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Fighting Free with Free: Freemium vs. Piracy*

Antoine Dubus[†] Christine Halmenschlager[‡] and Patrick Waelbroeck[§]

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Abstract

We analyze the optimal business model of a firm facing piracy. The firm either sells a premium version of its product, or also offers a free version along with the premium version. The firm can in turn impose restrictions on the use of the free version. Consumers can choose between the free and the premium version, but can also get an illegal digital copy. We show that freemium offers can reduce digital piracy by fighting free with free and that firms choose their optimal business model depending on the strength of copyright protection. Therefore, the strength of copyright protection can lead firms to choose a traditional business model rather than the new freemium model, impacting significantly the legal usage of the good.

Keywords: Online piracy, versioning, freemium, streaming, copyright, music.

JEL Classification: L12, L82, L86, O34.

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1 Introduction

Digital piracy has shaken up the pre-recorded music and movie industries ([Rob and Waldfogel, 2006](#)). Online piracy constitutes a significant threat to providers of digital content: pirated TV shows and films represent more than 230 billion views a year, inducing an expected loss of 11.58 billion USD for the TV and movie industries for the year 2022 in the US only.¹ Online content providers are therefore under huge pressure to design the best business models to deter online piracy.²

In reaction, cultural industries have experienced an increase in copyright protection through active online copyright enforcement such as graduated response schemes and the shutdown of illegal platform distributions, such as Napster, Kazaa or eMule, and of online servers of The Pirate Bay and Megaupload.

The strength of these measures varies greatly across countries. Most European countries have adopted a lenient view over copyright protection, resulting in a laissez-faire approach. On the contrary, to fight online piracy, the French government created Hadopi (Haute autorité pour la diffusion des oeuvres et la protection des droits sur Internet), an independent institution with the power to cut the connection of Internet users caught downloading copyrighted files from P2P networks (after several warnings). Similar measures have been enacted in many other countries including the U.S., New Zealand, Ireland, the United Kingdom, and South Korea.

What policymakers have sometimes failed to notice is that the Internet has also profoundly transformed how consumers access music. New ways of consuming digital music (streaming, download à la carte, cloud, transfer and synchronization with smartphones) have attracted new customers, therefore expanding demand. In addition to these demand-side effects, digitization has also transformed the supply side of digital industries by decreasing the costs of production, distribution and promotion of digital goods, as well as created new business models.

Freemium has become a mainstream way to provide cultural products to con-

¹18 Piracy Statistics To Keep You Away From Trouble in 2022, [Web Tribunal](#), last accessed December 12, 2022.

²See also [Givon et al. \(1995\)](#) for early estimations of losses due to piracy in the software industry.

sumers, not only in the pre-recorded music industry but also in digital video distribution, videogames and the media industry. Freemium services offer a free subscription service financed by ads and one or two premium services. Premium subscribers can enjoy the good without the ad nuisance, and they also may have access to additional services such as online communities and recommendation services.

Hence, freemium business models have become common in a wide range of industries. Indeed, most online music retailers and platforms propose freemium streaming services including large platforms such as Spotify, Apple and Amazon. Online movie rental companies have also widely adopted freemium business models, as companies such as Fandango at home and Rakuten TV offer the possibility to access a free version of a movie with advertising, or a paid version without ads.³ Online newspapers also offer free and premium content, as some articles can be previewed for free (with ads) and read in their totality at a monetary cost. Most video games on smartphone are also free and financed by ads, and the user can decide to make in-app purchases to improve their gaming experience.⁴ These firms also face intense competition from pirate websites, as users who are not ready to pay for premium content can also download pirated versions.

The main research question that we address in this article is the following: is an increase in copyright protection detrimental to the development of new business models that monetize free users? In turn, how does a stronger copyright protection regime impact the authorized usage of the good? These are important legal and economic questions. Indeed, the main justification for the implementation of graduated response schemes or active online copyright enforcement rests on the idea that without stronger copyright protection, the legal market would collapse and eventually disappear. However, this argument is only valid for a given business model. What if the strength of copyright protection also impacts the choice of cultural companies to invest in new business models? To give a preview of the model, we note that freemium models monetize consumers with low willingness to

³See [What is Fandango at Home?](#), Xumo; and [What is “Rakuten TV Free”?](#)

⁴Similarly, the free version of some online PC games (such as FIFA, Roblox, Smite, Brawlhalla as well DLC sponsored by brands in Fortnite and Mario Kart) are financed by ads and players can purchase premium content.

pay (through revenue from ads or cheap customer acquisition), who are exactly those who are more likely to consume pirated content. Therefore, by giving these consumers an opportunity to consume free content legally, freemium offers have the potential to reduce digital piracy. However, depending on the strength of copyright protection, it might be more profitable for firms to stick to purchase-only business models and to miss opportunities to adopt new business models based on freemium.

We analyze a theoretical model in which a firm sells a premium version of a good to consumers, and can also decide to offer a free version generating advertising revenues. The firm faces a competitive pressure from pirated versions of the content, for instance from Peer-to-Peer networks, from illegal online servers (such as the defunct Mega Upload website), or from illegal websites that offer free direct streaming services in the case of films and videos.⁵

We focus on the design of the free version offered by the firm, and analyze how the free and the premium versions of the good can deter piracy. The firm can lower the quality of the free version, for instance by restricting access to some of the content. The strategy of the firm is twofold: choosing the level of restrictions of the free version, as well as the price of the premium version. Restrictions on the free version constitute a central element of the design of the freemium business model, and we show that they have an important impact on the profitability of the firm. These restrictions include among other things the number of hours per month available for users of the free version, the size of the catalog available to free users, or the quality of the good in the case of videos and music.

Consumers face two types of costs when they acquire illegal digital goods. On the one hand, the copy might be of lower quality than the premium version, and this degradation cost increases with the taste for quality of a particular user. On the other hand, copying induces a fixed reproduction cost, for instance as piracy comes along with some fees (e.g. to have access to a digital locker, or private network), or because there are ethical and legal costs that people face when they get digital content from illegal sources. These two costs are impacted by the

⁵While piracy was initially mostly done by a small group of specialized pirates, [Reimers \(2016\)](#) provides empirical evidence showing that today, any consumer can use a pirated version.

strength of copyright protection, and we analyze how the optimal business model chosen by the firm changes with the degradation and reproduction costs.⁶ In particular, we show how these costs can represent property and liability rules used to enforce copyright protection.

We characterize the optimal business model of the firm as a response to the pressure exerted by the pirated version of the product. Introducing the free version of the good has two opposite effects on the profits of the firm. On the one hand, some of the users of the premium version are better off consuming the free version and this creates a cannibalization effect that reduces the profits of the firm. On the other hand, some consumers will choose the free version instead of the pirated version and this generates a market-expansion effect resulting from additional advertising revenues. By determining the level of restrictions of the free version and the price of the premium version, the firm can maximize the market-expansion effect while minimizing the cannibalization effect. The optimal price and level of restriction constitute the business model of the firm, which crucially depends on the properties of the pirated good and on copyright enforcement.

When the quality of the pirated good is low, the free version represents a credible alternative to the pirated version for some users, and can be used by the firm to fully deter piracy. In this business model, the firm benefits from advertising revenues generated from a large share of consumers who do not use the pirated version. On the contrary, when the quality of the pirated good is high, the firm must offer a high quality for the free good to deter piracy. The resulting cannibalization effect on the premium good is strong, and the firm might be better off offering only the premium version with a high price: by doing so, the firm extracts enough surplus from consumers with the highest willingness to pay for the product.

We show that freemium offers can indeed deter digital piracy by fighting free with free and that firms choose their optimal business model depending on the strength of copyright protection. Strengthening copyright protection can lead

⁶Bae and Choi (2006) have highlighted the importance of these two dimensions of piracy costs, and our framework encompasses as special cases models in which there is only one of these two dimensions (Novos and Waldman, 1984; Belleflamme and Picard, 2007). In a model with network effects, Volckmann (2023) provide conditions on the piracy costs for the pirated version to represent a credible threat for software developers.

firms to choose a traditional business model rather than the new freemium model. These results have important implications for policymakers who do not consider by default the impact of copyright protection on the adoption of new business models by a firm.

Our results also have implications for empirical research. Indeed, most empirical studies have ignored the development of freemium services when analyzing the impact of stronger copyright enforcement on sales. Finding a strong positive effect could result from stricter copyright protection or from the development of new business models that attract new consumers on the market, creating a potential omitted variable bias.

The article is organized as follows. In Section 2 we review the literature, and in Section 3 we provide an overview of legal responses to digital piracy. We present the setup in Section 4 and show how the firm adapts its business model to deter piracy in Section 5. We analyze the impacts of copyright protection on welfare in Section 6. We provide an interpretation of our model in terms of property and liability rules in Section 7. Section 8 concludes the article.

2 Literature

Previous literature has analyzed firms' responses to copyright laws within a given business model. For instance, [Bae and Choi \(2006\)](#) analyze the impact of copyright protection on the choice of the quality of a firm's product without versioning, while [Cho and Ahn \(2010\)](#) and [Lahiri and Dey \(2013\)](#) consider the choice of quality of different versions by the firm. The novelty of our analysis is to also endogenize the strategic choice of a business model by the firm, i.e., whether selling only a premium version of the product, or proposing also a free version financed by ads. In turn, our contribution is to show that copyright laws and enforcement can have a significant impact on the choice of business model selected by firms. In particular, we show that, when weaker copyright protection decreases the cost to copy, it can induce a firm to adopt a freemium business model in which all users consume a legal version of the good. In this case, total legal usage is maximized. This challenges the existing literature in which lower

copyright protection reduces the demand for the legal good. This result is due to the fact that the free version of the good allows the firm to capture the entire demand of consumers with a low willingness to pay, and could not be achieved without a freemium model.

Digital piracy. Our article contributes to the empirical literature on digital piracy by analyzing the development of freemium services as a response to piracy and to the implementation of stronger copyright enforcement. Most studies consider the substitution between pre-recorded music and digital piracy (Aguilar and Martens, 2016) and between box office sales and digital piracy (Peukert et al., 2017). Few articles study the impact of Subscription VOD (such as Netflix) on digital piracy. Lu et al. (2021) analyze the deployment of Netflix in South Asia.⁷ They find that the unavailability of Netflix in Indonesia led to more searches for pirated content, but the effect is relatively lower for original Netflix content.

Godinho de Matos et al. (2018) find a strong substitution between SVOD services such as Netflix and digital piracy. The level of substitution can be increased or decreased according to the size of the catalog and the number of restrictions on the premium version. Overall these studies do not consider consumers who can access digital content for free from legal streaming services financed by ads and how this changes their trade-off between the premium version and digital piracy. We argue in this article that research linking sales of legal products or services and digital piracy should account for the availability of freemium services to consumers.

Freemium. The literature on freemium mainly addresses the design of the freemium business model: the number of different versions; the number and the type of restrictions on the free version (difficulty or usage) (Aral and Dhillon, 2021; Gu et al., 2018); how to attract new customers (Belo and Li, 2022) or maximize the social engagement of premium users (Bapna et al., 2016). Few articles analyze the effect of ads on the trade-off between the free and the premium version (Huang et al., 2018; Lee et al., 2021) and on the optimality of freemium among other

⁷Netflix was available at the beginning of 2016 in 40 countries, but not in Indonesia.

business models (Sato, 2019).

Other articles study the impact of network effects on freemium design (Shi et al., 2019) and on competitors (Boudreau et al., 2021). Finally, Rietveld (2018) shows that in some cases, free versions of video games are less used and bring lower revenues than just selling a single premium version.

Finding the right product mix is a challenging task. In the beginning, Spotify offered 3 versions: a free version financed by ads, a desktop version at USD 4.99 and a premium version with mobile synchronization at USD 9.99. In 2010, the number of restrictions on the free version changed: a song could only be played 5 times, and a 10-hour limit restriction was imposed on the free version. Then Spotify respectively lifted these restrictions in 2013 and 2014. In 2014, Spotify revised its versioning strategy by offering only 2 versions: a free version and a mobile subscription at USD 9.99. While the question of choosing the right premium offer has been addressed in the literature (Gu et al., 2018), the literature on the freemium business ignores the effect of piracy on the choice of consumers and on the optimal product mix of the firm and we contribute to the literature on this point.

