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Chapter 8

**TREE TRUNK DIAMETER ESTIMATION
IN RAIN FOREST ECOLOGY:
ANALYSIS AND OPTIMAL CONTROL**

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Abstract

We discuss about a mechanical problem related to the evaluation of the dimensional variations of tree trunk diameter, in tropical forest, where some data is missing. In particular, the optimal control question is studied rigorously. Within a cost function, the control problem is formulated within the mechanical tree trunk variation model which requires boundary conditions to solve all equations. We give a characterization of the optimal measurement function (the optimal control) for the tree trunk problem.

Keywords: Tree trunk diameter variation model, optimal control, missing data

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1. Introduction

In this chapter, we study an important mechanical problem related to the evaluation of the dimensional variations of tree trunk diameter in tropical forest, where some data are missing. This estimation is very pertinent as it is related to the ecological tropical forest which represents an important natural planetary carbon storage, and so reduces the global warming that is bothering all the planet.

The tropical forest is a home of very dense wood species, pioneers in the capture of large amounts of carbon for a long time. This carbon capture is happening during the photosynthesis in which trees, through their leaves and for some species form their trunks, exchange gas with the atmosphere. The quantities of the captured carbon are important especially during the trees growth and they are stocked in the wood, and mostly in the trunk.

Since a larger part of a mature tropical forest acre is composed of trunks, tropical forests are highly qualified, as they represent an interesting carbon reservoir. In addition, the captured carbon remains in the wood even after the tree dies or its cutting; this highly qualifying wood seems to be a very interesting renewable resource used in construction and also as an ecological biomass energy.

In order to estimate the captured carbon quantities and to contribute to a sustainable forest management, quantifying the forest growth through the dimensional variation of tree trunk diameter is very important. A mathematical model is needed. It is based on continuum mechanical formulation, requiring equilibrium equations, constitutive law, and boundary conditions for the studied problem. In this chapter, an hypothetical regular, multilayer and coaxial cylindrical model is presented. In this case, displacement, strain and stress fields are only radial dependent. In addition, no shear stresses and no shear strains occur in the tree trunk. All these considerations simplify the mechanical equations so they can be handled analytically, but some boundary conditions are often missed.

In general, daily dimensional variations in wet season are considered for the trees trunk diameter estimation. So the problem is to evaluate the response of a cylindrical structure composed of different layers of orthotropic material, and subjected to an internal stress representing the hydric pressure in the inner bark and the sapwood. Boundary conditions are external constraints at the outer and the inner borders. In this chapter, we study a problem where missing data are

in the inner border, this concerns a great number of hollow species as the only known experimental data are given on the trees periphery, and no data inside. To overcome this difficulty, an optimal control problem is associated with the mechanical model.

Within a cost function, the control problem is formulated with a mechanical model that requires boundary conditions to solve all equations. We give a characterization of the optimal measurement function for the tree trunk problem.

We will recall the main equations of the mathematical model and some important definitions on anisotropic material. We then introduce the optimal control method and we give at the end the characterization of the optimal control.

The chapter is organized as follows. In sections 2 and 3, we explain the mechanical model and give the mathematical system considered for the control problem. We then introduce the optimal control analysis in section 3. In the last section, we give the characterization of the optimal control.

2. Preliminaries

For a mechanical analysis of a dimensional variation of a tree trunk, the complete model must include all the equations of the mechanics, especially equilibrium equations, constitutive law, and boundary conditions (where we generally find incomplete data). Under the internal and external actions considered in this paper, hydrostatic pressure principally, and an hypothetic regular cylindrical structure, the displacement, strain and stress fields generated are only radial dependent. In addition, no shear stresses and no shear strains occurs in the tree trunk. All these considerations simplify the mechanical equations.

The problem consists in computing the state of equilibrium of cylindrical structure composed of different layers of orthotropic material, and subjected to an internal stress representing the water pressure in the inner bark and the sapwood. Boundary conditions are external constraints at the outer and the inner borders. For this problem, missing data are in the inner border.

