according to von Weier's copy (and Robson's edition in GKAB, consulted in January 2016), but seems to be blank according to the photo (and Friberg 1993, p. 400).

³⁵ ¼ is expected; however, it seems to me that the space damaged could hardly contain "igi-4-gal₂".

³⁶ Idem.

However, these surfaces are not exactly those we read in section D. First, the measurement values are not always specified precisely. Indeed, there is no number before the units of length and surface in lines 2 and 4.³⁷ Second, in lines 6 and 7, the surfaces provided are not the surfaces expected (line 6, we read 1 *bur GAN* instead of 2 *bur GAN* expected, that is, 2 times less; line 7, we read 1 *šar GAN* instead of 3 *šar'u GAN* expected, that is, 30 times less). Are these discrepancies to be considered as mistakes of the scribes? If we take in account the observations above, another explanation is possible. It seems that the relationship between the sides and the surfaces of the squares in section D is not an exact correspondence, but just an indication of the broad orders of magnitude. Friberg probably had this idea in mind when he labelled this section as a “Range table for length and area”.³⁸ What may have been the function of such indication of the ranges?

The principal difficulty raised by the calculations of surfaces with ancient methods is the use of metrological tables for identifying the measurement value corresponding to a given number in SPVN, a notation without indication of the orders of magnitude: how to select the correct portion of the table to be used? The function of section D may be to help this selection. Indeed, we can observe that section D provides one or two broad orders of magnitude of surface per cycle of 60, which is exactly the information needed to make the correct selection. Perhaps the obscure lines 10–11, “Their multiplication (will be) correct. Their calculation will not be chopped”, refers to the fact that the section helps to avoid mistakes of order of magnitude of the surface after performing correctly the multiplication.

If we give some flesh to the skeletal narrative offered by section D, we might obtain something like the table below:

2. A square of some *šu-si* side has a surface to be found in the section of *še* in the metrological table for surfaces;
3. A square of about 1 *kuš* side has a surface to be found in the section of *gin* in the metrological table for surfaces;
4. A square of some *gi* side has a surface to be found in the section of *sar* in the metrological table for surfaces
5. A square of about 10 *ninda* side has a surface to be found in the section containing the area 1 *iku GAN* in the metrological table for surfaces
6. A square of about 1 *UŠ ninda* side has a surface to be found in the section containing the area 1 *bur GAN* in the metrological table for surfaces
7. A square of about 1 *danna* side has a surface to be found in the section containing the area 1 *šar GAN* in the metrological table for surfaces

With these landmarks in mind, the correct portion of table to be used for any surface of squares or quasi-squares can be selected. For quite elongated rectangles, a mental adjustment is necessary. Examples illustrating how this method may have been applied are suggested below in the discussion of text 5 (problem 5 in Sect. 4.2 and problems 6–7 in Appendix 2). In the Old Babylonian scribal schools, controlling the orders of magnitude used to be a basic technique taught in the elementary level and was mastered by the learned scribes.³⁹ This technique was apparently to be explained in written form to the far descendants of the Old Babylonian

³⁷ Note that in line 2 the place where we expect a number, namely 5, before the unit of surface *še* is blank according to the photo, and not damaged as suggested by von Weier’s copy.

³⁸ See Friberg (1993: 400, 401) about this section and a parallel section found in W 23281 (text 2), and similar observation in Robson (2008a: 230).

³⁹ Old Babylonian exercises of calculation of surfaces like Ist Ni 18 were probably used for such training (see Proust 2007: Sect. 6.4).

scholars. Finally, section D could be understood as a guide for the use of the metrological tables, that is, a kind of commentary of these tables which restores lost meanings.

Note that in section D, there is no reference to tables C-C' for vertical lines, which was used in Old Babylonian period for calculating volumes. We see later (Sect. 4) that the correspondences provided by tables C-C' were used in Late Babylonian Uruk not only for calculating volume in the Old Babylonian fashion, but also for adapting the traditional system to the new metrological practices based on the unit of length *kuš* (Sumerian sign *kuš*₃, Akkadian *ammatum*).

Non mathematical sections

Two sections (A and H) are different in nature from the others. Section A provides numbers associated with fourteen gods, and section H a shadow-length scheme. Both topics have nothing to do with metrological systems, but have some connection with the professional duties of the *āšipus* who compiled or gathered the scholarly collection found in room 4.

Some other striking features of text 1 must be underlined: no decimal numbers, no Akkadian name of the measurement unit,⁴⁰ and no “modernisation” of the name of the fractions appear in this text, unlike in other metrological tables from Late-Babylonian Nippur and Uruk. For example, in text 2, we observe the extensive use of decimal numbers and Akkadian phonetic orthography for writing the names of measuring units. Thereby, text 2 stands in stark contrast with text 1 from a stylistic point of view, even if the metrological systems are broadly the same.⁴¹ Text 1 adopts an old style, and testifies a purist approach of ancient traditions, while the style of text 2 is much more in tune with the time.

To sum up, text 1, and to some extent text 2, provide the information necessary to evaluate surfaces with the traditional methods. Text 3 is a table of reciprocals of large numbers beginning with 1, 2 and 3 and reflects a mathematical interest specific to the Late Babylonian periods: calculation (reciprocal and powers) with very large regular numbers.⁴² I come back to the reciprocal tables in Sect. 4.3.

3.2 Comparison with metrological tablets from Late-Babylonian Nippur

An interesting light on the approach of metrology in Achaemenid texts may come from Neo- or Late-Babylonian Nippur. A handful of mathematical tablets dated to the Neo- or Late Babylonian period probably or certainly come from Nippur. Five of them have a quite certain provenience, three containing metrological tables (CBS 8539, CBS 11032, and CBS 11019), and two being fragments containing lists of problems badly preserved (N 2694 and N 2873).⁴³ The date of the three metrological tables is not sure, as archaeological information is lacking and the tablets do not bear colophons. CBS 8539 was dated to the Neo-Babylonian period by

⁴⁰ John Steele (personal communication) notes: “An exception to this rule is found in the first line of the shadow scheme which gives the Akkadian equivalent of *kuš*₃. This adds further support to the idea that the shadow length material was copied onto this tablet from a different source text.”

⁴¹ Text 2 contains lists of length, surface and capacity units. The system of length units have the same general structure as their Old Babylonian counterpart, even if some units are added, and the surface units are the same as in Old Babylonian period. However, the capacity system is the one used in Late-Babylonian period. In addition, text 2 contains a table of reciprocals.

⁴² In other known tables of reciprocals from Achaemenid or Hellenistic period, the entries are (almost) all of the regular numbers beginning with 1, by 2, and sometimes by 3. See Ossendrijver’s chapter, this volume, for more details on Hellenistic reciprocal tables.

⁴³ See publication information on these tablets in the list of sources.

François Thureau-Dangin.⁴⁴ Indeed, the reference in the text to the “seed system”, attested in the sources after the 8th century,⁴⁵ makes it improbable that the document pre-dates the Neo-Babylonian period. The metrologies described in this text and the layout (inversion of the sub-columns in comparison with Old Babylonian tables) are the same as in text 1, dated with certainty to the Achaemenid period. There is no use of special sign for the “9” (reverse, col. i) as in Hellenistic texts, and no separation mark, which makes a Hellenistic date unlikely. Combining these pieces of information, it is therefore likely that the text dates to the Achaemenid period. The date of CBS 11019 and CBS 11032 is also quite uncertain. According to Sachs, “Paleographical and terminological considerations make it certain that they were written in the Neo-Babylonian period, and a more precise date in the Persian [Achaemenid] period seems rather reasonable.”⁴⁶ Friberg thinks that CBS 11032 and CBS 11019 were written by the same scribe. To sum up, it appears that little, if any, time separated the composition of these mathematical texts from Nippur and from Achaemenid Uruk. For the purpose of this chapter, it is interesting to compare the metrological tables from Nippur, especially the three tables noted on CBS 8539, with our text 1.

The text noted on CBS 8539 is composed of an alternation of metrological tables and short notices which specify the metrological systems used (relation between measurements of length and surface) and the correspondences adopted (relation between measurement values and SPVN) in the different tables, as shown in Table 7. The notices are of particular interest, because they may reflect, in the same way as in text 1, what explanations a reader in the Late Babylonian period needed in order to understand and use the metrological systems inherited from the Old Babylonian times.

⁴⁴ « Hilprecht [(1906)] place la rédaction de ce texte vers 1350, soit à l'époque Kassite; je ne sais sur quel fondement. L'écriture fait plutôt penser à l'époque néo-babylonienne. De plus, l'échelle ici employée pour les mesures de capacité est, comme à l'époque néo-babylonienne, $1/30$ (de *gur*) = 6 *qa* ... » (Thureau-Dangin 1909: 84) Actually, there is no « *qa* » in this table; Thureau-Dangin grounds the equivalence $1/30$ *gur* = 6 *qa* on a reconstruction of the calculation of a surface expressed as capacity of grain (the so-called “seed-system” – see below).

⁴⁵ Powell (1999: 483).

⁴⁶ Sachs (1947: 67).

#	Place	Notices (note: <i>ammatum</i> is the Akkadian name of the unit <i>kuš</i>).	Tables
A	Obv. <i>i-ii</i>		Metrological table for lengths (horizontal lines), traditional system and correspondence for horizontal lines $danna \leftarrow 30 - U\check{S} \leftarrow 6 - a\check{s}lu \leftarrow 2 - \check{s}uppan \leftarrow 5 - ninda \leftarrow 2 - (s.) ninda \leftarrow 6 - ku\check{s}_3$ 30 1 10 5 1 2 5
B	Obv. <i>ii</i>	1 st notice on metrological systems for lengths and surface: “This <i>šu-si</i> is such as 30 <i>šu-si</i> is 1 <i>kuš</i> . The <i>ammatum</i> (is for) <i>še-numun</i> or <i>gi-meš</i> ...”	
C	Obv. <i>ii</i>		Metrological table for lengths, traditional system and correspondence for vertical lines $\check{s}u-si$ 2
D	Obv. <i>iii</i>		Metrological systems for lengths and surface, Late-Babylonian system $ku\check{s}_3 \leftarrow 24 - \check{s}u-si$ 1 2.30
E	Obv. <i>iii</i>	2 nd Notice on metrological systems for lengths and surface: “This <i>šu-si</i> is such as 24 <i>šu-si</i> is 1 <i>kuš</i> . The <i>ammatum</i> (is for) <i>še-numun</i> and <i>gi-meš</i> . (The square) which length is 1 hundred <i>kuš</i> and width is 1 hundred <i>kuš</i> has (a surface of) 5 <i>ban</i> 3 <i>silá</i> 3 1/3 <i>GAR</i> of <i>še-numun</i> and ? <i>gi-meš</i> ”	
F	Obv. <i>iii</i> – rev. <i>i</i>		Metrological table for lengths, Late-Babylonian system $ku\check{s}_3 \leftarrow 24 - \check{s}u-si$ 36 1.30
G	Rev. <i>i</i>	3 rd Notice on metrological systems for lengths and surface, Late-Babylonian system: “[This] <i>šu-si</i> (is such as) 24 <i>šu-si</i> is 1 <i>kuš</i> . The <i>ammatum</i> (is for) <i>še-numun</i> . [The rectangle which length is 1 hundred <i>kuš</i> and] width is 1 hundred <i>kuš</i> has (a surface of) [...] <i>še-numun</i> ”	
H	Rev. <i>ii</i>		Metrological table for weights, traditional and Late-Babylonian systems identical $ma-na \leftarrow 60 - gin_2 \leftarrow 180 - \check{s}e$ 1 1 20
I	Rev. <i>ii-iii</i>		Metrological table for capacities, Late-Babylonian system $gur \leftarrow 5 - barig / pi \leftarrow 6 - ban_2 [\leftarrow 6 - sila_3 \leftarrow 10 -] GAR$ 5 1 10 1.40 10

Table 7: content of CBS 8539

Two different metrological systems for lengths are given: the traditional one, with 1 *kuš* = 30 *šu-si* (sections A, B), and the Late-Babylonian one, with 1 *kuš* = 24 *šu-si* (sections D, E, F, G). The contrast between both systems is underlined in the notices. For each system, traditional and Late Babylonian, several correspondences between measurement values and SPVN are adopted, breaking the ancient Old Babylonian coherence. Only the Late Babylonian units of

surface are mentioned in the notices. The weight system is the same in Old Babylonian and late periods. The capacity system in section I exhibits the factors used in Late Babylonian period: 1 *ban* = 6 *sila*, and 1 *sila* = 10 GAR, which differs from those that were used in the Old Babylonian period (see Table 8).

Late Babylonian capacity system (1 <i>sila</i> represents about 1 liter)				
<i>gur</i>	←5-	<i>barig</i>	←6-	<i>ban</i>
5		1		10
				←6-
				<i>sila</i>
				←10-
				GAR
				1.40
				10
Old Babylonian capacity system (1 <i>sila</i> represents also about 1 liter)				
<i>gur</i>	←5-	<i>barig</i>	←6-	<i>ban</i>
5		1		10
				←10-
				<i>sila</i>
				←60-
				<i>gin</i>
				1
				1

Table 8: LB and OB units of capacity and the corresponding sexagesimal place value numbers.

