



HAL
open science

Anthologie de la calculabilité

Paola Cantu

► **To cite this version:**

Paola Cantu. Anthologie de la calculabilité. History and Philosophy of Logic, 2023, pp.1-3.
10.1080/01445340.2023.2172303 . hal-03981158

HAL Id: hal-03981158

<https://hal.science/hal-03981158>

Submitted on 28 Feb 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

J. Mosconi and M. Bourdeau (eds.). *Anthologie de la calculabilité*. Paris : Cassini, 2022. xviii + 798 pp., €50. ISBN 978-2-84225-094-2.

Reviewed by P. Cantù, *Centre Gilles Gaston Granger, Aix-Marseille Université and CNRS, Aix-en-Provence, France*. paola.cantu@univ-amu.fr

The volume is an anthology of computability, published in French in the *Nouvelle Bibliothèque Mathématique* series by Editions Cassini under the direction of Jean Mosconi and Michel Bourdeau. The text contains a selection of 25 texts on computability covering a time span from 1842 to 1984 and dealing with a variety of topics, including the following: computing machines, algorithms, recursive functions, and logical models of computability. While some of the texts are available in English in the anthology *The Undecidable* edited by Martin Davis in 1965 (*Davis 1965*), the novelty lies in the temporal breadth of the contributions, the inclusion of some hard-to-find texts, Serge Grigorieff's rich preface offering a diachronic and synchronic overview of computability, and detailed historical introductory notes.

The anthology opens with excerpts from the French memoir 'Notions sur la machine analytique de M. Charles Babbage' published by Luigi Federico Menabrea in 1842 and excerpts from Charles Babbage's 1864 book *Passages from the Life of a Philosopher*. The introductory note by Jean Mosconi highlights an important aspect of Babbage's functionalist conception of machine, namely the relationship between technology and the theory of the organisation of human and mechanical labour.

Of particular interest is Behmann's little-known 1921 article 'Entscheidungsproblem und Algebra der Logik', preceded by an introductory note by Paolo Mancosu and Richard Zach, attesting to the centrality of Whitehead and Russell's *Principia Mathematica*, and the origin of the term 'Entscheidungsproblem', the problem of decision.

The analysis of primitive recursive functions is entrusted to a selection of texts by Skolem ('Begründung der elementaren Arithmetik durch die rekurrierende Denkweise...', 1923), Ackermann ('Zum Hilbertschen Aufbau der reellen Zahlen', 1928), Hilbert ('Über das Unendliche' 1925) and Gödel ('On undecidable propositions of formal mathematical systems', 1934), edited by Marcel Guillaume, Michel Bourdeau and Jean Fichot. These papers shed light on a variety of topics on recursive functions: dealing with elementary mathematics without quantifiers, solving the continuum problem, accounting for the iterative nature of arithmetic operations and many other classes of functions, up to the search for progressively more precise syntactic and semantic definitions of primitive recursive functions and general recursion, highlighting the computational aspect.

The topic of logical models of computability is addressed in a wide range of texts, including, in addition to two of the above-mentioned ones, the 1931 correspondence between Herbrand and Gödel, edited by Paul Egré, and a selection of texts (edited by Jean Mosconi, Marcel Guillaume, Pierre Wagner and Jean Fichot) by Kleene ('General recursive functions on natural numbers', 1936 and 'Recursive Predicates and Quantifiers', 1943), Hilbert-Bernays ('Eine Präzisierung des Begriffs einer berechenbaren Funktion, und der Satz von Church über das Entscheidungsproblem', 1939), Church ('An unsolvable problem of elementary number theory',

1936, 'A note on the Entscheidungsproblem', 1936), Rosser 'Extensions of some theorems by Gödel and Church', 1936), Schönfinkel ('Über die Bausteine der mathematischen Logik', 1924), and Gödel ('Über die Länge der Beweise', 1936). The Herbrand-Gödel correspondence, including a letter from Herbrand of 7 April 1931 and the draft of the reply sent on 25 July 1931, highlights the latter's contribution to the theory of general recursive functions. The relationship between recursive and computable functions is attested to in the other texts, which are now classics.