For predicting tree diameter variations, an analytical analysis based on a mechanical model by Alméras [3] and by Alméras and Gril [4] is performed. The model requires sufficient experimental data. This concerns a great number of hollow species where some data are missing as the only known experimental data are given on the trees periphery, and no data inside. To overcome this difficulty, an optimal control problem is associated with the mechanical model.

2.1. Mathematical Formulation with Functional Analysis

It is difficult (say impossible) to find an exact strong solution to the mechanical problem in its general form. We use the variational formulation to find a weak solution.

Strong Formulation

Let be Ω a sufficient smooth domain (a piece of wood under study) of \mathbb{R}^3 , of boundary:

$$\partial\Omega = \Gamma_0 \cup \Gamma_1 \quad \text{with} \quad \Gamma_0 \cap \Gamma_1 = \emptyset.$$

Then the initial boundary-value problem admits the following formulation:

Find the field displacement $\underline{u} \in \mathcal{C}^1(\bar{\Omega})$, where $\mathcal{C}^1(\bar{\Omega})$ is the set of regular functions of continuous derivatives on $\bar{\Omega}$, such that:

$$\text{div } \underline{\sigma}(\underline{u}) + \underline{f} = \underline{0} \quad (1)$$

$$\underline{\sigma}(\underline{u}) = \underline{\underline{D}} : \underline{\underline{\varepsilon}}(\underline{u}) \quad (2)$$

where \underline{f} admits three components $f_i \in L^2(\Omega)$.

The following boundary conditions have also to be satisfied:

$$\underline{u} = \underline{0} \quad \text{on } \Gamma_0 \quad \text{displacement equal to zero, imposed on } \Gamma_0 \quad (3)$$

$$\underline{\sigma} \cdot \underline{n} = \underline{g} \quad \text{on } \Gamma_1 \quad \text{imposed constraints } \Gamma_1 \quad (4)$$

where \underline{g} has three components $g_i \in L^2(\Gamma_1)$. The terms σ , ε , D ... are given below in their vector and matrices form.

This formulation is called the strong formulation, which gives a strong solution (that we can not calculate). We now give the weak formulation of the problem, called the *variational formulation*:

Variational Formulation

We define the Hilbert space V de Hilbert (subspace of $(H^1(\Omega))^3$) by:

$$V = \left\{ \underline{v} : \Omega \rightarrow \mathbb{R}^3; \quad v_i \in L^2(\Omega), \quad \frac{\partial v_i}{\partial x_j} = 0, \quad v_i = 0, \quad \text{on } \Gamma_0 \right\}, \quad (5)$$

V is the Sobolev space $(H^1(\Omega))^3$ of the vector fields of \mathbb{R}^3 which are equal to zero on one of the boundaries (say Γ_0).

The problem is now to:

Find the field displacement $\underline{u} \in V$, such that

$$a(\underline{u}, \underline{v}) = L(\underline{v}), \quad \forall \underline{v} \in V. \quad (6)$$

If $a(., .)$ is a bilinear, continuous and coercitive form on $V \times V$, and $L(.)$ is a linear continuous form on V , then there is a unique solution $\underline{u} \in V$ to the weak problem (6).

The details on $a(., .)$ and $L(.)$ are given by E. Robo-Petit in [10].

3. The Mechanical Coaxial-Cylindrical Model

For a complete description of the mechanical equations related to wood, we refer to [10] and the references therein. Let us consider an adequate cylindrical system (R, T, L) , where R , T and L are radial, tangential and longitudinal directions respectively.

A transverse section of tree trunk is composed of different layers (see Figure 1).

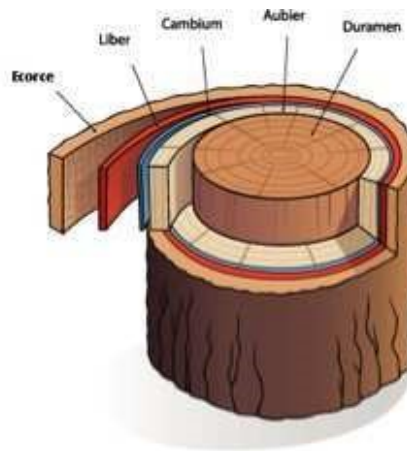


Figure 1. A transverse section of tree trunk.

