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Intermittency and electricity retailing: An incomplete market approach☆

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Abstract:
We analyze the integration of intermittent renewables-based technologies into an electricity mix comprising of conventional energy. Intermittency is modeled by a contingent electricity market and we introduce demand-side flexibility through the retailing structure. Retailers propose diversified electricity contracts at different prices, but in an insufficient number to cover intermittent production. These delivery contracts are modeled similarly to numeraire assets. We study the competitive equilibrium of the state-contingent wholesale electricity markets and the delivery contract markets. We also provide an analysis linking the delivery contracts to social welfare. Finally, we discuss the conditions under which changing the delivery contracts improve penetration of renewables and increases welfare. These provide useful insights for managing intermittency and achieving renewable capacity objectives.

Keywords: Electricity markets; Renewables; Intermittency; Incomplete markets

1. Introduction

The integration of renewables into the electricity mix is widely accepted as having a significant role to play in decarbonizing the electricity industry. Most particularly, the International Energy Agency reports on the deployment of energy sources as wind and solar helping in increasing renewables-based electricity. Still, their share in the global electricity mix remains quite modest, around 10% in 2021 (IEA 2021). A persisting obstacle to the adoption of these renewables lies in their variable and uncertain nature, collectively referred as *intermittent* (see Perez-Arriaga and Batlle 2012, Verzijlbergh et al. 2017).

Electricity from renewables varies significantly with natural and uncontrollable conditions.[3] These render the production process from renewable technologies not only intermittent but also inflexible, technically termed as *non-dispatchable*.[4] The integration of renewable electricity adds a new source of intermittency on the grid; demand intermittency is a long-existing phenomenon that is mainly managed with investment in dispatchable power plants[4] (IEA 2011). Hence, intermittency from renewables challenges the imperative of the electricity industry to constantly balance electricity supply and demand. Disruptions in this balance have both technical and economic impacts.[5] Contrary to demand intermittency, this new source of intermittency still needs to be tackled.

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3 As examples, electricity production from wind turbines fluctuates with wind speed and direction and that from solar photovoltaics with radiation intensity (Crawley 2013).

4 Renewable technologies such as wind turbines and solar photovoltaics are non-dispatchable as their output cannot be turned on, off, or adjusted according to variations in electricity demand. In contrast, conventional generators such as coal, nuclear, hydro, and gas power plants are controllable and dispatchable. See for instance IEA-ETSAP and IRENA (2015) on dispatchable and non-dispatchable technologies.

5 For example, mismatches between supply and demand can lead to frequency fluctuations that in turn cause brown-outs or blacks (see, for instance, Passey et al. 2011). These result in heavy economic losses such as manufacturing and sales losses, interruption of services, etc. See, for example, Küfeoğlu and Lehtonen (2015) on the economic consequences of power outages on service sector customers.

In the above context, introducing flexibility in the electricity market is recognized as a solution to managing renewables intermittency (Cochran et al. 2014, Eurelectric 2014, IEA 2011, IEA-ISGAN 2019). Flexibility can be implemented upstream of the market so that supply follows demand. For example, it can be through existing or new flexible power plants, storage capacities (Benitez et al. 2008, Green and Vasilakos 2012, Pommeret and Schubert 2022, Sioshansi 2011) and interconnection (Abrell and Rausch 2016, Yang 2022). It can also be developed downstream through demand-side flexibility that requires demand to follow supply. In this paper, we focus on the latter.

The main objective of our work is to provide an approach to manage intermittent supply by means of demand flexibility through the retail market. The importance of demand flexibility was first highlighted with the question of optimal investment in production capacities to meet intermittent demand. Borenstein and Holland (2005) and Joskow and Tirole (2007) study one weakness of the electricity market to derive optimal investment programs. They point out the disconnection between wholesale prices that vary with electricity provision and retail tariffs such as the flat tariff that do not reflect these changes.[6] Hence, consumers are unaware of varying wholesale market conditions including those where expensive power plants are run to meet peak demands. In a framework where demand is intermittent, Borenstein and Holland (2005) and Joskow and Tirole (2007) suggest implementing time-varying tariffs[7] to improve efficiency of the market in terms of capacity investment. In practice, time-varying tariffs that have been developed include Time-of-Use (ToU), Critical Peak Pricing (CPP), Variable CPP, and Real-Time-Pricing (RTP). These tariffs are able to shape demand thereby reducing peaks[8] and investments in expensive peaking power plants used only for a few hours during a year (IRENA 2019). Since these retail contracts can stabilize demand intermittency, we propose to tap into them and/or use more sophisticated ones to now adjust demand according to intermittent supply. Nevertheless, the contracts should not be too sophisticated, especially for households that do not necessarily react to non-linear electricity prices (see Ito 2014 and Shaffer 2020 for empirical evidence).

Can retail contracts be designed to unlock demand flexibility and ease the integration of intermittent renewable technologies? As part of their work, Ambec and Crampes (2012, 2021) and Rouillon (2015) address this question by studying the optimal investment in renewable capacity in a framework with intermittent supply. While Ambec and Crampes (2012) consider that consumers can use either a flat retail tariff or one that varies with the availability of the intermittent source of energy, Ambec and Crampes (2021) and Rouillon (2015) consider that there is a mix of both consumers. In this paper, we study a more general situation where retailers propose a large set of flexible delivery contracts.

To investigate the above question, we have in mind a theoretical framework of capacity investment and electricity production with two types of energy source: an intermittent, non-dispatchable source such as wind or solar and a non-intermittent, dispatchable source such as nuclear or fossil fuel. We propose that electricity production due to the integration of the intermittent renewable technology depends on conditions such as weather (e.g. “with”or “without”the intermittent source) or times of day (night, dawn, daytime, and dusk). We refer to these conditions as states of nature. Electricity production is therefore state-contingent. We consider that the wholesale market is a contingent one where contingent electricity is offered at contingent prices. We further propose that retailers introduce demand flexibility in the retail market through diversified electricity delivery contracts that are supplied at different prices. These allow consumers to choose their optimal electricity consumption based on their flexibility. The diversity of the delivery contracts is depicted through what we call *base state-contingent electricity delivery contracts*. Their structure is similar to the asset structure in the incomplete market theory (see, for instance, Magill and Quinzii 2002). These base delivery contracts can well generate contracts with flat and time-varying tariffs, but one can also think of more complex contracts depending on weather conditions or on the pressure on the wholesale markets. In this paper, we provide a general model for the structure of the base state-contingent electricity delivery contracts.

In a competitive setting, we study the equilibrium of the state-contingent wholesale and different retail markets where the optimal investment in intermittent renewable capacity is endogenous. We assume that the structure of the base state-contingent delivery contracts is not rich enough to generate retail delivery contracts that allow perfect adjustment of demand to variations in supply. This lack of richness can be explained, for instance, by variations in conditions and thereby electricity supply at a level of granularity that is too fine to incite a response from consumers. Nevertheless, we show that the electricity market equilibrium and social welfare are constraint efficient. The constraint is induced by the limited number of base delivery contracts that constrains electricity allocations. We are also able to determine the conditions under which changing the base delivery contracts improves (i) welfare, (ii) the degree of integration of the renewable capacity, and (iii) both. Ultimately, we find that it is impossible to find a change in the base delivery contracts structure that both increases investment in renewable capacity and reduces the production of conventional electricity in each state of nature. Nevertheless, it may occur that the average conventional production decreases.

The rest of the article is organized as follows. Section 2 presents the theoretical framework and the main assumptions. In Section 3, we describe the electricity contract markets and derive some useful properties of electricity demand. Section 4 studies the state-contingent electricity supply and describes the market equilibrium. Section 5 provides a welfare analysis. In Section 6, we study the impact of changing the base delivery contracts on social welfare, investment in intermittent renewable capacity, and conventional production. Section 7 concludes. Technical proves are relegated to the appendix.

2. The main assumptions

Our paper principally aims at addressing two crucial features of renewables: intermittency and the decision on optimal investment in capacity. For this purpose, we use a static framework. However, intermittency is not only a matter of physical conditions on which renewables depend, but also how these conditions change over time.[9] As such, there is a dynamic effect in renewable

[6] Retailers buy electricity at wholesale prices that vary with electricity provision (prices are low when power plants with low marginal costs of production are run and vice-versa.), with the commitment to supply electricity reliably to consumers for any level of demand and at fixed tariffs. The widely spread retail tariff is the flat tariff whereby consumers are charged the same price per unit of electricity all day.

[7] An extensive literature on time-varying tariffs can be found in the work of Borenstein et al. (2002).

[8] In France, for example, the majority of households have retail contracts with “peak/off-peak”hours tariffs where they pay a higher price during peak demand hours and a lower price during off-peak demand hours. This retail contract has been able to incentivize consumers to use electric water heaters to heat water during off-peak hours rather than during peak hours, thereby alleviating the peak demand for electricity due to water heating.

[9] For example, electricity production from wind turbines depends on wind speed which varies on all time scales, from sub-seconds to decades Widén et al. (2015).

electricity production that we capture by a set of states of nature $s \in \{1, \ldots, S\}$. Since electricity production is state-contingent and the wholesale market is organized in each state, we denote by $p(s)$ the contingent electricity price in state s and by $\mathbf{p} = (p_1, \ldots, p_S) \in \mathbb{R}^S$, the vector of all state-contingent prices. This choice allows us not to explicitly introduce probabilities nor to introduce a Von Neumann Morgenstern setting.

Another important feature of this framework is with respect to decision-making. Renewables-based producers behave *ex-ante*, i.e. before the realization of the state of nature. They choose how much to invest in intermittent renewable capacities by anticipating their future returns. Likewise, conventional electricity producers choose their dispatchable production strategy beforehand. Retailers and consumers *ex-ante* exchange retail contracts based respectively on their expected profits and utility for *ex-post* electricity deliveries, i.e. deliveries on the realization of the state of nature.

Keeping these two features in mind, we can now move to describe the agents of the electricity markets.

The *intermediary retailing structure* between electricity production and consumption is one of the novelties of the model. In fact, we assume that consumers do not have direct access to the wholesale electricity markets. They buy *electricity delivery contracts* from competitive retailers.

