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An Alternative Wavefront Reconstruction Method for Human Eyes

Damien Gatinel, MD, PhD; Radhika Rampat, MD; Laurent Dumas, PhD; Jacques Malet, PhD

ABSTRACT

PURPOSE: To expand upon and clinically demonstrate the results of a new polynomial decomposition method.

METHODS: To discuss the theoretical considerations comparing the qualitative and quantitative information produced by the Zernike coefficients and a new polynomial decomposition basis, in a comparative series of theoretical and clinical case studies.

RESULTS: These comparative studies validate the novel polynomial basis that decomposes the wavefront, with clear

segregation of the higher and lower aberrations. There is no artifactual reduction of some of the higher order aberration coefficients, providing a more clinically relevant retinal image quality prediction.

CONCLUSIONS: Some of the inherent limitations of the Zernike polynomials in clinical ophthalmic applications can be solved by a novel set of polynomials forming an alternative higher order basis. The new basis provides a clear separation between modes containing lower order terms versus higher order terms and offers clinicians a more clinically realistic wavefront analysis.

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Wavefront sensing has enabled clinicians to qualify and quantify the residual wavefront error that persists after best spectacle correction. This can be described using a specific aberration basis, which contains the “higher order aberrations” (HOAs). To aid the interpretation of wavefront error measurements, it is convenient to express wavefront data in polynomial form. Each polynomial describes a mathematical wavefront shape, and the associated coefficient gives the weight of that aberration in the total wavefront map. The output of a typical wavefront analysis is a list of coefficients, each weighing a specific aberration mode. It is expected that the taxonomy of the HOAs is clinically relevant and equipped with interesting mathematical properties.

Zernike polynomials were consensually adopted in 2002 to allow standardized interpretation of ocular wavefront data, whenever it was convenient to ex-

press it in polynomial form.¹ They were named after the work of Dutch physicist and Nobel prize winner Frits Zernike in 1934.² Zernike polynomials are orthonormal over circular pupils and some are related to classic aberrations such as defocus. With progress in wavefront technology, it is now becoming apparent that proper separation of the total wavefront into lower versus higher order wavefront components is essential for clinical tasks. These include customized ablation in refractive surgery, retinal image quality prediction, and perhaps even objective refraction.

In Zernike polynomials, lower order terms (eg, tilt, defocus, and second order astigmatism) are present within the analytical expression of some higher order modes. This suboptimal distinction between lower order aberrations and HOAs may cause some inaccuracies in the prediction of spherocylindrical refraction and the expected retinal image. These inadequacies

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have been noted previously in fitting the wavefront, in terms of refraction and optical quality.^{3,4} Klyce et al.⁵ demonstrated that the Zernike polynomials underestimated the amount of HOAs in certain cases, instead revealing that Fourier polynomials decomposed the wavefront more reliably. Unfortunately, this also does not present a clinically resilient solution because Fourier series are non-orthogonal and defined over rectangular pupils, not circular ones.⁶

This study aims to serve as supporting material for a recently published novel polynomial decomposition method,⁷ motivated by the need to provide clinicians with a better set of aberration descriptors that would overcome some limitations of Zernike polynomials. In the seminal article, we focused on calculating a new set of coefficients to weight the wavefront aberration modes. The current study conveys this information in a more tangible way to clinicians interested in wavefront analysis and its clinical applications.

CASE STUDIES: BACKGROUND

ZERNIKE POLYNOMIALS

Zernike polynomials are defined in a double index scheme Z_n^m and usually represented in a pyramid, where columns contain modes of the same azimuthal frequency, and rows contain the mode of the same radial order. In Z_n^m , n is a positive integer called the radial degree or order, and m is a negative or positive integer called the azimuthal or angular frequency. The radial order (n) corresponds to the value of the exponent of a function, which possesses r (the distance to the center of the unit pupil disk) as a variable. The normalization coefficient ensures that each mode is normalized, meaning that the square root of the sum of squared errors of each function equals one. Higher order modes are defined by a radial order (n), of 3 or above. The radial order of a Zernike mode is the highest order, but not the only one that may be contained within the analytical expression of that mode. One mode of order (n) may contain lower order terms such as $r^{(n-2)}$, $r^{(n-4)}$. The Zernike coma mode $Z_3^{-1} = 2\sqrt{2}(3r^3 - 2r^1) \sin\theta$ contains a lower order term in r^1 . Conversely, some other modes, such as trefoil $Z_3^{-3} = \sqrt{8}r^3 \sin 3\theta$ or quadrafoil $Z_4^{-4} = \sqrt{10}r^4 \cos 4\theta$ (in fact, all modes located on the edge of the pyramid), are “pure” in their maximal radial order. The presence of lower order terms in the radial polynomial of some higher order modes is due to the necessity of ensuring orthogonality with modes of lower radial degree, but of an identical azimuthal frequency.