Remark 1. *Due to the tree trunk geometry and to its constituent layers, it can be idealized as a coaxial multi-layer cylindrical structure as shown in Figure 1.*

At a given radial position r , the induced displacement vector and associated strains and stress are respectively represented by the following vectors:

$$\underline{u} = (u_R, u_T, u_L), \quad \underline{\varepsilon} = (\varepsilon_R, \varepsilon_T, \varepsilon_L), \quad \text{and} \quad \underline{\sigma} = (\sigma_R, \sigma_T, \sigma_L), \quad (7)$$

where strains components are given by

$$\varepsilon_R = \frac{du_R}{dr}(r) \quad \text{and} \quad \varepsilon_T = \frac{u_R}{r}(r), \quad (8)$$

u_R and ε_R being the radial displacement and strain and u_T and ε_T the tangential ones.

Wood is commonly modeled as an orthotropic linear elastic material [6]. Its 6 constituents are:

3 longitudinal elastic: E_R , E_T et E_L , in privileged directions,

3 Poisson moduli: V_{RT} , V_{RL} et V_{TL} .

In this case, wood behavior is represented by its flexibility matrix:

$$\underline{D} = \begin{bmatrix} \frac{1}{E_R} & \frac{-v_{TR}}{E_T} & \frac{-v_{LR}}{E_L} \\ \frac{-v_{RT}}{E_R} & \frac{1}{E_T} & \frac{-v_{LT}}{E_L} \\ \frac{-v_{RL}}{E_R} & \frac{-v_{TL}}{E_T} & \frac{1}{E_L} \end{bmatrix}$$

In our case, the constitutive law is restricted to this relation:

$$\underline{\sigma} = \underline{C} \cdot \underline{\varepsilon} + \underline{\beta} \quad \text{where} \quad \underline{C} = \begin{bmatrix} C_{RR} & C_{RT} & C_{RL} \\ C_{TR} & C_{TT} & C_{TL} \\ C_{LR} & C_{LT} & C_{LL} \end{bmatrix} \quad (9)$$

\underline{C} is the behaviour tensor and $\underline{\beta}$ is the initially induced stresses vector, related to hydrostatic pressure in the sapwood and inner bark. It is given by

$$\underline{\beta} = (\beta_R, \beta_T, \beta_L). \quad (10)$$

Equilibrium equations are restricted to the following equation:

$$\frac{d\sigma_R}{dr} + \frac{\sigma_R - \sigma_T}{r} = 0. \quad (11)$$

Finally, the equilibrium equation can be expressed in displacement term by combining behaviour law (9), strains expressions (8) together with the equilibrium equation (11). It is given by

$$r \frac{d^2 u_R}{dr^2} + \frac{du_R}{dr} - \alpha^2 \frac{u_R}{r} = \kappa, \quad (12)$$

where $\alpha = \sqrt{\frac{C_{RR}}{C_{TT}}}$ and $\kappa = \frac{\beta_T - \beta_R}{C_{RR}} + \frac{C_{TL} - C_{RL}}{C_{RR}} \varepsilon_L$ are positive constants.

We now give an important remark for the sequel of the analysis.

Remark 2. *We point out that the only displacement component is the radial displacement u_R , but tangential displacement is possible. For the same reason, no shear strain and stress fields are possible. All these considerations simplify the mechanical formulation.*

The general solution of equation (12) can be explicitly calculated and is given by:

$$u_R(r) = Ar^\alpha + Br^{-\alpha} + r \frac{\kappa}{1 - \alpha^2}, \quad (13)$$

where A and B are real constants. The strain field can then directly be derived from equation (8) and the associated stress field from the constitutive law (9).

Remark 3. *The real numbers A and B are integration constants determined using the boundary conditions of the problem. It is here where we find incomplete data.*

4. The Optimal Control

In some tropical wood species, the tree inners is hollow and the only available experimental data are in external border; here the initial data is unknown.

