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Response and damage identification by long-gauge strain sensor of beam-like bridges under vehicle's loads: a comparison of the analytical model and in situ measurement

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Abstract. Bridge assessment and monitoring are important subjects nowadays. While the monitoring of displacement can be challenging, the bridge monitoring based on strain measurement is preferred. The latter can be obtained directly by sensors such as fiber optics. This kind of sensors has some advantages such as precision, high sampling frequency, and durability. In order to analyze quickly the measurement results from this sensor, we have developed an analytical model of a bridge structure to study the influence of the vehicles (loads, velocity) on the behavior of the structure. This paper presents the aforementioned model which allows us to calculate rapidly the dynamic responses (displacement, average strain, acceleration...) of a beam-like structure under a vehicle load by using a transfer matrix of two successive beam segments. Moreover, a damage detection and localization method based on distributed long-gauge strain sensors is proposed. Next, the results of the model are compared with the measured data of a multi-girder prestressed concrete bridge located in France. The comparison shows that the model is efficient for fast computing of the bridge responses. Finally, numerical results of the aforementioned method will be discussed.

Introduction

Most bridges in the world have been built in the 20th century. Hence, bridge assessment and monitoring are necessary. One of the preliminary stages of bridge SHM (Structural Health Monitoring) is to construct a direct model that can predict the behavior of the structure under traffic loads. Many studies have been carried out about this problem: Ichikawa *et al.* [1] found the solution of a continuous beam subjected to a moving mass by eigenmodes superposition. Dugush *et al.* [2] used infinite polynomial series to describe the mode shapes of a continuous beam with non-uniform section. In another approach, Hoang *et al.* [3] used space-time Fourier transform to calculate the dynamic response of a periodically supported beam. Recently, Pham *et al.* [4] decoupled a continuous cracked beam into several segments. By introducing transfer matrices that connect these segments, the eigenvalue problem has been solved directly.

The presence of structural damage will modify the dynamic behavior of the beam. Many researchers have studied the effect of cracks on the natural frequency of a cantilever beam, and the identification of the location and the magnitude of the cracks [5, 6]. However, these methods require many sensors. A moving load is an excitation which depends not only on time, but also on space. From the response of the beam, we can find out its modal properties such as the natural frequencies, the mode shapes... Nowadays, bridges can be monitored continuously thanks to advanced technology. Unlike the traditionally electrical strain gauge, a long-gauge optical strand measures the average strain over its length. Hence, it has a higher probability to cover the damage. Moreover, Optical Strand sensors have many advantages such as high-sample strain measurements over long periods, high precision and durability [7]. The use of these sensors is not only restricted in civil engineering, but also in aviation, pipeline transport [8].

This paper presents firstly an analytical method that describes the response of a damaged continuous beam which is subjected to a multi-axle vehicle load. Based on the longitudinal distribution of strain on a simply supported beam, we propose a damage index which can be used to identify and localize the damage. By using this analytical method, a parametric study is carried out to study the dependence of the proposed index to the properties of the damage. Finally, the analytical method is applied to calculate the response of an existing structure, and the results are compared to the real signals obtained during the test.

Formulation of the problem

Consider a continuous beam of total length L , which has N_s spans. In the general case, we suppose that a damage is located in the k^{th} span. The damaged span is divided into 3 separated beam segments: the damaged zone of length L_e and two intact segments which are on the left and on the right of the damaged segment. The initial continuous beam can be modelled as a system of $N = N_s + 2$ beam segments. A vehicle of M -axles passes over the beam with constant velocity v . Let's denote d_i the distance between the i^{th} and $(i+1)^{\text{th}}$ axle and $D_{i-1} = \sum_{j=1}^{i-1} d_j$ is the distance between the first axle and the i^{th} axle ($i \geq 2$).

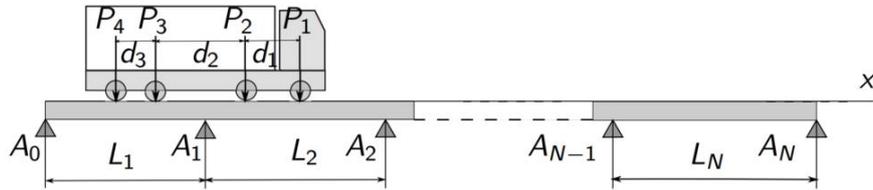


Figure 1. A continuous beam with N spans subjected to M moving loads.