These contracts can take several forms. For instance, the most common contract is the flat-rate contract. In this case, the consumer pays the same price for each unit of electricity, irrespective of the state of nature realized. In our model, it implies that the consumer chooses a quantity θ of contracts at a price q for a random electricity delivery of $e \in \mathbb{R}^S$ where e is the unit vector. In practice, there are also state-contingent contracts such as the "peak/off-peak"contract. In this case, the consumer pays q_p for 1 unit of electricity in a subset, S_p, of the states of nature. For the complementary even, he pays $q_{\bar{p}}$ for 1 unit of electricity in states $S_{\bar{p}}$. We interpret this contract as a 2-column matrix:

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} S_p \\ S_{\bar{p}} \end{array} & \begin{bmatrix} e_{S_p} & 0 \\ 0 & e_{S_{\bar{p}}} \end{bmatrix} \end{array}$$

where e_{S_p} and $e_{S_{\bar{p}}}$ stand for the unit vectors in $\mathbb{R}^{\#S_p}$ and $\mathbb{R}^{\#S_{\bar{p}}}$

We identify this "peak/off-peak"contract as the composition of two *base state-contingent delivery contracts*. The first base contract corresponds to the first column. The consumer buys quantity θ_p of this contract at price q_p. The second column is the second base contract with likewise interpretation. We assume that every existing retail contract can be decomposed into base contracts, perhaps different from the unit vectors. By repeating this operation for all existing contracts, we can extract a linearly independent subset of the base contracts. This subset represents the contract structure of our model.

More formally, we say that the holder of a unit base delivery contract k has the right to a random electricity consumption of $\mathbf{a}_k = (a_{1k}, \ldots, a_{Sk})$ where $a_{sk} \geq 0$ denotes electricity delivered in state s. We introduce a set $K = \{1, \ldots, K\}$ of unit contracts $\mathbf{a}_k$ that are offered on K competitive retail markets at price $\mathbf{q} = (q_1, \ldots, q_K)$ where q_k stands for the price of one unit of contract k. We summarize the electricity delivery in the (S, K) matrix $\mathbf{A} = [\mathbf{a}_k]_{k=1}^K \in \mathbb{R}_+^{SK}$. If $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_K) \in \mathbb{R}^K$ denotes a portfolio of contracts, then $\mathbf{A}\boldsymbol{\theta} \in \mathbb{R}^S$ describes the random electricity flow induced by the portfolio $\boldsymbol{\theta}$. We also introduce some assumptions on $\mathbf{A}$. Firstly, we say that $\mathbf{A}$ is of full rank, here $rank(\mathbf{A}) = K$. This simply means there is no redundant contract, i.e. a contract that can be obtained by a portfolio of the other contracts. Secondly, by suitable permutations of the states of nature, we say that $\mathbf{A}$ can be decomposed into $\begin{bmatrix} \mathbf{A}_{S-K} \\ \mathbf{A}_K \end{bmatrix}$ where $\mathbf{A}_{S-K}$ is a matrix of dimension $(S-K, K)$ and $\mathbf{A}_K$ is an invertible matrix of dimension K. Thirdly, we assume that $K < S$ since the retailers cannot propose a contract structure $\mathbf{A}$ that is rich enough to provide a perfect adjustment of demand to intermittent supply. In other words, $span(\mathbf{A}) < S$. Finally, we assume $\forall s, \exists k, a_{sk} > 0$, i.e. there always exists a contract that delivers electricity in a given state.

In this setting, the *retailers* work as intermediaries. They *ex-ante* sell a portfolio of delivery contracts $\boldsymbol{\theta}_r$ at price $\mathbf{q}$. They *ex-post* provide the contract owner with the required amount of electricity that they themselves buy on the wholesale markets. Hence, if a retailer sells a portfolio $\boldsymbol{\theta}_r$, his return is of $\mathbf{q}'\boldsymbol{\theta}_r$ while his expected cost due to electricity delivery is of $\mathbf{p}'\mathbf{A}\boldsymbol{\theta}_r$. His supply follows a competitive profit maximization strategy. Finally, let us observe that the transformation process from contract to electricity delivery is linear. We can therefore replace the set of all retailers with a representative one.

The *renewable energy technology*, in addition to being intermittent, is *non-dispatchable*. To describe non-dispatchability, we introduce an *ex-ante* capacity choice $\kappa \in \mathbb{R}_+$ and model intermittent electricity productivity as the random variable $\mathbf{g} = (g_1, \ldots, g_S) \in \mathbb{R}_{++}^S$. It describes the state-contingent production per unit of the renewable capacity. The cost of investing in capacity κ is given by $\mathcal{K}(\kappa)$ and we assume that $\mathcal{K}(0) = 0, \partial\mathcal{K}(\kappa) > 0$ and $\partial^2\mathcal{K}(\kappa) > 0$ as usually described in literature (e.g. Rouillon 2015).[10] The state-contingent renewable electricity production is equal to $\kappa\mathbf{g}$ and without loss of generality, we normalize short-run marginal cost of production to zero.[11] We finally assume that the optimal investment in capacity and thereby, the state-contingent production of electricity follows from a competitive profit maximizing behavior.

The *conventional energy technology* is an existing and fully-established one. Its purpose is to act as a back-up capacity of dispatchable electricity generation that compensates for fluctuations from the intermittent renewable energy technology. It ensures reliable electricity provision in the absence of demand flexibility.[12] We also assume that there is no capacity constraint.[13] As the intermittent renewable technology, electricity production from the conventional technology is state-contingent too. We denote it by $\mathbf{y} = (y_1, \ldots, y_S) \in \mathbb{R}_+^S$. The implementation of such a random production strategy is assumed to have an *ex-ante* production cost $\mathcal{C}(\mathbf{y})$. However, to capture the idea that the electricity generation is dispatchable, we assume that this cost is additively separable state by state, i.e. $\mathcal{C}(\mathbf{y}) = \sum_{s=1}^S c_s(y_s)$. Moreover, we say that inactivity is allowed in each state and the production cost is increasing at an increasing rate, i.e. $c_s(0) = 0$, $\partial c_s(y_s) > 0$ and $\partial^2 c_s(y_s) > 0$.[14] We also assume, as for the renewable energy sector, that the optimal production plan is derived from a competitive profit maximizing behavior.

The *consumption decision* is derived from utility maximization of a competitive representative agent. Within our partial equilibrium setting, we assume that this consumer derives his

[10] The strictly convex capacity investment cost function can be viewed as investment starting at the most productive site, e.g. in terms of weather conditions.

[11] The resource is "free"and variable costs such as operation and maintenance costs for solar and wind technologies tend to be typically lower than those of conventional technologies (IRENA 2018, Lazard 2018).

[12] In other words, costly conventional technologies with respect to the merit order remain operational. This is to avoid threats to the system reliability, in particular, when demand does not adjust to intermittent supply.

[13] Existing total capacities of conventional technologies are able to provide for electricity demand reliably. See, for instance, Rouillon (2015) and Twoney and Neuhoff (2010).

[14] The strictly convex production cost function can be explained by a variety of conventional power plants with exogenous production capacities and that are ordered by "merit", i.e. by increasing order of their marginal costs of production.

ex-ante utility from his random electricity consumption $\mathbf{x} = (x_1, \ldots, x_S) \in \mathbb{R}^S_+$ and from money $m \in \mathbb{R}$ that he does not spend for the consumption of this good. We assume, as usually in a partial equilibrium setting, that his utility is linear in money, i.e. $U(\mathbf{x}, m) = \mathcal{U}(\mathbf{x}) + m$ and that he owns a global amount of money $m_0 > 0$. The utility associated to the random electricity consumption is given by the continuously differentiable function $\mathcal{U}(\mathbf{x})$. It is increasing and strictly concave, i.e. $\partial\mathcal{U}(\mathbf{x}) \gg 0$ and $\partial^2\mathcal{U}(\mathbf{x})$ is negative definite. We also assume that electricity is a desired good in each state of nature. It means that for any sequence $\mathbf{x}_n$ of random electricity consumption, with the property there exists a state s for which the consumption becomes zero, the global marginal utility, measured as norm of the gradient, becomes large and reciprocally. More formally, we say that $\forall \mathbf{x}_n \to x_0$, with some $x_{0,s} = 0$, $\lim_n \|\partial\mathcal{U}(\mathbf{x}_n)\| \to \infty$. Finally, let us remember that the consumer has no direct access to the wholesale electricity markets but only to the set of contracts proposed by the retailer. Hence, his objective is to choose, at price $\mathbf{q} \in \mathbb{R}^K_+$, a portfolio $\boldsymbol{\theta}_c \in \mathbb{R}^K$ of contracts that induces an electricity consumption of $\mathbf{x} = \mathbf{A}\boldsymbol{\theta}_c$ and that maximizes his utility under a standard budget constraint.

To summarize this discussion, we now propose a definition of an equilibrium.

Definition 1. An equilibrium of the contingent electricity markets and the different delivery contract markets is given by a vector $\left(\boldsymbol{\theta}_c^*, m^*, \boldsymbol{\theta}_r^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*\right) \in \mathbb{R}^{K+1} \times \mathbb{R}^K \times \mathbb{R}^S_+ \times \mathbb{R}_+ \times \mathbb{R}^S_+ \times \mathbb{R}^K_+$ such that:

(i) the representative consumer maximizes his utility, i.e.

$$\left(\boldsymbol{\theta}_c^*, m^*\right) \in \underset{(\boldsymbol{\theta}_c, m) \in \mathbb{R}^{K+1}}{\arg\max} \quad \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c) + m \quad s.t. \quad \begin{cases} \mathbf{A}\boldsymbol{\theta}_c \geq 0 \\ (\mathbf{q}^*)' \boldsymbol{\theta}_c + m = m_0 \end{cases} \tag{1}$$

(ii) the retailer maximizes profit, i.e.

$$\boldsymbol{\theta}_r^* \in \underset{\boldsymbol{\theta}_r \in \mathbb{R}^K}{\arg\max} \quad \left((\mathbf{q}^*)' - (\mathbf{p}^*)' \mathbf{A}\right) \boldsymbol{\theta}_r \tag{2}$$

(iii) the conventional production plan maximizes profit, i.e.

$$\mathbf{y}^* \in \underset{\mathbf{y} \in \mathbb{R}^S_+}{\arg\max} \quad (\mathbf{p}^*)' \mathbf{y} - \sum_{s=1}^{S} c_s(y_s) \tag{3}$$

(iv) the investment in the renewable technology is profit maximizing, i.e.