Versioning. Given that information goods can be easily copied and modified at low marginal costs, it is often optimal to create several versions of a digital product (Shapiro and Varian, 1999). Freemium models can be seen as an extreme case of versioning in which the low-quality version is free and is therefore not monetized through a direct payment but indirectly through ad revenues. Freemium models encompass purely ad-based business models (YouTube) and premium-only subscription services (Netflix).

Offering a free version can be seen as a strategy similar to introducing damaged goods studied by Deneckere and Preston McAfee (1996). They show that it is profitable to introduce a damaged version of an existing (full-featured) product when the ratio of the valuation of the damaged good to the valuation of the full-featured good is decreasing in the valuation for the full-featured product.

We contribute to the literature on versioning by showing that freemium is optimal in the presence of a competitive fringe, regardless of the assumption on

the distribution of the taste parameter or on the costs of producing the different versions. Note that the competitive fringe could be anything that competes with the product line sold by the firm and is not limited to digital piracy: open source software, free video games, or even the existence of a second-hand market or parallel imports also represent a competitive fringe.

3 An Overview of Legal Responses to Piracy

Policymakers have adopted different approaches to copyright enforcement, combining property and liability rules (see [Shapiro \(2017\)](#) for a review of these rules in the case of patent infringement). Under property rules the copyright owners can exercise control over their work by seeking injunctive relief from a court, thereby requiring the infringing party to cease immediately their infringing activities. These injunctions are aimed at protecting the copyright owner's exclusive rights by preventing further infringement of their work.

On the contrary, under liability rules the infringer can still use the copyrighted work after being caught, under certain conditions – typically by paying a fine or monetary damages to the copyright owner.

In most countries, copyright laws include both responses and their application depends on the type of good pirated, and the view for each case of policymakers in charge of enforcing the law.

When it comes to the practical enforcement of these rules, several EU member states have implemented or considered three-strikes laws, also known as graduated response or “notice-and-notice” systems. These laws typically involve sending warning notices to Internet users accused of copyright infringement, with escalating penalties for repeated offenses.

France was among the first countries to adopt an institutional three-strikes law in 2009 with its Hadopi system, which allows rights holders to report copyright infringement to a government agency. Users receive warnings for first and second offenses. If a user is caught a third time or more, their case is analyzed by a special committee and the user can face a fine. Similar measures have been enacted in other parts of the world. In the United States, the Digital Millennium Copyright

Act (DMCA) of 1998 includes provisions to protect copyrighted works online and provides mechanisms for rights holders to address copyright infringement. Penalties for copyright infringement in the U.S. can include statutory damages ranging from \$750 to \$30,000 per work infringed, and up to \$150,000 for willful infringement. In some cases, infringers may also face criminal penalties, including fines and imprisonment. In the UK, copyright infringement is addressed under the Copyright, Designs and Patents Act 1988 and related legislation. Penalties for copyright infringement in the UK include civil remedies such as injunctions, damages, and account of profits. Criminal sanctions may also apply, with penalties including fines and imprisonment.

Japan's Copyright Act of 1970, amended to address digital copyright issues, provides protections against piracy and copyright infringement. Penalties for copyright infringement in Japan include civil remedies such as injunctions, damages, and account of profits. Criminal penalties may also apply, including fines and imprisonment. Japan takes measures to combat online piracy, including cooperation between rights holders, Internet service providers, and law enforcement agencies to identify and shut down websites hosting infringing content.

4 Description of the Model

We consider a monopolist firm providing a good to consumers. The firm can choose to either sell only a premium version of its good, or to also propose a free version financed by ads. Moreover, consumers can also obtain a pirated version of the good. Hence, consumers can choose between the three following options: use the free legal version, purchase the premium version or get a digital copy. The first two options are legal and generate income for the firm, while the latter is illegal. Hence, our analysis relates to models of vertical differentiation, pioneered by [Shaked and Sutton \(1987\)](#).

Qualities of the versions. The two legal versions can differ in quality. We denote by θ the baseline quality of the product sold, to which consumers have access if they choose to pay for the good. The firm then endogenously chooses the difference of quality between the free and the premium versions by determining a

level of restrictions R associated with the free version. In turn, the quality of the free version is equal to $(1 - R)\theta$. More restrictions reduce the utility of consumers choosing the free version and increase the number of premium users but also divert other users to piracy.

Piracy costs. Consumers who copy face two types of costs: a fixed cost $c \geq 0$ and a degradation cost $\delta < 1$ proportional to the quality of the good.

The fixed cost c can simply be the monthly fee that a user pays for direct download sites. For instance, users of Megaupload were paying a fee in USD per month (260 for a lifetime) to download files. It is also related to the ethical and technological costs to acquire illegal digital copies on the Internet. It takes time and knowledge to download files from specialized websites and P2P networks, especially in countries where there is close monitoring of file-sharing networks such as France or the US. Hence, this fixed reproduction cost associated with piracy can be interpreted as monetary, legal, ethical and technological costs that people face when they get digital content from illegal sources.

Parameter δ represents the possibility for digital copies to have a lower quality than that of the premium version. This cost can be related to the sound quality (320 Kbps or higher for the premium service vs. 128 Kbps for many pirated copies) for music files. For software, it is related to fewer features, while for video games, this cost translates the fact that connecting the legal services brings additional value such as connecting to a community. In terms of utility, this cost can be interpreted as a reduction of the quality of the pirated good compared to the premium version.

From a methodological point of view, it is important to introduce a fixed (reproduction) cost in our model. Indeed, without a fixed piracy cost, the freemium and pirated versions would compete head to head: among those who do not purchase the premium version, either all consumers use the free version, or they use the pirated version. When introducing the fixed cost to piracy, we will show that consumers are segmented in the following order: consumers with the lowest valuation for quality using the free version, then consumers copying illegal files, and finally consumers with the highest valuation for quality purchasing the premium

version.

Consumer utility. Let $p > 0$ denote the price of the premium version, R the level of restrictions, θ the willingness to pay for the good, and U_x the utility of consumers when they choose the option x . Consumers have the four following options:

1. Purchase the premium subscription at price p : $U_p = \theta - p$.
2. Use the free version with restrictions $0 \leq R \leq 1$: $U_f = (1 - R)\theta$.
3. Use a digital copy of lower quality $0 < \delta < 1$: $U_c = \delta\theta - c$.
4. Not consuming the good: $U_\emptyset = 0$.

A user of the free version generates an exogenous benefit $a > 0$ for the firm, which can be interpreted as an ARPU (average revenue per user) resulting from revenues from advertisements. For music and video streaming services, this ARPU is very small and is hardly negotiable with advertising companies, and can be considered exogenous. This assumption is supported by the literature on online advertising and in particular by [Gentzkow \(2014\)](#) who analyzes the value of targeted advertising by newspapers, and who argues that the price charged by online publishers is pinned-down by exogenous factors determining the willingness to pay of advertisers.⁸

We use the following tie-sorting condition: a consumer prefers the outside option and not to consume the good, instead of using the free version with the maximal number of restrictions $R = 1$.⁹

Assumptions. We make the two following assumptions

$$\text{Assumption A1. } a < \min\{1 - \delta, \frac{1}{4}\}$$

⁸[Crampes et al. \(2009\)](#) make a similar assumption, by considering digital companies making revenues from an exogenous return on the advertising displayed to consumers.

⁹This assumption guarantees that the firm does not make money from the segment of consumers with the lowest willingness to pay for quality (such that $0 < \theta < \frac{c}{a}$) who prefer the outside option to the free version with $R = 1$. Otherwise, the firm would generate a constant income of $\frac{ac}{\delta}$, which does not affect the optimum price and restrictions but changes the comparison between profits in the various cases that we analyze.

Assumption A2. $c < \min\{\delta, \frac{1}{2}\}$.

Assuming that $a < 1 - \delta$ ensures a non-negative demand for the premium version. Considering $a < \frac{1}{4}$ guarantees that a firm covering the unit segment with the free version and generating in this way a profit of a does not make more than the profits of $\frac{1}{4}$ of a monopoly without copies.

Overall, Assumption A1 requires that the ARPU is smaller than threshold values, which corresponds to the current state of online revenues generated by targeted advertising. For instance, Spotify has a quarterly ARPU around 1 USD.¹⁰ This value is especially small compared with the prices charged to Spotify premium service (29.97 USD/quarter).

Assumption A2 states that at least one consumer derives a strictly positive utility from the copy ($c < \delta$). Moreover, focusing on $c < \frac{1}{2}$ ensures that the cost of copying cannot be greater than the price of a monopoly facing the outside option.

5 Equilibrium

For the clarity of the exposition we assume that the consumer taste parameter θ is uniformly distributed on the segment $[0, 1]$. We show that the main results hold with a general distribution function in Appendix A.5.

5.1 Demand and Profit

The firm maximizes profits by choosing a couple (p, R) . Depending on its choices, some options are not available to consumers and others are dominated.

The consumers indifferent between the free and the premium versions, between the copy and the free version, and between the copy and the premium version are respectively:

$$\theta_{fp} = \frac{p}{R}, \quad \theta_{cf} = \frac{c}{R + \delta - 1}, \quad \theta_{cp} = \frac{p - c}{1 - \delta}.$$

When the number of restrictions is maximal, that is when $R = 1$, the consumers indifferent between the outside option and the copy version and between the out-

¹⁰Spotify Technology S.A. Announces Financial Results for First Quarter 2022, last accessed January 4, 2023.

side option and the premium version are respectively:

$$\theta_{\varnothing c} = \frac{c}{\delta}, \quad \theta_{\varnothing p} = p.$$

Different values of p and R will shift the utilities of consumers and change the location of the consumer indifferent between two options depending on the values of the primitives δ and c . In turn, the resulting demand for each option will change too. Hence, to characterize the profits of the firm and its equilibrium choice of business model, we must first understand how its choices of p and R impact consumer demand. There are two market configurations to consider, depending on the relative positions of the consumer indifferent between different options.¹¹

First, some consumers may prefer the digital copy to the free version, which happens when $\theta_{cp} \geq \theta_{fp} \geq \theta_{cf}$ (Case 1). In this case, some consumers use the copy, and the demand for the pirated good lies between the demands for the free and the premium versions, i.e. between the two legal versions. This brings a new element compared with classical models of end-user piracy where the pirated version is usually consumed by consumers with the lowest valuation for quality (Bae and Choi, 2006). The resulting demands are displayed in Figure 1:

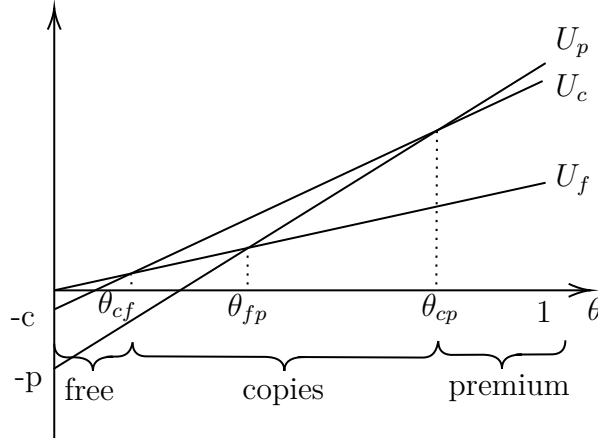


Figure 1: Demands for copies, free, and premium versions: Case 1.

Secondly, consumers may also choose between the free and the premium versions, without using the pirated version, which happens when $\theta_{fp} > \theta_{cp}$ (Case 2). This second configuration can happen in three different sub-cases: when $\theta_{cf} > \theta_{fp}$

¹¹See Appendix A.2 for a detailed characterization of the cases.

(Case 2A), when $0 \geq \theta_{cf}$ (Case 2B), and when $\theta_{cf} \geq 1$ (Case 2C). In this last case, note that we will have $0 \geq \theta_{cp}$, the free version allows the firm to fully deter piracy. The resulting demand configuration is depicted in Figure 2 (for Case 2B).