We can observe that, despite the fact that the Old Babylonian and the Late Babylonian *gur* differ, the number in sexagesimal place value notation which corresponds to 1 *gur* in our Nippur text is 5, which is the same number as in the Old Babylonian metrological tables. Yet, 1 *gur* is equivalent to 180 *sila* in Late Babylonian metrology, and to 300 *sila* in Old-Babylonian metrology, the absolute value of the unit *sila* being approximatively the same in both systems (about 1 liter). This means that the bridge between volumes and capacities on which the ancient Old-Babylonian system of correspondence was built is broken.⁴⁷

This unexpected correspondence between 1 *gur* (of 180 *sila*) and 5, as well as the multiplicity of correspondences between measurements of length and surface adopted in the metrological text CBS 8539, show that in this text some features of the ancient metrology are preserved, while the ancient general architecture is destroyed.

The three notices (sections B, E, G) are of special importance for understanding the mathematical problem texts found in room 4. These notices underline the metrological system for lengths adopted in the tables which precede them by pointing out their main features:

“This *šu-si* is such as 30 *šu-si* is 1 *kuš*.” (section B)

or

“This *šu-si* is such as 24 *šu-si* is 1 *kuš*.” (sections E and G).

The metrological system for surfaces is indicated in the three notices:

“The *ammatum* (is used for) seed-surface (*še-numun*) and reed-surface (*gi-meš*).”⁴⁸
(sections B, E and G).

In the second notice (section E), the relationship between length and seed-surface is specified: “(The square) which length is 1 hundred *kuš* and width is 1 hundred *kuš* has (a surface of) 5

⁴⁷ The number 5, which corresponds to 1 *gur* Old Babylonian metrology, is based on the equivalence of 60 *gur* with 1 *sar*-volume (1 *ninda* □ 1 *ninda* □ 1 *kuš*), that is about 18 m³. However, the Late-Babylonian *gur*, equivalent to 180 *sila* (about 180 liters), should correspond to 3, not to 5. For more details on the relation between volume and capacity in the Old Babylonian metrology, see Powell (1987-1990); for a detailed analysis of these relations in Old Babylonian mathematical texts, see Proust (forthcoming b).

⁴⁸ Translation by Thureau-Dangin (1909: 85): « Tel est le doigt, dont 30 font une coudée, coudée (qui sert à mesurer les superficies évaluées) en quantités de semence ou en cannes carrées ».

ban 3 sila 3 1/3 ninda of še-numun". This ratio between the square-*kuš*⁴⁹ and the seed-surface (*še-numun*) is equivalent to 30 square-*kuš* per GAR, that is, the "common seed measure" adopted in Late Babylonian southern Mesopotamia, but not at Uruk.⁵⁰ The third notice (section G) contains also a ratio between the square-*kuš* and the seed-surface, but its value is damaged; it may have been different than the one provided in the 2nd notice (section E). I will come back to these ratios in the discussion of text 5. The three notices draw attention to the key role of the length unit *kuš* (Akkadian *ammatum*) in Late Babylonian metrology.

We will see in Sect. 4 that the calculations of surfaces in the mathematical problems found in room 4 are based on the metrology conveyed both by this metrological document from Nippur, which confronts traditional and Late Babylonian systems, and by the Urukian metrological tables found on text 1, which present only the traditional systems.

4 The evaluation of surfaces: bridging different metrological systems

Texts 4 and 5 are strongly related. Both are systematic lists of problems dealing with the evaluation of surfaces and include detailed procedures for solving the problems. Text 4 addresses essentially the issue of relationships between traditional metrological systems for surfaces and the Late Babylonian "seed system", while text 5 focuses on relationships between traditional metrological systems for surfaces and the Late Babylonian "reed system".

I provide here an overview of the general structure of the texts, and a detailed discussion of some of the problems, the transliteration and translation of which are given in Appendix 2. For the complete edition of the texts, with transliteration, translation, interpretation and elucidation of the metrological systems, see Friberg, Hunger, and Al-Rawi (1990) and Friberg (1997).

4.1 Seed-surface and *sar*-surface in text 4 (W 23291 = SpTU 4, 175)

Text 4 is a highly organized list of problems dealing with the evaluation of surfaces of squares, rectangles and triangles. The calculations are performed with Late Babylonian metrology ("seed system") in problems 1–15, with both traditional and Late Babylonian metrology in problem 17, and with traditional metrology in problems 19–21. The seed system is based on five different coefficients which represent different ratios between the standard surface (length multiplied by width) and the seed-surface (expressed as a capacity of seed). The calculation itself is based on four different systems of correspondence between the unit *kuš* and numbers in sexagesimal place value notation (1 *kuš* corresponds in turn to 5, 1, 36, and perhaps 6). The content of text 4 is summarized in Table 9.⁵¹

⁴⁹ The unit of surface square-*kuš*, termed as "small *kuš*" (*kuš₃ tur*) in problem 19 of text 5, is used mainly in mathematical texts. However, Heather Baker has shown that this "small *kuš*" is attested in economic texts, for example in a text from Borsippa dated to 517 BCE and in a text from Babylon dated to the same year (Baker 2004, 2011: 310). It is striking that, in this latter text, the unit is termed literally as a "square-*kuš*" (*labag-kuš₃*) (Baker 2011: 310).

⁵⁰ Powell (1984: 35).

⁵¹ In Table 9 and in the following, I represent the correspondence between measurement values and SPVN by an arrow, for example "1 *kuš* → 5" means "1 *kuš* corresponds to 5 in SPVN", "1 *kuš* → 1" means "1 *kuš* corresponds to 1 in SPVN", etc.

#	Theme	Metrology	Correspondences
1-7	Relations between sides and surface - Calculation of the surface (1) - Calculation of the length, with surface and width given (2-3) - Quadratic problems (4-7)	Length: <i>kuš</i> (counted in decimal notation) Surface: square- <i>kuš</i> ; seed expressed as measure of capacity. Capacity: Late-Babylonian system	1 <i>kuš</i> → 5, coefficient of the seed 48 1 <i>kuš</i> → 1, coefficient of the seed 20
8-9	Conversion of surface into seed-surface with five different coefficients of the seed	idem	1 <i>kuš</i> → 5, coefficient of the seed 43.12 1 <i>kuš</i> → 1, coefficient of the seed 18
10-11			1 <i>kuš</i> → 5, coefficient of the seed 51.50.24 1 <i>kuš</i> → 1, coefficient of the seed 21.36 1 <i>kuš</i> → 36, coefficient of the seed 1
12-13			1 <i>kuš</i> → 5, coefficient of the seed 57.36 1 <i>kuš</i> → 1, coefficient of the seed 24 (restoration Friberg)
14-15			1 <i>kuš</i> → 5, coefficient of the seed 41.40 1 <i>kuš</i> → 1, coefficient of the seed 36 1 <i>kuš</i> → 6 (?), coefficient of the seed 1
17			Length: traditional metrology (<i>ninda</i>) Surface: traditional metrology (<i>GAN</i>) and seed-surface.
19-21	Calculation of surface of triangles in «traditional» units	Length: traditional metrology (<i>UŠ</i> , <i>ninda</i>) Surface: traditional metrology (<i>GAN</i>)	

Table 9: content of text 4 (W 23291 = SpTU 4, 175)

The first problem being destroyed, let us examine the process in problem 2, which is damaged but easy to restore thanks to the highly standardized style and the systematic organization of the text.

Problem 2

The text of problem 2 runs as follows (see complete edition in Appendix 2):

- 2'. [1 hundred *kuš* is the width. How much is the length] such as (the surface) is 1 *gur* of seed?
 3' [As you do not know: the reciprocal] of the width to the surface you raise
 4' [and you multiply by the reciprocal] of the coefficient of the seed.
 5' And by the seed of your plot you multiply: the length you see.
 6' If 5 is your *ammatum*, 8.20 is 1 hundred *kuš*. The reciprocal of 8.20 is 7.12.
 7' [7.12] by 1.15' multiply: it is 9. 9 by 5 multiply: it is 45.
 8' 45 as the length of your surface you set.
 9' If 1 is your *ammatum*, 1.40 is 1 hundred *kuš*. The reciprocal of 1.40 is 36.
 10' 36 by 3 multiply: it is 1.48. 1.48 by 5 multiply, it is 9.
 11' '5 *me* (?)¹ 40 *kuš*, as the length you put down.

A surface, expressed as a capacity of seed, and a width are given, and the length is asked for (line 2’). The surface is implicitly supposed to be rectangular. The procedure must have been introduced by the conventional formula “As you do not know” (mu nu-zu- u_2), as in other sections (see problem 3 in Appendix 2), but the text is broken at this place. The procedure is first given in general terms: divide the surface by the width and multiply by the coefficient of the seed (igi-gub- be_2-e še-numun), lines 3’–4’. After that, the procedure is exemplified with the actual quantities specified in the statement. Two options are offered, based on two different correspondences between measurements of length and SPVN. The first option is “if 5 is your *ammatum*”, that is, 1 *kuš* (Akkadian 1 *ammatum*) corresponds to 5, and the second option is “if 1 is your *ammatum*”, that is, 1 *kuš* (Akkadian 1 *ammatum*) corresponds to 1.

Procedure implemented with the correspondence “5 is your *ammatum*” (1 *kuš* → 5).

In the same manner as in Old Babylonian mathematical texts, the calculation begins with the transformation of measurement values into SPVN, and the procedure is implemented only in SPVN. The text specifies that the side of the rectangle, 1 hundred *kuš*, corresponds to the sexagesimal place value number 8.20 (line 6’). Indeed, 1 *kuš* (Akkadian 1 *ammatum*) corresponds to 5 as indicated by the condition “if 5 is your *ammatum*”, and the decimal number 1 hundred corresponds to the sexagesimal number 1.40, thus 1 hundred *kuš* corresponds to the product 1.40×5 , which is 8.20.⁵² In line 7’, the text specifies that the surface 1 *gur* corresponds to the sexagesimal place value number 5. This correspondence is explicitly stated in the metrological table from Nippur CBS 8539, section I, rev. col. *ii* (see Table 7).

In the procedure, the operations prescribed by the text are the following: first, the number 1.15, the reciprocal of the coefficient of the seed (45), is multiplied by the number 7.12, the reciprocal of the width (8.20); second the result is multiplied by the seed-surface (5, which corresponds to 1 *gur*):

$$\begin{aligned} &\text{recip}(45) \times \text{recip}(8.20), \text{ that is, } 1.15 \times 7.12, \text{ produces } 9 \\ &9 \times 5 \text{ produces } 45 \end{aligned}$$

The text concludes that the length is 45 (line 8’). The text does not specify which actual measurement of length the sexagesimal place value number 45 corresponds to, unlike in problem 3. According to the metrological tables for lengths (see for example text 1, section B, obv. col. *i* and *ii*), 45 may correspond to 9 *kuš* (1/2 *ninda* 3 *kuš*) or to 540 *kuš* (45 *ninda*). As the length must be greater than the width (100 *kuš*), the measurement of the length is 540 *kuš*.

The explanation of the calculation is not detailed. Why is the seed-surface (5) multiplied by the reciprocal of the coefficient of the seed (48) and by the reciprocal of the width (8.20)? The calculation makes sense if we assume that the “coefficient of the seed” (igi-gub- be_2-e še-numun) represents a ratio between the standard surfaces and the capacities of seed it corresponds to, as explained in sections E and G of the Nippur table (see Table 7). Then, the seed-surface is obtained by multiplying the standard surface by the seed coefficient.⁵³ We can

⁵² For checking the calculations, the reader is invited to use MesoCalc, the Mesopotamian Calculator developed by Baptiste Méléès (<http://baptiste.meles.free.fr/site/mesocalc.html>).

⁵³ The standard surface is obtained by multiplying the length by the width, these magnitudes being represented by sexagesimal place value numbers according to tables of text 1 (see Sect. 3 and Appendix 1). The standard surface is termed “sar” surface in text 5, problem 5, obv. col. *ii*, line 11. Here, the scribe works with the correspondences 1 *kuš* → 5, so that the standard surface of 1 *kuš*-side square corresponds to 25 in SPVN.

now explain the calculation in modern notation as follows: If S is the standard surface, S' the seed-surface, L the length, w the width, and c the coefficient of the seed (that is, S'/S), we have:

$$S = L \times w$$

$$L = S / w = (S' / c) / w = S' \times \text{recip}(c) \times \text{recip}(w)$$

We recognize the steps of calculation described lines 6'-8', each of the magnitudes being represented by their values in SPVN according to tables B and E of text 1.⁵⁴

The relation between the standard surface and the seed-surface in the first option "If 5 is your *ammatum*" (that is, 1 *kuš* corresponds to 5) can be summarized in Table 10.

Standard surface (length multiplied by width) with the correspondence 1 <i>kuš</i> → 5	$\xrightarrow{\times 48}$ $\xleftarrow{\times 1.15}$	Seed-surface
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Table 10: Coefficient of the seed in problem2 with the correspondence 1 *kuš* → 5

Procedure implemented with the correspondence "1 is your *ammatum*" (1 *kuš* → 1)

The problem is solved again with the second option, "if 1 is your *ammatum*" (1 *kuš* corresponds to 1), in exactly the same way (lines 9'-11'). I just summarize the steps:

- 1 hundred *kuš* corresponds to 1.40 (indeed, 1 *kuš* corresponds to 1 and 1 hundred corresponds to 1.40)
- The reciprocal of 1.40 is 36
- 36×3 produces 1.48 (3 is the reciprocal of 20, the coefficient of the seed in the option "if 1 is your *ammatum*")
- 1.48×5 produces 9 (1 *gur* corresponds to 5, the seed-surface)

Thus the length is 9. The actual measurement of the length is not specified in the text, but it is easy to find that the number 9 corresponds to 45 *ninda* (540 *kuš*) according to the metrological table of text 1, section C, rev. col. v and a control of the order of magnitude. The result 540 *kuš* seems to be given in line 11' (see comment on this problem in Appendix 2).