$$\kappa^* \in \underset{\kappa \in \mathbb{R}_+}{\arg\max} \quad \kappa (\mathbf{p}^*)' \mathbf{g} - \mathcal{K}(\kappa) \tag{4}$$

(v) the retail contract and contingent wholesale electricity markets clear, i.e.

$$\boldsymbol{\theta}_r^* = \boldsymbol{\theta}_c^* \quad \text{and} \quad \mathbf{y}^* + \kappa^* \mathbf{g} = \mathbf{A}\boldsymbol{\theta}_r^* \tag{5}$$

3. Contract markets and the demand for electricity

Let us first remember that electricity delivery is indirectly obtained through our base contracts that work like financial assets. Hence, the existence of an equilibrium requires a no-arbitrage condition. Loosely speaking, it implies that there exists no portfolio of contracts that costs nothing and furnishes electricity in at least one state. Otherwise, everybody will purchase this portfolio which is in contradiction with market clearing. To construct this no-arbitrage condition, we define the $(S+1, K)$ matrix $\mathbf{W}$ given by $\left[\begin{smallmatrix} -\mathbf{q}' \\ \mathbf{A} \end{smallmatrix}\right]$. If there exists a portfolio $\boldsymbol{\theta} \in \mathbb{R}^K$ with the property that $\mathbf{W}\boldsymbol{\theta} \geq \mathbf{0}$ with at least one strict inequality, we can conclude that there exists an arbitrage portfolio. This portfolio pays back either money or electricity in at least one state $s_0 \in S$ without generating a cost in terms of money or electricity in the other states. Thus, the existence of an equilibrium requires some additional restrictions. This is where Farkas' Lemma enters the story and claims that:

$$\nexists \boldsymbol{\theta} \in \mathbb{R}^K, \mathbf{W}\boldsymbol{\theta} \geq \mathbf{0} \text{ (at least one strict)} \Leftrightarrow \exists \boldsymbol{\beta} \in \mathbb{R}^{S+1}, \boldsymbol{\beta} \gg \mathbf{0} \text{ and } \boldsymbol{\beta}'\mathbf{W} = \mathbf{0} \tag{6}$$

Moreover, if such a $\boldsymbol{\beta}$ exists, the same must be true for the vector $\left(\begin{smallmatrix} 1 \\ \boldsymbol{\beta}_1 \end{smallmatrix}\right)$ where $\boldsymbol{\beta}_1 \in \mathbb{R}^S_{++}$ is composed of the S last components of $\boldsymbol{\beta}$ divided by its first one. Owing to the block decomposition of $\mathbf{W}$, the no-arbitrage condition becomes:

$$\exists \boldsymbol{\beta}_1 \in \mathbb{R}^S, \boldsymbol{\beta}_1 \gg \mathbf{0} \text{ and } \mathbf{q} = \mathbf{A}'\boldsymbol{\beta}_1 \tag{7}$$

If we now study the consumer's demand for contracts, we need to restrict the set of contract prices to the open set Q given by:

$$\mathbf{q} \in Q = \left\{\mathbf{q} \in \mathbb{R}^K : \mathbf{q} = \mathbf{A}'\boldsymbol{\beta}_1 \text{ with } \boldsymbol{\beta}_1 \in \mathbb{R}^S_{++}\right\} \tag{8}$$

Moreover, from Eq. (1) of Definition 1, the demand for contracts, $\boldsymbol{\theta}_c(\mathbf{q})$, can be rewritten as:

$$\boldsymbol{\theta}_c(\mathbf{q}) = \underset{\boldsymbol{\theta} \in \mathbb{R}^K}{\arg\max} \quad \mathcal{U}(\mathbf{A}\boldsymbol{\theta}) - \mathbf{q}'\boldsymbol{\theta} \text{ s.t. } \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \tag{9}$$

The existence of a solution of this unbounded problem is not really an issue if the no-arbitrage condition is satisfied (see proof of Proposition 1). We can even claim that the constraint $\mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$ is never binding since we have assumed that the marginal utility becomes "large"as the consumption in one state goes to zero. Finally, since $\mathcal{U}$ is strictly concave, we can say that the demand for contracts solves the following first-order condition:

$$\mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}) - \mathbf{q} = \mathbf{0} \tag{10}$$

Moreover, by applying the Implicit Function Theorem to the above equation, one can compute the Jacobian of $\boldsymbol{\theta}_c(\mathbf{q})$ and show that this matrix is symmetric and negative definite. This means (i) the effect of the change of the price of contract k on the demand of contract k' is the same as the change of the price of contract k' on the demand of contract k and (ii) the demand for a contract is decreasing with its own price. We can even characterize the boundary behaviors of demand for contracts. The first boundary behavior follows from the no-arbitrage condition. If we take a sequence $\mathbf{q}_n$ of contract prices free of arbitrage and that converges to an arbitrage price, we can expect that the sequence of the optimal portfolio choice $\boldsymbol{\theta}_c(\mathbf{q}_n)$ solving Eq. (1) of Definition 1 is unbounded, i.e. $\|\boldsymbol{\theta}(\mathbf{q}_n)\| \to \infty$. The explanation is simple: at the limit, the consumer is willing to buy an infinite amount of the arbitrage portfolio. The second boundary behavior follows from the boundary condition on the marginal utility and that is associated with the first-order condition (see Eq. (10)). It says that if some contract prices q_k become very high, i.e. $\|\mathbf{q}_n\| \to \infty$, the same must be true, by Eq. (10), for the marginal utility which induces that the electricity consumption in some states become zero. The next proposition summarizes these results.

Proposition 1. *The demand of contract is a differentiable function $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$ with the property that:*

(i) $\partial\boldsymbol{\theta}_c(\mathbf{q}) = \left(\mathbf{A}'\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}))\mathbf{A}\right)^{-1}$ is a symmetric and negative definite matrix.

(ii) If $\forall n$, $\mathbf{q}_n \in Q$ and $\mathbf{q}_n \to \mathbf{q}_0$ with $\mathbf{q}_0 \in bd(Q)$, then $\|\boldsymbol{\theta}_c(\mathbf{q}_n)\| \to \infty$.

(iii) If $\forall n$, $\mathbf{q}_n \in Q$ and $\|\mathbf{q}_n\| \to \infty$, then $\exists s_0 \in S$, $(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))_{s_0} \to 0$.

Let us now move to the supply of contracts. From Definition 1, we know that the portfolio supplied by the representative retailer solves $\max_{\boldsymbol{\theta}_r \in \mathbb{R}^K} \left(\mathbf{q}' - \mathbf{p}'\mathbf{A}\right)\boldsymbol{\theta}$. Since this program is linear in $\boldsymbol{\theta}$, we

can immediately assert that a finite solution exists if and only if $\mathbf{q} = \mathbf{A}'\mathbf{p}$. Moreover, his profit is zero at that portfolio price and he is indifferent between any kind of supply $\boldsymbol{\theta}_r$. Therefore, under market clearing, he supplies exactly what is asked by the consumer. In other words, the equilibrium quantity of the contract markets is $\boldsymbol{\theta}_r(\mathbf{p}) = \boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})$ and is now a function of the state-contingent electricity prices. However, trading an amount of $\boldsymbol{\theta}(\mathbf{p})$ contracts also engages the retailer to provide the consumer with the required amount of electricity in each state of nature. This state-contingent demand of electricity $\mathbf{D} : \mathbb{R}^S_{++} \to \mathbb{R}^S$ is given by:

$$\mathbf{D}(\mathbf{p}) = \mathbf{A}\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p}) \tag{11}$$

The demand for electricity is directly deduced from the consumer's demand for contracts and hires several properties induced by Proposition 1. First of all, we notice that this demand for electricity is only defined for strictly positive prices, $\mathbf{p} \in \mathbb{R}^S_{++}$, otherwise the no arbitrage condition given by Eq. (7) is violated. Likewise, if at least one of the contingent prices becomes zero, i.e. $\mathbf{p} \to \mathbf{p}_0$ with some $\mathbf{p}_{0,s} = 0$, the contract price vector $\mathbf{q}$ is no more free of arbitrage. The consumer will take advantage of this opportunity to increase his electricity consumption by an adequate portfolio choice. One can therefore expect that the retail electricity demand that fulfills these contracts becomes large, i.e. $\|\mathbf{D}(\mathbf{p})\| \to \infty$. *A contrario*, if some contingent prices become large, i.e., $\|\mathbf{p}\| \to \infty$, we know from $\mathbf{A} \geq 0$ and $\forall s, \exists k$, $a_{sk} > 0$, that the contract prices must become large, $\|\mathbf{q}\| \to \infty$. It immediately follows from (iii) of Proposition 1 that the electricity consumption and therefore the electricity demand of the retailer must go to zero in at least one state, i.e. $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$. We can finally deduce the Jacobian of $\mathbf{D}(\mathbf{p})$ from that of $\boldsymbol{\theta}_c(\mathbf{q})$. This one is:

$$\partial\mathbf{D}(\mathbf{p}) = \mathbf{A}\partial\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})\mathbf{A}' \tag{12}$$

This matrix is again symmetric and induces the same interpretation as earlier but it is only negative semi-definite. The explanation for this is quite obvious. If one considers a state-contingent price change that does not affect the contract prices, i.e. a price change in the ker $(\mathbf{A}')$ of dimension $S-K$, the demand for contracts does not change and therefore, so does the electricity demand of the retailer. The next proposition summarizes this discussion.

Proposition 2. *The state-contingent demand of electricity is a differentiable function $\mathbf{D} : \mathbb{R}^S_{++} \to \mathbb{R}^S$. Under the no-arbitrage condition, it is defined for strictly positive state-contingent electricity prices $\mathbf{p}$ with the property that:*

(i) $\partial\mathbf{D}(\mathbf{p}) = \mathbf{A}\partial\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})\mathbf{A}'$ is a symmetric and negative semi-definite matrix.

(ii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $\mathbf{p}_{0,s} = 0$, then $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$.

(iii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$, then $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$.