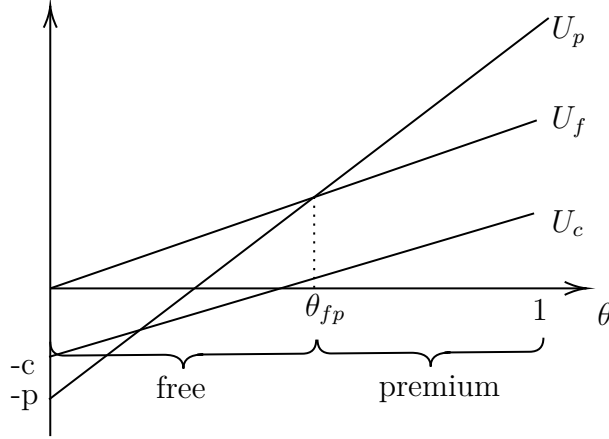


Figure 2: Demands for copies, free, and premium versions: Case 2.

The different cases depend on the value of p and R , and we provide below a summary of the range of values for which each case takes place:

	$p \leq c$	$p > c$
$R \leq 1 - \delta$	Case 2B	
$R > 1 - \delta$	Case 2C	$R < \frac{p(1-\delta)}{p-c}$: Case 2A
		$R \geq \frac{p(1-\delta)}{p-c}$: Case 1

The availability of digital copies exerts a constraint on the range of prices and restrictions that the firm can choose. As we move from Case 1 to Cases 2A, 2B, and 2C, the intensity of the competition from illegal copies decreases, and therefore the strategy that the firm can use to deter or accommodate piracy will also change.

We now characterize the profits in each configuration.

First configuration (Case 1). The demand for the premium and the free services are respectively:

$$\begin{aligned} D_{premium} &= 1 - \theta_{cp} = 1 - \frac{p-c}{1-\delta} \\ D_{free} &= \theta_{cf} = \frac{c}{R+\delta-1}. \end{aligned}$$

Given that $p > c$, we have $\frac{p}{p-c} > 1$, and it is straightforward to see that condition $R \geq \frac{p(1-\delta)}{p-c}$ is more restrictive than $R > 1 - \delta$, the limit case when the cost to copy is equal to zero. The firm maximizes profits

$$\Pi = a \frac{c}{R+\delta-1} + p \left[1 - \frac{p-c}{1-\delta} \right] \text{ subject to } p > c \text{ and } R \geq \frac{p(1-\delta)}{p-c}. \quad (1)$$

In this case, the pricing of the premium version does not depend on the introduction of the free version, as the firm is only constrained by the pirated version. Introducing the free version can be considered as a source of extra profits and will always be profitable for the firm.

Second configuration (Case 2). When consumers do not use the pirated version, the demand for the premium and the free services are respectively:

$$\begin{aligned} D_{premium} &= 1 - \theta_{fp} = 1 - \frac{p}{R} \\ D_{free} &= \theta_{fp} = \frac{p}{R}. \end{aligned}$$

The firm maximizes

$$\Pi = a \frac{p}{R} + p \left[1 - \frac{p}{R} \right],$$

subject to:¹²

- Case 2A: $\min\{1, \frac{p(1-\delta)}{p-c}\} > R > 1 - \delta$, $R \geq p$, and $p > c$,
- Case 2B: $R \geq p$ and $R \leq 1 - \delta$,
- Case 2C: $R \geq p$, $p \leq c$ and $R > 1 - \delta$.

¹²The conditions characterizing the different cases are provided in Appendix A.2.

5.2 Optimal Business Models

There exist four business models that can be optimal for the firm, depending on the primitives of the model a , c , and δ : the freemium, freemium+, limit-pricing, and monopoly business models. We briefly explain the main differences between these business models, and we then characterize each business model.

In both the freemium and the freemium+ business models, the firm proposes a free version of its product, along with a premium version. In the two other business models, the firm can deter the copy without versioning strategies and offers only a premium version of its product.

In the monopoly business model, the firm can charge its monopoly price as if there was no pirated version, and consumers either use the premium version, or do not consume.¹³ However, if the utility provided by the copy is high enough, some consumers would use the pirated version with this unconstrained price exerting a pressure on the firm. In this case, the firm can achieve higher profits and fully deter the copy with a limit-pricing strategy, where it lowers its price down to a point where all consumers buy the premium version.¹⁴

To understand the trade-off between the different business models, it is useful to analyze the impact of introducing a free version on the profits of a firm. Introducing the free version has two main opposite effects:¹⁵

- A market-expansion effect: consumers who were previously copying now use the free version; the firm increases the lower part of the demand function that was previously lost to copies and fights free with free. Note that if the copy is not a threat, a freemium business model may not be optimal.
- A cannibalization effect: the firm loses demand for its premium version to the free version with restrictions. Consumers who were previously purchasing the premium version now only generate an average revenue of a per consumer.¹⁶

¹³In this case, it is as if the copy was blockaded by the premium version since it provides consumers with a very low utility.

¹⁴Note that our use of the name “Monopoly” business model is a slight abuse of terminology, as in equilibrium the firm will act as a monopolist in any business model. What we mean here is that the firm can behave as if it was not facing any competitive pressure from the copy, and therefore charge an unconstrained monopoly price.

¹⁵Introducing a free version will also change the equilibrium price of the premium version.

¹⁶See Belleflamme (2005) and Siebert (2015) for a review.

Proposition 1 summarizes the equilibrium restrictions, prices, and profits for the different business models.

Proposition 1.

The optimal strategies lead to the following business models:

- *Freemium:* $R^* = 1 - \delta$, $p^* = \frac{a+1-\delta}{2}$, $\Pi^* = \frac{(a+1-\delta)^2}{4(1-\delta)}$.
- *Freemium+:* $R^* = \frac{(a+c+1-\delta)(1-\delta)}{a-c+1-\delta}$, $p^* = \frac{a+c+1-\delta}{2}$, $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)}$.
- *Limit-pricing:* $R^* = 1$, $p^* = \frac{c}{\delta}$, $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$.
- *Monopoly:* $R^* = 1$, $p^* = \frac{1}{2}$, $\Pi^* = \frac{1}{4}$.

Proof: see Appendix A.3.¹⁷

Optimal business model and copyright protection. The optimal business models depending on the characteristics of the copy and on the ARPU a are represented in Figure 3.¹⁸

¹⁷The proof proceeds in two steps: first, we determine optimal profits for cases 1 and 2; and secondly, we compare profits in each area.

¹⁸Note that the business model chosen by the firm changes with different values of the parameters, in particular, within the different cases characterized in Section 5.1. For instance, the firm chooses the freemium+ business model in Cases 1 and 2A.

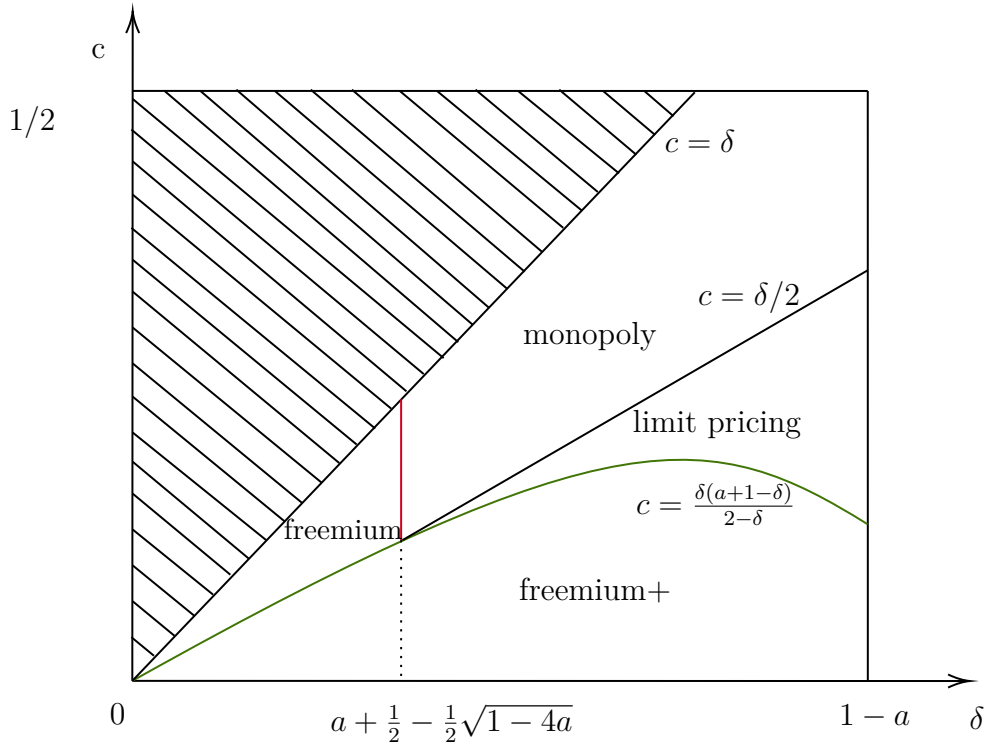


Figure 3: Demands for copies, free, and premium versions: $a = 0, 1$.

The shaded area above the diagonal line corresponds to a situation in which $c > \delta$. In this area, consumers can no longer get a strictly positive utility from the copy. This shaded area is ruled out of our analysis by Assumption A2. Below the diagonal, the firm deters the copy by offering a free version of its product in the freemium and freemium+ business models.

In the freemium model, the firm chooses restrictions such that the quality of the free version is just equal to that of the illegal copy ($R = 1 - \delta$). On the contrary, with the freemium+ business model, the firm offers consumers a free version of lower quality than the freemium model, i.e., with stronger restrictions.

By doing so, the firm can achieve higher profits than with the freemium model in the situation where copying is the biggest threat – when δ is high and c is low (the southeast part of Figure 3). Indeed, since the quality of the copy in this zone is high, the freemium model diverts too many consumers from the premium version, leading to excessive cannibalization of the premium demand. With the freemium+ business model, the firm can capture consumers with a low θ , who do not copy due to the cost c , even if c is low in this zone. On this part of the

demand, the firm earns the ARPU a .

When copies exert a slightly weaker threat (c remains low but δ is small, southwest quadrant of Figure 3), freemium+ remains the optimal business model compared to freemium. However, when the cost of copying c increases, and the firm cannot further increase its restrictions, the freemium strategy takes over, with restrictions $R = 1 - \delta$.

The alternative to these models for the firm is to adopt either a monopoly or a limit pricing business model. The firm using the limit pricing strategy sets a price $p = \frac{c}{\delta}$ just necessary to eliminate illegal copying and only offers the premium version.

When c is low and δ is high (the southeast part of Figure 3), the optimal value of p is too low for the limit pricing strategy to be interesting. Hence, the freemium+ strategy is also more profitable than the limit pricing strategy when c is lower than $\delta \frac{a+1-\delta}{2-\delta}$. In this case, by adjusting the price and the level of restrictions, illegal copying is eliminated with the freemium+ strategy by making the position of the indifferent consumer between the free version and the copy equal to the position of the indifferent consumer between the copy and the premium version.

This freemium+ model does not necessarily remain optimal when the cost of copying c increases. Indeed, as c increases, the restrictions of the free version rise in response, up to a certain threshold $R = 1$. This increases in turn the demand for the premium good. At this point, the firm switches to the limit pricing strategy. As c becomes even higher, the competitive pressure exerted by the copy softens, allowing the firm to charge an unconstrained monopoly price in the monopoly business model.

It should be noted, however, that for small values of δ , freemium models are always more profitable than a monopoly, as long as the ARPU a is sufficiently high, regardless of c . Indeed, the firm can always offer a free version of its product with significant restrictions to combat illegal copying, without cannibalizing its premium version. In particular, we can note that even for arbitrarily small values of a , the firm benefits from adopting a freemium business model as long as c is small enough.

Finally, we can interpret our results in terms of an increase in copyright pro-

tection, measured in our model by parameters c and δ . Increasing c increases the level of restrictions and can be seen as a reduction of the competitive pressure exerted by illegal copying. In Figure 3, this corresponds to a shift towards the north of the figure. There, an increase in copyright protection leads the firm to change its business model from freemium+ to limit pricing, and then towards monopoly.

On the other hand, a decrease (increase) in copyright protection can also be captured by an increase (decrease) in the quality of the copy δ . Changes in δ have a non-monotonous effect on the choice of business model. Indeed, consider a value of c above $\frac{\delta}{2}$ and δ at the right of $a + \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4a}$. There, the monopoly business model is optimal, but as δ increases copies represent a stronger threat and the firm first adopts a limit pricing business model for higher values of δ , and then fights free with free by adopting a freemium+ business model.