In the same way as previously, the relation between the standard surface and the seed-surface in the second option "If 1 is your *ammatum*" (that is, 1 *kuš* corresponds to 1) is summarized in Table 11.

Standard surface (length multiplied by width) with the correspondence 1 <i>kuš</i> → 1	$\xrightarrow{\times 20}$ $\xleftarrow{\times 3}$	Seed-surface
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Table 11: Coefficient of the seed in problem2 with the correspondence 1 *kuš* → 1

⁵⁴ In fact, in the cuneiform text, the order of multiplication is $\text{recip}(c) \times \text{recip}(w) \times S'$ (read from left to right). It is striking that this order does not allow the reader to make sense to the intermediary steps $\text{recip}(c) \times \text{recip}(w)$. Indeed, the reciprocal of the product of the length by the coefficient of the seed (9) does not represent any actual magnitude. At the opposite, the calculations performed in the order $S' \times \text{recip}(c) \times \text{recip}(w)$ would have made more sense since the intermediate step $S' \times \text{recip}(c)$ represents the standard surface. The fact that a formal calculation was preferred to a calculation whose steps make sense probably reflects the way in which the procedure prescribed in problem 1 was conceptualized in order to be reversed. But as problem 1 is destroyed, we cannot go further in this speculation. For a discussion of such phenomena, see Proust (forthcoming a).

Note that the coefficients of the seed in the second option results directly from that of the first option. Indeed, as the ratio between the numbers corresponding to the lengths in the two options is 5, the ratio between the surfaces is 25. Thus the coefficient of the seed in the first option is 25 times the coefficient of the seed in the second option (48×25 produces 20). We shall see that this factor 25 appears again in text 5, problem 4.

The following sections (problems 3-7) deal with linear and quadratic problems based on the same definition of the seed-surfaces. In each procedure, the two options for the number corresponding to 1 *kus* are offered.

Note that in problem 3 (see Appendix 2), unlike in problem 2, the measurements of length corresponding to the results in SPVN are provided. In the first option the result is given as following:

16' 20 each (side) is what you take. 20 *ninda* each (side) is the square.

In the second option, the result is given as following:

18' 4 each (side) is what you take. 2 hundred 40 *kuš* each (side) is the square.

It is striking that the measurement of length is given as 20 *ninda* in the first option, and 240 *kuš* in the second (of course, as 1 *ninda* = 12 *kuš*, the two measurements of length are equal). The first option reflects the Old Babylonian style, and the second option reflects a Late Babylonian style. I come back below on the specific role played by the unit of length *kuš* in Late Babylonian metrology.

Similar methods are adopted in problems 8–17, but the coefficient of the seed varies. We see below that these different coefficients of the seed may reflect different local seed systems adopted in Babylon, Uruk and other cities (see Table 12), which may reflect different tax systems. Moreover, something new goes on: the correspondences between measurement values and SPVN not only envisage the two previous options (1 *kuš* → 5 and 1 *kuš* → 1), but also others. Indeed, problems 10–11 offer the option 1 *kuš* → 36, also found also in table F of Nippur's metrological table, and problems 14-15 seem to offer the option 1 *kuš* → 6. The flexibility of the correspondence between measurement values and SPVN adopted in text 4 strongly links it to the Nippur metrological table CBS 8539.

From this brief discussion of text 4 and related metrological tables, it appears that two important problems were addressed by the authors of the mathematical texts found in room 4: the problem of the correspondence between measurement values and SPVN, and the problem of the coefficient of the seed-surface. Let us examine more closely these two problems.

Correspondences

As explained with the example of problem 2, the evaluation of the surface is based on the multiplication of two linear dimensions (length by width for the rectangles, or side by side for the squares, or base by height for the triangles). The multiplication is performed in SPVN, which supposes that the linear measurement values have been transformed into SPVN. In all the problems, two options are offered for such a transformation (and sometimes additional options). The first option is “if 5 is your *ammatum*” (1 *kuš* corresponds to 5, as provided by tables B–B' of text 1), and the second option is “if 1 is your *ammatum*” (1 *kuš* corresponds to 1, as provided by tables C–C' of text 1). The former reproduces the standard correspondence

found in Old Babylonian metrological tables for horizontal lines, and the latter reproduces the standard correspondence found in Old Babylonian metrological tables for vertical lines. However, in the context of text 4, there is no reference to vertical lines, or to volume. Here, all the lines are horizontal or undetermined. The meaning of the correspondence $1 \text{ kuš} \rightarrow 1$ appears to be new in comparison with the highly coherent Old Babylonian system. It is as if the old techniques were reused, but their significance reshaped.⁵⁵ Actually, in Late Babylonian calculations of surfaces, the second option “if 1 is your *ammatum*” (1 *kuš* corresponds to 1) tends to predominate. This preference for the second option reflects the fact that the unit *kuš* is the bridge between the units of length and surface in the Late-Babylonian metrological systems, in which the lengths tend to be expressed as a decimal number of *kuš*, and the surfaces tend to be expressed as a decimal number of square-*kuš*, as shown for example in problem 3. The articulation between the traditional and the Late Babylonian metrological systems is facilitated by a significant coincidence: despite the fact that the systems of length differ (for example, $1 \text{ kuš} = 30 \text{ šu-si}$ in the former, and $1 \text{ kuš} = 24 \text{ šu-si}$ in the latter), the absolute value of the unit *kuš* seems to be the same in both systems, namely, about 50 cm. In a way, the unit *kuš* is the pivot between the old and new metrologies.

Coefficients of the seed

After the standard surface is obtained in SPVN by “multiplying the line by the line”, this result is multiplied in turn by the “coefficient of the seed-surface” (“*igi-gub-be₂-e še-numun*”, text 4, problem 2, obv. *i l. 4'*) in order to produce the seed-surface, that is, the surface expressed as a capacity of seed. What is the meaning of the coefficient of the seed? The statement of problem 2 shows that the coefficient of the seed (48 in the first option, and 20 in the second) corresponds to the ratio 54000 square-*kuš* per *gur* (or, equivalently, 30 square-*kuš* per *GAR* in the Late-Babylonian capacity system). This coefficient is not a mathematical invention, but the coefficient that was adopted in administrative practices in Late Babylonian cities other than Uruk.⁵⁶ The same coefficient is adopted in problems 3–7. This coefficient appears also in the second notice of the Nippur’s metrological tablet CBS 8539.⁵⁷ However, another coefficient is adopted in problems 10–11: it is 51.50.24 in the first option, and 21.36 in the second option. Here again, this coefficient is not a mathematical invention, but the coefficient that was adopted in administrative practices in Late Babylonian Uruk. Other coefficients are used in problems 8–9 and 12–15; to date, we do not know cities which adopted them, but further studies of administrative texts may change the picture.

⁵⁵ Note that in the Hellenistic text from Uruk AO 6484, calculations of volumes revive the old signification of these correspondences, with explicit references to horizontal and vertical lines.

⁵⁶ Powell (1984: 34), Baker (2011: 312). The system for seed-surfaces based on this coefficient is termed as “common seed measure” by Friberg (1997: 273). This coefficient is evaluated as $1/3 \text{ bariga}$ for 60×60 square-*kuš* by Friberg (1997: 273), and as $36 \text{ gi } 1 \text{ nikkas } 1 \text{ kuš } 15 \text{ } 3/7 \text{ šu-si}$ for 1 *bariga* by Baker (2011: 312) (I use my own conventions for representing the quantities). These evaluations are of course equivalent to the ratio 30 square-*kuš* per *GAR* mentioned above.

⁵⁷ Notice 2 of CBS 8539 (obv. col. *iii*) provides the coefficient of the seed as follows: “(The rectangle) whose length is 1 hundred *kuš* and width is 1 hundred *kuš* has (a surface of) $5 \text{ ban } 3 \text{ sila } 3 \text{ } 1/3 \text{ GAR } 39$ of *še-numun*”. This surface represents 10000 square-*kuš* for $333 \text{ } 1/3 \text{ GAR}$ (in LB capacity metrology), that is, 30000 square-*kuš* for 1000 *GAR*, that is, 30 square-*kuš* per *GAR*.

#	Coefficient of the seed with the correspondence 1 <i>kuš</i> → 5	Coefficient of the seed with the correspondence 1 <i>kuš</i> → 1	Actual ratio	Seed system of
1-7	48	20	30 square- <i>kuš</i> per <i>GAR</i>	Babylon, Borsippa, and other places except Uruk
8-9	43.12	18	33 1/3 square- <i>kuš</i> per <i>GAR</i>	
10-11	51.50.24	21.36	27 7/9 square- <i>kuš</i> per <i>GAR</i>	Uruk
12-13	57.36	24	25 square- <i>kuš</i> per <i>GAR</i>	
14-15	41.40	36	16 2/3 square- <i>kuš</i> per <i>GAR</i>	

Table 12: Coefficients of the seed in text 4

To sum up, problem 3 and related metrological tables show how different metrological systems were generated by the combination of different choices for the correspondence between measurement values and SPVN, and for the coefficient of the seed. However, the sophisticated mathematical elaboration which results from these combinations are not pure speculations. It reflects administrative practices attested in different cities of Babylonia. The ancient and new metrological systems are not only juxtaposed. The ancient systems seem to have been understood differently in the Old and Late Babylonian periods, and some of its essential elements have been reconfigured. For example, the correspondence “if 1 is your *ammatum*” (1 *kuš* corresponds to 1) is not reserved to the measurement of vertical lines, as in the Old Babylonian period, but appears as a possible correspondence among others. The option “If 1 is your *ammatum*” tended to be privileged contra the others because this correspondence resonates with the role of bridge between the units of length and surface that the unit *kus* acquired in the Late Babylonian period. Another example of this reconfiguration is that the text adopts the Late Babylonian metrological system for capacities, even if the Old Babylonian correspondence 1 *gur* → 5, which does not make sense regarding the definition of the new *gur*, is preserved. One further significant example of this “modernization” is that numbers are decimalized in many instances.

The knowledge of the authors or compilers of text 4 and related metrological tables encompassed not only the learned traditions transmitted by communities of scholars but also current practices for evaluating surfaces with seed system in various cities throughout the Babylonian province of the Achaemenid Empire.

4.2 Reed-surfaces and other definitions of surfaces in text 5 (W 23291-x)

Text 5 was owned by Šamaš-Iddin, the father of Rīmūt-Anu who copied texts 1 and 2; it is not impossible that Rīmūt-Anu was also the copyist of texts 4 and 5. Text 5, like text 4, is devoted to the evaluation of surfaces in different systems, and to the articulation of ancient and new metrologies. However, while text 4 focuses on the seed system, text 5 focuses on another system based on the unit of length *gi* (meaning “reed”), equal to ½ *ninda*, and to the associated unit of surface *gi*, which is equivalent to a 1 *gi*-side square. As with text 4, text 5 offers two options for solving the different problems, first “if 5 is your *ammatum*” (1 *kuš* corresponds to 5), second “if 1 is your *ammatum*” (1 *kuš* corresponds to 1). The content of text 5 is summarized in Table 13.

#	Theme	Metrology	Correspondences
1-3	Surfaces of crescents, circles and coronas, with diagrams.	Length (traditional) uš ←60– ninda ←2– gi ←6– kuš ₃ ←30– šu-si Surface (traditional) (system G) gan ₂ ←100– sar ←60– gin ₂ ←180– še Bridge between length and surface units (traditional) 1 sar = 1 ninda □ 1 ninda	1 kuš → 5 1 kuš → 1
4-7	Surfaces of rectangles and squares. #4 provides the general procedure of #5-7	Idem	1 kuš → 5 1 kuš → 1
8-11; 12?	Surfaces of rectangles and squares. #8 provides the general procedure of #9-11	Length (traditional) ninda ←2– gi ←6– kuš ₃ Bridge between length and surface units 1 gi = 6 kuš □ 6 kuš = 1 gi □ 1 gi	1 kuš → 5 1 kuš → 1
13-17	The price of a house	Idem	
18	Damaged		
19	Surfaces of rectangles and squares in “small” (tur) system	Length (Late-Babylonian) gi ←7– kuš ₃ ←24– šu-si Bridge between length and surface units 1 gi = 7 kuš □ 7 kuš = 1 gi □ 1 gi Surfaces gi ←7– kuš ₃ ←24– šu-si	1 kuš → 5 1 kuš → 1
20	Extension of the reed system	Length (Late-Babylonian) gi ←7– kuš ₃ ←24– šu-si 3 different systems based on 3 different bridges between length and surface units. 1 gi (surface) = 1 gi □ 1 gi 1 kuš (surface) = 1 kuš □ 1 kuš 1 šu-si (surface) = 1 šu-si □ 1 šu-si	
21-22	Conversion of surface from gi (reed-surface) to še-numun (seed-surface) and vice versa	Surface (Late-Babylonian) gi (reed-surface) and še-numun (seed-surface)	

Table 13: content of text 5 (W 23291-x)

This set of problems exemplify three different metrological systems for lengths and surfaces, which are explicitly defined in sections 4, 8 and 19. Moreover, in section 20, several variants of the reed-surface are defined. These definitions specify the bridge between the units of length and surface.