ORTHOGONALITY

Because all Zernike modes are orthogonal, the total root mean square (RMS) of the wavefront can be

computed as the root square of the sum of all squared coefficients. The RMS allows a clinician to compute the magnitude of a group of aberrations into a single number in microns. It is calculated by summing the squared coefficients for each mode, and then taking the square root of that sum. However, to satisfy orthogonality conditions, some of the higher order Zernike modes (all modes located in the five central columns) must have lower order terms embedded within them, causing major issues for human eye wavefront analysis.

LOWER AND HIGHER DEGREE POLYNOMIAL DECOMPOSITION: CLINICAL CONSIDERATIONS

To avoid mixing of lower and higher degree (LD/HD) terms within some of the higher order modes of Zernike polynomials, a new expansion, allowing a clear-cut separation between higher and lower order wavefront components was developed.⁷ The mathematical derivation of this basis is beyond the scope of this article and can be found elsewhere.^{7,8} In this new LD/HD classification, the modes can be arranged in a pyramid with a double index scheme $G(n,m)$ where n and m have the same meaning, as in the Zernike classification (**Figure 1A**). All higher order $G(n,m)$ modes located in the five central columns are free of lower order terms. Although the higher order $G(n,m)$ modes ($n \geq 3$) are orthogonal within themselves and normalized, they are not orthogonal to the lower order modes ($n \leq 2$) (**Table 1**). Hence, the computation of the higher order RMS can be done either individually or grouped arbitrarily between higher order modes. However, the computation of the total RMS wavefront error cannot be made by adding the squared $G(n,m)$ coefficients of the lower and higher modes as with the Zernike modes (lower and higher order), which are all orthonormal.

It is important to note that the coefficients weighting the new polynomials can be directly computed analytically from the coefficients weighting a Zernike expansion for the same fit order, without the need of a new wavefront reacquisition. The mathematical process to obtain the new coefficients can be applied after the Zernike coefficients have been obtained from the slopes of a typical Hartmann-Shack or any other kind of aberrometer. From a given set of Zernike coefficients, a unique solution for the coefficients of the new expansion is obtained. This rearrangement does not affect the quality of the initial fit, but might improve the correlation between the coefficients' weight distribution and the clinical situation. As for the Zernike coefficients, the new LD/HD coefficients change with the pupil analysis diameter and the order used for the fit.