More precisely, the model describes the trunk daily variation under a water flow through the trees inner bark and the sapwood, and an optimal control

function for the trunk diameter measurement in presence of incomplete data is used.

Back to the problem (12). We consider the radial set in the half space $[r_0, r_1]$ where $0 < r_0 < r_1 < \infty$. We then complete by the initial data as follows:

$$r \frac{d^2 u_R}{dr^2} + \frac{du_R}{dr} - \alpha^2 \frac{u_R}{r} = \kappa + v \quad (14)$$

$$u_R(r_0) = g \quad (15)$$

$$u_R(r_1) = 0 \quad (16)$$

where g is unknown and where $v = v(r)$ is the control function. We here suppose that the displacement u_R in the external radial bound r_1 is null as in [4]. The solution to (14) has the form (13). The question that we address is the following: *Even if the constants A and B are unknown, can we make $u_R(r)$ as close as possible to some nominative \widetilde{u}_R given by experiments (or observation), along the interval $[r_0, r_1]$?*

We denote by $u_R = u_R(r; v, g) := u_R(v, g)$, the displacement function. We want to choose a control v such that $|u_R - \widetilde{u}_R| = \min$. One method is the optimal control method. We define the *cost function*:

$$J(v, g) = \int_{r_0}^{r_1} |u_R(v, g) - \widetilde{u}_R|^2 dr + N \int_{r_0}^{r_1} |v|^2 dr, \quad N > 0. \quad (17)$$

Remark 4. *Note that the cost function is quadratic. The main reason is of computational order. Another convex function can be considered too.*

4.1. Adapted Functional Spaces

The above cost function can be justified by the following: The gradient du_R/dr must exist, moreover u_R has to be bounded. This, with (13) we can suppose that:

$$u_R \in H^1(]r_0, r_1[) = \left\{ u_R \in L^2(]r_0, r_1[), \quad \text{s.t.} \quad \frac{du_R}{dr} \in L^2(]r_0, r_1[) \right\}, \quad (18)$$

where

$$L^2(]r_0, r_1[) = \left\{ u_R \text{ measurable, s.t.} \quad \int_{r_0}^{r_1} |u_R(r)|^2 dr < +\infty \right\}. \quad (19)$$

The perturbation $g \in \mathbb{R}$ which is an \mathbb{R} -vectoriel space. The constants \widetilde{u}_R and the reals g are clearly in $L^2(]r_0, r_1[)$.

4.2. The Optimal Control Problem

We refer to Lions [8] (see also Dorville et al. [5]) to see that the problem:

$$\inf_{v \in L^2(]r_0, r_1[)} J(v, g) \quad \forall g \in \mathbb{R} \quad (20)$$

is impossible, since $g \in \mathbb{R}$ is an infinite set. One idea is then to consider the no-regret control problem:

$$\inf_{v \in L^2(]r_0, r_1[)} \left(\sup_{g \in \mathbb{R}} [J(v, g) - J(0, g)] \right)$$

which selects controls v better than $v = 0$ (better than no control on the problem) for any perturbation $g \in \mathbb{R}$.

Remark 5. *We notice that we have*

$$u_R(v, g) = u_R(v, 0) + u_R(0, g) - u_R(0, 0).$$

We immediatly deduce that

$$\begin{aligned} J(v, g) - J(0, g) &= J(v, 0) - J(0, 0) \\ &+ 2 \int_{r_0}^{r_1} (u_R(0, g) - u_R(0, 0)) (u_R(v, 0) - u_R(0, 0)) dr. \end{aligned}$$

We should evaluate the integral

$$\int_{r_0}^{r_1} (u_R(0, g) - u_R(0, 0)) (u_R(v, 0) - u_R(0, 0)) dr. \quad (21)$$

We introduce the adjoint linear function $\xi = \xi(r; v, 0) := \xi(v)$ such that

$$r \frac{d^2 \xi}{dr^2} - \frac{d\xi}{dr} - \alpha^2 \frac{\xi}{r} = u_R(v, 0) - u_R(0, 0) \quad (22)$$

$$\xi(v)(r_0) = 0 \quad (23)$$

$$\xi(v)(r_1) = 0 \quad (24)$$

then we have the following Lemma:

Lemma 6. *Using the adjoint problem (22) we obtain:*

$$\int_{r_0}^{r_1} (u_R(0, g) - u_R(0, 0)) (u_R(v, 0) - u_R(0, 0)) dr = -r_0 g \frac{d\xi}{dr}(r_0). \quad (25)$$

Proof. In the (21) we set

$$u_R(v, 0) - u_R(0, 0) = r \frac{d^2\xi}{dr^2} - \frac{d\xi}{dr} - \alpha^2 \frac{\xi}{r}. \quad (26)$$

Then we multiply this equality by $u_R^g := u_R(0, g) - u_R(0, 0)$ and we integrate by parts as the following:

$$\begin{aligned} & \int_{r_0}^{r_1} (u_R(v, 0) - u_R(0, 0)) u_R^g dr \\ &= \int_{r_0}^{r_1} \left(r \frac{d^2\xi}{dr^2} - \frac{d\xi}{dr} - \alpha^2 \frac{\xi}{r} \right) u_R^g dr \\ &= \int_{r_0}^{r_1} \xi \left(r \frac{d^2 u_R^g}{dr^2} + \frac{d u_R^g}{dr} - \alpha^2 \frac{u_R^g}{r} \right) dr + r_1 \frac{d\xi}{dr}(r_1) u_R(r_1) \\ & \quad - r_0 \frac{d\xi}{dr}(r_0) u_R(r_0) - r_1 \xi(r_1) \frac{d u_R}{dr}(r_1) + r_0 \xi(r_0) \frac{d u_R}{dr}(r_0). \end{aligned}$$

Now we use the boundary conditions of systems (14) and (22), and we obtain immediatly (25). ■

5. Characterization: The Low-Regret Control

As in [5], for any $\gamma > 0$, we introduce the low-regret control solution to the following perturbed system :

$$\inf_{v \in L^2(\lceil r_0, r_1 \rceil)} \left(\sup_{g \in \mathbb{R}} [J(v, g) - J(0, g) - \gamma |g|^2] \right).$$

We obtain the classical control problem :

$$\inf_{v \in L^2(\lceil r_0, r_1 \rceil)} \mathcal{J}^\gamma(v) \quad (27)$$

where

$$\mathcal{J}^\gamma(v) = J(v, 0) - J(0, 0) + \frac{r_0^2}{\gamma} \left| \frac{d\xi}{dr}(v)(r_0) \right|^2, \quad (28)$$

and where ξ is defined by (22).

The problem (27) admits a unique solution $v = u_\gamma$. Then the necessary condition of first order of Euler on $L^2(]r_0, r_1[)$ for every $w \in L^2(]r_0, r_1[)$ writes

$$(u_R(u_\gamma, 0) - \widetilde{u}_R, u_R(w, 0) - u_R(0, 0)) + (Nu^\gamma, w) + \left(\frac{r_0^2}{\gamma} \frac{d\xi}{dr}(r_0)(u_\gamma), \frac{d\xi}{dr}(r_0)(w) \right) \geq 0. \quad (29)$$

We have the following proposition.

Proposition 7. *The low-regret control u_γ solution to (27)–(28), is characterized by the unique $\{u_R^\gamma, \xi_\gamma, \rho_\gamma, p_\gamma\}$ solution to :*

$$\begin{cases} Au_R^\gamma = \kappa + u_\gamma, & A^*\xi_\gamma = 0, & A\rho_\gamma = 0, & A^*p_\gamma = 0, \\ u_{R|\Gamma}^\gamma = 0, & \xi_{\gamma|\Gamma} = u_R^\gamma - u_R(0, 0), & \rho_{\gamma|\Gamma} = \frac{1}{\gamma}\xi_\gamma, & \partial p_{\gamma|\Gamma} = u_R^\gamma - \widetilde{u}_R + \rho_\gamma, \end{cases}$$