Assume that the beam follows the Euler-Bernoulli beam theory and has a linear elastic behavior. The governing equation of the beam which describes the vertical displacement $u(x, t)$ is [9]:

$$EI \frac{\partial^4 u}{\partial x^4} + EI^* \frac{\partial^5 u}{\partial t \partial x^4} + c \frac{\partial u}{\partial t} + \rho A \frac{\partial^2 u}{\partial t^2} = f(x, t), \quad 0 \leq x \leq L. \quad (1)$$

where EI is the flexural stiffness, ρ is the mass density and A is the cross-sectional area of the beam. In Eq. 1, the familiar term $c(\partial u / \partial t)$ represents the external damping whereas the term $EI^*(\partial^5 u / \partial t \partial x^4)$ represents the internal damping. The force applied by the vehicle can be expressed by:

$$f(x, t) = P_1 \delta(x - vt) + P_2 \delta(x - vt + D_1) + \dots + P_M \delta(x - vt + D_{M-1}) = \sum_{i=1}^M P_i \delta(x - vt + D_{i-1}).$$

where $\delta(x)$ is the Dirac distribution.

The solution of the initial equation can be decoupled into two terms: the time-dependent term $q(t)$ and the mode shapes $\phi(x)$:

$$u(x, t) = \phi(x)q(t). \quad (2)$$

Substituting Eq. 2 into Eq. 1, setting $f(x, t)$ to be zero to obtain the free vibrations and by separating the variables, we have:

$$\frac{\phi''''(x)}{\phi(x)} = -\frac{c\dot{q} + \rho A \ddot{q}}{EI^* \dot{q} + EIq} = \lambda^4. \quad (3)$$

Natural frequencies and mode shapes.

From Eq. 3, the mode shapes of the r^{th} beam segment are the solution of the following differential equation:

$$\frac{d^4 \phi_r(x_r)}{dx_r^4} - \lambda^4 \phi_r(x_r) = 0, \quad (4)$$

where $\phi_r(x_r)$ is the mode shape of the r^{th} beam segment. The form of $\phi_r(x_r)$ can be expressed explicitly as:

$$\phi_r(x_r) = A_r \sin \lambda x_r + B_r \cos \lambda x_r + C_r \sinh \lambda x_r + D_r \cosh \lambda x_r. \quad (5)$$

Configuration of the Beam. By the same concept of transfer matrices as introduced by Pham *et al.* [4], we search the transfer matrix \mathbf{M} such that $X_r = \mathbf{M}X_{r-1}$. Here $X_r = [A_r, B_r, C_r, D_r]^T$ is the vector which contains the coefficients of the mode shapes $\phi_r(x_r)$ for the r^{th} beam segment. This 4x4 matrix ensures the continuity conditions at the intermediate support points and at the interface between the intact segment and the damaged segment.

The intermediate supports. The intermediate supports are assumed to be infinitely rigid, so the displacement is blocked at these points. Moreover, the slope of the beam and the bending moment are unchanged across the support. Considering the $(r-1)^{\text{th}}$ support which is found between the $(r-1)^{\text{th}}$ span and the r^{th} span, the above conditions can be written as:

$$\begin{aligned} u_r(0, t) &= u_r(L_r, t) = 0, \\ u'_r(0, t) &= u'_{r-1}(L_{r-1}, t), \\ u''_r(0, t) &= u''_{r-1}(L_{r-1}, t). \end{aligned} \quad (6)$$

We obtain $\mathbf{S}(L_{r-1}, L_r)$ the transfer matrix at the intermediate support, such that: $X_r = \mathbf{S}_{4 \times 4} X_{r-1}$. This matrix can be expressed as:

$$\mathbf{S}_{4 \times 4} = \frac{1}{2} \begin{bmatrix} -\kappa.s + 2\eta.c & -\eta.s - \kappa.c & \kappa.sh + 2\eta.ch & 2\eta.sh + \kappa.ch \\ s & c & -sh & -ch \\ \kappa.s + 2\varepsilon.c & -2\varepsilon.s + \kappa.c & -\kappa.sh + 2\varepsilon.ch & 2\varepsilon.sh + \kappa.ch \\ -s & -c & sh & ch \end{bmatrix} \quad (7)$$

in which:

$$\begin{aligned} s &= \sin \lambda L_{r-1}, c = \cos \lambda L_{r-1}, sh = \sinh \lambda L_{r-1}, ch = \cosh \lambda L_{r-1}, \\ \kappa &= (\cos \lambda L_r - \cosh \lambda L_r) / (\sin \lambda L_r - \sinh \lambda L_r), \eta = -\sinh \lambda L_r / (\sin \lambda L_r - \sinh \lambda L_r), \varepsilon = 1 - \kappa. \end{aligned}$$