4. The state-contingent electricity supply and market equilibrium

The production level of the conventional sector is obtained by solving the profit maximization program given by Eq. (3) of Definition 1. This sector has the ability to adjust its production plan in each state since the contingent cost is additively separable. It follows that the optimal production level in each state simply equates the state-contingent price to the marginal cost in state s:

$$\forall s \in S,\ p_s = \partial c_s(y_s) \Leftrightarrow y_s = (\partial c_s)^{-1}(p_s) \tag{13}$$

The contingent supply of the conventional sector is therefore given by $\mathbf{Y} : \mathbb{R}^S_{++} \to \mathbb{R}^S_+$ with $\mathbf{Y}(\mathbf{p}) = \left((\partial c_s)^{-1}(p_s)\right)_{s\in S}$. Under our assumption on the different cost function $c_s(y)$, we can claim, as usual, that the supply in each state, $y_s(p_s)$, is increasing with the state-contingent price because

$$\frac{dy_s(p_s)}{dp_s} = \frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)} > 0 \tag{14}$$

The supply also satisfies usual boundary conditions: for a low price, the supply goes to zero and for a high price, it becomes infinite, i.e. $\lim_{p_s\to+\infty} y_s(p_s) = +\infty$. Moreover, since the production decision is taken independently state by state, we can see that the Jacobian of this contingent supply, $\partial\mathbf{Y}(\mathbf{p})$, is a diagonal matrix denoted by $\mathcal{D}$ and with the sth diagonal term being $\frac{dy_s(p_s)}{dp_s} > 0$. Since this quantity is positive, we can even say that $\partial\mathbf{Y}(\mathbf{p})$ is positive definite.

Contrary to the conventional sector, the intermittent renewable energy sector is not able to adjust its production state by state. It chooses, *ex-ante*, a production capacity that provides a random production level $\mathbf{g}$ per unit of the installed capacity. This capacity choice is obtained by solving the profit maximization program given by Eq. (4) of Definition 1. Since $\mathbf{p}$ are state-contingent prices, the optimal capacity choice is simply obtained by equating the expected additional return, $\mathbf{p}'\mathbf{g}$ of a new unit of capacity, to its marginal cost, i.e.

$$\partial\mathcal{K}(\kappa) = \mathbf{p}'\mathbf{g} \Leftrightarrow \kappa = (\partial\mathcal{K})^{-1}\left(\mathbf{p}'\mathbf{g}\right) \tag{15}$$

Consequently, the state-contingent supply of the intermittent renewable energy sector, $\mathbf{I} : \mathbb{R}^S_{++} \to \mathbb{R}^S_+$, is:

$$\mathbf{I}(\mathbf{p}) = \left((\partial\mathcal{K})^{-1}\left(\mathbf{p}'\mathbf{g}\right)\right)\mathbf{g} \tag{16}$$

This contingent production is therefore not very reactive to price changes since the capacity choice is only responsive to the change of the expected returns. This fact can be easily observed by computing the Jacobian of $\mathbf{I}(\mathbf{p})$ given by:

$$\partial\mathbf{I}(\mathbf{p}) = \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p})\right)}\mathbf{g}'\mathbf{g} \tag{17}$$

This is a matrix of rank 1 since every price change that is orthogonal to the random production plan $\mathbf{g}$ does not modify the investment and this set of prices is typically a subset of dimension $S-1$. Nevertheless, we can observe that this matrix remains semi-positive definite since $\partial^2\mathcal{K} > 0$. More precisely:

$$\forall\mathbf{v} \neq 0,\ \mathbf{v}'\partial\mathbf{I}(\mathbf{p})\mathbf{v} = \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p})\right)}\left(\mathbf{g}'\mathbf{v}\right)^2 \geq 0 \tag{18}$$

This state-contingent intermittent production has another interesting property. As long as one assumes that the production of an installed unit is always strictly positive and even very small, i.e. $\mathbf{g} \gg \mathbf{0}$, the production level, under our standard assumptions on $\mathcal{K}$, becomes infinite in each state when the prices $\mathbf{p} \in \mathbb{R}^S_{++}$ become large, i.e. $\|\mathbf{p}\| \to \infty$.

At this point, we can construct the contingent electricity supply $\mathbf{S}(\mathbf{p}) = \mathbf{Y}(\mathbf{p}) + \mathbf{I}(\mathbf{p})$ and summarize our results in the next proposition.

Proposition 3. *The contingent electricity supply $\mathbf{S} : \mathbb{R}^S_{++} \to \mathbb{R}^S_+$, given by $\mathbf{S}(\mathbf{p}) = \mathbf{Y}(\mathbf{p}) + \mathbf{I}(\mathbf{p})$, is a differentiable function with the property that:*

(i) $\partial\mathbf{S}(\mathbf{p}) = \mathcal{D} + \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p})\right)}\mathbf{g}'\mathbf{g}$ with $\mathcal{D}$ a diagonal matrix of the generic term $\frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)}$

(ii) $\partial\mathbf{S}(\mathbf{p})$ is positive definite.

(iii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$, then $\forall s \in S$, $(\mathbf{S}(\mathbf{p}))_s \to +\infty$.

(iv) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $\mathbf{p}_{0,s} = 0$, $\mathbf{S}(\mathbf{p}_n) \to \mathbf{S}(\mathbf{p}_0) \geq \mathbf{0}$ and is finite.

We now can move to market clearing as given by Eq. (5) of Definition 1 and we first consider the state-contingent electricity markets. From Proposition 2, we conclude that the state-contingent demand for electricity as depicted by the function $\mathbf{D}(\mathbf{p})$ is monotonically decreasing while supply, given by the function $\mathbf{S}(\mathbf{p})$ from Proposition 3, is monotonically increasing. Subject to the boundary behaviors of $\mathbf{D}(\mathbf{p})$ and $\mathbf{S}(\mathbf{p})$, we conclude that there exists a unique set of prices $\mathbf{p}^* \in \mathbb{R}^S_{++}$ that brings demand and supply on the different state-contingent electricity markets into equilibrium. Finally, by recalling the relationship between the free-arbitrage retail contract prices $\mathbf{q}$ and state-contingent wholesale prices $\mathbf{p}$, we deduce that the different retail contract markets are cleared at price $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^* \in \mathbb{R}^K_{++}$. The next proposition is subsequently self-contained:

Proposition 4. *There exists a unique contingent price vector $\mathbf{p}^* \in \mathbb{R}^S_{++}$ that clears the different state-contingent electricity markets and an associated electricity delivery contract price vector $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^* \in \mathbb{R}^K_{++}$ that is free of arbitrage and clears the different retail contract markets.*

5. Welfare analysis

Up to now, we have shown that one can consider an equilibrium structure in which sophisticated state-contingent delivery contracts may exist. The basic questions that remain open concern the richness of the available base contracts and the effect of their change on the equilibrium of the electricity markets. Our aim, here and in the next section, is to point out that the contract structure matters as long as this one is unable to provide an effective adjustment of demand to intermittent supply. In our model, this directly follows from the number of base contracts, K, being smaller than S, the number of states of nature. This restricts the potential contingent electricity consumptions to the linear subspace generated by the columns of $\mathbf{A}$, i.e. $\mathbf{x} \in span(\mathbf{A})$. In some sense, the markets are incomplete (with $rank(\mathbf{A}) < S$).

To gain a better understanding of this problem, let us come back to the consumer's program ((i) of Definition 1). His portfolio choice is essentially motivated by the electricity consumption that he obtains. If he faces two retail contract structures $\mathbf{A}$ and $\mathbf{A}'$ with the property that $span(\mathbf{A}) = span(\mathbf{A}')$, he simply adjusts his demand for the contracts in order to maintain the same state-contingent electricity consumption. In other words, we can say that $\mathbf{A}$ is equivalent to $\mathbf{A}'$ in terms of electricity demand, i.e. $\mathbf{A} \sim_e \mathbf{A}'$. Since $\sim_e$ is an equivalence relation, we can even restrict our attention to contracts induced by a representative element of each indifference class of $\sim_e$. To identify this element, remember that $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{S-K} \\ \mathbf{A}_K \end{bmatrix}$, where $\mathbf{A}_K$ is an invertible matrix of dimension K. Under this assumption, by simply changing the portfolio to $\boldsymbol{\theta}_\mathbf{B} = \mathbf{A}_K\boldsymbol{\theta}_\mathbf{A}$, any electricity allocation, $\mathbf{x} = \mathbf{A}\boldsymbol{\theta}$, can be obtained with the equivalent contract structure:

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \text{with } \mathbf{B} = \mathbf{A}_{S-K}\left(\mathbf{A}_K\right)^{-1} \tag{19}$$

This observation clearly suggests that we can take a matrix $\left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{I}_K \end{smallmatrix}\right]$ as a representative element of a family of contract structure given by:

$$\mathbf{A} = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix} \text{with } \mathbf{C} \text{ any invertible matrix of dimension } K \tag{20}$$

This observation has another consequence. If we restrict our attention to delivery contracts given by $\left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{I}_K \end{smallmatrix}\right]$, we can say that the contingent electricity consumption $\mathbf{x}$ satisfies:

$$\mathbf{x} = \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}\boldsymbol{\theta} \Leftrightarrow \begin{cases} \boldsymbol{\theta} = (x_s)_{s=S-K+1}^K \\ \begin{bmatrix} \mathbf{I}_{S-K} & -B \end{bmatrix}\mathbf{x} = \mathbf{0} \end{cases} \tag{21}$$

This clearly says, for $K < S$, the potential electricity consumption that the consumer can obtain from the delivery contract structure is limited to the set of all $\mathbf{x}$ that satisfies

$$\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix}\mathbf{x} = \mathbf{0} \tag{22}$$

It means the existence of a limited amount of delivery contracts reduces the trade capacity of the consumer and restricts the adjustment capacity of electricity demand to fluctuating supply. To summarize, we can say:

Proposition 5. *Under our assumptions on the base contract structure $\mathbf{A}$:*

(i) The equivalent contract structure as given in Eq. (19) provides the same electricity allocation at the same contingent prices. Only contract prices and the portfolio adjust. They are given by $(\mathbf{q}_\mathbf{B}, \boldsymbol{\theta}_\mathbf{B}) = \left(\left(\mathbf{A}_\mathbf{K}^{-1}\right)'\mathbf{q}_\mathbf{A}, \mathbf{A}_K\boldsymbol{\theta}_\mathbf{A}\right)$.

(ii) Reciprocally, the electricity allocation and prices induced by the contract structure as given in Eq. (19) remain the same for every element of the family given by Eq. (20). The associated contract price and portfolio are $(\mathbf{q}_\mathbf{A}, \boldsymbol{\theta}_\mathbf{A}) = \left(\mathbf{C}'\mathbf{q}_\mathbf{B}, \mathbf{C}^{-1}\boldsymbol{\theta}_\mathbf{B}\right)$.