In section 4 (obv. ii, l. 8), the bridge connects the unit of length *ninda* with the units of surface *sar*:

Reeds which the length is a reed of 1 *ninda* and the width is a reed of 1 *ninda* are 1 *sar*.
(gi-meš ša₂ gi 1 ninda uš gi 1 ninda sag 1 sar)

It means that the surface of a 1 *ninda*-side square is 1 *sar* (in modern notations: 1 *sar* = 1 *ninda* □ 1 *ninda*). This is the traditional system which goes back to the Old Babylonian period and, actually, is attested as early as the mid-third millennium.

In section 8 (obv. ii, line 22), the bridge connects the unit of length *gi* (6 *kuš*) with the unit of surface of the same name:

A *gi*-surface which length is 6 *kuš* the width is 6 *kuš* (that is) 1 *gi*
(^rgi¹-meš ša 6 kuš₃ uš 6 kuš₃ sag 1 gi).

This system borrows the metrological factors for the units of length to the traditional one (1 *ninda* = 12 *kuš*, or equivalently, 1 *gi* = 6 *kuš*), and borrows the definition of the unit of surface "gi", a 1 *gi*-side square, to the Late-Babylonian "reed system" (1 *gi*-surface = 1 *gi* □ 1 *gi* = 6 *kuš* □ 6 *kuš*). This system is a hybrid between the Old and the Late Babylonian metrologies.

In section 19 (rev. ii line 14), the bridge connects the unit of length *gi* (7 *kuš*) with the unit of surface of the same name:

[A reeds surface which] 7 *kuš* is the length and 7 *kuš* is the width. 1 *gi* according to the small (system) is the interior (surface).
([gi-meš ša₂] 7 kuš₃ uš 7 kuš₃ sag 1 gi kuš₃-meš ša tur *qe₂-e*)

Here is defined the Late Babylonian "reed system", termed as the "small" (tur) system. In the "reed system", the metrological factors for the length units differ to the traditional one (1 *gi* = 7 *kuš*), and the surface unit "gi" is, as previously, a 1 *gi*-side square (1 *gi*-surface = 1 *gi* □ 1 *gi* = 7 *kuš* □ 7 *kuš*).

Section 20 does not contain a problem with its solution but instead a table which defines variants of the reed-system, moving the bridge between the units of length and surface from the unit of length *gi* to the unit *kuš*, and then to the unit *šu-si* (see Appendix 3). The last problems of text 5 are not clear; they seem to deal with the conversion of seed-surfaces into reed-surfaces and vice-versa.

Text 5 appears to be a large overview of the metrological systems used to evaluate surfaces in ancient tradition and in contemporary practices. In the following, I examine in detail problems 4 and 5 of text 5 to analyze how the data provided by metrological tables such as those in text 1 are incorporated in the calculation.

Problem 4 gives the general procedure for evaluating surfaces of a square with the traditional method, and then, problems 5, 6 and 7 exemplify the method with specified data (see complete text in Appendix 3). The text of problem 4 is extremely terse. It runs as follows (obv. col. ii, lines 8–10):

8. Reeds which the length is a reed of 1 *ninda* and the width is a reed of 1 *ninda* (have a surface of) 1 *sar*. If
9. 5 is your *ammatum*, the line times the same (line) and (if 1 is your *ammatum*, the line times the same line), by
10. 25 you multiply. (Reciprocally) times 2.24, each side (i.e. the square root) (you take).

Line 8, already quoted, indicates which metrology is adopted in the problem (see above). This metrology is also described completely by tables B–B' of text 1.

As in text 4, two options are offered for the calculation of the surface. First option, "if 5 is your *ammatum*", the surface is obtained by multiplying the side by itself. Second option, "if 1 is your *ammatum*" (this option is not provided in problem 4, but it is in problems 5, 6, and 7), the text prescribes to multiply the side by itself, and then, to multiply by the coefficient 25, the

coefficient already encountered.⁵⁸ The ultra-terse final instruction in line 10, “2.24-am₃”, which can be understood “(Reciprocally) times 2.24, each side (i.e. the square root) (you take)”, refers to the reciprocal problem, which is exemplified in problem 6.⁵⁹ The relations with the metrological tables such as those in text 1 underlined in the discussion of text 4 are again observed here. Indeed, the two options point to sections B–B’ and C–C’ of text 1, and the coefficient 25 points to the use of metrological table for surfaces such as that exhibited in sections E–E’ of text 1.

Problem 5 implements, for specified values, the general procedure given in problem 4 (obv. col. ii, lines 11-14).

11. A *šuppān* the length, a *šuppān* the width. How much is the *sar*-(surface)? If
12. 5 is your *ammatum*, 5 is the *šuppān*. 5 by 5 multiply: it is 25. The *sar*-(surface) is 25 *sar*.
13. If 1 is your *ammatum*, 1 is the *šuppān*. 1 by 1 multiply: it is
14. 1. By 25 multiply: it is 25.

A square which side is one *šuppān* is given. The unit of length *šuppān* is defined in text 2 (W 23281), obv. col. i, line 13, which provides the equivalency:

$$10 \text{ gi} = 1 \text{ šuppān}$$

Thus 1 *šuppān* is 10 *gi*, that is, 5 *ninda*, a “chain” of about 30 m. The surface, termed as “sar”, that is, the standard surface, is asked for. As usual, the problem is solved first with the correspondence 1 *kuš* → 5, and then with the correspondence 1 *kuš* → 1.

The calculation with the correspondence 1 *kuš* → 5 follows the same scheme as in Old Babylonian mathematical texts. The measurement values given in the statements are transformed into SPVN: 1 *šuppān* corresponds to 5. This correspondence is given by the Nippur table CBS 8536, section A, obv. col. i. The standard surface in SPVN is obtained by multiplying the side by the side (5×5 produces 25), and the result is transformed into *sar* thanks to a metrological table for surfaces such as sections E or E’ of text 1 (rev. col. ii), which indicates that 25 corresponds to 25 *sar*. To control the order of magnitude, the author or reader of the text may have been guided by a notice such as section D of text 1. Indeed, the line 4 of this section, “*gi* to [blank?] *sar*” can be understood as an indication that “a square of some *gi* side has a surface to be found in the section of *sar* in the metrological table for surfaces” (see Sect. 3.1).

The calculation with the correspondence 1 *kuš* → 1 is similar. It can be summarized as follows:

- 1 *šuppān* → 1 (see the item related to 5 *ninda* in text 1, section C, obv. col. iv)
- 1×1 produces 1
- In order to convert into *sar*-surface, that is, to use the metrological table for surfaces, the coefficient 25 must be applied:
- 1×25 produces 25
- The last step, the conversion of 25 into *sar*-surface, not indicated, is the same as in *ninda*-system.
- 25 → 25 *sar* (see text 1, section E, rev. col. ii)

⁵⁸ As explained previously, the coefficient 25 comes from the fact that, in the second option, the SPVN corresponding to 1 *kuš* is divided by 5 (1 *kuš* corresponds to 1 instead of 5), thus, the surface is divided by 25. In order to use the metrological table for surfaces such as section E of text 1, the right value must be restored by multiplying by 25.

⁵⁹ Friberg et al. (1990: 508–509).

Note that the application of the coefficient 25 allows the use of the same metrological table for surfaces such as the section E of text 1 in both systems of correspondence.

Once again, we see how the calculation is based on metrological tables for lengths and surfaces such as those found in sections B-B', C-C', and E-E' of text 1, and the notice D of text 1 which helps to control the orders of magnitude.

4.3 Procedures and tables

Texts 4 and 5 are organized in symmetrical way. In text 4, first calculations involving the new systems (reed-surface) are explained, second, problems shifting from old to new metrologies are solved, and third, calculations of surface involving the traditional systems are offered. Text 5 adopts the reverse plan: first come the calculations of surface involving the traditional systems, second, the problems shifting from old to new metrologies, and third, the calculations involving the new systems (reed-surface and its variants). Finally, Text 5 seems to end with a procedure for converting reed-surface into reed-surface and vice versa, which appears to be a conclusion to the whole composed material found on text 4 and text 5. Thus, Text 5 appears to be the continuation of text 4, or perhaps, the reverse,⁶⁰ both of them being a systematic and coherent presentation of the old and new methods for calculating surfaces. Texts 4 and 5 cover not only the calculations attached to surfaces of rectangles and squares, but also of circular fields, coronas, and triangles. The direct problems (evaluate the surface knowing its sides), the reverse problems (evaluate a side knowing the other sides and the surface), as well as related quadratic problems are treated.

We have analyzed how the metrological tables are used in each steps of the calculation.

- For transforming measurement values into SPVN in the beginning of the procedure, tables such as sections B, and E of text 1 have been used “if 5 is your *ammatum*”, and sections C and E of text 1 may have been used “if 1 is your *ammatum*”.
- For transforming the SPVN in measurement values, tables such as sections B', and E' of text 1 have been used “if 5 is your *ammatum*”, sections C' and E' of text 1 may have been used “if 1 is your *ammatum*”.
- For controlling the order of magnitudes, notices such as section D of text 1 may have been used.

What is the role of the numerical tables in solving the problems?. The arithmetical operations to be performed are mainly multiplications and reciprocals (for divisions). The use of multiplication tables such as that noted on W 22715-2 (SpTU 4, 177) is quite evident. Let us examine closer the relationship between the problem texts 4 and 5, and the reciprocal tables provided by text 3 and the last section of text 2. Table 14 below shows that for the nine entries which may correspond to a preserved part of one of the two tables at least, and which belong to the range of the table (text 3 contains only reciprocal of numbers beginning by 1, 2 and 3), the reciprocals needed in texts 4 and 5 are provided by the tables. Thus, the reciprocals tables found on the mathematical tablets of room 4 appear to have played the role of a tool for the divisions required by reverse problems and the use of coefficients in texts 4 and 5.

⁶⁰ As suggested by Robson (2008a: 230) and Friberg (1997: 251).

Reciprocal pair		Used in	Provided by text 3	Provided by text 2, section K
1	1	Text 5 #14	Broken	Broken
1.12	50	Text 4 #6 Text 5 #22	Broken	Broken
1.15	48	Text 5 #7 Text 5 #21	Broken	No
1.40	32	Text 4 #2	Broken	Broken
2	30	Text 5 #10	Yes	Broken
2.15	26.40	Text 5 #17	Yes	Yes
2.24	25	Text 5 #14	Yes	Broken
2.30	24	Text 5 #10	Yes	Broken
3	20	Text 5 #21	Yes	Broken
3.7.30		Text 5 #18	No	Yes
3.20	18	Text 5 #15	Yes	Broken
3.45	16	Text 5 #13	Yes	Broken
5.24	11.6.40	Text 5 #17	Out of range	Perhaps
6	10	Text 4 #6	Out of range	No
6.40	9	Text 5 #13	Out of range	No
7.12	8	Text 4 #2	Out of range	Yes

Table 14: reciprocals used in texts 4 and 5

5- The *āšipus* and mathematics

We can now come back to the questions asked in the introduction: Why were the keepers or users of the scholarly archives, among them possibly the *āšipus* Šamaš-iddin and his sons Rīmūt-Anu, interested by the mathematical problems treated in tablets found in room 4? Why did the *āšipus* consider it important to re-work and transmit an obsolete metrology inherited from a distant past, and to confront them to the metrologies of their own time? Friberg considered that the goal of these mathematical problems was educational. The role of transmission of knowledge played by the scholarly documents found in the Urukean scholarly archives can hardly be contested. But beyond this general goal, can we detect some mathematical needs that are specific to the milieu of the *āšipus*?

Two different kinds of concerns seem to emerge from the examination of the mathematical texts found in room 4. On the one hand, texts 1 and 2 echo the professional commitment of the *āšipus*. Indeed, in text 1, sections on gods and shadow-length scheme are inserted alongside the metrological tables. Shadow-length schemes appear in texts containing schematic astronomy dated to the same period.⁶¹ Sections H, I and J of text 2 too seem to have some relation with divination or the astral sciences. Indeed, section H of text 2 establishes a correspondence between a number of days and the size of an unborn baby.⁶² Such data on the growth of a fetus is found also in another tablet dated to a later phase of the “House of the *āšipus*”, W 22646 (SpTU 2, 43) an astronomical text belonging to the “Iqiša Library”.⁶³ Section J, which provides a correspondence between measurements of length, weights and capacity, may be linked to measures of time in astronomical observations.

On the other hand, the mathematical problems treated in texts 4 and 5 have little connection, if any, with astronomical calculation. They show a strong interest in the calculation of surfaces, while the metrology of surface is almost completely absent from astrological and astronomical

⁶¹ Steele (2013).

⁶² Hunger (1994).

⁶³ Hunger (1996); see also Hunger’s chapter in this volume.

texts.⁶⁴ All aspects of Old and Late Babylonian metrologies of surfaces are explored in a systematic way in the mathematical collection found in room 4. The different metrological systems of surface are not simply juxtaposed. The main effort consists to connect the different methods of evaluations of surfaces, whether traditional, seed, reed or other systems, the ones with the others. The puzzling question is the reason of such cohabitation and articulations of many different systems, leading to an impressive virtuosity in the art of switching from one system to another, which may have appeared as quite difficult to understand for a practitioner.