However, on the left of Figure 3, the value of δ is low and therefore copies do not represent a strong threat (and the level of restrictions is high). Consider the case where the firm adopts a monopoly business model. When c is not too high, decreasing δ (increasing copyright protection) makes the firm switch from the monopoly to the freemium business model, generating an increase in authorized usage (see next paragraph) and in advertising revenues from a large share of consumers who do not use the pirated version.

We can understand this result by considering the effects that we have identified above: market expansion vs. cannibalization. When the quality of the pirated good δ is low, the free version represents a credible alternative to the pirated version for users with a low value of θ , and can be introduced to deter piracy. In this freemium business model, the firm benefits from advertising revenues generated from a large share of consumers who do not use the pirated version. On the contrary, when the quality of the pirated good δ is high, the firm must offer a high quality for the free good to deter piracy. The resulting cannibalization effect on the premium good is strong, and the firm is better off offering only the premium version with a high price, thereby extracting more surplus from consumers with a high θ .

Authorized usage. Our results allow policymakers to understand how copyright protection impacts the authorized usage of the good and can make legal digital products available to a wider audience. The equilibrium demands are derived in the Appendix and given in Lemma 2.

Lemma 2. *The demands for the legal versions in each business model are:*

- *Freemium:* $D_{\text{freemium}} = \frac{a+1-\delta}{2(1-\delta)}$, $D_{\text{premium}} = \frac{1-\delta-a}{2(1-\delta)}$.
- *Freemium+:* $D_{\text{freemium}} = \frac{a-c+1-\delta}{2(1-\delta)}$, $D_{\text{premium}} = \frac{c-a+1-\delta}{2(1-\delta)}$.
- *Limit-pricing:* $D_{\text{premium}} = 1 - \frac{c}{\delta}$.
- *Monopoly:* $D_{\text{premium}} = \frac{1}{2}$.

We can see that for a given business model, all consumers use authorized versions with the freemium and freemium+ business models, but only part of the demand is served by the firm with the limit pricing and monopoly business models, corresponding to consumers with the highest values for θ . When copyright protection becomes stronger, its impact on authorized usage does not change in the monopoly business model, while in the limit pricing business model when c increases and δ decreases, the demand for the premium good decreases. Indeed, consider a value of c above $\frac{\delta(a+1-\delta)}{2-\delta}$, a slight decrease in c induces the firm to switch from the limit pricing to the freemium+ plus business model. In this case, total legal usage is maximized as consumers with the lowest values of θ can consume the free version of the product.

On the other hand, whenever the firm moves from the monopoly business model to the freemium business model, the legal usage moves from half of the consumer demand (with the highest values of θ) to the whole demand of consumers, with those having the lowest values of θ consuming the free version.

6 Welfare

We characterize social welfare, defined as the sum of consumer surplus and of the profits of the firm. Proposition 3 provides the expressions of welfare – denoted by SW – with the different business models.

Proposition 3.

- *Freemium*: $SW = \frac{1}{8} \frac{(a-\delta)(3a+\delta+2)+3}{1-\delta}$.
- *Freemium+*: $SW = \frac{1}{8} \frac{(a-c)(3a-c)-(\delta-1)(2a+\delta+3)}{1-\delta}$.
- *Limit-pricing*: $SW = \frac{1}{2} (\delta - c) \frac{c+\delta}{\delta^2}$.
- *Monopoly*: $SW = \frac{3}{8}$.

Proof: see Appendix A.4.

We can analyze how changes in parameters δ and c impact social welfare.¹⁹ Overall, we show that stronger copyright protection will impact welfare by increasing the profits of the firm, and by reducing the surplus of consumers.

The impact of stronger copyright protection on welfare depends on whether it changes the fixed or variable costs of consuming the copy. On the one hand, when copyright protection reduces the value of δ , the loss of consumer surplus outweighs the benefits for the firm, and social welfare decreases.

On the other hand, when copyright protection increases the fixed copy cost c , we show that in general, an increase in c reduces social welfare. Social welfare can also increase with c when $a < \delta \frac{1-\delta}{4-3\delta}$ and $c > 2a$ in the Freemium+ business model. In this last case, the gains of profits from a higher c for the firm dominate the loss for consumers.

These results are summarized in the table below.

Business Models	$\frac{\partial SW}{\partial \delta}$	$\frac{\partial SW}{\partial c}$
Freemium	> 0	\emptyset
Freemium+	> 0	< 0 if $a > \delta \frac{1-\delta}{4-3\delta}$
	> 0	< 0 if $a < \delta \frac{1-\delta}{4-3\delta}$ and $c < 2a$
	> 0	> 0 if $a < \delta \frac{1-\delta}{4-3\delta}$ and $c > 2a$
Monopoly	$= 0$	$= 0$
Limit pricing	> 0	< 0

¹⁹The variations of welfare are formally analyzed in Appendix A.4.

7 Discussion

Our model can be interpreted in terms of property and liability rules described in Section 3. In this section, we show how the primitives of our model can be translated into parameters capturing changes in these rules.

Throughout this section, the baseline utility of a consumer using the copy without being caught by the authorities is equal to θ , and reductions in utility come from the enforcement of copyright laws. We denote by β the probability of being caught by authorities, which characterizes how stringent are copyright laws. This probability will be identical with property and liability rules, and we describe below how the copyright regime changes the utility of consumers using the copy.

Liability Rule: On the one hand, under the liability rule consumers can keep using the copy even if they are caught by the authorities, by paying a fine equal to t (under certain conditions, this fine can be transferred to the firm, but for simplicity we focus here on a fine transferred to the authority). In this case, the expected utility of consuming the pirated good is:

$$U_c = (1 - \beta)\theta + \beta(\theta - t) = \theta - \beta t$$

Property Rule: On the other hand, under property rules, consumers cannot consume the pirated good if they are caught by the authorities. In this case, the expected utility of a consumer choosing to consume the pirated good can be written:

$$U_c = (1 - \beta)\theta + \beta \cdot 0 = (1 - \beta)\theta.$$

We can write a general utility function where β is the probability of being caught, and that accounts for the relative weight λ of property rules and $1 - \lambda$ the weight of liability rules:

$$U_c = \lambda(1 - \beta)\theta + (1 - \lambda)(\theta - \beta t) = (1 - \lambda\beta)\theta - (1 - \lambda)\beta t$$

This formulation is equivalent to our baseline model where $\delta = (1 - \lambda\beta)$ and $c = (1 - \lambda)\beta t$. This allows us to understand the variations in strength of the

different copyright protection rules on the incentives of the firm to choose a given business model.

On the one hand, an increase in the value of the fine t increases the fixed cost of use c . On the other hand, an increase in the probability of being caught increases the value of β which shifts upward the value of c and downward the value of δ . However, the enforcement of copyright laws can come at a cost for the regulators and for society.²⁰ In this case, lowering the value of β , and in turn, the resulting cost of enforcement, can be socially desirable as it would also allow freemium or freemium+ business models to be adopted by the firm.

Finally, the overall cost to copy depends on the relative weight of the liability and property rules, which we capture with parameter λ . In particular, all other things equal, an increase in the value of λ reduces the reproduction cost but also the effective quality of the pirated good.

8 Conclusion

We have analyzed a model of online piracy where a firm strategically chooses its business model as a response to the threat posed by the pirate product: the firm can stick to a traditional business model where it sells its good for a price; or, along with the pay version the firm can also choose to offer a free version of its product, financed by ads, where the firm uses restrictions as a strategic variable to reach a specific group of consumers. We have shown that even when free, pirate digital content is available online, the firm completely deters online piracy in equilibrium, by offering a free version with a low level of restriction along with the premium version. The incentives of the firm to adopt such a strategy depend on the strength of the copyright enforcement regime. Hence, our results stress the importance for policymakers to anticipate how copyright laws and enforcement will impact the choice of business model of the firm.

Therefore, our results have important policy implications. The recent European successes of Spotify and Deezer suggest that freemium models are excellent

²⁰We would like to thank an anonymous referee for pointing this issue. See for instance the long going debates about the enforcement cost of three-strike laws in France, which amounts to around 10 millions USD per year ([The cost of French HADOPI, BFM, August 3, 2020](#)).

market-based alternatives to fight online piracy. In the digital video industry, consumers with a low willingness to pay can stream online content for free and pay a monthly subscription fee for premium services such as Video on Demand. Similarly, in the video games industry, free-to-play models offer players new ways to try out new games for free, and they only purchase premium services and additional content. There are thus market solutions based on freemium models that can fight free with free by better segmenting consumers and audiences according to their willingness to pay for digital content. These results therefore challenge the view that strong copyright laws are necessary to fight online piracy.²¹ A policymaker who does not anticipate the response of firms to piracy in terms of choice of business model only has an incomplete picture of the market, as we show that business model innovations can complement the legal enforcement of intellectual property rights.

Further research could explore the interaction between the choice of business models by digital firms, innovation strategies and the design of copyright laws by policymakers. In particular, our analysis could be extended to consider long-term dynamics and the incentives of firms to develop new products depending on the legal environment (Dam, 1995).

References

- Aguiar, L. and Martens, B. (2016), ‘Digital music consumption on the internet: Evidence from clickstream data’, *Information Economics and Policy* **34**, 27–43.
- Aral, S. and Dhillon, P. S. (2021), ‘Digital paywall design: Implications for content demand and subscriptions’, *Management Science* **67**(4), 2381–2402.
- Bae, S. H. and Choi, J. P. (2006), ‘A model of piracy’, *Information Economics and Policy* **18**(3), 303–320.
- Bapna, R., Ramaprasad, J. and Umyarov, A. (2016), ‘Monetizing freemium communities: Does paying for premium increase social engagement?’, *Available at SSRN 2885681* .

²¹See for instance Landes and Posner (1989).

- Belleflamme, P. (2005), ‘Versioning in the information economy: theory and applications’, *CEifo Economic Studies* **51**(2-3), 329–358.
- Belleflamme, P. and Picard, P. M. (2007), ‘Piracy and competition’, *Journal of Economics & Management Strategy* **16**(2), 351–383.
- Belo, R. and Li, T. (2022), ‘Social referral programs for freemium platforms’, *Management Science* .
- Boudreau, K. J., Jeppesen, L. B. and Miric, M. (2021), ‘Competing on freemium: Digital competition with network effects’, *Strategic Management Journal* .
- Cho, W.-Y. and Ahn, B.-H. (2010), ‘Versioning of information goods under the threat of piracy’, *Information Economics and Policy* **22**(4), 332–340.
- Crampes, C., Haritchabalet, C. and Jullien, B. (2009), ‘Advertising, competition and entry in media industries’, *The Journal of Industrial Economics* **57**(1), 7–31.
- Dam, K. W. (1995), ‘Some economic considerations in the intellectual property protection of software’, *The Journal of Legal Studies* **24**(2), 321–377.
- Deneckere, R. J. and Preston McAfee, R. (1996), ‘Damaged goods’, *Journal of Economics & Management Strategy* **5**(2), 149–174.
- Gentzkow, M. (2014), ‘Trading dollars for dollars: The price of attention online and offline’, *American Economic Review* **104**(5), 481–88.
- Givon, M., Mahajan, V. and Muller, E. (1995), ‘Software piracy: Estimation of lost sales and the impact on software diffusion’, *Journal of Marketing* **59**(1), 29–37.
- Godinho de Matos, M., Ferreira, P. and Smith, M. D. (2018), ‘The effect of subscription video-on-demand on piracy: Evidence from a household-level randomized experiment’, *Management Science* **64**(12), 5610–5630.
- Gu, X., Kannan, P. and Ma, L. (2018), ‘Selling the premium in freemium’, *Journal of Marketing* **82**(6), 10–27.