with the adjoint state:

$$p_\gamma + Nu_\gamma = 0 \quad \text{in } L^2(]r_0, r_1[),$$

and with the notations

$$A = r \frac{d^2}{dr^2} + \frac{d}{dr} - \frac{\alpha^2}{r} id \quad \text{and} \quad A^* = r \frac{d^2}{dr^2} - \frac{d}{dr} - \frac{\alpha^2}{r} id,$$

and

$$\Gamma = \{r_0, r_1\}, \quad u_R^\gamma \in L^2(]r_0, r_1[) \quad \text{and} \quad \xi_\gamma, \rho_\gamma, p_\gamma \in H^{3/2}(]r_0, r_1[).$$

Proof. For a proof, see the article by Y. Aimène et al. [2]. ■

6. Concluding Remarks

In some tropical wood species, tree inner layers are hollow and the only available experimental data are in external border; here the inner (and not initial) data is unknown.

The modeling of the daily tree trunk dimensional variation during the wet season is examined on tropical species. It is the result of different phenomena happening at different layers that constitute the transverse section of the trunk. The mechanical formulation associated to this problem represents the trunk with

a cylindrical coaxial layers where the variations are due to hydric pressure only. The symmetry of the problem gives rise to an ordinary differential equation for which analytical solution is available, when all boundary conditions are available. In the case of hollow trunks, one boundary condition is missing and the solution of this problem is impossible.

To circumvent this problem of missing boundary condition, we used the optimal control theory to propose an alternative to the current formulation and overcome the difficulty of solving the ordinary differential equation with the missing boundary condition. Our development will focus on one layer of the trunk. The extension to the whole cylindrical trunk will be achieved by means of superposition and boundary conditions continuity between layers.

We associated an optimal control problem to the mechanical formulation in order to propose an alternative to the current formulation and to overcome the difficulty of solving the ordinary differential equation. Within a cost function, the control problem is formulated with a mechanical model that requires boundary conditions to solve all equations.

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References

- [1] Aimene Y., Stahl C., Bonal D., Thibaut B. Modelling of the trunk daily diameter variation during wet season in a neotropical rain forest of French Guiana, 6th *Biomechanics Plant Conference* (2009).
- [2] Aimene Y., Dorville R., Omrane A. Optimal control for trees trunk diameter in rain forest ecology, *Applied Mathematical Sciences*, Vol. 7, No 17, pp 807–816 (2013).
- [3] Alméras T. Mechanical analysis of the strains generated by water tension in plant stems. Part II: Strain in wood and bark and apparent compliance, *Tree Physiol.* 28: 1513–1523 (2008).

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- [4] Alméras T., Gril J. Mechanical analysis of the strains generated by water tension in plant stems. Part I: stress transmission from water to the plant material at the cell level, *Tree Physiol.* 27: 1505–1516 (2007).
- [5] Dorville R., Nakoulima O., Omrane A Low-regret control for the backward heat equation. *Applied Mathematics Letters*, Vol. 17, 5, pp 549–552 (2004).
- [6] Guitard D. *Mécanique du matériau bois et composites*. Cepadues Édition, Toulouse, France, (1987).
- [7] Irvine J., Grace J. Continuous measurements of water tensions in the xylem of trees based on the elastic properties of wood. *Planta* 202, pp 455–461 (1997).
- [8] Lions J. L. Contrôle à moindres regrets des systèmes distribués. *C. R. Acad. Sc. de Paris*, t. 315, Série I, pp 1253-1257 (1992).
- [9] Prévost M.-F., Puig H. Accroissement diamétral des arbres en Guyane: Observations sur quelques arbres de forêt primaire et de forêt secondaire. *Bulletin du Muséum National d'Histoire Naturelle* 3(2): 147–171 (1981).
- [10] Robo-Petit E. Analyse mathématique et numérique d'un problème en mécanique du bois, Mémoire de Master 2, *Mathématiques et Modélisation (MAMO)*, Université des Antilles et de la Guyane, IUFM de Guyane (2012).