The study of the implementation of computability by machines is illustrated by a selection of texts, edited by Jean Mosconi and Maurice Margenstern, and translated by Laurent Mazliak and Andrei Rodin, including: Turing ('On computable numbers with an application to the Entscheidungsproblem', 1936, 'Solvable and unsolvable problems', 1954), Post ('Finite combinatory processes. Formulation 1', 1936 and 'Recursive unsolvability of a problem of Thue', 1947), Markov ('Teoriya algoritmov, 1951), and Kolmogorov & Uspensky ('k opredeleniyu algoritma', 1958). This broadening of the perspective to Russian texts allows one to fully assess the complex origin of alternative definitions of algorithm: for example, Markov in the text discussed here, which predates his well-known 1954 book, refers to the demonstrative requirements of algebra and topology and raises the problem of constructivism and abstraction. Kolmogorov & Uspensky seek a more general definition of an algorithm, which also allows a comparison of the advantages and limitations of the different notions used so far: general recursion, Church's lambda-definability, Kleene's mu-recursion, Turing and Post's machines, Markov normal algorithms.

The relation between computability and logic, between computation and demonstration, and in particular the study of total functions and Curry-Howard isomorphism, i.e., the correspondence between proofs defined in the natural deduction system and programs defined in the lambda-calculus, is further illustrated by Howard's text ('The Formulae-as-Types Notion of Construction', 1980), translated by Pierre Wagner and edited by Jean Fichot. The relationship between demonstration theory and computability theory in turn refers to the discussion on constructivism and the possibility of using certain types of computable functions to prove the non-contradiction of arithmetic with non-purely finite methods, as shown in Gödel's posthumous text ('On an extension of finitary mathematics which has not yet been used', 1972).

The questions of decidability and undecidability and the proof of the algorithmic non-solvability of Hilbert's tenth problem (the decision problem of resolvability in integers of Diophantine equations) proved by Matiyasevich in 1970, are also present in the anthology, which includes a text from 1984 ('On the researches concerning some algorithmic problems of algebra and number theory'), edited by Jean Fichot and Jean Mosconi with a translation by Maurice Margenstern. Matiyasevich's text closes the anthology, attesting to the fruitfulness of computability theory in solving a classical mathematical problem.

There are several reasons why this anthology is a novelty not only on the French but also on the international scene, constituting a useful research tool for different types of audiences. Students can for the first time have access to a wider selection of classical texts on computability, wider than Davis's selection, both chronologically (Davis's anthology goes from Gödel to Post, and thus from 1931 to the end of the 1950s) and thematically (by including Russian authors, a more general reflection on the notion of algorithm is added, and the question of constructivism

is emphasised). Researchers interested in the historical dynamics of the constitution of computability theory can find texts attesting to a rich and non-linear dialectic between classical mathematical problems (e.g., Hilbert's tenth problem), logical questions about the foundations of mathematics (e.g., constructivism), questions related to the construction of machines, and connections between different views (e.g., the Curry-Howard correspondence). Philosophers can disclose, thanks to the richness of the introductory apparatus, unexplored bridges between texts that are far apart chronologically or lexically. French readers can profit from the introduction of a thoughtful French terminology, one of the main goals that the editors have set for themselves and which we feel has been achieved, thanks to extremely accurate translations and a flawless editing. Furthermore, Serge Grigorieff's introduction offers a good cartography of the anthology, combining a diachronic and synchronic view on computability, without sacrificing the technicality required by the topic.

One flaw in the volume is the absence of some (female) figures central to the development of the notion of algorithm and recursive function, such as Ada Lovelace and Róza Peter. Although Jean Mosconi mentions Ada Lovelace's notes in the introduction to Menabrea's text and Marcel Guillaume mentions Róza Peter in the introductory note to Ackermann's text, it would have been more interesting to read the authors' texts directly. However, the temporal and thematic width and the wealth of historical notations make the volume a valuable contribution that promises to promote the historical and philosophical study of computability and the dialectic between the construction of logical theories and the development of computing machines.

References:

Davis, M. 1965, *The Undecidable: Basic Papers on Undecidable Propositions, Unsolvability Problems and Computable Functions*, Hewlett, N.Y.: Raven Press.