LD/HD polynomial basis

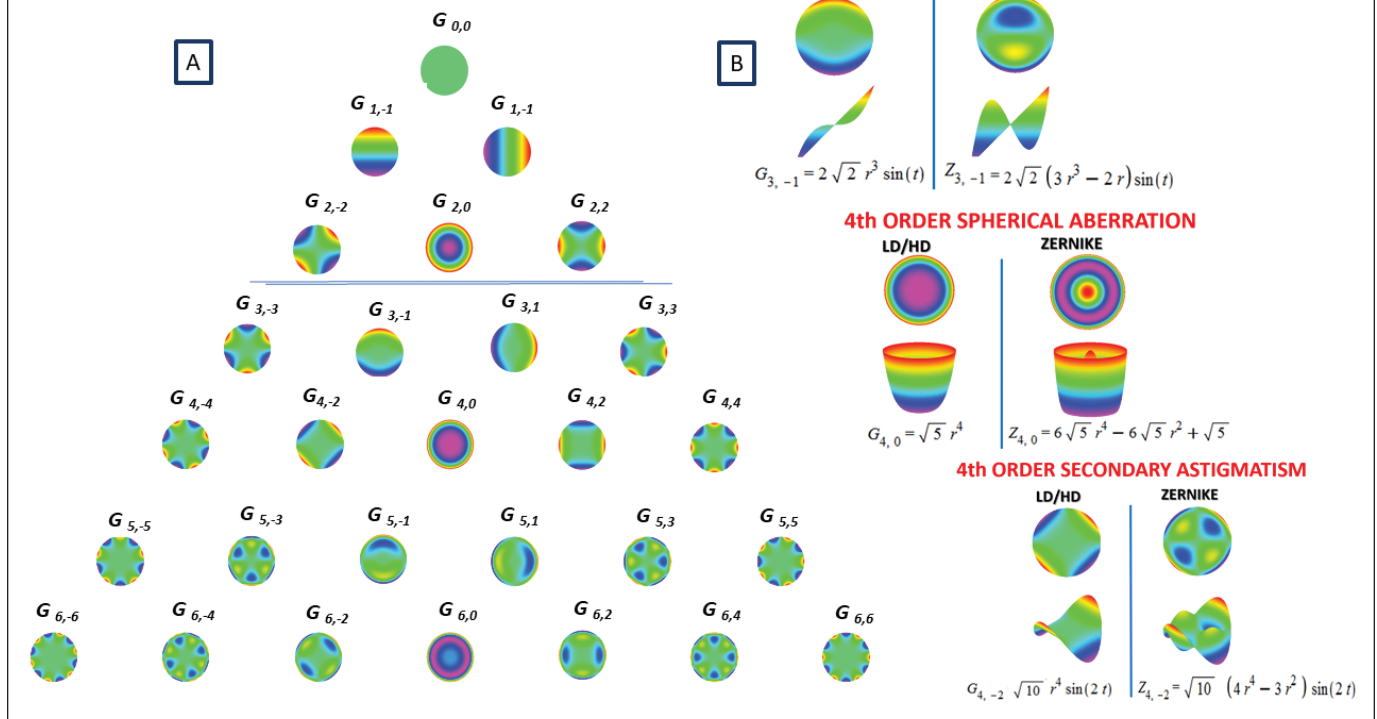


Figure 1. (A) Pyramid of the lower degree/higher degree (LD/HD) modes with depiction of wavefront surfaces. (B) This allows visualization of the main differences between Zernike and the new LD/HD basis wavefront shapes for modes of clinical importance (including coma, spherical aberration, and secondary astigmatism).

TABLE 1

Analytical Expression in Polar Coordinates of the New Higher Order G(n,m) Modes Up to the 6th Radial Order on a Unit Circular Pupil Domain of Radius r and Polar Angle t

Mode	Value
G(3,-1)	$2\sqrt{2}r^3 \sin(t)$
G(3,1)	$2\sqrt{2}r^3 \cos(t)$
G(4,-2)	$\sqrt{10}r^4 \sin(2t)$
G(4,0)	$\sqrt{5}r^4$
G(4,2)	$\sqrt{10}r^4 \cos(2t)$
G(5,-1)	$2\sqrt{3} (5r^5 - 4r^3) \sin(t)$
G(5,1)	$2\sqrt{3} (5r^5 - 4r^3) \cos(t)$
G(6,-2)	$\sqrt{14} (6r^6 - 5r^4) \sin(2t)$
G(6,0)	$\sqrt{7} (6r^6 - 5r^4)$
G(6,2)	$\sqrt{14} (6r^6 - 5r^4) \cos(2t)$

It is possible to express two components for the total wavefront by grouping the modes according to their radial order:

- The LD component of the wavefront: This can be described with an expansion of weighted low order modes of the same analytical structure as their Zernike counterparts. The value of the coefficients weighting these modes of radial order 2 should correlate with the spectacle correction of the measured eye.
- The HD component of the total wavefront: This component should correspond to the residual optical deficits that persist, after prescribing the best spectacle correction. It is expressed as an expansion of the new higher order modes, which are all devoid of lower order terms.

CASE STUDIES: THEORETICAL EXAMPLES

Figure 1B allows the visualization of the differences between Zernike coefficients and the new LD/HD basis. To understand this in depth, the theoretical examples below are helpful.

TILT IN ZERNIKE COMA

In **Figure 2A**, we consider an eye suffering from pure normalized Seidel-like coma aberration with a coefficient of 1 μm . However, the use of a Zernike coma mode to reconstruct such a wavefront adds the need for a non-null tilt coefficient to compensate for the tilt term present