- Huang, J., Reiley, D. and Riabov, N. (2018), ‘Measuring consumer sensitivity to audio advertising: A field experiment on pandora internet radio’, *Available at SSRN 3166676* .
- Lahiri, A. and Dey, D. (2013), ‘Effects of piracy on quality of information goods’, *Management Science* **59**(1), 245–264.
- Landes, W. M. and Posner, R. A. (1989), ‘An economic analysis of copyright law’, *The Journal of Legal Studies* **18**(2), 325–363.
- Lee, S., Zhang, J. and Wedel, M. (2021), ‘Managing the versioning decision over an app’s lifetime’, *Journal of Marketing* **85**(6), 44–62.
- Lu, S., Rajavi, K. and Dinner, I. (2021), ‘The effect of over-the-top media services on piracy search: evidence from a natural experiment’, *Marketing Science* **40**(3), 548–568.
- Novos, I. E. and Waldman, M. (1984), ‘The effects of increased copyright protection: An analytic approach’, *Journal of political economy* **92**(2), 236–246.
- Peukert, C., Claussen, J. and Kretschmer, T. (2017), ‘Piracy and box office movie revenues: Evidence from megaupload’, *International Journal of Industrial Organization* **52**, 188–215.
- Reimers, I. (2016), ‘Can private copyright protection be effective? evidence from book publishing’, *The journal of law and economics* **59**(2), 411–440.
- Rietveld, J. (2018), ‘Creating and capturing value from freemium business models: A demand-side perspective’, *Strategic Entrepreneurship Journal* **12**(2), 171–193.
- Rob, R. and Waldfogel, J. (2006), ‘Piracy on the high c’s: Music downloading, sales displacement, and social welfare in a sample of college students’, *The Journal of Law and Economics* **49**(1), 29–62.
- Sato, S. (2019), ‘Freemium as optimal menu pricing’, *International Journal of Industrial Organization* **63**, 480–510.

- Shaked, A. and Sutton, J. (1987), ‘Product differentiation and industrial structure’, *The Journal of Industrial Economics* pp. 131–146.
- Shapiro, C. (2017), ‘Property rules vs. liability rules for patent infringement’, *Berkeley working paper* .
- Shapiro, C. and Varian, H. (1999), *Information rules: A strategic guide to the network economy*, Harvard Business Press.
- Shi, Z., Zhang, K. and Srinivasan, K. (2019), ‘Freemium as an optimal strategy for market dominant firms’, *Marketing Science* **38**(1), 150–169.
- Siebert, R. (2015), ‘Entering new markets in the presence of competition: Price discrimination versus cannibalization’, *Journal of Economics & Management Strategy* **24**(2), 369–389.
- Volckmann, W. M. (2023), ‘The effects of market size, wealth, and network effects on digital piracy and profit’, *European Journal of Law and Economics* **55**(1), 61–85.

A Appendix

A.1 Discussion of the Assumption $\delta < 1$

We relax in this appendix the assumption that the quality of the copy is below that of the premium good, by considering $\delta > 1$. This case differs from previous analysis when it comes to the decision of consumers to use the premium version or the copy. Indeed, we have then:

$$U_c > U_p \iff \delta \theta - c > \theta - p \iff \theta > \theta_{cp} = \frac{c - p}{\delta - 1}$$

Hence, either $p > c$, and the firm does not sell the premium version and makes zero profits with this version of its good. Or $p \leq c$ and consumers with the highest valuation use the copy, while those using the premium version have a lower valuation for the good.

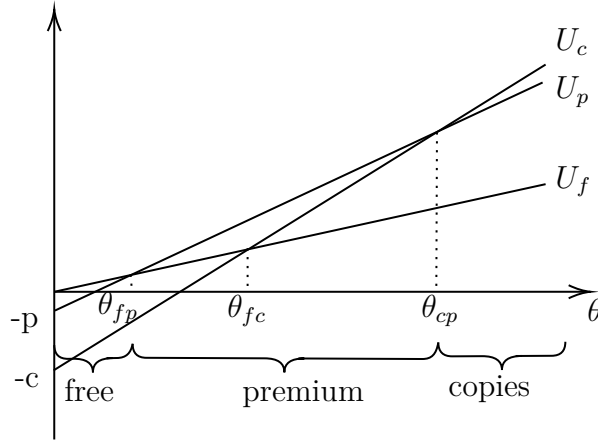


Figure 4: Demands for copies, free, and premium versions: $\delta > 1$.

If $\delta = 1$, U_c and U_p have the same slope, so they are either identical or they are parallel and one dominates the other for all θ .

Overall, we rule out this scenario from our baseline analysis as this would imply that the quality of the copied good is greater than that of the original version. We could justify this configuration by assuming that consumers can have a taste for piracy. For instance, if they enjoy the thrill of using a pirated version, and they do so every time they consume. Yet, this effect is unlikely to occur for the bulk of the population and we focus on $\delta < 1$ in the main analysis.

A.2 Characterization of Cases 1, 2A, 2B and 2C

We need to analyze four cases according to whether the digital copy represents a credible threat to the firm.

Case 1: $\theta_{cf} \leq \theta_{fp}$ and $p > c$. This condition can be written as $R \geq \frac{p(1-\delta)}{p-c}$. The number of restrictions on the free version is higher than a threshold. There exists a set of parameter values for which some consumers prefer the digital copy. This situation is depicted in Figure 5.

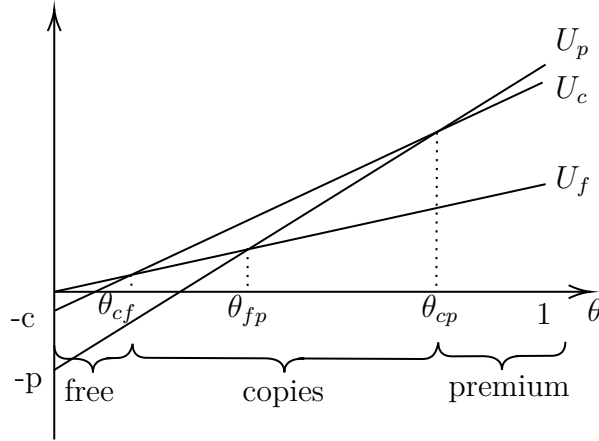


Figure 5: Case 1

It is easy to show that $\theta_{cf} \leq \theta_{fp}$ implies $\theta_{fp} \leq \theta_{cp}$.

Case 2A: $\theta_{cf} > \theta_{fp}$ and $p > c$. The number of restrictions is below the threshold defined in Case 1. Consumers choose between the free and the premium version. It is straightforward to show that $\theta_{cf} > \theta_{fp}$ implies $\theta_{fp} > \theta_{cp}$.

Moreover, the constraint $\theta_{cf} > \theta_{fp}$ can be written as:

$$R < \frac{p(1-\delta)}{p-c}$$

Thus, adding the condition on the non negativity of the premium demand, the number of restrictions is such that:

$$\min\left\{1, \frac{p(1-\delta)}{p-c}\right\} > R > 1-\delta, R \geq p, \text{ and } p > c$$

This configuration of the parameters is represented in Figure 6.

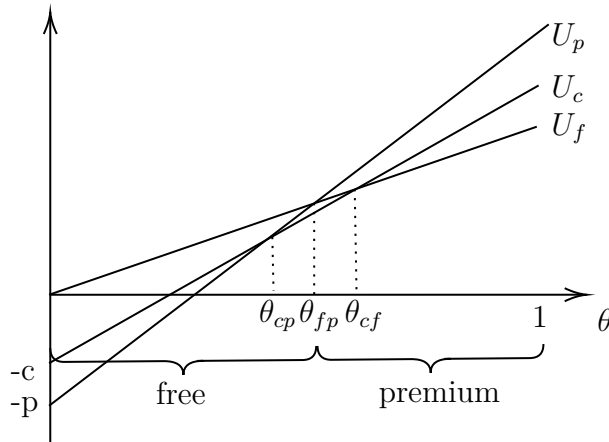


Figure 6: Case 2A

Case 2B: $R \leq 1 - \delta$. The quality of the copy is always lower than the “quality” of the free version. The conditions characterizing this situation are:

$$R \geq p \text{ and } R \leq 1 - \delta.$$

This case is depicted in Figure 7.

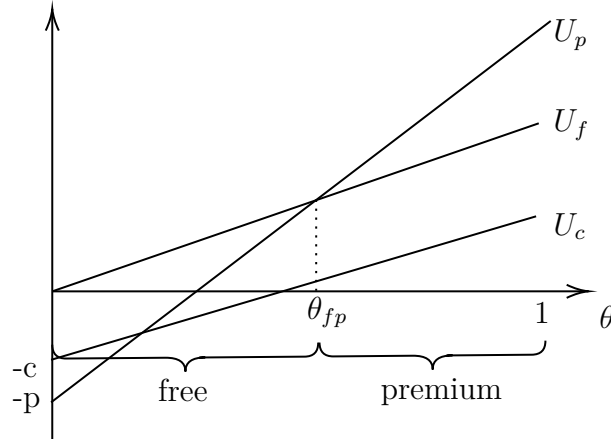


Figure 7: Case 2B

The copy is always dominated by the free version because the slope of U_f is greater than the slope of U_c , and U_f intersects the y-axis above the intersection of U_c and the y-axis.

Case 2C: $R > 1 - \delta$ and $p \leq c$. The quality of the copy is higher than the “quality” of the free version, but the fixed cost of the copy is higher than the price of the premium version. The conditions are:

$$R \geq p, p \leq c \text{ and } R > 1 - \delta.$$

This configuration of the parameters is depicted in Figure 8.

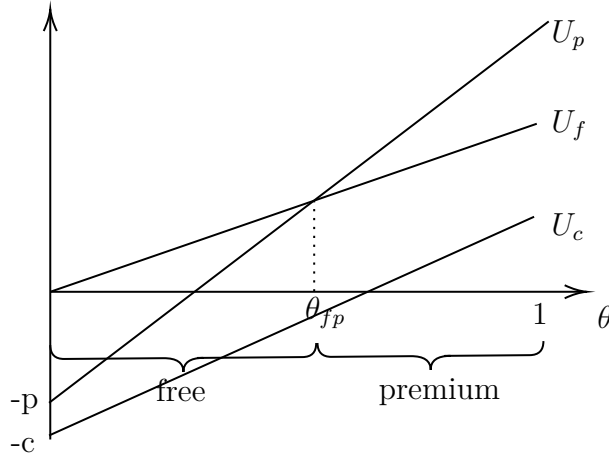


Figure 8: Case 2C

The consumer prefers the free version to the copy up to the intersection of the lines U_f and U_c , which is always to the right of the point where the consumer is indifferent between the free version and the premium version. Therefore, the copy is always dominated by the free version.

A.3 Proof of proposition 1

The optimal strategies lead to the following business models:

- (A) $\frac{\delta(a+1-\delta)}{2-\delta} \leq c < \frac{1}{2}$ and $0 < \delta < a + \frac{1}{2} - \frac{1}{2}\sqrt{(1-4a)}$: the freemium model is optimal with

$$R^* = 1 - \delta, \quad p^* = \frac{a + 1 - \delta}{2}, \quad \text{and } \Pi^* = \frac{(a + 1 - \delta)^2}{4(1 - \delta)}$$

- (B) $0 < c < \frac{\delta(a+1-\delta)}{2-\delta}$: the freemium+ model is optimal, with

$$R^* = \frac{(a+c+1-\delta)(1-\delta)}{a-c+1-\delta}, \quad p^* = \frac{a+c+1-\delta}{2}, \quad \text{and } \Pi^* = \frac{(1-\delta)^2 + (a-c)^2 + 2(1-\delta)(a+c)}{4(1-\delta)}$$

- (C) $\frac{\delta(a+1-\delta)}{2-\delta} \leq c < \frac{\delta}{2}$ and $a + \frac{1}{2} - \frac{1}{2}\sqrt{1-4a} < \delta < 1 - a$: the limit-pricing model is optimal, with

$$R^* = 1, \quad p^* = \frac{c}{\delta}, \quad \text{and } \Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$$

(D) $\frac{\delta}{2} \leq c < \frac{1}{2}$ and $a + \frac{1}{2} - \frac{1}{2}\sqrt{1-4a} < \delta < 1 - a$: the monopoly model is optimal with

$$R^* = 1, p^* = \frac{1}{2}, \text{ and } \Pi^* = \frac{1}{4}$$

We proceed in two steps. First, we determine optimal profits for cases 1 and 2. Secondly, we compare profits in each area.