(iii) A contract structure $\mathbf{A}$ equivalent to the structure as given in Eq. (19) induces a restriction on electricity trades given by $\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix}\mathbf{x} = \mathbf{0}$.

From this proposition, we will mainly restrict our attention to the contract structure given by $\left[\begin{smallmatrix} \mathbf{B} \\ \mathbf{I}_K \end{smallmatrix}\right]$ in the rest of this paper. Also, point (iii) of Proposition 5 shows another aspect of introducing intermittent renewables into the electricity mix. As long as the set of contracts proposed to the consumer is not large enough, i.e. $K < S$, his potential state-contingent consumptions are constrained by Eq. (22). Hence, it is impossible to reach a first-best allocation (unless this one satisfies the set of restrictions). In other words, even in our competitive setting, Pareto-optimality is out of reach and it is always interesting to see how changes in the contract structure affect welfare. To address this question, we first introduce a notion of constrained efficiency:

Definition 2. An electricity production plan and allocation $\left(\tilde{\mathbf{y}}, \tilde{\kappa}, \tilde{\mathbf{x}}\right) \in \mathbb{R}^{S+1}_+ \times \mathbb{R}^S_+$ is constrained efficient iff it solves:

$$SW(\mathbf{B}) = \max_{(\mathbf{y},\kappa,\mathbf{x})\in\mathbb{R}^{2S+1}_+} \mathcal{U}(\mathbf{x}) - \mathcal{C}(\mathbf{y}) - \mathcal{K}(\kappa)\ s.t. \begin{cases} \mathbf{x} - \mathbf{y} - \kappa\mathbf{g} = \mathbf{0} \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix}\mathbf{x} = \mathbf{0} \end{cases} \tag{23}$$

If we denote by $\boldsymbol{\lambda} = (\lambda_s)_{s=1}^S$ and $\boldsymbol{\mu} = (\mu_i)_{i=1}^{S-K}$ Lagrangian multipliers associated respectively with the first and second set of constraints, we obtain the following first-order conditions:

$$\begin{cases} \partial\mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda} - \begin{bmatrix} \mathbf{I}_{S-K} \\ -\mathbf{B}' \end{bmatrix} \cdot \boldsymbol{\mu} = \mathbf{0} \\ -\partial\mathcal{C}(\mathbf{y}) + \boldsymbol{\lambda} = \mathbf{0} \\ -\frac{d\mathcal{K}(\kappa)}{d\kappa} + \mathbf{g}' \cdot \boldsymbol{\lambda} = 0 \end{cases} \tag{24}$$

If we identify, as usually, the Lagrangian multiplier $\boldsymbol{\lambda}$ to the contingent price vector $\mathbf{p}$, we observe that the second and the third conditions of Eq. (24) are exactly the same as the profit maximization conditions for, respectively, the conventional sector (see Eq. (13)) and the intermittent sector (see Eq. (15)). A same observation can be made for the first condition of Eq. (24) but it requires a little transformation. In fact,

$$\partial\mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda} - \begin{bmatrix} \mathbf{I}_{S-K} \\ -\mathbf{B}' \end{bmatrix} \cdot \boldsymbol{\mu} = \mathbf{0} \Leftrightarrow \begin{cases} \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}' (\partial\mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda}) = \mathbf{0} \\ \boldsymbol{\mu} = \left(\partial_{x_s}\mathcal{U}(\mathbf{x}) - \lambda_s\right)_{s=1}^{S-K} \end{cases} \tag{25}$$

The first condition of Eq. (25) now becomes very close to the consumer's first-order condition given by Eq. (10). One simply has to remember that (i) we work with an equivalent contract structure and (ii) by the no-arbitrage condition and the identification of $\boldsymbol{\lambda}$ to $\mathbf{p}$, we must have $\mathbf{q} = \left[\begin{smallmatrix}\mathbf{B}\\ \mathbf{I}_K\end{smallmatrix}\right]'\boldsymbol{\lambda}$. The second condition defines $\boldsymbol{\mu}$ whose interpretation is non-trivial in our framework. Remember that $\boldsymbol{\mu}$ is tied to the set of constraints on electricity allocation as defined by condition 2 of Eq. (23). Thus, one can expect that $\boldsymbol{\mu} \neq \mathbf{0}$, otherwise all constraints on trades become ineffective. The latter implies that the first-best consumption allocation satisfies the restrictions on trades that are induced by the contract structure; a situation that surely does not resist to a slight perturbation of the utility function. In the next section in which we consider the effect of changing contracts, we will even sometimes assume that all components of $\boldsymbol{\mu}$ are different from $\mathbf{0}$. Loosely speaking, it means that the direction to the first-best is a non-zero linear combination of all the restrictions on trades (i.e. the different lines of $[\,\mathbf{I}_{S-K}\ \ -\mathbf{B}\,]$).

In any case, the above largely suggests that:

Proposition 6. *Any competitive electricity production plan and allocation issued from Definition 1 is a constrained efficient allocation given by Definition 2 and reciprocally.*

6. Changing contracts

As long as welfare is concerned and at least one constraint on trade is effective (i.e. $\boldsymbol{\mu} \neq \mathbf{0}$), it is obvious that adding a base contract to an existing contract structure $\mathbf{A}$ will be welfare improving as long as this one is linearly independent of the existing ones. The intuition is clear. If we move from a matrix $\mathbf{A}_0$ of dimension (S, K) to $\mathbf{A}_1$ of dimension $(S, K+1)$ by adding a new column, the different electricity consumption profiles $\mathbf{x} = \mathbf{A}_0\boldsymbol{\theta}$ that were initially available, remain reachable by simply not purchasing the new contract. Hence, $span(\mathbf{A_0}) \subset span(\mathbf{A_1})$. This clearly means if one remembers the definition of a constraint efficient allocation (see Definition 2), that moving from a contract structure $\mathbf{A}_0$ to $\mathbf{A}_1$ must be welfare improving.

Instead of adding a new base contract, we now identify which changes of the existing contracts can be Pareto improving. Let us start with a given contract structure $\mathbf{A}$ or its equivalent matrix $\mathbf{B}$. By applying a standard Envelope Theorem to the optimization problem of Definition 2, we can show that (see proof of Proposition 7): $\forall s = 1, \ldots, S-K$, $k = 1, \ldots, K$,

$$\partial_{b_{sk}} SW(\mathbf{B}) = \mu_s x_{S-K+k} = \left(\partial_{x_s}\mathcal{U}(\mathbf{x}) - \lambda_s\right) x_{S-K+k} \tag{26}$$

By considering that the first-best electricity consumption does not satisfy the trade constraints, i.e. $\boldsymbol{\mu} \neq \mathbf{0}$ (see our discussion in the previous section), we identify state s and contract k for which $|\mu_s|\, x_{S-K+k}$ is maximal. The precedent equation says that the largest effect on welfare can be obtained by increasing or decreasing b_{sk}, whenever $\mu_s > 0$ or < 0.

We summarize this discussion by:

Proposition 7. *Concerning the welfare effect of a change of the contract structure* $\mathbf{A}$*, we can say that as long as* $\boldsymbol{\mu} \neq \mathbf{0}$*,*

(i) any addition of a new contract linearly independent of the existing ones improves welfare.

(ii) the best welfare improving option is to change the coefficient $(s, k)_0$ *of the equivalent matrix* $\mathbf{B}$ *for which* $(s, k)_0 = \arg\max_{(s,k)} \{|\mu_s|\, x_{S-K+k}\}$.

We now move to the effect of changing $\mathbf{B}$ on the equilibrium investment in renewable capacity, $\kappa^* = (\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p}^*)$ (see Eq. (15)). First, it is important to understand how changes in $\mathbf{B}$ affect the contingent equilibrium prices $\mathbf{p}^*$ since

$$d\kappa^* = \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p}^*)\right)}\mathbf{g}'\partial_{\mathbf{B}}\mathbf{p}^* d\mathbf{B} \tag{27}$$

For this exercise, we reformulate the definition of an equilibrium price. We start by replacing $\mathbf{A}'$ with the equivalent contract structure $\left[\begin{array}{cc}\mathbf{B}' & \mathbf{I}_K\end{array}\right]$ in the consumer's first-order condition given by Eq. (10). By market clearing, (i) the consumption will be equal to the supply $\mathbf{S}(\mathbf{p})$ and (ii) the contract price must be given by $\mathbf{q} = \left[\begin{array}{cc}\mathbf{B}' & \mathbf{I}_K\end{array}\right]\mathbf{p}$. If we add to the story the idea that the supply $\mathbf{S}(\mathbf{p})$ must satisfy the restrictions on trades imposed by the structure of the contract matrix, we can construct a function $f : \mathbb{R}^S_+ \times \mathbb{R}^{(S-K)K} \to \mathbb{R}^S$ given by:

$$f(\mathbf{p}, \mathbf{B}) = \begin{cases} \left[\begin{array}{cc}\mathbf{B}' & \mathbf{I}_K\end{array}\right](\partial\mathcal{U}(\mathbf{S}(\mathbf{p})) - \mathbf{p}) \\ \left[\begin{array}{cc}\mathbf{I}_{S-K} & -\mathbf{B}\end{array}\right]\mathbf{S}(\mathbf{p}) \end{cases} \tag{28}$$

The zero of Eq. (28) is our state-contingent equilibrium price vector $\mathbf{p}^*$. The effect of a change of $\mathbf{B}$ on $\mathbf{p}^*$ can therefore be obtained by applying the Implicit Function Theorem to $f(\mathbf{p}, \mathbf{B}) = \mathbf{0}$.

Lemma 1. *We observe that:*

(i) $\partial_p f(\mathbf{p}, \mathbf{B})$ *is invertible and by the Implicit Function Theorem:*

$$\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B}) = \left[\partial_p f(\mathbf{p}, \mathbf{B})\right]^{-1}\left[\begin{array}{c}\mathbf{I}_K \otimes \left(\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}\right)' \\ \left(-\left(\mathbf{S}_{S-K+k}(\mathbf{p})\right)_{k=1}^{K}\right)' \otimes \mathbf{I}_{S-K}\end{array}\right] \tag{29}$$

($\otimes$ denotes the tensor product)

(ii) if only one component of $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ *is different from* 0*, then* $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ *is of rank* $S-1$.