The interface between intact and damaged beam segments. In the literature, a crack can be modelled by a spiral spring or by a reduction of the bending stiffness in the concerned zone [10]. In this paper, we use the second model to describe the damage. We denote $(EI)_1$ and L_1 respectively the bending stiffness and the length of the segment located on the left of the interface. By the same way, $(EI)_2, L_2$ are the bending stiffness and the length of the right segment. Across the interface, the traverse displacement, the slope of the beam, the bending moment and the shear force are unchanged. Therefore the continuity conditions at the interface are:

$$\begin{aligned} u_2(0, t) &= u_1(L_1, t), & u'_2(0, t) &= u'_1(L_1, t), \\ (EI)_2 u''_2(0, t) &= (EI)_1 u''_1(L_1, t), & (EI)_2 I u'''_2(0, t) &= (EI)_1 I u'''_1(L_1, t). \end{aligned} \quad (8)$$

The transfer matrix at the interface $\mathbf{T}(k, L_1)$ is such that $X_2 = \mathbf{T}(k, L_1)X_1$:

$$\mathbf{T}_{4 \times 4} = \frac{1}{2} \begin{bmatrix} (1+k)c & -(1+k)s & (1-k)ch & (1-k)sh \\ (1+k)s & (1+k)c & (1-k)sh & (1-k)ch \\ (1-k)c & -(1-k)s & (1+k)ch & (1+k)sh \\ (1-k)s & -c(1-k)c & (1+k)sh & (1+k)ch \end{bmatrix} \quad (9)$$

in which:

$$k = \frac{(EI)_2}{(EI)_1},$$

$$s = \sin \lambda L_1, c = \cos \lambda L_1, sh = \sinh \lambda L_1, ch = \cosh \lambda L_1.$$

By assembling all these aforementioned matrices together and taking into account the boundary conditions of the continuous beam, we deduce an eigenvalue problem to solve for λ . Once the values of $\{\lambda\}$ are found, by integrating both sides of the initial equation, we solve the same initial differential

equation for the general coordinates $q(t)$, segment by segment. Finally, by superposition of the modes, we have the analytical solution of the problem.

Damage identification using distributed long-gauge strain sensors

Method. For a simply supported beam, it can be shown that the maximum and the area values of the static macro-strain follow a parabolic distribution [11]. Some damage indices based on this property were proposed in [11, 12]. Moreover, the parabolic shape of the area values is still correct in case where the total strain is measured by a long-gauge sensor, which is the sum of static and dynamic strain. For illustration, it is supposed that a sequence of 10 Optical Strands sensors of equal length $l_g = 1 \text{ m}$ placed under a simply supported beam of 10 m length. The sensors are named as CO-1 – CO-10, from left to right. Fig. 2a presents the numerical results of total macro-strain time histories obtained by these Optical Strands, subjected to a 2-axle vehicle which passes over the beam at constant speed $v = 5 \text{ m/s}$. The front load and the rear load are 1 kN and 2 kN, respectively. The distance between these two loads is 3m. The beam is made of steel ($E = 210 \text{ GPa}$, $\rho = 7800 \text{ kg/m}^3$). The Fig. 2b shows the distribution of the area of these responses. The orange line is the regression parabola \hat{f} .