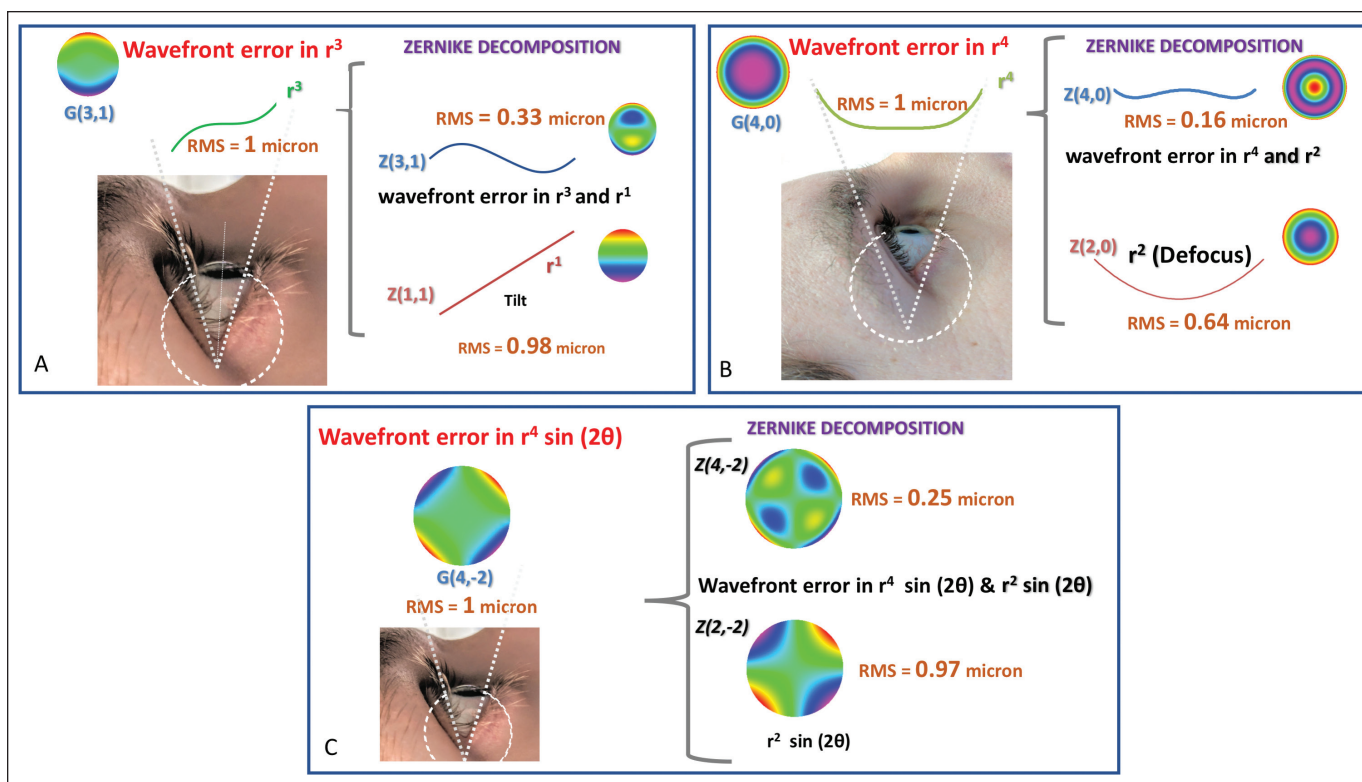


Figure 2. Higher order, higher degree (HD) modes of clinical importance (root mean square [RMS] = 1 μm) and their corresponding Zernike decomposition. The induction of non-null, lower order coefficients for Zernike tilt or defocus is caused by the necessity to compensate for the presence of corresponding lower order terms with the analytical expression of these higher order Zernike modes. This also results in a reduction of the value of the affected Zernike coefficient (A) coma, (B) spherical aberration, and (C) secondary astigmatism.

within the Zernike coma mode. Quantitatively, if one expresses this wavefront in Zernike modes, the coefficient of the coma mode is divided by 3, and an almost equivalent tilt coefficient (0.98 μm) is induced. This tilt is an artifact, caused by the need to cancel the tilt introduced by the Zernike coma mode. In eyes with increased levels of odd order coefficients, such as Zernike coma, some concomitant tilt elevation is always observed.

DEFOCUS IN ZERNIKE SPHERICAL ABERRATION

Figure 2B represents a cross-sectional depiction of a wavefront error, acquired in mesopic conditions, which is limited to a normalized Seidel-like spherical aberration error and refers to an error in r^4 in the wavefront decomposition. This wavefront error is typical of eyes operated on with myopic laser in situ keratomileusis, where emmetropia is achieved, but with a far too narrow functional optical zone. However, the reconstruction of the spherical aberration in r^4 with a Zernike spherical aberration mode will add unwanted defocus in r^2 , which must be compensated for. However, this compensation will appear in the Zernike decomposition as defocus within the lower order wavefront component, reducing the correlation between the clinical refractive defocus and the value of the coefficient weighting the Zernike defocus.

PRIMARY ASTIGMATISM IN ZERNIKE SECONDARY ASTIGMATISM

Similarly, the presence of primary astigmatism in Zernike secondary astigmatism may reduce the pertinence of the refractive astigmatism evaluation when performed based on a Zernike coefficient expansion (**Figure 2C**). For 1 μm of “pure” normalized 4th order astigmatism wavefront error, the magnitude of secondary astigmatism is reduced by 4 in the Zernike coefficient expansion. Additionally, almost the same magnitude (0.97 μm) of lower order astigmatism is introduced in the decomposition.

CASE STUDIES: CLINICAL EXAMPLES ZERNIKE VERSUS LD/HD POLYNOMIAL DECOMPOSITION

To illustrate the differences between Zernike coefficients and the new method of aberration analysis in real eyes, we have chosen two clinical examples. In the clinical setting, wavefront aberrometry currently expresses the wavefront as a sum of the Zernike weighting coefficients. From these values, one can calculate the values of the new coefficients of the LD/HD basis, by adequately splitting the terms obtained from the lower and higher Zernike modes into their respective wavefront components. The lower and higher wavefront compo-