(iii) if at least two components of $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ *are different from* 0*, then* $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ *is of full rank S. This implicitly requires that* $1 < K < S-1$.[15]

We can now move to understanding the effect of a change in $\mathbf{B}$ on investment in renewable capacity. From Eq. (27), we observe that the induced price change $\mathbf{dP} = \partial_{\mathbf{B}}\mathbf{p}^* d\mathbf{B}$. The set of all reachable $\mathbf{dP}$ being a linear space, we can conclude, still from Eq. (27), that if there exists a $\mathbf{dP} \subset g^{\perp}$ with $g^{\perp}$ the $S-1$ dimensional hyperplane orthogonal to $\mathbf{g}$, then investment in renewable capacity can be improved by a contract change $d\mathbf{B}$. This occurs in case *(iii)* of Lemma 1 since $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ is of full rank. As a surjective mapping, we can even define the most efficient direction of the induced price change that solves $\max_{\mathbf{dP}} \mathbf{g}'\frac{\mathbf{dP}}{\|\mathbf{dP}\|}$ and is given by any $\mathbf{dP}$ collinear to $\mathbf{g}$. If we now have in mind that $\mathbf{dP} : \mathbb{R}^{K(S-K)} \longrightarrow \mathbb{R}^S$, then there even exists, by the Rank Theorem, a subset of contract change associated to this most efficient direction of price change. This subset of $d\mathbf{B}$ is of dimension $K(S-K) - S$. We can therefore say:

Proposition 8. *If at least two components of* $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ *are different from* 0 *and* $1 < K < S-1$*, all the directions of price changes that improve investment in renewables can be reached, especially the one that is collinear to* $\mathbf{g}$ *and that "maximizes"the penetration of renewables. Moreover, each of these improving directions can be obtained by a subset of dimension* $(K(S-K) - S)$ *of changes in* $\mathbf{B}$.

Now, what does at least two components of $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ are different from 0 mean? To answer this question, let us recall Proposition 6 and Eq. (25). Proposition 6 says that a competitive equilibrium is a constrained efficient allocation obtained by solving the optimization program given by Definition 2. As for Eq. (25), it says that at least two constraints on trade are effective in this optimization program. This rather generic case

[15] If $K = S-1$, the vector $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ contains only one component while for $K = 1$, $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ only contains $S-1$ columns.

is in fact depicted by the restriction of Proposition 8. Of course, if this restriction is not fulfilled, then the result on investment in renewable capacity holds only if there exists a **dP** that does not belong to $g^{\perp}$.

Proposition 8 also suggests that there are many ways to change the contract structure in order to improve investment in renewable capacity. This brings two additional questions. The first one is: can we improve both welfare and investment in renewables? The answer to this question is quite obvious. Let us come back to the vectors $\partial_B \kappa^*$ (Eq. (27)) and $\partial_B SW$ (Eq. (26)) of dimension $(1, K(S-K))$. These two gradients point towards the best-improving direction of investment in renewables and social welfare respectively. Hence, as long as these two vectors do not point in exactly two opposite directions, we can conclude (at least intuitively) that there exist changes in the contract structure that improve both investment and welfare.

The second question is: can we ensure that an increase in investment in renewables reduces production of the conventional sector in each state of nature? The answer is unfortunately no. Since the conventional sector adjusts its production level to the observed (*ex-post*) contingent price that he equates to the marginal cost, production in state s only decreases if p_s decreases. Thus, a lower conventional production level in each state requires lower prices in each state as well. Consequently, this reduces the expected returns $\mathbf{g}'\mathbf{p}$ of unit investment in renewables and thereby total investment. Nevertheless, it may occur that the average conventional production decreases. We simply claim that at least one component of this average measure should increase to ensure an increase in renewables. This observation drives the following result.

More formally, by using the Gordan's form of Farkas' Lemma, we can say:

Proposition 9.

(i) As long as $\partial_B \kappa^*$ and $\partial_B SW$ are not collinear with a negative coefficient, there exist changes in **B** *that strictly improve welfare and investment in renewables.*

(ii) It is nevertheless impossible to find a change in **B** *that both increases investment in renewables and reduces production of conventional electricity in each state.*

7. Conclusion

The deployment of intermittent renewables introduces pressure on the grid that calls for more flexibility in electricity markets. Focusing on demand-side flexibility, we have addressed the question if diversified retail contracts at different prices can ease the penetration of intermittent renewables. We have modeled intermittency by contingent electricity markets and diversified retail contracts by a set of base state-contingent electricity delivery contracts. First, we have studied the normative properties of the competitive equilibrium of the state-contingent wholesale electricity markets and the delivery contract markets. Secondly, assuming a limited number of base delivery contracts that constrain electricity allocation, we have been able to find that the electricity market equilibrium and social welfare are constraint efficient. Finally, we have described the conditions under which changing the structure of the base contracts can improve welfare, the degree of integration of renewable capacity, and both. We have also found that it is impossible to find a change in the contracts that both increases investment in renewable capacity and reduces production of conventional electricity in each state of nature. Nevertheless, it may occur that the average conventional production decreases.

The results of the paper firstly provide insights on how the role of retailers can be redefined, for example, in proposing diverse retail contracts. The base delivery contracts modeled here can be a tool for retailers to propose different contracts that can trigger demand-side flexibility. Secondly, the results also highlight the importance of accounting for intermittency in order to achieve renewable capacity objectives.

Several extensions of this model can be expected. The first one has to do with the accounting of carbon emissions. The model can be used to design an economic model for transitioning to a decarbonized electricity mix by considering both intermittency of renewables and carbon emissions from fossil fuels. Ambec and Crampes (2019) examine how the presence of policy instruments (e.g. carbon tax, feed-in tariffs and renewable portfolio standards) affect the socially efficient energy mix with intermittent renewables. This literature can be complemented by investigating how, in the context of a contingent market with renewables, the decision strategy between an *ex-ante* Pigouvian tax and *ex-post* trade of carbon emissions permits matters (see Neerunjun 2022).

A second extension can be to include supply flexibility in the model through the storage of electricity. However, this question cannot be directly addressed with the static model considered here since storage is intrinsically a dynamic one. By extending the model to a dynamic framework, it may be interesting to determine, in the context of incomplete markets, the optimal decision strategy to store and deliver stored electricity given the intermittent nature of renewables. This is in line with a recent literature such as Pommeret and Schubert (2022). The authors focus on a social planner's problem and propose one of the first dynamic models of optimal transition from fossil-fueled technologies to renewables-based that includes intermittency of renewables together with storage.

Thirdly, throughout this paper, we have made the assumption that the wholesale and retail markets are perfectly competitive. It may therefore be worthwhile to investigate how the model behaves with market power. Joskow and Tirole (2007) and Rouillon (2015), by considering demand and supply intermittency respectively, show that when conventional producers own transmission and distribution networks, investment in renewables become less attractive. With the idea that sophisticated retail contracts can be a solution to manage supply intermittency, we can think of tapping into a contract instrument to address the question of market power distortions in electricity markets.

Appendix A. Proof of Proposition 1

(o) Existence and uniqueness of the contract choice $\boldsymbol{\theta}_c(\mathbf{q})$

Let us study the consumer program given by Eq. (1) of Definition 1 for all $\mathbf{q} \in Q$. First observe that the global utility $\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_\mathbf{c}) + m$ is increasing in m. We can therefore say that the optimal solution necessarily belongs to:

$$B = \left\{(\boldsymbol{\theta}, m) \in \mathbb{R}^{K+1} : \ \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \text{ and } \mathbf{q}'\boldsymbol{\theta} + m = m_0\right\} \tag{A.1}$$

This set is obviously non-empty and closed. So if we show that B is also bounded, hence compact, we know that this program has a solution. To verify this property, let us first observe that $\forall(\boldsymbol{\theta}, m) \in B$, m is bounded from above. In fact since $\mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$, we can say by the no-arbitrage condition (see Eq. (7)) that $\boldsymbol{\beta}_1'\mathbf{A}\boldsymbol{\theta} = \mathbf{q}'\boldsymbol{\theta} \geq \mathbf{0}$ (remember that $\boldsymbol{\beta}_1 \gg 0$). It follows that $m = m_0 - \mathbf{q}'\boldsymbol{\theta}$ is bounded from above by m_0. Let us now show that $\forall(\boldsymbol{\theta}, m) \in B$, $\boldsymbol{\theta}$ is bounded. Assume the contrary, i.e., there exists a sequence $(\boldsymbol{\theta}_n, m_n) \in B$ with the property that $\|\boldsymbol{\theta}_n\| \to \infty$ and define $\boldsymbol{\vartheta}_n = \frac{\boldsymbol{\theta}_n}{\|\boldsymbol{\theta}_n\|}$. Since $\boldsymbol{\vartheta}_n$ belongs to the unit circle of $\mathbb{R}^K$ which is a compact set, $\boldsymbol{\vartheta}_n$ admits a converging subsequence whose limit is $\boldsymbol{\vartheta}_0$. Because $(\boldsymbol{\theta}_n, m_n) \in B$, we can also say that $\forall n$, $\mathbf{A}\boldsymbol{\vartheta}_n \geq \mathbf{0}$ and $\mathbf{q}_n'\boldsymbol{\vartheta} + \frac{m_n}{\|\boldsymbol{\theta}_n\|} \leq \frac{m_0}{\|\boldsymbol{\theta}_n\|}$ and since m_n is bounded from above and m_0 finite, we deduce that $\mathbf{A}\boldsymbol{\vartheta}_0 \geq \mathbf{0}$ and $\mathbf{q}'\boldsymbol{\vartheta}_0 \leq 0$. By the no-arbitrage condition, neither one component of $\mathbf{A}\boldsymbol{\vartheta}_0$ nor of $\mathbf{q}'\boldsymbol{\vartheta}_0$

can be strictly positive otherwise $\boldsymbol{\vartheta}_0$ is an arbitrage portfolio. It follows in particular that $\mathbf{A}\boldsymbol{\vartheta}_0 = 0$ and since $\mathbf{A}$ is of full rank, this implies that $\boldsymbol{\vartheta}_0 = 0$. But this is the desired contradiction since by construction $\|\boldsymbol{\vartheta}_0\| = 1$. Finally, since $m = m_0 - \mathbf{q}'\boldsymbol{\theta}$ with $\boldsymbol{\theta}$ bounded, we can say that m is not only bounded from above but also from below.