The traditional metrology inherited from the Old Babylonian period remained extensively used in mathematical texts until the end of the Hellenistic period, as shown for example by the mathematical tablet AO 6484 which offers a wide overview of mathematical methods applied to various topics. The traditional metrology survived in the scholarly milieu, and was probably transmitted through the clans involved in the development and maintenance of the scholarly archives. However, the texts found in room 4 which present and explain traditional metrology exhibit interesting differences from their ancient precursors. The first difference is that some elements of the ancient systems had to be explicitly explained for a Late Babylonian audience. The explanatory sections inserted in text 1 (section D) and in the Nippur table (sections B, E and G, see Table 7) evoke commentaries addressed to users who had lost whole chapters of knowledge of ancient cultures of computation and quantification, in particular the art of controlling the orders of magnitudes when transforming the SPVN into measurement values. The inversion of the order of the signs in the items of tables B', C', and E' may reflect the need of the Late Babylonian practitioners to have two different tables for the two directions of reading; and the writing of the measurement unit as entry may have served to facilitate the selection of the correct order of magnitude. These features may illustrate the loss of ancient metrological skills by Late Babylonian scholars.

The second difference is that the correspondence “if 5 is your *ammatum*” (1 *kuš* corresponds to 5) for horizontal lines, which is the corner stone of the ancient system, is only one among several possible choices in Late Babylonian metrologies. Further, the correspondence “if 1 is your *ammatum*” (1 *kuš* corresponds to 1) tends to become preponderant. This change reflects the fact that, in the traditional system, the bridge between the measurement of length and surface connects the length 1 *ninda* to the surface 1 *sar*, while in the reed system, it connects the length 1 *kuš* (Akkadina *ammatum*) to the surface 1 square-*kuš*. Thus the traditional system is not only presented, explained, and transmitted, but also adapted.

However, this tradition appeared to be in conflict with the practices of quantification developed in administrative or legal milieus in the different cities of Babylonia, at least those using cuneiform writing. The practices of quantification in late periods are much more diverse than in the Old-Babylonian period. The two main systems of units of surfaces are, as we have seen in texts 4 and 5, the seed system (with different coefficients according to the cities) and the reed system. Both are attested in administrative and legal texts.⁶⁵ What are the respective fields of use of these two main systems? Powell thought that the seed system was used for surfaces of larger dimensions than those evaluated with reed system.⁶⁶ However, Baker recently argued that both systems were used for different kinds of surfaces:

⁶⁴ Among the exceptions is the use of the surface of a trapezoid to model the displacement of a celestial body moving with a variable speed attested in texts dated to the Hellenistic period (Ossendrijver 2016). However, in these texts, the metrological facet of the evaluation of the surface is absent.

⁶⁵ Baker (2011: 313-317).

⁶⁶ Powell (1984: 36)

Concerning the measurement of areas, Powell distinguished between the reed system, which was said to be used for smaller plots, and the seed system, used for larger areas. [...] The degree of overlap in size between properties measured by the “seed” and “reed” systems is thus greater than previously suspected, and it now seems clear that the distinction was not governed by size alone but rather by the nature of the property in question, that is, between urban real estate on the one hand and agricultural land on the other. (Baker 2011: 312)

In this respect, it is interesting to point out the presence, in room 4 of level IV, of an important private archive which deals mainly with the garden prebends owned by members of the Gimil-Nana family.⁶⁷

These observations shed an interesting light on the motivation of the *ašipus* of Achaemenid Uruk, in particular Šamaš-iddin and his sons Rīmūt-Anu, who seem to have been highly interested in quantifying urban real estate and agricultural lands. The *ašipus* of Achaemenid Uruk, or scholars linked to them, seem to have produced mathematical tools to calculate surfaces in new metrology with ancient methods, namely, to have applied a sophisticated ancient mathematical knowledge to their “modern” business of land. The mathematical texts found in room 4 betray two concerns of their authors, compilers, copyists or users: on the one hand transmitting ancient mathematical knowledge, and on the other hand providing technical tools for quantifying, buying and selling land, perhaps in connection with the management of garden prebends. The preservation of ancient traditions appears to be not simply antiquarianism but a pragmatic tentative of updating ancient methods in order to improve the methods of evaluation of surfaces with a new metrology.

⁶⁷ Kessler (2003). See also Jimenez and Gabbay’s chapter in this volume.

Appendix 1: translation of text 1 (W 23273 = SpTU 4, 172)

Only the translation, necessary for the understanding of the chapter, is provided here. It is based upon von Weiher's copy (SpTU 4, 172), the black and white photograph taken by the excavators, the partial edition (detailed description of the content and structure of the metrological systems) published in Friberg 1993, and the edition of section H by Steele (2013). This translation benefited from the kind collaboration of Hermann Hunger. A complete edition is published in Friberg and Al-Rawi (2017). The conventions adopted for translating numbers and measurement values are detailed in Appendix 4.

Section A: table of gods

Obverse i

[1]	Anum	25	5 <i>kuš</i>
[2]	Enlil	30	$\frac{1}{2}$ <i>ninda</i>
[3]	Ea	35	$\frac{1}{2}$ <i>ninda</i> 1 <i>kuš</i>
[4]	Sîn	40	$\frac{1}{2}$ <i>ninda</i> 2 <i>kuš</i>
[5]	[...]	45	$\frac{1}{2}$ <i>ninda</i> 3 <i>kuš</i>
[6]	Enki	50	$\frac{1}{2}$ <i>ninda</i> 4 <i>kuš</i>
[7]	Its Seven Gods	55	$\frac{1}{2}$ <i>ninda</i> 5 <i>kuš</i>
8	Igigi	1	1 <i>ninda</i>
9	Annunaki	1.30	1 $\frac{1}{2}$ <i>ninda</i>
10	Bel	2	2 <i>ninda</i>
20	Šamaš	2.30	2 $\frac{1}{2}$ <i>ninda</i>
30	Sîn	3	3 <i>ninda</i>
40	Ea	3.30	3 $\frac{1}{2}$ <i>ninda</i>
50	Enlil	4	4 <i>ninda</i>

Section B: table for lengths, "if 5 is your *ammatum*" (1 *kuš* corresponds to 5)

[10]	1 <i>šu-si</i>	6	6 <i>ninda</i>
20	2 <i>šu-si</i>	6.30	6 $\frac{1}{2}$ <i>ninda</i>
30	3 <i>šu-si</i>	7	7 <i>ninda</i>
40	4 <i>šu-si</i>	7.30	7 $\frac{1}{2}$ <i>ninda</i>
50	5 <i>šu-si</i>	8	8 <i>ninda</i>
1	6 <i>šu-si</i>	8.30	8 $\frac{1}{2}$ <i>ninda</i>
1.10	7 <i>šu-si</i>	9	9 <i>ninda</i>
1.20	8 <i>šu-si</i>	9.30	9 $\frac{1}{2}$ <i>ninda</i>
1.30	9 <i>šu-si</i>		
1.40	$\frac{1}{3}$ <i>kuš</i>		
2.30	$\frac{1}{2}$ <i>kuš</i>		
3.20	$\frac{2}{3}$ <i>kuš</i>		
5	1 <i>kuš</i>		
6.40	1 $\frac{1}{3}$ <i>kuš</i>		
7.30	1 $\frac{1}{2}$ <i>kuš</i>		
8.20	1 $\frac{2}{3}$ <i>kuš</i>		
10	2 <i>kuš</i>		
15	3 <i>kuš</i>		
20	4 <i>kuš</i>		

Obverse ii

10	[10 <i>ninda</i>]
15	[15 <i>ninda</i>]
20	[20 <i>ninda</i>]
25	[25 <i>ninda</i>]
30	[30 <i>ninda</i>]
35	[35] <i>ninda</i>
40	[40] <i>ninda</i>
45	[45] <i>ninda</i>
50	[50] <i>ninda</i>
55	[5(u)]5 <i>ninda</i>
1	1 UŠ
1.10	1×60+10 <i>ninda</i>
1.20	1×60+20 <i>ninda</i>
1.30	1×60+30 <i>ninda</i>

1.40	1×60+40 <i>ninda</i>
1.50	1×60+50 <i>ninda</i>
2	2 UŠ
3	3 UŠ
4	4 UŠ
5	5 UŠ
6	6 UŠ
7	7 UŠ
8	8 UŠ
9	9 UŠ
10	10 UŠ
11	11 UŠ
12	12 UŠ
13	13 UŠ
14	14 UŠ
15	15 UŠ
16	16 UŠ
17	17 UŠ
18	18 UŠ
19	19 UŠ
20	1/3 <i>danna</i>
25	1/3 <i>danna</i> 5 UŠ
30	1 <i>danna</i>
35	1 <i>danna</i> 5 UŠ
40	2 <i>danna</i>
45	1 1/2 <i>danna</i>
50	1 2/3 <i>danna</i>
55	1 2/3 <i>danna</i> 5 UŠ
1	2 <i>danna</i>
1.30	3 <i>danna</i>
2	4 <i>danna</i>
2.30	5 <i>danna</i>
3	6 <i>danna</i>
3.30	7 <i>danna</i>
4	8 <i>danna</i>
4.30	9 <i>danna</i>
5	10 <i>danna</i>
5.30	11 <i>danna</i>
6	12 <i>danna</i>
6.30	13 <i>danna</i>
7	14 <i>danna</i>
7.30	15 <i>danna</i>
8	16 <i>danna</i>
8.30	17 <i>danna</i>
9	18 <i>danna</i>
9.30	19 <i>danna</i>
10	20 <i>danna</i>
15	30 <i>danna</i>
20	40 <i>danna</i>
25	50 <i>danna</i>

30 1 sixty *danna*

Obverse iii

35	1×60+10 <i>danna</i>
40	1×60+20 <i>danna</i>
45	1×60+30 <i>danna</i>
50	1 hundred <i>danna</i>

Section B': similar to section B, with inversion of the order of the signs

<i>šu-si</i>	1	10
<i>šu-si</i>	2	20
[<i>šu-si</i>	3]	30
<i>šu-si</i>	4	40
<i>šu-si</i>	5	50
<i>šu-si</i>	6	1
<i>šu-si</i>	7	1.10
<i>šu-si</i>	8	1.20
<i>šu-si</i>	9	1.30
<i>kuš</i>	1/3	1.40
<i>kuš</i>	1/2	2.30
<i>kuš</i>	2/3	3.20
<i>kuš</i>	1	5
<i>kuš</i>	1 1/3	6.40
<i>kuš</i>	1 1/2	[7.30]
<i>kuš</i>	1 [2/3	8.20]
<i>kuš</i>	[2	10]
[<i>kuš</i>	3	15]
[<i>kuš</i>	4]	20
[<i>kuš</i>]	5	25
<i>ninda</i>	1/2	30
<i>ninda</i> 1/2 <i>ninda</i>	1 <i>kuš</i>	35
<i>ninda</i> 1/2 <i>ninda</i>	2 <i>kuš</i>	40
<i>ninda</i> 1/2 <i>ninda</i>	3 <i>kuš</i>	45
<i>ninda</i> 1/2 <i>ninda</i>	4 <i>kuš</i>	50
<i>ninda</i> 1/2 <i>ninda</i>	5 <i>kuš</i>	55
<i>ninda</i>	1	1
<i>ninda</i>	1 1/2	1.[30]
<i>ninda</i>	2	[2]
<i>ninda</i>	[2 1/2]	2.30
<i>ninda</i>	[3]	3
<i>ninda</i>	3 1/2	3.30
<i>ninda</i>	4	4
<i>ninda</i>	4 1/2	4.30
<i>ninda</i>	5	5
<i>ninda</i>	5 1/2	5.30
<i>ninda</i>	6	6
<i>ninda</i>	6 1/2	6.30
<i>ninda</i>	7	7

<i>ninda</i>	7 ½	7.30
<i>ninda</i>	8	8
<i>ninda</i>	8 ½	8.30
<i>ninda</i>	9	9
<i>ninda</i>	9 ½	9.30
<i>ninda</i>	10	10
<i>ninda</i>	15	15
<i>ninda</i>	20	20
<i>ninda</i>	25	25
[<i>ninda</i>]	30	30
[<i>ninda</i>]	35	35
[<i>ninda</i>]	40	40
[<i>ninda</i>]	45	45
[<i>ninda</i>]	50	50
[<i>ninda</i>]	5(u)5	55
UŠ	1	1
<i>ninda</i>	1×60+10	1.10
<i>ninda</i>	1×60+20	1.20
<i>ninda</i>	1×60+30	1.30
<i>ninda</i>	1×60+40	1.40

Obverse iv

<i>ninda</i>	1×60+50	1.50
UŠ	2	2
UŠ	3	3
UŠ	4	4
UŠ	5	5
UŠ	6	6
UŠ	7	7
UŠ	8	8
UŠ	9	9
UŠ	10	10
UŠ	11	11
UŠ	12	12
UŠ	13	13
UŠ	14	14
UŠ	15	15
UŠ	16	16
UŠ	17	17
UŠ	18	18
[UŠ	19	19]
[<i>danna</i>	2/3	20]
[<i>danna</i>	1	30]
[-----]		

Section C: table for lengths, “if 1 is your ammatum” (1 *kuš* corresponds to 1)

2		[10 <i>šu-si</i>] ⁶⁸
4		[20] <i>šu-si</i>
6		30 <i>šu-si</i>
8		40 <i>šu-si</i>
10		50 <i>šu-si</i>
12		[1 sixty <i>šu-si</i>]
14		[1×60+10 <i>šu-si</i>]
[16		1×60+20 <i>šu-si</i>]
18		1×60+30 <i>šu-si</i>
20		1×60+40 (=) 1/3 <i>kuš</i>
30		2×60+30 (=) 1/2 <i>kuš</i>
40		3×60+20 (=) 2/3 <i>kuš</i>
1	5	1 <i>kuš</i>
1.20	6.40	1 1/3 <i>kuš</i>
1.30	7.30	1 1/2 <i>kuš</i>
1.40	8.20	1 2/3 <i>kuš</i>
2	10	<2> <i>kuš</i>
3	15	<3> <i>kuš</i>
4	20	<4> <i>kuš</i>
5	25	<5> <i>kuš</i>
6	30	½ <i>ninda</i>
7	35	<7> <i>kuš</i>
8	40	<8> <i>kuš</i>
9	45	<9> <i>kuš</i>
10	50	<1(u)> <i>kuš</i>
11	55	<11> <i>kuš</i>
12		1 <i>ninda</i>
18		1 ½ <i>ninda</i>
24		2 <i>ninda</i>
30		2 ½ <i>ninda</i>
36		3 <i>ninda</i>
42		3 ½ <i>ninda</i>
48		4 <i>ninda</i>
54		4 ½ <i>ninda</i>
1		5 <i>ninda</i>
1.6		5 ½ <i>ninda</i>
1.12		6 <i>ninda</i>
1.18		6 ½ <i>ninda</i>
1.24		7 <i>ninda</i>
1.30		7 ½ <i>ninda</i>
1.36		8 <i>ninda</i>

⁶⁸ The unit *šu-si* in Table C is equivalent to 10 *šu-si* in tables A and B, that is, a ‘small’ *šu-si*.