EMMETROPIA BUT HALOS POST MYOPIC LASIK

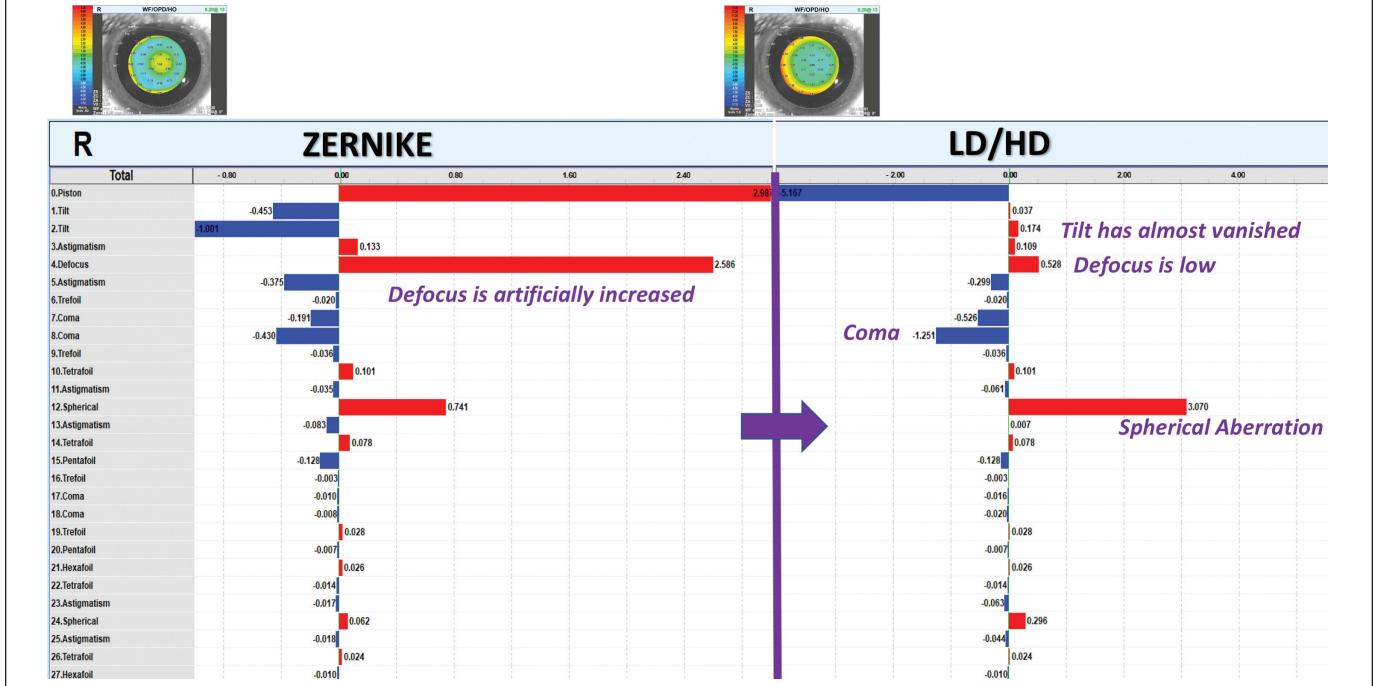


Figure 3. In the lower degree/higher degree (LD/HD) decomposition, there is no artificial reduction of the coma and spherical aberration coefficients, which are clearly highlighted in the coefficients list. The tilt and defocus coefficients have significantly reduced.

nents are then expanded into the LD and HD modes series, respectively. We obtained the LD/HD wavefront coefficients by using a beta-software coded to convert the Zernike coefficients obtained from the usual fitting of the wavefront error recorded on a OPD-scan III wavefront aberrometer (Nidek, Gamagori, Japan). The case studies were conducted in accordance with the tenets of the Declaration of Helsinki.

Example 1. A 26-year-old man experiences mild halos at night after laser in situ keratomileusis for the correction of myopia (-5.00 diopters [D]). His right eye is emmetropic and has an uncorrected distance visual acuity of 20/15. The Zernike decomposition of the total wavefront on a 6-mm pupil has a mixture of lower order aberration and HOA coefficients, with the most prominent being defocus, despite the fact that the eye is emmetropic. Consequently, the Zernike predicted spherical equivalent (-2.00 D) is affected by the defocus coefficient, which correlates with the need for compensating for lower order terms embedded in the Z_4^0 mode (**Figure 3**). The clinician may falsely think that some myopia remains. The weighted coefficient for spherical aberration Z_4^0 is reduced by a factor of 6, compared to that of the “pure” normalized Seidel-like spherical aberration mode in r^4 .

Due to the presence of the defocus term in r^2 in the Zernike higher order mode, the higher order point

source function (PSF) light spread is exaggerated (**Figure 4**). The concentric annulus of light energy visible in the Zernike higher order PSF plot is caused by the interaction of (unwanted) r^2 and r^4 terms within the Zernike spherical aberration mode. The presence of defocus explains the marked reduction of the retinal image sharpness, when compared to the alteration incurred by pure HOA modes. For each micron of Zernike spherical aberration, we must take a square root of 15, so nearly 4 μm of additional defocus is required to cancel the term in r^2 , in the presence of a pure higher order mode in r^4 . The Snellen chart retinal image simulation, obtained via convolutional techniques from the PSF function, suggests an exaggerated visual blur and a best spherocylindrical visual acuity of less than 20/50. The LD/HD decomposition has negligible tilt and defocus, with no artificial reduction of the coma and spherical aberration coefficients compared to their Zernike counterparts. The truly clinically significant HOAs are highlighted within the novel decomposition. The PSF and simulated Snellen chart retinal image are in line with the patient’s actual visual performance.