Let us now move to the uniqueness issue. We have seen from Eq. (9) that the optimal portfolio choice can be obtained by solving the equivalent program given by:

$$\max_{\boldsymbol{\theta}\in\mathbb{R}^K} \left\{ \underbrace{\mathcal{U}(\mathbf{A}\boldsymbol{\theta}) - \mathbf{q}'\boldsymbol{\theta}}_{=f(\boldsymbol{\theta},\mathbf{q})} \text{ s.t. } \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \right\} \tag{A.2}$$

Since the set of feasible solutions ($\mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$) is convex, if $f(\boldsymbol{\theta})$ is strictly concave, we know that the solution is unique and is given by the continuous function $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$. So let us verify that the Hessian of $f(\boldsymbol{\theta}, \mathbf{q})$ with respect to $\boldsymbol{\theta}$ is negative definite. By computation $\forall \mathbf{v} \in \mathbb{R}^K$ and $\mathbf{v} \neq 0$

$$\begin{aligned}\mathbf{v}'\partial^2_{\boldsymbol{\theta},\boldsymbol{\theta}} f(\boldsymbol{\theta}, \mathbf{q})\mathbf{v} &= \mathbf{v}'\left(\mathbf{A}'\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta})\mathbf{A}\right)\mathbf{v} \\ &= (\mathbf{A}\mathbf{v})'\,\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta})(\mathbf{A}\mathbf{v})\end{aligned} \tag{A.3}$$

Let us now remember that $\mathbf{A}$ is a (S, K) matrix of full rank with $K < S$. It follows that for $\mathbf{v} \neq 0$, $\mathbf{h} = \mathbf{A}\mathbf{v} \neq 0$ and since $\mathcal{U}$ is strictly concave, we can conclude that $\mathbf{v}'\partial^2 f(\boldsymbol{\theta})\mathbf{v} < 0$.

(i) $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$ is differentiable and its Jacobian $\partial\boldsymbol{\theta}_c(\mathbf{q}) = \left(\mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q}))A\right)^{-1}$ is negative definite

Let us first show that the constraints $\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}) \geq \mathbf{0}$ are non binding at an optimal solution of Eq. (A.2). Assume the contrary. This means $\exists \mathbf{q}_0 \in Q$ and $\boldsymbol{\theta}_c(\mathbf{q}_0)$ with the property that for at least one s, the sth component $(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_0))_s = 0$. Moreover, since the optimization problem given by Eq. (A.2) is differentiable, it should also satisfy the Karush–Kuhn–Tucker conditions. So if $\boldsymbol{\lambda}_0 \geq 0$ denotes the Lagrangian multipliers, we should have:

$$\mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_0)) - \mathbf{q}_0 + \mathbf{A}'\boldsymbol{\lambda}_0 = 0 \tag{A.4}$$

But let us now define a sequence $\boldsymbol{\theta}_n \to \boldsymbol{\theta}_c(\mathbf{q}_0)$ such that $\forall n$, $\mathbf{A}\boldsymbol{\theta}_n \gg 0$ and construct the sequence

$$\mathbf{t}_n = \mathbf{A}'\left(\underbrace{\frac{\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\|} + \frac{\boldsymbol{\lambda}_0}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\|}}_{=\mathbf{y}_n}\right) - \frac{\mathbf{q}_0}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\|} \tag{A.5}$$

Since $\mathbf{A}\boldsymbol{\theta}_n \to \mathbf{A}\boldsymbol{\theta}_0$ which contains a zero component, we know, by assumption on the boundary behavior of the utility function, that $\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\| \to \infty$. Now observe that $\frac{\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))\|} \to \mathbf{z} \geq \mathbf{0}$ at least for a subsequence and $\mathbf{z}$ belongs to the unit circle of $\mathbb{R}^S$, i.e., with at least one strictly positive component, say, $s' \in S$. Because $\boldsymbol{\lambda}_0 \geq 0$, we deduce that $\exists N$, $\forall n > N$, $(\mathbf{y}_n)_{s'} \geq z_{s'} > 0$. Let us now remember that $\mathbf{A} \geq \mathbf{0}$ and $\forall s$, $\exists k$, $a_{sk} > 0$. This means $\forall n > N$ at least one component of $\mathbf{A}'\mathbf{y}_n$, say component k, verifies $\left(\mathbf{A}'\mathbf{y}_n\right)_k \geq a_{sk}z_{s'} > 0$. Moreover $\frac{\mathbf{q}_{0,k}}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))\|} \to \mathbf{0}$ which means that $\forall \varepsilon \in (0, a_{sk}z_{s'})$, there exists $N' > N$ with the property $\forall n > N'$ $\left(\mathbf{A}'\mathbf{y}_n\right)_k - \frac{\mathbf{q}_{0,k}}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))\|} \geq \varepsilon > 0$. In other words, the kth condition of Eq. (A.4) cannot be satisfied at the limit, otherwise this contradicts optimality.

From the previous observation, we deduce that the necessary and sufficient condition for optimality of Eq. (A.2), is given by:

$$\boldsymbol{\phi}(\boldsymbol{\theta}_c, \mathbf{q}) = \mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c) - \mathbf{q} = 0 \tag{A.6}$$

Now observe that $\partial_{\boldsymbol{\theta}_c}\boldsymbol{\phi}(\boldsymbol{\theta}_c, \mathbf{q}) = \partial^2_{\boldsymbol{\theta},\boldsymbol{\theta}} f(\boldsymbol{\theta}, \mathbf{q})$ (see Eq. (A.3)) is negative definite and therefore of full-rank. It follows by the Implicit Function Theorem that $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$ is differentiable and its Jacobian is:

$$\partial\boldsymbol{\theta}_c(\mathbf{q}) = \left(\mathbf{A}'\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q}))\mathbf{A}\right)^{-1} \tag{A.7}$$

Moreover $\partial\boldsymbol{\theta}_c(\mathbf{q})$ is the inverse of a symmetric and negative definite matrix and therefore shares this last property.

(ii) If $\forall n$, $\mathbf{q}_n \in Q$ and $\mathbf{q}_n \to \mathbf{q}_0$ with $\mathbf{q}_0 \in bd(Q)$ then $\|\boldsymbol{\theta}(\mathbf{q}_n)\| \to \infty$

Assume the contrary, i.e., $\exists K > 0$, $\forall n$, $\|\boldsymbol{\theta}(\mathbf{q}_n)\| < K$, this means for at least one subsequence, $\boldsymbol{\theta}(\mathbf{q}_n) \to \boldsymbol{\theta}_0$. By continuity of the optimization problem given by Eq. (A.2), $\boldsymbol{\theta}_0$ should therefore be a solution at price $\mathbf{q}_0$. But remember that $\mathbf{q}_n \to \mathbf{q}_0$ with $\mathbf{q}_0 \in bd(Q)$. This means by Eq. (6) that there exists, for $\mathbf{q}_0$, an arbitrage portfolio $\boldsymbol{\theta}_1$ with the property that $\mathbf{A}\boldsymbol{\theta}_1 \geq 0$ and $(\mathbf{q}_0)'\boldsymbol{\theta}_1 \leq 0$ with at least one strict inequality. It follows that the portfolio $\boldsymbol{\theta}_2 = \boldsymbol{\theta}_0 + \boldsymbol{\theta}_1$ satisfies the constraint of problem (A.2) since $\mathbf{A}\boldsymbol{\theta}_2 = \mathbf{A}\boldsymbol{\theta}_0 + \mathbf{A}\boldsymbol{\theta}_1 \geq 0$ and, under the assumption that $\partial\mathcal{U}(x) \gg 0$,

$$\begin{aligned}\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_2) - (\mathbf{q}_0)'\boldsymbol{\theta}_2 &= \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_0 + \mathbf{A}\boldsymbol{\theta}_1) - (\mathbf{q}_0)'\boldsymbol{\theta}_0 \\ &\quad - (\mathbf{q}_0)'\boldsymbol{\theta}_1 > \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_0) - (\mathbf{q}_0)'\boldsymbol{\theta}_0\end{aligned} \tag{A.8}$$

which contradicts the fact that $\boldsymbol{\theta}_0$ is a solution at price $\mathbf{q}_0$.

(iii) If $\forall n$, $\mathbf{q}_n \in Q$ and $\|\mathbf{q}_n\| \to \infty$ then $\exists s_0 \in S$, $(\mathbf{A}\boldsymbol{\theta}(\mathbf{q}_n))_{s_0} \to 0$

From the optimality condition given by Eq. (A.6), we know $\forall n$, $\mathbf{q}_n = \mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q_n}))$. So if $\|\mathbf{q}_n\| \to \infty$, we can then say that $\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q_n}))\| \to \infty$. From our assumption on the boundary behavior of the utility function, we conclude that $\exists s_0 \in S$, $(\mathbf{A}\boldsymbol{\theta}(\mathbf{q_n}))_{s_0} \to 0$.

Appendix B. Proof of Proposition 2

(i) $\partial\mathbf{D}(\mathbf{p}) = \mathbf{A}\partial\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})\mathbf{A}'$ is symmetric and negative semi-definite matrix

Computing $\partial\mathbf{D}(\mathbf{p})$ is a simple exercise. This matrix is symmetric since $\partial\boldsymbol{\theta}_c(\mathbf{q})$ is symmetric. But it is only negative semi-definite even if $\partial\boldsymbol{\theta}_c(\mathbf{q})$ is negative definite since $dim\left(\ker\left(\mathbf{A}'\right)\right) = S - K > 0$. This induces that $\forall \mathbf{h} \in \ker\left(\mathbf{A}'\right)$, $\mathbf{h}'\partial\mathbf{D}(\mathbf{p})\mathbf{h} = 0$

(ii)If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $\mathbf{p}_{0,s} = 0$ then $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$

If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$, we know that $\mathbf{q}_n = \mathbf{A}'\mathbf{p}_n \in Q$ but at the limit $\mathbf{q}_0 \in bd(Q)$. It follows, from (ii) of Proposition 1, that $\|\boldsymbol{\theta}(\mathbf{q}_n)\| \to \infty$. Now observe that $\|\mathbf{D}(\mathbf{p}_n)\| = \|\boldsymbol{\theta}(\mathbf{q}_n)\| \left\|\mathbf{A}\frac{\boldsymbol{\theta}(\mathbf{q}_n)}{\|\boldsymbol{\theta}(\mathbf{q}_n)\|}\right\|$. Since $\frac{\boldsymbol{\theta}(\mathbf{q}_n)}{\|\boldsymbol{\theta}(\mathbf{q}_n)\|} \in S^k$, the unit sphere and A is of full rank, $\mathbf{A}\frac{\boldsymbol{\theta}(\mathbf{q}_n)}{\|\boldsymbol{\theta}(\mathbf{q}_n)\|} \to z \neq 0$. It follows that $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$.