A.3.1 Step1. Optimal profits

A.3.1.1 Case 1: $1 \geq R \geq \frac{p(1-\delta)}{p-c}$ and $p > c$ In this case, the firm maximizes profits

$$\Pi = a \frac{c}{R + \delta - 1} + p \left[1 - \frac{p-c}{1-\delta} \right] \text{ subject to } 1 \geq R \geq \frac{p(1-\delta)}{p-c}.$$

The first order condition with respect to R is $\frac{\partial \Pi}{\partial R} = -\frac{ac}{(R+\delta-1)^2} < 0$. Therefore, the firm sets R at its minimum level $R^* = \frac{p(1-\delta)}{p-c}$.

Result 1. The optimal profits are:

$$(i) \text{ If } 0 < c < \frac{\delta(a+1-\delta)}{2-\delta}, \Pi^* = \frac{(1-\delta)^2 + (a-c)^2 + 2(1-\delta)(a+c)}{4(1-\delta)};$$

$$(ii) \text{ If } \frac{\delta(a+1-\delta)}{2-\delta} \leq c < 1/2, \Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta} \right).$$

Proof. Using this level of restriction R^* , the firm maximizes profits

$$\Pi = a \frac{p-c}{1-\delta} + p \left[1 - \frac{p-c}{1-\delta} \right] \quad (2)$$

The unconstrained optimum is $p^* = \frac{a+c+1-\delta}{2}$.²² The associated unconstrained optimal number of restrictions is

$$R^* = \frac{(a+c+1-\delta)(1-\delta)}{a-c+1-\delta}$$

Note that R^* is increasing with c : when the copy becomes more costly, the free version becomes more attractive and the firm can increase the number of restrictions on the free version. There are two cases to analyze: when $R^* < 1$ (the utility

²²The level of restrictions is such that the firm faces two segments of the total demand that are separated by $\frac{p}{R}$, instead of the three initial segments (free/copy/premium).

of the free version is positive) and when $R^* = 1$ (the utility of the free version is 0).

First, R^* is strictly smaller than 1 if

$$c < \frac{\delta(a+1-\delta)}{2-\delta} \quad (3)$$

When this condition (3) is satisfied, we also have $p^* > c$ (since $R^* < 1$ is equivalent to $p^* > \frac{c}{\delta}$ and that $\delta < 1$) so that the condition on the price of Case 1 is satisfied.

The optimal profit is

$$\Pi^* = \frac{(1-\delta)^2 + (a-c)^2 + 2(1-\delta)(a+c)}{4(1-\delta)} \quad (4)$$

This proves (i).

Secondly, when condition (3) is not satisfied, the firm sets $R^* = 1$. Given our tie-sorting assumption, the free version is dominated by the outside option and the consumer chooses between the premium version and a copy. In Case 1, condition $R \geq \frac{p(1-\delta)}{p-c}$ becomes $\frac{p-c}{1-\delta} \geq p$ or $p \geq \frac{c}{\delta}$. Recall that $\frac{c}{\delta} = \theta_{\varnothing c}$ and $p = \theta_{\varnothing p}$. Then, the indifferent consumer between the copy and the outside option has to be located to the left of the indifferent consumer between the outside option and the premium version. The latter has to be located to the left of the indifferent consumer between the copy and the premium version. The profit of the firm is then: $\Pi = p \left[1 - \frac{p-c}{1-\delta} \right]$. The optimal unconstrained price, $p^* = \frac{c+1-\delta}{2}$, is greater than $\frac{c}{\delta}$ if $c < \frac{\delta(1-\delta)}{2-\delta}$. However, when (3) is not satisfied, we must have $c \geq \frac{\delta(a+1-\delta)}{2-\delta}$. Since $a > 0$, these two inequalities are mutually incompatible, and therefore the firm sets a constrained price $p^* = \frac{c}{\delta}$ (indeed, $\frac{\partial \Pi}{\partial p} \Big|_{p=\frac{c}{\delta}} < 0$ for $c > \frac{\delta(a+1-\delta)}{2-\delta}$ and $\frac{c}{\delta} > c$) and there are no copies in equilibrium. This proves (ii). \square

A.3.1.2 Case 2A: $R < \min\{1, \frac{p(1-\delta)}{p-c}\}$, $R > 1 - \delta$ and $p > c$ In this case, recall that the profit function is given by:

$$\Pi = a \frac{p}{R} + p \left[1 - \frac{p}{R} \right] \text{ subject to } 1 - \delta < R \leq \min\left\{1, \frac{p(1-\delta)}{p-c}\right\}$$

The first order condition with respect to R yields

$$\frac{\partial \Pi}{\partial R} = \frac{p(p-a)}{R^2} \quad (5)$$

There are two cases to distinguish according to whether (A) $a \leq c$ or (B) $a > c$.

(A) When $a \leq c$, condition $p > c$ defining Case 2A implies $p > a$. Thus (5) is positive. The firm sets R to its maximal value, i.e $R^* = \min\{1, \frac{p(1-\delta)}{p-c}\}$.

Result 2. The optimal profits are:

(i) If $a \leq c < \frac{\delta(a+1-\delta)}{2-\delta}$ and $\delta \geq 2a$, $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)}$;

(ii) If $\frac{\delta(a+1-\delta)}{2-\delta} \leq c \leq \frac{\delta}{2}$ and $\delta \geq 2a$, $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$;

(iii) If $(\frac{\delta}{2} \leq c < \frac{1}{2}$ and $\delta \geq 2a$) or $(a \leq c < \frac{1}{2}$ and $\delta \leq 2a)$, $\Pi^* = \frac{1}{4}$.

Proof. There are two cases to analyse according to whether $R^* = \min\{1, \frac{p(1-\delta)}{p-c}\} = \frac{p(1-\delta)}{p-c}$ or $R^* = \min\{1, \frac{p(1-\delta)}{p-c}\} = 1$.

In the first situation, the firm chooses an optimal price p^* such that $\frac{p^*(1-\delta)}{p^*-c} \leq 1$, that is $p^* \geq \frac{c}{\delta}$. The unconstrained optimum price is $p^* = \frac{a+c+1-\delta}{2}$, that is greater than $\frac{c}{\delta}$ if $c \leq \frac{\delta(a+1-\delta)}{2-\delta}$. When this condition is satisfied, the associated unconstrained optimal number of restrictions is $R^* = \frac{(a+c+1-\delta)(1-\delta)}{a-c+1-\delta} \leq 1$, and the optimal profit is either $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)}$ when $R^* < 1$ or $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$ when $R^* = 1$. On the contrary, when $c > \frac{\delta(a+1-\delta)}{2-\delta}$, the optimal price is constrained to $p^* = \frac{c}{\delta}$ with $R^* = 1$ and the optimal profit is $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$.

In the second situation, the firm chooses an optimal price p^* such as $\frac{p^*(1-\delta)}{p^*-c} \geq 1$, that is $p^* \leq \frac{c}{\delta}$. The firm sets $R^* = 1$, and the free version is dominated by the outside option. Furthermore, by $p^* \leq \frac{c}{\delta}$, the copy is dominated by the premium. The consumer chooses between the premium and the outside option. The profit becomes $p(1-p)$. The unconstrained optimal price is $p^* = \frac{1}{2}$, which is less than $\frac{c}{\delta}$ if $c \geq \frac{\delta}{2}$. When this condition is satisfied, the optimal profit is $\Pi^* = \frac{1}{4}$. When it is not satisfied, that is when $c < \frac{\delta}{2}$, $p^* = \frac{c}{\delta}$ and the optimal profit is $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$.

We determine the choice of the firm by comparing the different optimal profits. We have first to notice that the thresholds on c are such that $\frac{\delta(a+1-\delta)}{2-\delta} \geq (\leq) \frac{\delta}{2} \iff \delta \leq (\geq) 2a$ and that $\frac{\delta(a+1-\delta)}{2-\delta} \geq (\leq) a \iff \delta \geq (\leq) 2a$. It is straightforward to show that for $\delta \geq 2a$, when $c < \frac{\delta(a+1-\delta)}{2-\delta}$, the optimal profit $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)} > \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$ and when $c \geq \frac{\delta}{2}$, the optimal profit $\Pi^* = \frac{1}{4} > \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$. Finally, when $\frac{\delta(a+1-\delta)}{2-\delta} \leq c \leq \frac{\delta}{2}$, the optimal profit is $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$. And, for $\delta \leq 2a$, we have $\frac{\delta}{2} \leq \frac{\delta(a+1-\delta)}{2-\delta} \leq c \leq a$. Thus, the only optimal profit is $\Pi^* = \frac{1}{4}$. This proves (i), (ii), (iii). \square

(B) When $a > c$, there are 3 sub-cases to analyze: (B1) $p > a > c$, (B2) $p = a$, (B3) $c < p < a$.

We first show that Case (B3) is impossible. Suppose on the contrary that $c < p < a$. Expression (5) is negative. The firm chooses the lowest possible value for R .²³ We have to make sure that the demand for the premium version is positive, that is $p \leq R$. As R increases, R converges to $\max\{p, 1 - \delta\}$ given that $R > 1 - \delta$ by the definition of Case 2A. By Assumption A1 ($a < 1 - \delta$), we know that $p < a < 1 - \delta$, so the firm sets $R^* \rightarrow 1 - \delta$ and a price $p^* \rightarrow \frac{a+1-\delta}{2}$. However, this price is strictly greater than a , by Assumption A1. This is a contradiction.

In subcase (B1), expression (5) is positive. The firm sets $R^* = \min\{1, \frac{p(1-\delta)}{p-c}\}$. This case can be analyzed in a similar way as Case 2A (A), but with $a > c$. We have three possible prices according to Assumption A1:

- $p^* = \frac{a+c+1-\delta}{2} > a$,
- $p^* = \frac{1}{2} > a$,
- $p^* = \frac{c}{\delta}$, this price is possible only for $\frac{c}{\delta} > a$, if not, the optimal price is constrained $p^* = a$, and the analysis is similar to Subcase (B2).

In Subcase (B2), where $p = a$, the firm can choose any value of R^* such that $R^* \in \left(1 - \delta, \min\left(1, \frac{a(1-\delta)}{a-c}\right)\right)$. The optimal profit is $\Pi^* = a$.

Results of Case 2A (B) are summarized in Result 3.

Result 3. The optimal profits are:

(i) If $(0 < c < a$ and $\delta \geq 2a)$ or $(0 < c < \frac{\delta(a+1-\delta)}{2-\delta}$ and $\delta \leq 2a)$, $\Pi^* = \frac{(1-\delta)^2 + (a-c)^2 + 2(1-\delta)(a+c)}{4(1-\delta)}$,

(ii) If $\frac{\delta(a+1-\delta)}{2-\delta} \leq c < a$ and $\delta \leq 2a$, $\Pi^* = \frac{1}{4}$.

Proof. When $\delta \geq 2a$, we have $c < a < \frac{\delta(a+1-\delta)}{2-\delta} < \frac{\delta}{2} < \frac{1}{2}$. Thus, in subcase (B1), the optimal profit is $\Pi^* = \frac{(1-\delta)^2 + (a-c)^2 + 2(1-\delta)(a+c)}{4(1-\delta)}$. This profit is greater than a , the optimal profit in subcase (B2). When $\delta \leq 2a$, the different thresholds are such that $\frac{\delta}{2} < \frac{\delta(a+1-\delta)}{2-\delta} < a < \frac{1}{2}$. In subcase (B1), when $0 < c < \frac{\delta}{2}$, the optimal profit

²³If $p = a$, we have already shown that the profit evaluated at this price is $\Pi^* = a$.

$\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)} > \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$, and when $\frac{\delta}{2} \leq c < \frac{\delta(a+1-\delta)}{2-\delta}$, the optimal profit is still $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)} > \frac{1}{4}$. Finally, when $\frac{\delta(a+1-\delta)}{2-\delta} \leq c < a$, the optimal profit is $\Pi^* = \frac{1}{4} > \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$. Both optimal profits of subcase (B1) are greater than a , the optimal profit in subcase (B2). This proves (i'), (ii''). \square

Bringing together Result 2 and Result 3 gives us the following conclusion for Case 2A.