Obverse v

1.42	8 ½ <i>ninda</i>
1.48	9 <i>ninda</i>
1.54	9 1/2 <i>ninda</i>
2	10 <i>ninda</i>
3	15 <i>ninda</i>
4	20 <i>ninda</i>
5	25 <i>ninda</i>
6	30 <i>ninda</i>
7	35 <i>ninda</i>
8	40 <i>ninda</i>
9	45 <i>ninda</i>
10	50 <i>ninda</i>
11	55 <i>ninda</i>
12	1 [UŠ]

Section C': similar to section C, with inversion of the order of the signs

<i>šu-si</i>	[10	2]
<i>šu-si</i>	20	4]
<i>šu-si</i>	30	6]
[<i>šu-si</i>	40	8]
[<i>šu-si</i>	50	10]
[<i>šu-si</i>	1 šu	12]
[<i>šu-si</i>	1×60+10	1]4
[<i>šu-si</i>	1×60+20	1]6
<i>šu-si</i>	[1×60+30	1]8
<i>kuš</i>	[1×60+40 1/3]	20
[<i>kuš</i>	2×60+30 ½]	30
[<i>kuš</i>	3×60+20 2/3]	40
[<i>kuš</i>	1	1]
[<i>kuš</i>	1 1/3	1].20
[<i>kuš</i>	1 ½]	1.30
[<i>kuš</i>	1 2/3]	1.40
<i>kuš</i>	2	2
<i>kuš</i>	3	3
<i>kuš</i>	4	4
<i>kuš</i>	5	5
<i>ninda</i>	½	6
<i>ninda</i>	½ 1 <i>kuš</i>	7
<i>ninda</i>	½ 2 <i>kuš</i>	8
<i>ninda</i>	½ 3 <i>kuš</i>	9
<i>ninda</i>	½ 4 <i>kuš</i>	10
<i>ninda</i>	½ 5 <i>kuš</i>	11

<i>ninda</i>	1	12
<i>ninda</i>	1 ½	18
<i>ninda</i>	2	24
<i>ninda</i>	2 ½	30
<i>ninda</i>	3	36
<i>ninda</i>	3 ½	42
<i>ninda</i>	4	48
<i>ninda</i>	4 ½	54
<i>ninda</i>	5	1
<i>ninda</i>	5 ½	1.6
<i>ninda</i>	6	1.12
<i>ninda</i>	6 ½	1.18
<i>ninda</i>	7	1.24
<i>ninda</i>	7 ½	1.30
<i>ninda</i>	8	1.36
<i>ninda</i>	8 ½	1.42
<i>ninda</i>	9	1.48
<i>ninda</i>	9 1/2	1.54
<i>ninda</i>	10	2 ⁶⁹
<i>ninda</i>	15	3
<i>ninda</i>	20	4
<i>ninda</i>	25	5
<i>ninda</i>	30	6
<i>ninda</i>	35	7
<i>ninda</i>	40	8
<i>ninda</i>	45	9
<i>ninda</i>	50	10
<i>ninda</i>	55	11
UŠ	1	12

Section D: orders of magnitudes of squares

Reverse i

Kingship, destiny
šu-si to *še*
 1 *kuš* to [x] *gin*
gi to [x] *sar*
 10 *ninda* to 1 *iku GAN*
 1 UŠ to *buru GAN*
 1 *danna* to 1 *šar GAN*
 From 6 UŠ to 1 *šar [GAN]*
 You do [...]
 Their multiplication (will be) correct
 Their calculation will not be chopped (?).

⁶⁹ According to the copy, the text reads *ninda* 1(u) 1 / *ninda* 2(u) 2 / etc. / *ninda* 5(u) 5(*diš*) 12 (the list of number in SPVN is shift one). This cannot be checked with the photo, because the right sub-

column is on the shadow. The last item is *ninda* 1(*diš*) 12 (expected UŠ 1(*diš*) 12) according to the copy, but seems to not exist according to the photo. (Collation needed).

Section E: table for “sar-surfaces”

10	$\frac{1}{2}$ še	
20	1 še	
30	1 $\frac{1}{2}$ še	
40	2 še	
50	2 $\frac{1}{2}$ še	
1	3 še	
1.20	4 še	
1.40	5 še	
2	6 še	
2.20	7 še	
2.30	7 $\frac{1}{2}$ še	
2.40	8 še	
3	9 še	
3.20	10 še	
3.40	11 še	
4	12 še	
4.20	13 še	
4.40	14 še	
5	15 še	
5.20	16 še	
5.40	17 še	
6	18 še	
6.20	19 še	
6.40	20 še	
7	21 še	
7.20	22 še	
[7].30	22 $\frac{1}{2}$ še	$\frac{1}{8}$ <gin>
[7].40	23 še	
8	24 še	
8.20	25 še	
8.40	26 še	
9	27 še	
9.20	28 še	
9.40	29 še	
10	$\frac{1}{6}$ <gin>	
11.40	< $\frac{1}{6}$ > gin 5 še	
13.20	< $\frac{1}{6}$ > gin 10 še	
15	< $\frac{1}{4}$ > gin	
16.40	< $\frac{1}{4}$ > gin 5 še	
18.20	< $\frac{1}{4}$ > gin 10 še	
20	$\frac{1}{3}$ <gin> še	
30	$\frac{1}{2}$ <gin> še	
40	$\frac{2}{3}$ <gin> še	
50	$\frac{5}{6}$ <gin> še	
1	1 gin	
1.10	1 gin $\frac{1}{6}$	
1.15	1 gin $\frac{1}{4}$	
1.20	1 gin $\frac{1}{3}$ še	
1.30	1 gin $\frac{1}{2}$ še	

1.40	1 gin $\frac{2}{3}$ še
1.50	1 gin $\frac{5}{6}$ še
2	2 gin
3	3 gin
4	4 gin
5	5 gin
6	6 gin
7	7 gin

Reverse ii

8	8 gin
9	9 gin
10	10 gin
11	11 gin
12	12 gin
13	13 gin
14	14 gin
15	1 gin $\frac{1}{4}$ sar
16	16 gin
17	17 gin
18	18 gin
19	19 gin
20	$\frac{1}{3}$ sar
30	$\frac{1}{2}$ sar
40	$\frac{2}{3}$ sar
50	$\frac{5}{6}$ sar
1	1 sar
1.10	1 sar 10 gin
1.15	1 sar gin $\frac{1}{4}$ sar
1.20	1 sar $\frac{1}{3}$ sar
1.30	1 sar $\frac{1}{2}$ sar
1.40	1 sar $\frac{2}{3}$ sar
1.50	1 sar $\frac{5}{6}$ sar
2	2 sar
3	3 sar
4	4 sar
5	5 sar
6	6 sar
7	7 sar
8	8 sar
9	9 sar
10	10 sar
11	11 sar
12	12 sar
13	13 sar
14	14 sar
15	15 sar
16	16 sar
17	17 sar
18	18 sar

19	19 sar
20	20 sar
30	30 sar
40	40 sar
50	1/2 iku GAN
1	1×60 sar*
1.10	1×60+10 sar
1.20	1×60+20 sar
1.30	1×60+30 sar
1.40	1 iku GAN
2.30	1 1/2 iku GAN
3.20	2 iku GAN
4.10	2 1/2 iku GAN
5	3 iku GAN
5.50	3 1/2 iku GAN
6.40	4 iku GAN
7.30	4 1/2 iku GAN
8.20	5 iku GAN
9.10	5 1/2 iku GAN
10	1 eše GAN
11.40	1 eše 1 iku GAN
13.20	1 eše 2 iku GAN
15	1 eše 3 iku GAN
16.40	1 eše 4 iku GAN
18.20	1 eše 5 iku GAN
20	2 eše GAN
<i>Reverse iii</i>	
21.40	2 eše [1] GAN
23.20	2 eše 2 iku GAN
25	2 eše 3 iku GAN
26.40	2 eše 4 iku GAN
28.20	2 eše 5 iku GAN
30	1 bur GAN
40	1 bur 1 eše GAN
50	1 bur 2 eše GAN
1	2 bur GAN
1.30	3 bur GAN
2	4 bur GAN
2.30	5 bur GAN
3	6 bur GAN
3.30	7 bur GAN
4	8 bur GAN
4.30	9 bur GAN
5	1 bur'u GAN
5.30	1 bur'u 1 bur GAN
6	1 bur'u 2 bur GAN
6.30	1 bur'u 3 bur GAN
7	1 bur'u 4 bur GAN
7.30	1 bur'u 5 bur GAN

8	1 bur'u 6 bur GAN
8.30	1 bur'u 7 bur GAN
9	1 bur'u 8 bur GAN
9.30	1 bur'u 9 bur GAN
10	2 bur'u GAN
15	3 bur'u GAN
20	4 bur'u GAN
25	5 bur'u GAN
30	1 šar GAN
35	1 šar 1 bur'u GAN
40	1 šar 2 bur'u GAN
45	1 šar 3 bur'u GAN
50	1 šar 4 bur'u GAN
55	1 šar 5 bur'u GAN
1	2 šar GAN
1.30	3 šar GAN
2	4 šar GAN
2.30	5 šar GAN
3	6 šar GAN
3.30	7 šar GAN
4	8 šar GAN
4.30	9 šar GAN
5	1 šar'u GAN
5.30	1 šar'u 1 šar GAN
6	1 šar'u 2 šar GAN
6.30	1 šar'u 3 šar GAN
7	1 šar'u 4 šar GAN
7.30	1 šar'u 5 šar GAN
8	1 šar'u 6 šar GAN
8.30	1 šar'u 7 šar GAN
9	1 šar'u 8 šar GAN
9.30	1 šar'u 9 šar GAN

E': similar to section E, with inversion of the order of the signs and an additional section for weights (last two lines), or table of weights

še	1/2	10
še	1	20
še	1 1/2	30
še	2	40
še	2 1/2	50
še	3	1
<i>Reverse iv</i>		
še	4	1.20
še	5	1.40
še	6	2
še	7	2.20

še	7 ½	2.30
še	8	2.40
še	9	3
še	10	3.20
še	11	3.40
še	12	4
še	13	4.20
še	14	4.40
še	15	5
še	16	5.20
še	17	5.40
še	18	6
še	19	6.20
še	20	6.40
še	21	7
še	22	7.20
še	22 ½ = 1/8 (gin)	[7].30
še	23	[7].40
še	24	8
še	25	8.20
še	26	8.40
še	[27	9]
še	[28	9.20]
še	[29	9.40]
še	[30 = 1/6 gin	10]
še	4[5 = 1/4 gin	15]
še	[60 = 1/3 gin	20]
še	60+[30 = ½ gin	30
še	2×60 = 2/3 gin	[40]
še	2×60+30 [= 5/6 gin	50]
še	3×60 = 1 gin	[1]
še	3×3600 = 1 ma-[na	1]
še	3×60×3600 = 1 gun	[1]

Section F: catch line

1 *gin* grain comes after it⁷⁰

Section G

Finished

Section H: shadow scheme⁷¹

1 *ammatum* month IV

[...] *kuš* month V and month III the same

[...] *kuš*] month VI and month II the same

[...] *kuš* month VII and] month I the same

[...] *kuš* month VII and month XII] the same

[...]

[...]

Month V 15 the shadow is delayed

Month VI 30 the shadow is delayed

Month VII 45 the shadow is delayed

Month VIII 1 the shadow is delayed

Month IX 1.15 the shadow is delayed

Month X 1.30 the shadow is delayed

I 1.12 shadow 1.40 *danna* a day after it

Colophon⁷²

According to a tablet, original of

Babylon, Rīmūt-Ani,

[son of] Šamaš-iddin, descendant of Šangi-

Ninurta,

[wrote and] checked it.