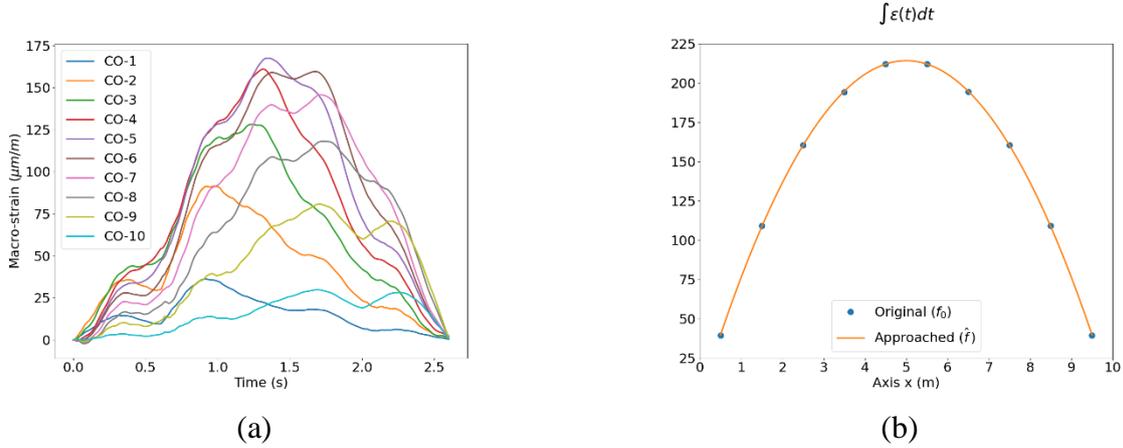


Figure 2. Macro-strain time histories of a simply supported beam and the distribution of their area.

We propose a damage index (DI) which measures the relative error between the approached parabola and the measured points:

$$EG = 100 \times \frac{\|f - f_0\|_2}{\|f_0\|_2}. \quad (10)$$

in which: $\|\mathbf{x}\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$ is the Euclidian norm of vector \mathbf{x} .

In order to localize the damaged zone, we propose an additional criterion, which is based on the above damage index EG. It is calculated by the same way of the criterion EG, but the considered point ε_i is excluded:

$$EL_i = EG^i = 100 \times \frac{\|f^i - f_0^i\|_2}{\|f_0^i\|_2}. \quad (11)$$

Results. To illustrate the above criteria, we take the same steel beam and same vehicle load as presented on the previous subsection. We create a damaged zone at the center of the beam. The length of the damage varies from 0.5 m to 3.0 m and the reduction of bending stiffness on this zone varies from 0% (intact) to 50%. As a parabola is defined by 3 parameters, at least 4 measure points are required to determine the damage index EG. Hence, to identify and localize the damage, at least 5 measured points are required. In this parametric study, we use 5 Optical Strands with 2 metre-length each, covering the whole length of the beam. The response of the beam is calculated by the current analytical model.

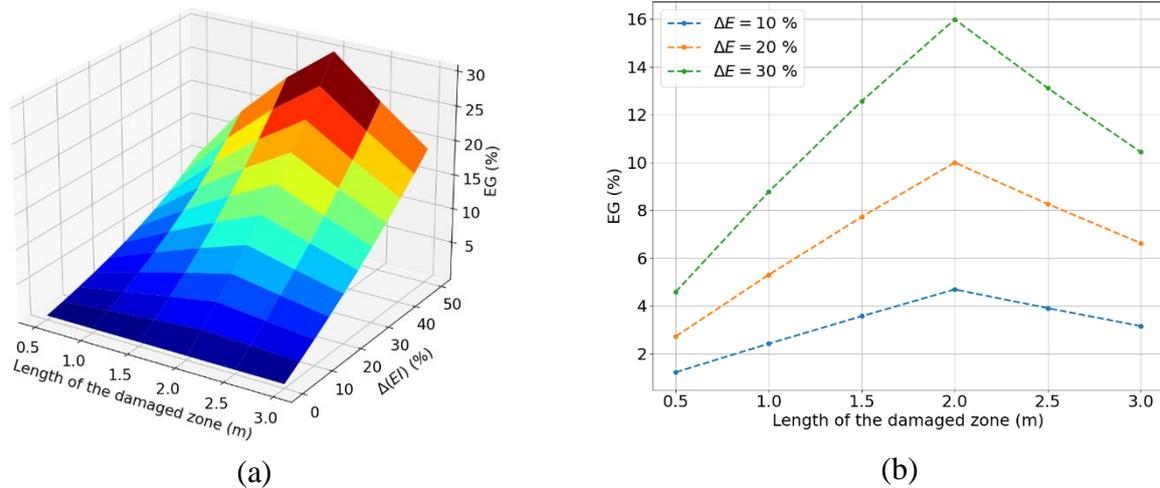


Figure 3. Damage index EG in function of damage's properties

Fig. 3 presents the dependence of EG as the function of the length and of the reduction of bending stiffness of the damaged zone.

From Fig. 3a, we see that EG varies linearly with $\Delta(EI)$. Fig. 3b shows that the damage index EG get its maximal value while the damage's length is equal to 2m. The relation between EG and the size of the damage is also linear. The global damage index is exceeded 5 % for a damaged zone of length 1.5 m and a reduction of bending stiffness $\Delta E = 10\%$.

To localize the damage, the local criterion EL is calculated in case the size of the damage is 2 m, for different levels of reduction of bending stiffness. We see that this index EL vanishes at the sensor which corresponds to the damage (Fig. 4). For a damage of $\Delta E = 10\%$, the average value of EL is over 4 %.