Example 2. A 28-year-old man with keratoconus complains of a right-sided visual disturbance. The subjective refraction is +0.50 -5.00 \times 75° and the corrected distance visual acuity is 20/20. There is a reduc-

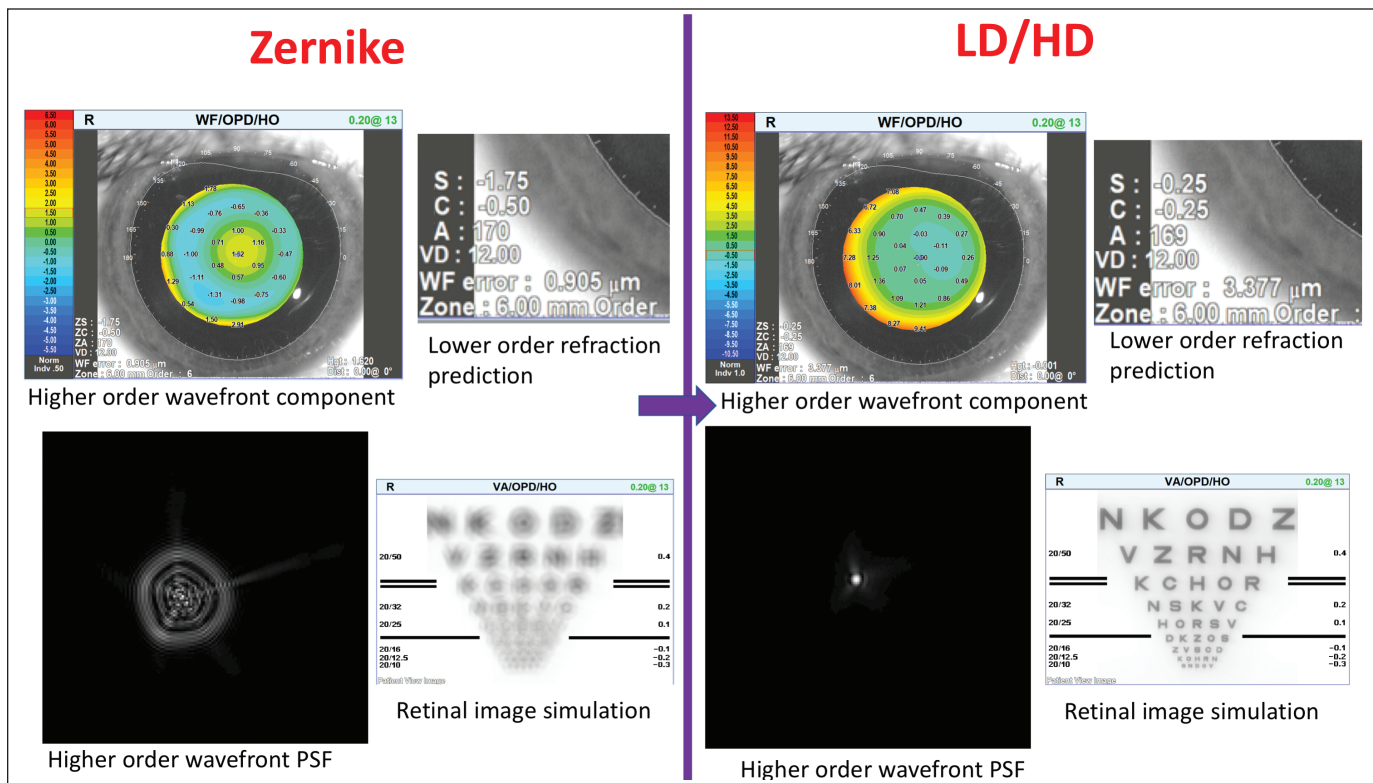


Figure 4. Comparison of the higher order wavefront components, predicted refraction, higher order point spread function, and retinal image in the Zernike versus lower degree/higher degree (LD/HD) wavefront decompositions (6-mm pupil) of a patient with halos after laser in situ keratomileusis. PSF = point spread function

tion of tilt aberration and an increase in significance of some HOAs in the LD/HD method (**Figure 5**). The low degree component of the wavefront, on a 6-mm pupil, provides a better estimation of the subjective refraction with the LD/HD method (**Figure 6**). The higher order retinal image prediction seems better correlated with the corrected distance visual acuity in the LD/HD method because the higher order wavefront component is free from lower order terms. The Zernike predicted refraction is biased by the defocus and astigmatism coefficients, which correspond to artifactual compensation for lower order terms embedded in the Z_4^0 and $Z_4^{\pm 2}$ modes. The presence of defocus explains the reduction of the retinal image sharpness, when compared to the alteration incurred by the pure HOA modes.

DISCUSSION

One can see from these examples that the new basis allows for better segregation of the pure lower and higher order wavefront components, allowing for a clinically relevant wavefront analysis.

Because its higher order modes are free from linear and quadratic terms, our new basis function can be used to better fit the higher order element of the wave-

front. The collection of all quadratic defocus terms, which are parceled in lower and higher order modes of azimuthal frequency 2, corresponds to paraxial curvature matching of the wavefront aberration map. This method has been shown to be one of the most accurate for determining the spherical equivalent error, whereas least-squares fitting of the wavefront is one of the least accurate.⁹ We can also infer that the objective refraction calculated with the novel polynomials will be more accurate for eyes exhibiting an increase in HOAs.