⁷⁰ Steele 2013 identifies this section as a catch line. Hunger reads the last two signs *egir-su₂* (“comes after it”) and thus recognize also a catch line (personal communication, June 1st 2016).

⁷¹ Steele 2013.

⁷² Transliteration and translation by Hunger (personal communication April 2015).

Appendix 2: extracts of text 4 (W 23291 = SpTU 4, 175)

The following transliterations and translations rely on the copy by von Weiher (1993) and the edition of the text by Friberg (1997). I did not have access to either the photograph or to the tablet itself, which is held at the Baghdad Museum.

Only the problems discussed in § 4.1 are provided here. Some explanations on problem 3, which are not detailed in § 4.1, are added.

Problem 2

Obv. *i*

- 2' [1 *me* *kuš*₃ *sag* *uš* *en*] *ḫe*₂ *KUR*^{sic} (*gid*₂!)-*ma* *ḫe*₂ 1(*aš*) *gur* *še*-*numun*
 3' [*mu* *nu*-*zu*-*u*₂ *igi*-*u*₂] *ša* *sag* *a*-*ša*₃ *il*₂-*ma*
 4' [*u*₃ *a*-*ra*₂ *igi*-*u*₂] ¹*ša* *igi*-*gub*-*be*₂-*e* *še*-*numun* *du*-*ak*
 5' ¹*u*₃ *še*-*numun* *ša* *e*-*ka*⁷³ *du*-*ma* *sag* *igi*-*mar*
 6' [*šum*]-*ma* 5 *am*-*mat*-*ka* 8.20 1 *me* *kuš*₃ *igi* 8.20 7.12
 7' [7.12] *a*-*ra*₂ 1.12^{sic} (1.15¹) *du*-*ma* 9 9 *a*-*ra*₂ 5 *du*-*ma* 45
 8' ¹45¹ *a*-*na* *šid*-*du* *a*-*ša*₃-*ka* *ta*-*šak*-*kan*
 9' *šum*-*ma* 1 *am*-*mat*-*ka* 1.40 1 *me* *kuš*₃ *igi* 1.40 36
 10' 36 *a*-*ra*₂ 3 *du*-*ma* 1.48 1.¹48¹ *a*-*ra*₂ 5 *du*-*ma* 9
 11' ¹5¹ *me*⁷⁴ 40 ¹*kuš*₃ *a*-¹*na*¹ *šid*-*du* *ta*-*šak*-*kan*

Translation

- 2'. [1 hundred *kuš* is the width. How much is the length] such as (the surface) is 1 *gur* of seed?
 3' [As you do not know: the reciprocal] of the width to the surface you raise
 4' [and you multiply by the reciprocal] of the coefficient of the seed.
 5' And by the seed, what was said to you, multiply: the length you see.
 6' If 5 is your *ammatum*, 8.20 is 1 hundred *kuš*. The reciprocal of 8.20 is 7.12.
 7' [7.12] by 1.15¹ multiply: it is 9. 9 by 5 multiply: it is 45.
 8' 45 as the length of your surface you set.
 9' If 1 is your *ammatum*, 1.40 is 1 hundred *kuš*. The reciprocal of 1.40 is 36.
 10' 36 by 3 multiply: it is 1.48. 1.48 by 5 multiply, it is 9.
 11' 540 (corresponding to the width in) *kuš*, (this) length you put down.

Explanations: see §4.1

⁷³ As in line 5', *ša e-ka* is probably *ša qabû-ka* (Hunger, personal communication June 1st 2016).

⁷⁴ On his copy, in the beginning of line 11', von Weiher had drawn the signs “ŠA DIŠ 40”, read “¹*ša*¹ 1.40” by Friberg (1997: 260). However, these signs could be read as well ¹5 *me*¹ 40, which makes more sense. The translation should be “5 hundred 40 *kuš*, to the length you put down”. This would be consistent with:

- the expected result
- the use of decimal system for counting *kuš*, as used in the same context in problem 3, and as used in Late Babylonian habits;
- the fact that the syntax “SPVN + measurement unit” is unlikely,
- the fact that *a-na* is recognizable, and more probable than *a-ra*₂ as restored by Robson in GKAB.
- the space between DIŠ and 40 on the copy.

Problem 3

Obv. *i*

- 12' a-ša₃ en am₃ lu-[mah]-hir-ma ħe₂ 1(aš) gur 2(ban₂) še-numun
 13' mu nu-zu-u₂ še-numun ša e-ka⁷⁵ e (?)
 14' 'ša¹ igi-gub-be₂-e še-'numun¹ du-ak x il₂'-ma ši-id-du¹
 15' šum-ma 5 am-mat-ka 5.20 a-ra₂ 1.15⁷⁶ du-'ma¹ 6.'40¹-e
 16' 20-am₃ ti-qe₂ 20 ninda-am₃ tu-'mah¹-ħar
 17' šum-ma 1 am-mat-ka 5.20 a-ra₂ 3 du-ma 16-e
 18' '4¹-am₃ ti-qe₂ 2 me 40 kuš₃-am₃ tu-'mah¹-ħar

Translation

- 12' A surface. How much is each (side) such as I make it square and the seed is 1 gur 2 ban?
 13' As you do not know: the seed, what I said to you,
 14' by the coefficient of the seed multiply. [...] the side.
 15' If 5 is your *ammatum*, 5.20 by 1.15 multiply: it is 6.'40¹.
 16' 20 each (side) you take. 20 *ninda* each (side) is the square.
 17' If 1 is your *ammatum*, 5.20 by 3 multiply: it is 16.
 18' 4 each (side) is what you take. 2 hundred 40 *kuš* each (side) is the square.

Problem 3 states that the surface of a square field is 1 *gur* 2 *ban*. The side of the square is asked for. The problem has the same structure as problem 2: statement (line 12'), conventional formula which opens the procedure (line 13'), general procedure (lines 13'-14'), specific procedures with numerical values. As in problem 2, two options are offered to solve the problem: 1 *ammatum* (Sum. kuš₃) corresponds to 5 and the coefficient of the seed is 48 (lines 15'-16'); 1 *ammatum* corresponds to 1 and the coefficient of the seed is 20 (lines 17'-18').

Here is a synthetic explanation of the specific procedures with numerical values, according to the two options.

First option: "If 5 is your *ammatum*" (1 *kuš* corresponds to 5)

In this case, the coefficient of the seed is 48, reciprocal 1.15.

- 1 *gur* 2 *ban* corresponds to 5.20 (according to a metrological table for capacities similar to the Nippur metrological text CBS 8539, section I, rev. *ii*)
- 5.20×1.15 produces 6.40
- 6.40 is the square of 20

The side 20 corresponds to 20 *ninda* according to a metrological table for lengths such as text 1, section B, obv. *ii* (broken but easy to restore) or section B', obv. *iii*, and a control of the orders of magnitude.

Second option: "If 1 is your *ammatum*" (1 *kuš* corresponds to 1)

In this case, the coefficient of the seed is 20, reciprocal 3.

- 1 *gur* 2 *ban* → 5.20
- 5.20×3 produces 16
- 16 is the square of 4

The side 4 corresponds to 20 *ninda* (240 *kuš*) according to a metrological table for lengths such as text 1, section C or C', obv. *v*, and a control of the orders of magnitude. Interestingly, in the

⁷⁵ *ša e-ka* is probably *ša qabû-ka* "what was said to you" (Hunger, personal communication June 1st 2016).

⁷⁶ 1.16 on the copy ; 1.15 expected. Modern copy error of von Weiher? According to the photo, 1.15 is possible (Hunger, personal communication June 1st 2016).

second option, the result is expressed as a number of *kuš* in decimal system in a Late Babylonian fashion, while in the first option, it is expressed as a number of *ninda* in a traditional fashion.

Appendix 3: extracts of text 5 (W 23291x = BagM 21, 554-557)

The following transliteration and translation rely on the copy by von Weiher (1993) and the edition of the text by Friberg, Hunger, and Al-Rawi (1990). I did not have access to either the photograph nor to the tablet itself.

Only the problems discussed in § 4.2 are provided here. I added some comments on problems 6-7 and 19-20 insofar as the explanations are not detailed in § 4.2.

Problem 4

Obverse *ii*

8. gi-meš $\check{s}a_2$ gi 1 ninda uš gi 1 ninda sag 1 sar $\check{s}um$ - $\check{r}ma^1$
9. 5 *am-mat-ka mi- $\check{h}i$ -il-tu $_4$* a-ra $_2$ ki-min u_3 a-ra $_2$
10. 25 du-*ak* a-ra $_2$ 2.24-am $_3$

Translation

8. Reeds which the length is a reed of 1 *ninda* and the width is a reed of 1 *ninda* (have a surface of) 1 *sar*. If
9. 5 is your *ammatum*, the line times the same (line) and (if 1 is your *ammatum*, the line times the same line), by
10. 25 you multiply. (Reciprocally) times 2.24, each side (i.e. the square root) (you take).

Explanations: see §4.2.

Problem 5

Obverse *ii*

11. $\check{r}\check{s}u^1$ -*up-pan* uš u_3 *šu-up-pan* sag en sar-me $\check{s}um$ -*ma*
12. $\check{r}5^1$ *am-mat-ka* 5 *šu-up-pan* 5 a-ra $_2$ 5 du-*ma* 25 25 sar sar
13. $\check{r}\check{s}um^1$ -*ma* 1 *am-mat-ka* 1 *šu-up-pan* 1 a-ra $_2$ 1 du-*lak-ma*
14. $\check{r}1^1$ a-ra $_2$ 25 du-*ma* 25

Translation

11. A *šuppān* the length, a *šuppān* the width. How much is the *sar*-(surface)? If
12. 5 is your *ammatum*, 5 is the *šuppān*. 5 by 5 multiply: it is 25. The *sar*-(surface) is 25 *sar*.
13. If 1 is your *ammatum*, 1 is the *šuppān*. 1 by 1 multiply: it is
14. 1. By 25 multiply: it is 25.

Explanations: see §4.2.

Problem 6

Obverse *ii*

15. $\check{r}e_2^1$ 25 sar-meš ur-a $\check{h}e_2$ -en $\check{s}um$ -*ma* 5 *am-mat-ka*
16. $\check{r}am_3$ 25 ti-*qe $_2$* $\check{s}um$ -*ma* 1 *am-mat-ka* 25 a-*na*
17. $\check{r}2^1$.24 du-*ma* 1-e am_3 ti-*qe $_2$* *šu-up-pan* ur- $\check{r}a^1$

Translation

15. A house of 25 *sar*. How much is the equal-side? If 5 is your *ammatum*,
16. each side (the square root) of 25 you take. If 1 is your *ammatum*, 25 by
17. '2'.24 you multiply, it is 1. Each side (the square root of 1) you take. A *šuppān* is the equal-side.

This problem is the reciprocal of the previous one: the surface of a square is given, find its side. The square has the same dimensions as in problem 5. The procedure is applied in the quantities specified in the statement. As usual, the measurement values are transformed into SPVN, and for that, as in the other problems of texts 3 and 4, two options are offered.

First option: “If 5 is your *ammatum*” (1 *kuš* corresponds to 5)

25 *sar* corresponds to 25 according to the metrological table for surfaces (see for example text 1, section E, rev. *ii*). The square root of 25 is 5, the side. The number 5 is transformed into measurement of length (1 *šuppān*) only at the end of the problem (see below).

Second option: “If 1 is your *ammatum*” (1 *kuš* corresponds to 1)

The coefficient 2.24 (reciprocal of 25) is to be applied to the surface 25, as stated in problem 4.

25×2.24 produces 1
The square root of 1 is 1, the side is 1.

The last step, not detailed in the text, is the transformation of 1 into measurement value. 1 corresponds to 5 *ninda* according to the metrological table for lengths in the second option (see for example text 1, table C), or, equivalently, to 1 *šuppān*. The equivalency between 5 *ninda* (that is, 10 *gi*) and 1 *šuppān* is stated in text 2 (W 23281 = SpTU 173), obv. *i*, line 13: “10 *gi* are 1 *šuppān*”. This observation reveals a thread more that binds together the texts of room 4. Here again, to control the order of magnitude, the author or reader of the text may have been guided by a notice such as section D of text 1, line 4, which indicates that the surface of a square of some *gi* side has a surface of some *sar*, and that, by reverse reading, that a square of some *sar* surface has a side of some *gi*.

Problem 7

Obverse *ii*

18. [sag] 4 uš en ḥe₂-gid₂-da ḥe₂ 20 sar sar *šum-ma* [5]
19. [am]-mat-ka igi 4-gal₂-la 15 15 a-ra₂ 20 du-[ma 5]
20. [1 *šu*]-up-pan gid₂^{ad} *šum-ma* 1 am-mat-ka igi '48'-[gal₂-la] 1.15
21. 1.15 a-ra₂ 2.24 du-ma 3 3 a-ra₂ [20 du-ma 1]

Translation

18. [The width is] 4. How much is the long side so that the *sar*-surface is 20 *sar*? If [5]
19. is your *ammatum*, (4 is the width). The reciprocal of 4 is 15. 15 by 20 multiply: it is [5].
20. [1] *šuppān* is the long side. If 1 is your *ammatum*, (48 is the width). The reciprocal of 48 is 1.15.
21. 1.15 by 2.24 multiply: it is 3. 3 by [20 multiply: it is 1].