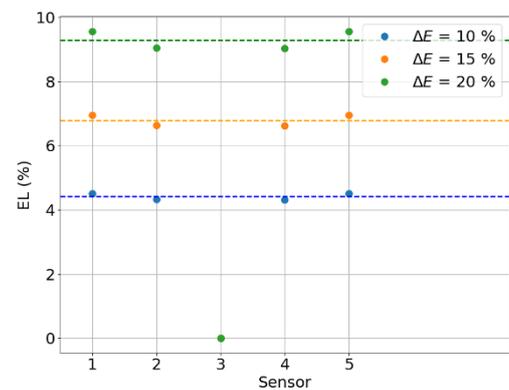


Figure 4. Local criterion for localizing the damage.

Application to bridge structure

The analytical method is now applied to calculate the strain response of an existing highway overpass in France. This prestressed concrete girder bridge was built in the 1960s. The bridge has 4 spans, where each of those is composed of 10 prestressed concrete beams. The structure has been monitored since 2011 by the Optical Strands sensors of length 1 m (red) and 2 m (blue). Fig. 5 shows the instrumented plan of the bridge with the selected sensors. There was a calibration test on the bridge in 2019.

One of the test trucks is a 5-axle of 44 tons. From the first axle, the distances between two consecutive axles is: $d_1 = 3.8$ m, $d_2 = 3.0$ m, $d_3 = d_4 = 1.25$ m. The axle loads P_1, P_2, P_3, P_4, P_5 are respectively 6.94 t, 10.62 t, 9.2 t, 8.64 t and 8.16 t. The vehicle passes over the bridge in direction S-N at constant speed 50 km/h, so it enters firstly into the Span 3, then the Span 2. In the analytical model, the bridge is considered as a set of T-beams in parallel. The general cross section of the T-beam is calculated for its properties (second moment of area, area...). The density of concrete is 2500 kg/m^3 and its Young modulus is 36.6 GPa. Because the forces are also distributed in the transverse direction, a coefficient of 0.14 is taken into account for the response of the largest signal. The other signals are re-constructed from the load distribution factor obtained for this girder bridge.

Fig. 6 shows the results of the analytical method comparing with the signal obtained from the in-situ test. The solid lines present the results calculated by the present analytical method, while the dashed lines are the measured strain during the test. The calculated and measured signals are coherent.

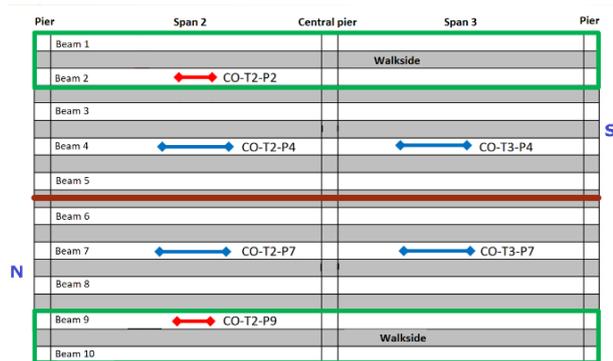


Figure 5. Instrumentation plan on the central spans of the bridge.

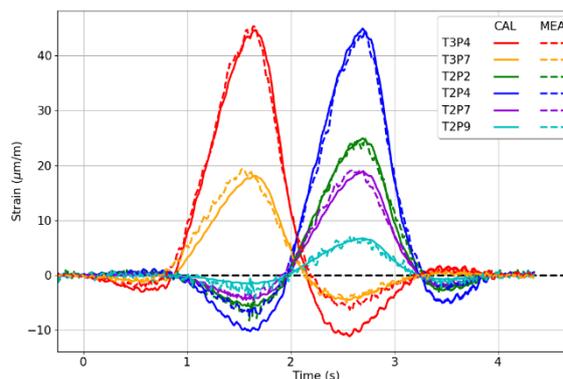


Figure 6. Comparison of the analytical results and in situ measurement.

Conclusion

The analytical method gives us quickly the dynamic response of the bridge structure. It takes under 10 seconds to calculate the presented parametric study, whereas the same model calculated by FEM costs more than 15 minutes in total. From the distribution of the area of strain time history along the beam, the proposed index can detect the presence of a damage and localized its position. The coherence between the simulated results and the real signals gives an opportunity for the analytical method on modelling of the bridge, in order to obtain rapidly the total response of the structure.

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