The lack of orthogonality between the lower and higher order components may not be too detrimental in ophthalmic optics, where clinical interpretation considers lower order aberrations and HOAs separately. Clinicians generally prefer to estimate the impact of the LD component of the wavefront through the classic expression of spherocylindrical refraction into diopters. In clinical optics, it is not common to calculate the total RMS values including a mixture of both LD and HD coefficients. Using the LD/HD decomposition still allows computation of total or grouped RMS of the HOAs.

Due to the presence of lower order terms of negative sign in their analytical expression, the normalization coefficients of some of the higher order Zernike

KERATOCONUS

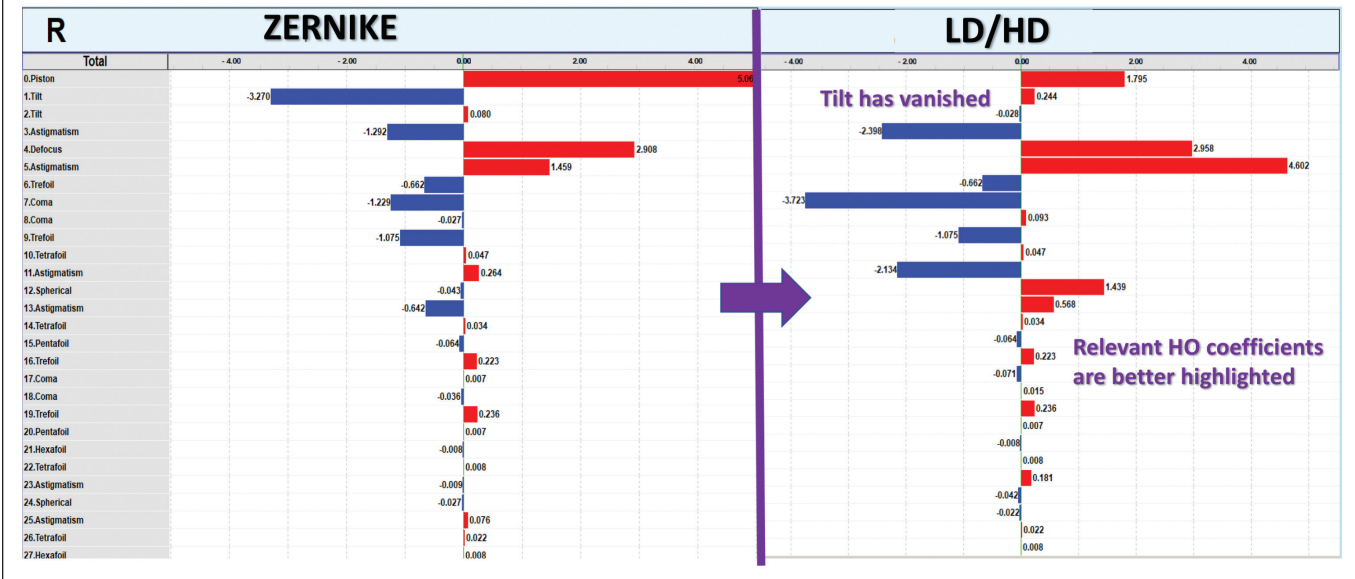


Figure 5. In the lower degree/higher degree (LD/HD) decomposition, there is no artificial reduction of LD of the coma, spherical aberration, and secondary astigmatism coefficients. HO = higher order