Result 4. The optimal profits of Case 2A are:

- (i) If $0 < c < \frac{\delta(a+1-\delta)}{2-\delta}$, $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)}$;
- (ii) If $\frac{\delta(a+1-\delta)}{2-\delta} \leq c \leq \frac{\delta}{2}$ and $\delta \geq 2a$, $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$;
- (iii) If $\left(\frac{\delta(a+1-\delta)}{2-\delta} \leq c < \frac{1}{2} \text{ and } \delta \leq 2a\right)$ or $\left(\frac{\delta}{2} \leq c < \frac{1}{2} \text{ and } \delta \geq 2a\right)$, $\Pi^* = \frac{1}{4}$.

We can easily bring together Result 1 of Case 1 and Result 4 of Case 2A (remember that $\frac{c}{\delta} \left(1 - \frac{c}{\delta}\right) < \frac{1}{4}$):

Result 5. Suppose $p > c$, $R > 1 - \delta$.

- (R5.i) If $0 < c < \frac{\delta(a+1-\delta)}{2-\delta}$, the optimum is $p^* = \frac{a+c+1-\delta}{2}$, $R^* = \frac{(a+c+1-\delta)(1-\delta)}{a-c+1-\delta}$, $\Pi^* = \frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)}$;
- (R5.ii) If $\frac{\delta(a+1-\delta)}{2-\delta} \leq c \leq \frac{\delta}{2}$ and $\delta \geq 2a$, the optimum is $p^* = \frac{c}{\delta}$, $R^* = 1$, $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$;
- (R5.iii) If $\left(\frac{\delta(a+1-\delta)}{2-\delta} \leq c < \frac{1}{2} \text{ and } \delta \leq 2a\right)$ or $\left(\frac{\delta}{2} \leq c < \frac{1}{2} \text{ and } \delta \geq 2a\right)$, the optimum is $p^* = \frac{1}{2}$, $R^* = 1$, $\Pi^* = \frac{1}{4}$.

Result 5 shows that the profits and strategies of Case 1 are the same as those of Case 2A.

A.3.1.3 Case 2B: $R \leq 1 - \delta$ Profits are given by:

$$\Pi = a \frac{p}{R} + p \left(1 - \frac{p}{R}\right) \text{ subject to } R \geq p \text{ and } R \leq 1 - \delta$$

The first order condition with respect to R gives $\frac{\partial \Pi}{\partial R} = \frac{p(p-a)}{R^2}$. Therefore if the firm chooses a price $p < a$, the firm can set $R^* = p$ and there is no demand

for the premium version. If the firm sets $p = a$ then $R^* \in (a, 1 - \delta)$. In both cases, profits are equal to $\Pi^* = a$. If the firm chooses a price $p > a$, the firm sets $R^* = 1 - \delta$: $p^* = \frac{a+1-\delta}{2}$, $\Pi^* = \frac{(a+1-\delta)^2}{4(1-\delta)}$. It is easy to show that $\frac{(a+1-\delta)^2}{4(1-\delta)} > a$. Result 6 summarizes Case 2B.

Result 6. (*R6*) Suppose $R \leq 1 - \delta$. $\forall c, a, \delta$, the optimum is $p^* = \frac{a+1-\delta}{2}$, $R^* = 1 - \delta$, $\Pi^* = \frac{(a+1-\delta)^2}{4(1-\delta)}$.

A.3.1.4 Case 2C: $p \leq c$, $R > 1 - \delta$ The firm chooses p and R in order to maximize:

$$\Pi = a\frac{p}{R} + p\left(1 - \frac{p}{R}\right) \text{ subject to } R \geq p, p \leq c \text{ and } R > 1 - \delta.$$

The first derivative of Π with respect to R is $\frac{\partial \Pi}{\partial R} = \frac{p(p-a)}{R^2}$. We break down the analysis of the sign of this derivative according to whether $a \leq c$ or $a > c$.

First, when $a > c$, $p \leq c$ implies $p < a$ and $\frac{\partial \Pi}{\partial R} < 0$ so that $R^* \rightarrow \max\{1-\delta, p\} = 1 - \delta$ using Assumption A1 with $p < a < 1 - \delta$. The unconstrained optimal price is $\frac{a+1-\delta}{2} > c$. The set-up of Case 2C constrains the price to $p^* = c$ and the profits are then $\Pi^* = \frac{c(a+1-\delta-c)}{1-\delta}$.

Secondly, when $a \leq c$, there are 3 cases to analyze: (A) $p < a$, (B) $p = a$, (C) $p > a$.

Case (A): $p < a$. We use the same argument as above. The price is now constrained by a so that $p^* = a$, with profit $\Pi^* = a$.

Case (B): $p = a$. We have already shown that $\Pi^* = a$ with that price.

Case (C): $p > a$; $\frac{\partial \Pi}{\partial R} > 0$. The firm sets $R^* = 1$. In this situation, the consumer chooses either the outside option or the premium version, the firm's profit is then equal to $p(1-p)$. The unconstrained optimal price is $\frac{1}{2}$. However, given that $c < \frac{1}{2}$ implied by Assumption A2, the price is constrained to $p^* = c$ and the firm's profit is given by $\Pi^* = (1-c)c$.

We now compare $\Pi^* = a$ to $\Pi^* = (1-c)c$. The difference between $(1-c)c$ and a can be written as the polynomial in c , $-c^2 + c - a$. The two roots are $c_1 = \frac{(1-\sqrt{1-4a})}{2}$ and $c_2 = \frac{(1+\sqrt{1-4a})}{2}$. It is easy to show that these roots are such that $a < c_1 < \frac{1}{2} < c_2$. Thus for c such that $a \leq c < \frac{1}{2}$, if $c < c_1$, $a > c(1-c)$ and if $c > c_1$, $a < c(1-c)$.

Result 7 summarizes Case 2C.

Result 7. Suppose $p \leq c$, $R > 1 - \delta$.

(R7.i) If $0 < c < a$, the optimum is $p^* = c$, $R^* \rightarrow 1 - \delta$, $\Pi^* \rightarrow \frac{c(a+1-\delta-c)}{1-\delta}$;

(R7.ii) If $a \leq c \leq \frac{(1-\sqrt{1-4a})}{2}$, the optimum is $p^* = a$, $R^* \in (1 - \delta, 1)$, $\Pi^* = a$;

(R7.iii) If $\frac{(1-\sqrt{1-4a})}{2} < c < \frac{1}{2}$, the optimum is $p^* = c$, $R^* = 1$, $\Pi^* = c(1 - c)$.

A.3.2 Step2. Comparison between profits

In the second step, we compare all profits in order to determine the equilibrium behavior of the firm.

Result 8. The profit from Case 2B (R6) is greater than the profits from Case 2C (R7) for all values of the model.

Proof. (R6) vs (R7.i): $\frac{(a+1-\delta)^2}{4(1-\delta)} - \frac{c(a+1-\delta-c)}{1-\delta} = \frac{(2c-a+\delta-1)^2}{4(1-\delta)} > 0$.

(R6) vs (R7.ii): $\frac{(a+1-\delta)^2}{4(1-\delta)} - a = \frac{(a+\delta-1)^2}{4(1-\delta)} > 0$.

(R6) vs (R7.iii): $\frac{(a+1-\delta)^2}{4(1-\delta)} - c(1 - c)$ has the same sign as the following polynomial in c : $P_1(c) = 4(1 - \delta)(1 - c)c - (\delta - a - 1)^2$ which is negative for $c = 0$. The roots c_1 and c_2 are such that: $0 < c_1 < \frac{(1-\sqrt{1-4a})}{2} < c < \frac{1}{2} < c_2$. Thus, $\frac{(a+1-\delta)^2}{4(1-\delta)} > c(1 - c)$ when $\frac{(1-\sqrt{1-4a})}{2} < c < \frac{1}{2}$. \square

It remains to compare profits from Case 1 and Case 2A with the profit from Case 2B.

(R5.i) vs (R6): $\frac{(1-\delta)^2+(a-c)^2+2(1-\delta)(a+c)}{4(1-\delta)} - \frac{(a+1-\delta)^2}{4(1-\delta)} = \frac{(c+2(1-a-\delta))c}{4(1-\delta)} > 0$ by Assumption A1 ($a < 1 - \delta$). This proves Proposition 1 (B).

(R5.iii) vs (R6): $\frac{(a+1-\delta)^2}{4(1-\delta)} - \frac{1}{4}$ has the same sign as the following polynomial in δ : $P_2(\delta) = \delta^2 - \delta(2a + 1) + a(a + 2)$, which is positive for $\delta = 0$. The roots δ_1 and δ_2 are such that: $0 < 2a < \delta_1 < 1 - a < \delta_2 < 1$, where $\delta_1 = a + \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4a}$. Thus, when $0 < \delta < \delta_1$ then $\frac{(a+1-\delta)^2}{4(1-\delta)} > \frac{1}{4}$ and when $\delta_1 < \delta < 1 - a$ then $\frac{(a+1-\delta)^2}{4(1-\delta)} < \frac{1}{4}$. Finally, the optimal profit is $\frac{1}{4}$ when ($\delta > \delta_1$) and ($\frac{\delta(a+1-\delta)}{2-\delta} \leq c \leq \frac{1}{2}$ and $\delta \leq 2a$) or ($\frac{\delta}{2} \leq c \leq \frac{1}{2}$ and $\delta \geq 2a$). This proves Proposition 1 (D).

The comparison (R5.iii) vs (R6) gives us also the following intermediate result: if ($\frac{\delta(a+1-\delta)}{2-\delta} \leq c < \frac{1}{2}$ and $\delta \leq 2a$) or ($\frac{\delta}{2} \leq c < \frac{1}{2}$ and $2a < \delta < a + \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4a}$) then the optimal profit is $\frac{(a+1-\delta)^2}{4(1-\delta)}$.

We have to compare now (R5.ii) vs (R6): $\frac{(a+1-\delta)^2}{4(1-\delta)} - \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$ has the same sign as the following polynomial in c : $P_3(c) = \delta^2(\delta - a - 1)^2 - 4c(1 - \delta)(\delta - c)$, which is positive for $c = 0$. When $2a < \delta < a + \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4a}$, $P_3(c)$ has no roots and is always positive. In this case $\frac{(a+1-\delta)^2}{4(1-\delta)} > \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$. Combining this with the intermediate result above proves Proposition 1 (A).

When $a + \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4a} < \delta < 1 - a$, $P_3(c)$ has two roots c_1 and c_2 such that $0 < c_1 < \frac{\delta}{2} < c_2$. The difficult point here is that the root c_1 may be lower or greater than $\frac{\delta(a+1-\delta)}{2-\delta}$. More precisely, $0 < c_1 < \frac{\delta(a+1-\delta)}{2-\delta}$ when $a < \underline{a}$ where $\underline{a} = \frac{(4-\delta)(1-\delta)\delta}{\delta^2+8(1-\delta)}$. In order to simplify the result, we consider an ARPU a that is relatively small, that is $a < \underline{a}$.²⁴ Thus, $\max\left\{\frac{c}{\delta} \left(1 - \frac{c}{\delta}\right), \frac{(a+1-\delta)^2}{4(1-\delta)}\right\} = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$. This concludes the proof of Proposition 1.

A.4 Proof of Proposition 3

We examine the impact of changes in δ and c parameters on welfare in the different business models, accounting for how these changes impact the choice of business model by the firm.

A.4.1 Monopoly

In the monopoly business model we have: $p^* = \frac{1}{2}$, $R^* = 1$, and $\Pi^* = \frac{1}{4}$. According to the proof of Proposition 1, the Monopoly is optimal only in Case 2A. The demand function for the premium version is therefore:

$$D_{premium} = 1 - \theta_{fp} = 1 - \frac{p}{R} = \frac{1}{2}.$$

With $R^* = 1$, $U_f = 0$ and according to our hypothesis of sorting condition, the demand function for the free version is $D_{free} = 0$.

Finally, the utility $U_c = \delta\theta - c < 0$ for consumers to the left of the consumer θ_{fp} ($\theta < \theta_{fp} = \frac{1}{2}$) since the monopoly business model is optimal when $\frac{\delta}{2} \leq c$, thus:

$$D_{copy} = 0.$$

The consumer surplus is therefore:

$$CS^M = \int_{\frac{1}{2}}^1 (t - p) dt = \int_{\frac{1}{2}}^1 \left(t - \frac{1}{2}\right) dt = \frac{1}{8}.$$

²⁴When $a > \underline{a}$ there exists a small area in which the freemium is preferred to the limit-pricing monopoly.