This problem is a variant of the previous ones: the surface and the width of a rectangle is given, find its length (the “long side”). As usual, the measurement values are transformed into SPVN. Here again, two options are offered.

First option: “If 5 is your *ammatum*” (1 *kuš* corresponds to 5)

The width is given directly in SPVN, 4. 20 *sar* corresponds to 20 according to the metrological table for surfaces (see for example text 1, section E, rev. *ii*).

The length is obtained by dividing the surface (20) by the width (4), that is, multiplying 20 by 15, the reciprocal of 4:

$$20 \times 15 \text{ produces } 5$$

As in the previous problem, the length corresponding to 5 is 1 *šuppān*, using the data of texts 1 and 2 and the control of the order of magnitude according to section D of text 1 (see details above).

Second option: “If 1 is your *ammatum*” (1 *kuš* corresponds to 1)

The width 4 *ninda* correspond to 48 according to the metrological table for lengths with the correspondence 1 *kuš* → 1 (see text 1, section C, obv. *iv*).

The coefficient 2.24 (reciprocal of 25) is to be applied to the surface 20, as stated in problem 4. The length is then obtained by dividing the surface (20×2.24) by the width (48), that is, multiplying 20×2.24 by 1.15, the reciprocal of 48:

$$(20 \times 2.24) / 48 \text{ produces } 20 \times 2.24 \times 1.15 \text{ which produces } 20 \times 3, \text{ that is, } 1$$

The length which corresponds to the number 1 is not given in this problem, but can be easily be found as 5 *ninda*, that is 1 *šuppān* using tables in texts 1 and 2 and section D of text 1.

Problem 19

Reverse *ii*

- 14. [gi-meš *ša*] 7 *kuš*₃ uš 7 *kuš*₃ sag 1 gi *kuš*₃-meš *ša* tur *qe*₂-*e*⁷⁷
- 15. [mi-*hi*]-^ril¹-*tu*₄ a-ra₂ ki-min *u*₃ 1.12 du-*ma* *ša*₂ a-na igi-ka *e*₁₁-a
- 16. x mi-*nu-ti* gi-meš *kuš*₃-meš tur-meš *te-eš-šip* 1.10 *kuš*₃ ur-a
- 17. ^rhe₂¹-en gi-meš 1.10 a-ra₂ 1.10 1.21.40 1.21.40 a-ra₂ 1.12
- 18. 1.38 gi-meš 1.38 *kuš*₃-meš tur-meš 2 gi-meš 2 gi-meš a-na
- 19. 1.38 *te-eš-šip-ma* 1 me gi-meš

Translation

- 14. [Reeds which] the length is 7 *kuš* is and the width is 7 *kuš* (have) the interior (surface equal to) 1 *gi* according to the small (system).
- 15. The line by the same (itself) and by 1.12 you multiply: then, to what comes up before you,
- 16. a number of *gi* (according to the system) of small *kuš*, you add (to adjust). 1.10, the *kuš*, is the equal-side.
- 17. How much is the *gi*-surface? 1.10 times 1.10 is 1.21.40. (Then), 1.21.40 times 1.12 is
- 18. 1.38, the (number) of *gi*. (To) 1.38 *gi*¹, according to the small (system), 2 *gi* (is to be added). 2 *gi* to
- 19. 1.38 you add (to adjust). The surface is 1 hundred *gi*.

Problem 19 applies the traditional method to calculate the surface: transform the measurement values into SPVN, calculate with SPVN, and come back to the measurement values. However, there is a difficulty: this method was conceived for a metrology which uses only regular factors. But it is applied here to calculation of surface with Late Babylonian metrology, where the factor 7 appears (1 *gi* = 7 *kuš*). The traditional method applied on an inappropriate metrology leads to the thorny problem of reciprocal of non-regular numbers. Problem 19 deals with this issue.

⁷⁷ Friberg, Hunger, and Al-Rawi (1990) transliterate: x e; Hungers reads: *qe*₂-*e*.

The line 14 defines a new unit of surface: 1 *gi* is the surface of a 7 *kuš*-side square. It seems that the corresponding system is labelled as the “small” one (*tur*). Lines 15-16 provide the general procedure to calculate the surface in the new “small system”: multiply the side by itself. Then, multiply the result by a coefficient 1.12, which is the reciprocal of 50. Thus, the square of the side is divided by 50 instead to be divided by 49, as expected, with the correspondence 1 *kuš* → 1 (implicitly assumed). Indeed:

The surface of the 7 *kuš*-side square with the correspondence 1 *kuš* → 1 is
 $7 \times 7 = 49$

The surface of the 7 *kuš*-side square in the small system is 1 *gi*. Thus, the number of *gi* is obtain by dividing by 49 the surface with the correspondence 1 *kuš* → 1.

Dividing by 50 instead of 49 is easier because 50 is regular, while 49 is not. However, this division introduces an error. Line 16, this error is corrected by adding an adjustment (*ešēpu*). It is not explained how to calculate this adjustment.

“1.10 *kuš*”, that I translate “1.10, *kuš*” must represent 10 *gi* (70 *kuš*) for sake of consistency of the problem. The text provides the SPVN counterpart of the measurement value, which is given as 1.10, and an indication on the metrological system: the calculations are performed with the correspondence 1 *kuš* → 1. Other interpretation should be to translate “1.10 *kuš*” as “1(*geš*₂) 10 *kuš*”. With these elements in mind, the problem can be re-formulated as follows: if the surface of a 7 *kuš*-side square is 1 *gi*, the surface of a 70 *kuš*-side square is how many *gi*? A straightforward solution appears to the modern reader (and would have appeared to the ancient reader): the sought surface is 100 *gi*, as stated in line 19. The very interesting feature of the problem is not the solution, but the procedure leading to the solution.

In application of the procedure (lines 15-16), we expect that the number of *gi* is obtained by:

- Converting the side in SPVN with the correspondence 1 *kuš* → 1.
- Multiplying the side by the side in SPVN
- Dividing the obtained surface by 49 (see diagram below)

Side	Side in SPVN	Surface	Surface in SPVN	Number of <i>gi</i>
1 <i>kuš</i>	1	1 <i>kuš</i> □ 1 <i>kuš</i>	1	
7 <i>kuš</i>	7	7 <i>kuš</i> □ 7 <i>kuš</i>	49	1
70 <i>kuš</i>	1.10	70 <i>kuš</i> □ 70 <i>kuš</i>	1.21.40	x

Number of *gi* in problem 19

Problem 20

Reverse *ii*

20. 1 *gi* a-ra₂ 1 *gi* 1 *gi* 1 *gi* a-ra₂ 1 *kuš*₃ 1 *kuš*₃
21. 1 *gi* a-ra₂ 1 šu-si 1 šu-si 1 *kuš*₃ a-ra₂ 1 *gi* 1 *kuš*₃
22. [1] *kuš*₃ a-ra₂ 1 *kuš*₃ 1 *kuš*₃-tur-*tu*₂ 1 *kuš*₃ a-ra₂ 1 šu-si 1 [še¹
23. [1] šu-si < a-ra₂> 1 *gi* 1 šu-si 1 šu-si a-ra₂ 1 *kuš*₃ 1 [še]
24. [1 šu]-si a-ra₂ 1 šu-si tur^{*tu*₂} 24 šu-si-¹meš¹ tur-meš
25. [1] še 7 še-meš 1 šu-si 24 šu-si-meš 1 *kuš*₃ 7 *kuš*₃-meš 1 *gi*
26. 3 su-si 3 še 1 *kuš*₃-tur^{*tu*₂} 7 *kuš*₃ tur-meš 1 *kuš*₃

This section contains two tables (here again, I follow Friberg, Hunger, and Al-Rawi 1990: 538–539). Lines 20-24 contain a table of metrological “multiplication” in the “small” system, that is, with surface expressed as numbers of *gi*. It is a kind of generalization of problem 19. Lines

24-26 contains relations between surface units in the « small » system. I present the two tables in tabular format, with explanations on the metrology in the last columns.

First table (lines 20-24)

Line	Side	Side	Surface	Metrology
20	1 gi	1 gi	1 gi	Length (Late-Babylonian) gi ←7– kuš ₃ ←24– šu-si
	1 gi	1 kuš ₃	1 kuš ₃	
21	1 gi	1 šu-si	1 šu-si	Surface (Late-Babylonian) gi ←7– kuš ₃ ←24– šu-si Bridge between length and surface units (Late-Babylonian) 7 kuš □ 7 kuš = 1 gi □ 1 gi = 1 gi
	1 kuš ₃	1 gi	1 kuš ₃	
22	[1] kuš ₃	1 kuš ₃	1 kuš ₃ -tur ^{tu₂}	1 kuš □ 1 kuš = 1 kuš-tur (1 square-kuš)
	1 kuš ₃	1 šu-si	1 'še'	
23	[1] šu-si	1 gi	1 šu-si	
	1 šu-si	1 kuš ₃	1 [še]	
24	[1 šu]-si	1 šu-si	tur ^{tu₂} (?)	1 šu-si □ 1 šu-si = 1 šu-si-tur (1 square-šu-si)

This table is based on the following length system:

$$gi \leftarrow 7- kuš_3 \leftarrow 24- šu-si$$

Three surface systems are defined from this, based successively on the square-gi (= 1 gi □ 1 gi), the square-kuš (= 1 kuš □ 1 kuš), and the square-šu-si (= 1 šu-si □ 1 šu-si).

Second table (lines 24-26)

24-25	24 šu-si-'meš' tur-meš	=	[1] še	Length (Late-Babylonian) gi ←7– kuš ₃ ←24– šu-si ←7– še ←24 – šu-si tur
25	7 še-meš	=	1 šu-si	
	24 šu-si-meš	=	1 kuš ₃	
	7 kuš ₃ -meš	=	1 gi	
26	3 su-si 3 še	=	1 kuš ₃ -tur ^{tu₂}	Length (Late-Babylonian) kuš ₃ ←7 – kuš ₃ -tur←24– še ←24 – šu-si tur
	7 kuš ₃ tur-meš	=	1 kuš ₃	

This second table is represented by a unique diagram in Friberg, Hunger, and Al-Rawi 1990: 540 as follows:

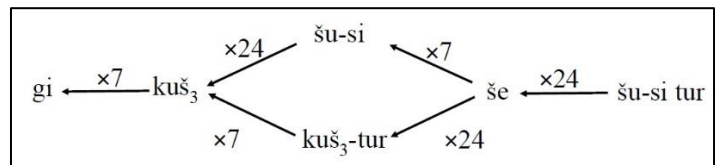


Figure 3: Metrological system in the second table of #20, after Friberg, Hunger, and Al-Rawi 1990: 540

Appendix 4: conventions

In this chapter, the measurement units are not translated in English, but represented in italics by their Akkadian or Sumerian name according to the original text.

Examples:

Sumerogram	Akkadian	Translation
kuš ₃	<i>ammatum</i>	<i>kuš</i> or <i>ammatum</i>
ninda		<i>ninda</i>
silā ₃	<i>qa</i>	<i>silā</i> or <i>qa</i>
barig	<i>pi</i>	<i>barig</i> or <i>pi</i>

References

List of sources

(IM-- = National Museum of Iraq, Baghdad, unknown number)

Museum number	Excavation number	Publication number	CDLI number	Text in this chapter	Secondary publication
IM --	W 23273	SpTU 4, 172	P348765	1	Friberg 1993: 400
IM --	W 23281	SpTU 4, 173	P348766	2	Friberg 1993: 401-2
IM --	W 23283 + W 22905	SpTU 4, 174	P348767	3	
IM --	W 23291	SpTU 4, 175	P348768	4	Friberg 1997
IM 75985	W 23291x	BagM 21, 554-557	P430090	5	Friberg, Hunger, and Al-Rawi 1990
IM --	W 23021	SpTU 4, 176	P348769		
IM --	W 22715-2	SpTU 4, 177	P348770		
IM --	W 22260a	SpTU 1, 101	P348522		Friberg 1993: 395-6
IM --	W 22309a+b	SpTU 1, 102	P348523		Friberg 1993: 392-3
IM --	W 22656-1	SpTU 4, 178	P348771		
IM --	W 22661-3a+b	SpTU 5, 317	P348899		
IM --	W 23016	SpTU 5, 316	P348898		
AO 6484		TU, 33	P254387		
CBS 8539		BE 20/1, 30	P230041		Friberg 1993: 299
CBS 11019		Sachs 1947, 69-70	P266196		
CBS 11032		Sachs 1947, 68	P266208		
HS 241		BE 20/1, 42	P388160		
HS 249		Proust 2008, 3	P388149		
N 2873		Robson 2000: n°20	P277942		
N 2694		Unpublished?	P277762		

Abbreviations / websites

- CDLI Cuneiform Digital Library Initiative. Robert Englund, University of California, Los Angeles, University of Oxford, and Max Planck Institute for the History of Science, Berlin. <http://cdli.ucla.edu/>
- GKAB Geography of Knowledge Corpus. Eleanor Robson, Department of History and Philosophy of Science (HPS), University of Cambridge. <http://oracc.museum.upenn.edu/cams/gkab/>
- MesoCalc The Mesopotamian Calculator developed by Baptiste Méléès. <http://baptiste.meles.free.fr/site/mesocalc.html>

(Other abbreviations: see Proust Steele 2019: ix-x)

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