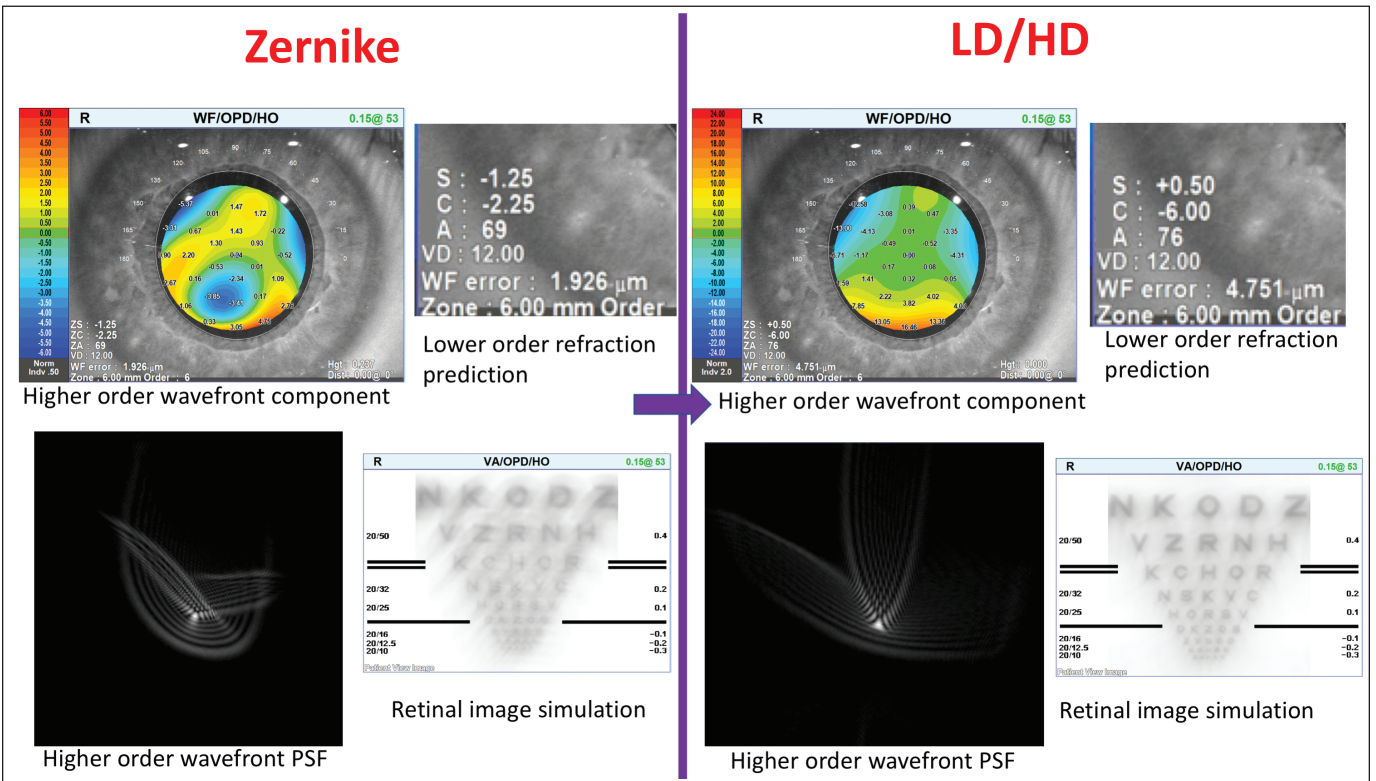


Figure 6. Comparison of the higher order wavefront components, predicted refraction, higher order point spread function (PSF), and retinal image in the Zernike versus lower degree/higher degree (LD/HD) wavefront decompositions (6-mm pupil) of a patient with keratoconus.

modes have higher magnitudes than their corresponding HD modes, which are free of lower order terms.

This results in the minimization of the magnitude of some Zernike coefficients of clinical importance, such

as $Z_3^{\pm 1}$, Z_4^0 , and $Z_4^{\pm 2}$, compared to their counterparts in the LD/HD basis (Figure 2). This may reduce the apparent contribution of the higher order wavefront modes in the total wavefront expansion.

The LD/HD polynomial basis allows the calculation of optical metrics that are more realistic compared with the Zernike classification, to evaluate the impact of high-level aberrations on the quality of the retinal image (Figures 4 and 6). The computation of the higher order PSF and simulated retinal image from the HD mode coefficients is no longer affected by the presence of LD terms in the higher order wavefront component.

When planning customized laser corrections, better inferences can be made by refractive surgeons to give realistic clinical results, without the need for compensatory treatment programming. Undesirable refractive results can occur from corrections planned using Zernike wavefront analysis, if the surgeon does not anticipate the unwanted interactions between lower order terms present within the higher order wavefront components. The presence of lower order terms in higher order modes increases the risk of surplus laser ablation when correcting spherocylindrical and higher order optical errors, and the LD/HD wavefront decomposition reduces this risk. Although the biomechanical and wound healing effects of removing tissue will still occur, it is expected that a profile of ablation truly restricted to higher order wavefront errors would incur less unpredictable effects. Additionally, compensation of presbyopia using multifocal corneal laser ablations may benefit from a better understanding of the selective impact of higher order terms on the depth of focus.¹⁰⁻¹³

Some of the problems of the Zernike polynomials and in particular of the standards defined by the OSA Taskforce for measuring and reporting aberrations for the human eye have been discussed before.^{14,15} Alternative representations to the Zernike polynomials have been applied in visual sciences,¹⁶ including the unsuitable Fourier series.¹⁷

We have expanded on the concept and application of the novel polynomial series, which keep most of the desirable properties of the Zernike basis. This allows current ophthalmic practitioners to understand and adopt a more clinically relevant method of ocular wavefront analysis for their patients. The comparison of the Zernike and LD/HD sets of reconstruction polynomials on a larger dataset of eyes is required to confirm this working hypothesis.

AUTHOR CONTRIBUTIONS

Study concept and design (DG, LD, JM); data collection (DG, LD, JM); analysis and interpretation of data (DG, RR, LD, JM); writing the manuscript (DG, RR);

critical revision of the manuscript (DG, RR, LD, JM); administrative, technical, or material support (DG, RR, LD, JM); supervision (DG)

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