Turning to social welfare:

$$SW^M = CS^M + \Pi^M = \frac{3}{8}.$$

It is constant and therefore does not vary with either δ or c .

A.4.2 Limit pricing

In the area where the limit pricing model is optimal we have, $R^* = 1$, $p^* = \frac{c}{\delta}$ and $\Pi^* = \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right)$. According to the proof of Proposition 1, the limit pricing is optimal in Cases 1 and 2A. The demand functions for the premium version are:

$$D_{premium} = 1 - \theta_{cp} \quad (\text{in Case 1}) \quad \text{or} \quad D_{premium} = 1 - \theta_{fp} \quad (\text{in Case 2A}).$$

Moreover, $\theta_{cp} = \frac{p-c}{1-\delta} = \frac{c}{\delta}$ and $\theta_{fp} = \frac{p}{R} = \frac{c}{\delta} = \theta_{cp}$.

This implies that:

$$D_{premium} = 1 - \theta_{cp} = 1 - \theta_{fp} = 1 - \frac{c}{\delta}.$$

We also note that with $R = 1$ and $p = \frac{c}{\delta}$, we have $\theta_{fp} = \theta_{cp} = \theta_{cf} = \theta_{\emptyset c} = \theta_{\emptyset p} = \frac{c}{\delta}$.

With our hypothesis of sorting condition – a consumer prefers the outside option to the free version with the maximum number of restrictions $R = 1$ – we have:

$$D_{free} = 0 \quad (\text{and} \quad D_{copy} = 0 \quad \text{since} \quad \theta_{\emptyset c} = \theta_{cp}).$$

Consumer surplus is therefore:

$$CS^{LP} = \int_{\frac{c}{\delta}}^1 (t - p) dt = \int_{\frac{c}{\delta}}^1 \left(t - \frac{c}{\delta}\right) dt = \frac{1}{2} \frac{(c - \delta)^2}{\delta^2},$$

$$SW^{LP} = CS^{LP} + \Pi^{LP} = \frac{1}{2} \frac{(c - \delta)^2}{\delta^2} + \frac{c}{\delta} \left(1 - \frac{c}{\delta}\right) = \frac{1}{2} (\delta - c) \frac{c + \delta}{\delta^2} > 0 \quad (A2).$$

SW^{LP} depends on both c and δ :

$$\frac{\partial(SW^{LP})}{\partial\delta} = \frac{c^2}{\delta^3} > 0,$$

$$\frac{\partial(SW^{LP})}{\partial c} = -\frac{c}{\delta^2} < 0.$$

SW^{LP} increases with δ and decreases with c , meaning that the higher the δ and the lower the c , the higher the welfare in the limit pricing area.

A.4.3 Freemium

In the area where the freemium model is optimal, $R^* = 1 - \delta$, $p^* = \frac{a+1-\delta}{2}$ and $\Pi^* = \frac{(a+1-\delta)^2}{4(1-\delta)}$. According to the proof, freemium is optimal only in Case 2B. The demand functions are therefore:

$$D_{premium} = 1 - \theta_{fp} = 1 - \frac{p}{R} = \frac{1}{2} \frac{1-a-\delta}{1-\delta} > 0 \quad (A1 : a < 1 - \delta),$$

$$D_{free} = \theta_{fp} = \frac{1}{2} \frac{1+a-\delta}{1-\delta} > 0.$$

$$\begin{aligned} CS^F &= \int_0^{\theta_{fp}} (1-R)t \, dt + \int_{\theta_{fp}}^1 (t-p) \, dt \\ &= \int_0^{\frac{1}{2} \frac{a-\delta+1}{1-\delta}} \delta t \, dt + \int_{\frac{1}{2} \frac{a-\delta+1}{1-\delta}}^1 \left(t - \frac{a+1-\delta}{2} \right) dt \\ &= -\frac{1-2a+2\delta+2a\delta-3\delta^2+a^2+1}{8(\delta-1)}, \end{aligned}$$

this implies that:

$$SW^F = CS^F + \Pi^F = \frac{1}{8} \frac{(a-\delta)(3a+\delta+2)+3}{1-\delta} \quad (A1).$$

In this area, SW^F does not depend on c .

$$\frac{\partial(SW^F)}{\partial\delta} = \frac{1}{8} \frac{(\delta-1)^2 + 3a^2}{(1-\delta)^2},$$

$\frac{\partial(SW^F)}{\partial\delta}$ has the sign of $(\delta-1)^2 + 3a^2$ and is therefore positive. SW^F increases with δ , that is, the higher δ , the higher the welfare in the freemium area.

A.4.4 Freemium+

Finally, in the area of the freemium+ model, the results are:

$$R^* = \frac{(a+1)\delta - c}{\delta} \quad \text{and} \quad p^* = \frac{a+1}{2},$$

$$\Pi^* = \frac{(a+1)^2}{4} - \frac{c(a+1)\delta - c}{\delta}.$$

According to the proof of Proposition 1, Freemium+ is optimal in Case 3. Demand functions are therefore:

$$D_{premium} = 1 - \theta_{fp} = 1 - \frac{p}{R} = 1 - \frac{\frac{a+1}{2}}{\frac{(a+1)\delta - c}{\delta}} = \frac{\delta - c}{(a+1)\delta - c} \quad (A2),$$

$$D_{free} = \theta_{fp} = \frac{a+1 - \frac{c}{\delta}}{2} = \frac{a\delta + \delta - c}{2\delta} > 0 \quad (A1 : a < 1 - \delta).$$

$$\begin{aligned} CS^{F+} &= \int_0^{\theta_{fp}} (1-R)t \, dt + \int_{\theta_{fp}}^1 (t-p) \, dt \\ &= \int_0^{\frac{a\delta + \delta - c}{2\delta}} \frac{\delta(c-\delta)}{\delta(a+1) - c} t \, dt + \int_{\frac{a\delta + \delta - c}{2\delta}}^1 \left(t - \frac{a+1}{2}\right) \, dt \\ &= -\frac{(a+1)(\delta-1) + c(2-\delta)}{8\delta} \left(\frac{\delta-c}{(a+1)\delta-c}\right). \end{aligned}$$

$$\begin{aligned} SW^{F+} &= CS^{F+} + \Pi^{F+} \\ &= -\frac{(a+1)(\delta-1) + c(2-\delta)}{8\delta} \left(\frac{\delta-c}{(a+1)\delta-c}\right) + \left[\frac{a^2 + 2a + 1}{4} - \frac{c(a+1)\delta-c}{\delta}\right] \\ &= \frac{(a+1)\delta-c}{8\delta}. \end{aligned}$$

In this area, SW^{F+} depends on c and δ :

$$\begin{aligned} \frac{\partial(SW^{F+})}{\partial\delta} &= \frac{a+1}{8\delta} > 0, \\ \frac{\partial(SW^{F+})}{\partial c} &= -\frac{1}{8\delta} < 0. \end{aligned}$$

SW^{F+} increases with δ and decreases with c , meaning that the higher the δ and the lower the c , the higher the welfare in the freemium+ area.

However, if $a < \hat{a}$, there are two cases depending on whether $c < \hat{c} = 2a$, then $\frac{\partial(SW^{F+})}{\partial c} < 0$, and if $c > \hat{c} = 2a$, then $\frac{\partial(SW^{F+})}{\partial c} > 0$.

A.5 Extension: General Distribution of the Taste Parameter

We now discuss how our results change with a general distribution of the taste parameter θ . Let $F(\theta)$ be the distribution of θ with support $[\theta_0, \theta_1]$ and density $f(\theta)$.

We argue that the qualitative features of Figure 3 do not change with a general distribution of θ . We will show that the firm will always prefer the freemium and freemium+ models to a monopoly model without discrimination when it is feasible to do so. We establish this result in Proposition 4.

When the firm is either unconstrained – and charges a monopoly price – or uses limit pricing without introducing the free version, the optimal business model will not change with a general distribution function.

Hence, we focus in the remainder of the analysis on cases where the freemium model is achievable. We show that in these areas the freemium model dominates the monopoly business model. Since the freemium+ model always yields a higher profit than the freemium model, it will dominate both business models when it is achievable.

The main trade-off identified in the previous section (market expansion vs cannibalization) is still valid with a general distribution of the taste parameter, and Proposition 4 establishes that, in equilibrium, the market-expansion effect when there is no cost of versioning always dominates the cannibalization effect. Therefore, the freemium model dominates the premium version only.

Consider first the profits of the firm when proposing only the premium version, but facing copies in areas A or B. The consumer who is indifferent between the copy and the premium version is defined by:

$$\theta_{cp} = \frac{p - c}{1 - \delta}.$$

The demand for the premium service is $1 - F\left(\frac{p-c}{1-\delta}\right)$, and the firm's profit is

$$\Pi_p = \left(1 - F\left(\frac{p - c}{1 - \delta}\right)\right)p. \quad (6)$$

Consider now the introduction of the free version. When the copy is not initially dominated by the free version, the indifferent consumers between the copy and the free version is located at $\frac{c}{R+\delta-1} = \theta_{fc} \leq \theta_{fp}$, where the consumer indifferent between the free version and the premium version is defined by:

$$\theta_{fp} = \frac{p}{R}.$$

It is clear that the firm can increase its profits by lowering R , thereby poaching consumers who use the copy. Hence, in equilibrium, we have $\theta_{fp} \leq \theta_{fc}$. This gives a demand for the premium version equal to $1 - F\left(\frac{p}{R}\right)$ and a demand for the free version of $F\left(\frac{p}{R}\right)$. The profit is

$$\Pi_f = aF\left(\frac{p}{R}\right) + \left(1 - F\left(\frac{p}{R}\right)\right)p. \quad (7)$$

As no consumer uses the copy, the condition $\theta_{cp} \leq \theta_{fp}$ is equivalent to having $R \leq \min\{1, \frac{p(1-\delta)}{p-c}\}$. As we focus on parameter values where some consumers use the free version, we always have $\frac{p(1-\delta)}{p-c} < 1$, so that $R < 1$, and the condition boils down to $R \leq \frac{p(1-\delta)}{p-c}$.

Finally, let us denote (p^*, R^*) the optimal price and level of restrictions when the firm introduces the free version, which maximize $\Pi_f(p, R)$. Similarly, we introduce $p^{**} = \arg \max_{p>0} \{\Pi_p(p)\}$. Proposition 4 establishes that the freemium and freemium+ models generate more profits than offering a single product.

Proposition 4.

Offering a free version is strictly optimal: $\Pi_f(p^) > \Pi_p(p^{**})$.*

Proof. By definition, we know that

$$\begin{aligned} \Pi_f(p^*, R^*) &\geq \Pi_f\left(p^{**}, \frac{p^{**}(1-\delta)}{p^{**}-c}\right) \\ &\geq aF\left(\frac{p^{**}-c}{1-\delta}\right) + \left(1 - F\left(\frac{p^{**}-c}{1-\delta}\right)\right)p^{**} \\ &\geq aF\left(\frac{p^{**}-c}{1-\delta}\right) + \Pi_p(p^{**}) > \Pi_p(p^{**}). \end{aligned}$$

□

The term $aF\left(\frac{p^{**}-c}{1-\delta}\right)$ is the positive market-expansion effect on profits. The following Proposition 5 establishes that introducing the free version always increases the price of the premium service.

Proposition 5.

The price of the premium service is higher with the free version: $p^ \geq p^{**}$.*

Proof. By contradiction, assume that $p^* < p^{**}$. In this case, there exists $R^{**} > R^*$ such that $\frac{p^*}{R^*} = \frac{p^{**}}{R^{**}}$. Hence, setting the prices and restrictions (p^{**}, R^{**}) does not impact the demand of the firm, but increases its revenue per consumer generated by the premium version. This yields strictly greater profits, which contradicts the optimality of p^* . This establishes the proof. □

Finally, we can analyze the impact of an increase in the ARPU a on the business models of the firm. A higher value of a increases the revenues per consumer for

the free product, which enhances the benefits from reaching a higher demand with this version of the good. Hence, an increase in a decreases the optimal level of restrictions R in the freemium and freemium+ business models. The cannibalization effect is fiercer as the free version serves a larger demand, and the optimal price for the premium good increases with a , as only consumers with the highest valuation will consume this version. The overall impact on profit is positive.