(iii)If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$ then $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$

Notice that when $\|\mathbf{p}_n\| \to \infty$, then $\|\mathbf{q}_n\| = \left\|\mathbf{A}'\mathbf{p}_n\right\| \to \infty$ since $\mathbf{p}_n \geq \mathbf{0}$, $\mathbf{A} \geq \mathbf{0}$ and $\forall s$, $\exists k$, $a_{sk} > 0$. This result follows directly from (iii) of Proposition 1.

Appendix C. Proof of Proposition 3

(i) $\partial\mathbf{S}(\mathbf{p}) = \mathcal{D} + \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p})\right)}\mathbf{g}'\mathbf{g}$ with $\mathcal{D}$ a diagonal matrix of generic term $\frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)}$

This follows directly from Eqs. (14) ad (17).

(ii) $\partial\mathbf{S}(\mathbf{p})$ is positive definite

$\partial\mathbf{Y}(\mathbf{p})$ is a diagonal matrix $\mathcal{D}$ with positive terms and is therefore positive definite. $\partial\mathbf{I}(\mathbf{p})$ is positive semi-definite (see Eq. (18)). Their sum is therefore positive definite.

(iii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$ then $\forall s \in S$, $(\mathbf{S}(\mathbf{p}))_s \to +\infty$

If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$, we know from the property of the intermittent production that $\forall s \in S$, $(\mathbf{I}(\mathbf{p}))_s \to +\infty$.

This is sufficient to show that $\forall s \in S$, $(\mathbf{S}(\mathbf{p}))_s \to +\infty$ since $\mathbf{S}(\mathbf{p}) = \mathbf{Y}(\mathbf{p}) + \mathbf{I}(\mathbf{p})$ and $\mathbf{Y}(\mathbf{p}) \geq \mathbf{0}$.

(iv) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$, $\mathbf{S}(\mathbf{p}_n) \to \mathbf{S}(\mathbf{p}_0) \geq \mathbf{0}$ and is finite

If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$, we know that for the conventional sector $\lim_{p_s \to 0} y_s(p_s) = 0$. This results in $\mathbf{Y}(\mathbf{p}_n) \to \mathbf{Y}(\mathbf{p}_0) \geq \mathbf{0}$ and is finite. As for the intermittent sector, with some $\mathbf{p}_{0,s} = 0$, we have $\mathbf{I}(\mathbf{p}_n) \to \mathbf{I}(\mathbf{p}_0) \geq \mathbf{0}$ and is finite. Consequently, $\mathbf{S}(\mathbf{p}_n) \to \mathbf{S}(\mathbf{p}_0) \geq \mathbf{0}$ and is finite.

Appendix D. Proof of Proposition 4

(i) Existence

The proof is essentially based on a homotopy argument. An intuitive presentation can be found in Eaves and Schmedders (1999) (for a more detailed argument see Villanacci et al. (2002) ch.7 or Hirsch (1976) ch.5). Following this presentation, a complex equation system $\mathbf{f}(\mathbf{p}) = \mathbf{0}$, here the function $\mathbf{f} : \mathbb{R}^S_{++} \to \mathbb{R}^S$ with $\mathbf{f}(\mathbf{p}) = \mathbf{S}(\mathbf{p}) - \mathbf{D}(\mathbf{p})$, has a solution if there exists (i) a simple equation system $\mathbf{g}(\mathbf{p}) = \mathbf{0}$, here the function $\mathbf{g} : \mathbb{R}^S_{++} \to \mathbb{R}^S$ with $\mathbf{g}(\mathbf{p}) = \mathbf{p} - \hat{\mathbf{p}}$, $\hat{\mathbf{p}} \gg \mathbf{0}$ given, and (ii) a homotopy $\mathbf{H} : [0, 1] \times \mathbb{R}^S_{++} \to \mathbb{R}^S$ given by $\mathbf{H}(\mathbf{p}, \lambda) = \lambda \mathbf{f}(\mathbf{p}) + (1 - \lambda)\, \mathbf{g}(\mathbf{p})$ with the property that:

- $\mathbf{g}(\mathbf{p})$ admits a unique and regular solution. This is the case here since (i) $\mathbf{g}(\mathbf{p}) = \mathbf{0} \Leftrightarrow \mathbf{p} = \hat{\mathbf{p}}$ and (ii) $\partial g|_{\mathbf{p}=\hat{\mathbf{p}}} = \mathbf{I}_S$, the identity matrix of dimension S, a matrix obviously of full rank.
- $\mathbf{0}$ is a regular value of $\mathbf{H}$, meaning that for all $(\mathbf{p}, \lambda) \in H^{-1}(\mathbf{0})$, $\partial \mathbf{H}|_{(\mathbf{p},\lambda)}$ is a surjection. This is for instance the case if $\partial \mathbf{H}$ is of full rank. Here, the sub-matrix $\partial_p \mathbf{H}$ of $\partial \mathbf{H}$ is given by:

$$\partial_p \mathbf{H} = \lambda \mathbf{I}_S + (1 - \lambda)\,(\partial \mathbf{S}(\mathbf{p}) - \partial \mathbf{D}(\mathbf{p})) \tag{D.1}$$

  Since $\partial \mathbf{S}(\mathbf{p})$ is positive definite (see point (ii) of Proposition 3) and $\partial \mathbf{D}(\mathbf{p})$ negative semi-definite (see point (i) of Proposition 2), $\partial_p \mathbf{H}$ is also positive definite and therefore of rank S. It follows for all $(\mathbf{p}, \lambda) \in H^{-1}(0)$, $\partial \mathbf{H}|_{(\mathbf{p},\lambda)}$ is a surjection.
- $\mathbf{H}^{-1}(\mathbf{0})$ is a compact subset of $[0, 1] \times \mathbb{R}^S_{++}$

It simply remains to check this last point. Assume the contrary there exists a sequence $(\mathbf{p}_n, \lambda_n) \in \mathbf{H}^{-1}(\mathbf{0})$ with the property that either $\|\mathbf{p}_n\| \to \infty$ or $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$. If $\lambda_n \to \lambda_0 = 0$, the only point in $\mathbf{H}^{-1}(\mathbf{0})$ is $(\mathbf{p}_0, \lambda)$. So let us assume in the rest of the argument that $\lambda_0 > 0$. In the first case, we know by (iii) of Proposition 2 that $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$ and by (iii) of Proposition 3 that $\forall s \in S$, $(\mathbf{S}(\mathbf{p}_n))_s \to +\infty$. It implies that $\exists s_0 \in S$, $(\mathbf{H}(\mathbf{p}_n, \lambda_n))_{s_0} \to +\infty$. It therefore exists, for each $K > 0$, a rank N such that $\forall n > N$, $(\mathbf{H}(\mathbf{p}_n, \lambda_n))_{s_0} > K > 0$ which is the desired contradiction. In the second case, we know by (ii) of Proposition 2 that $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$. But remember that $\mathbf{p}_n \to \mathbf{p}_0$ finite, this means that $\mathbf{S}(\mathbf{p}_0)$ as well as $\mathbf{g}(\mathbf{p})$ are finite. Hence $\exists s_0 \in S$, $(\mathbf{H}(\mathbf{p}_n, \lambda_n))_{s_0} \to -\infty$, which is again the desired contradiction.

(i) Uniqueness

This argument is mainly based on the degree theory (see Hirsch (1976) ch.5, or Villanacci et al. (2002) ch.7). In fact, from the previous point, we also deduce that the two maps $\mathbf{f}$ and $\mathbf{g}$ have the same degree, i.e. $\deg(\mathbf{f}) = \deg(\mathbf{g})$. Moreover, if one defines for a given regular map $\mathbf{h}$ the quantity $ind_{\mathbf{h}}(\mathbf{p}) = \frac{\det(\partial \mathbf{h}|_{\mathbf{p}})}{|\det(\partial \mathbf{h}|_{\mathbf{p}})|}$, we know that $\deg(\mathbf{h}) = \sum_{\mathbf{p} \in \mathbf{h}^{-1}(0)} ind_{\mathbf{h}}(\mathbf{p})$. It follows, by computation, that the degree of our simple map $\mathbf{g}$ is 1, hence $\sum_{\mathbf{p} \in \mathbf{f}^{-1}(0)} ind_{\mathbf{f}}(\mathbf{p}) = 1$. Let us now remember that $\partial \mathbf{f}(\mathbf{p}) = \partial \mathbf{S}(\mathbf{p}) - \partial \mathbf{D}(\mathbf{p})$ is a positive definite matrix. Its determinant is therefore always positive. It implies in particular that $\forall \mathbf{p} \in \mathbf{f}^{-1}(0)$, $ind_{\mathbf{f}}(\mathbf{p}) = 1$ and since these quantities sum to 1, the solution is unique.

Appendix E. Proof of Proposition 5

(i) Let $\left(\boldsymbol{\theta}^*_c, m^*, \boldsymbol{\theta}^*_r, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*\right)_{\mathbf{A}}$ be an equilibrium with the contract structure $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{S-K} \\ \mathbf{A}_K \end{bmatrix}$ and $\mathbf{A}_K$ invertible satisfying Definition 1. Now define

$$\begin{aligned} &\left(\boldsymbol{\vartheta}^*, m^*, \boldsymbol{\vartheta}^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*\right)_{\mathbf{B}} \\ &\quad = \left(\mathbf{A}_K \boldsymbol{\theta}^*_{c,\mathbf{A}}, m^*_{\mathbf{A}}, \mathbf{A}_K \boldsymbol{\theta}^*_{r,\mathbf{A}}, \mathbf{y}^*_{\mathbf{A}}, \kappa^*_{\mathbf{A}}, \mathbf{p}^*_{\mathbf{A}}, \left(\mathbf{A}_{\mathbf{K}}^{-1}\right)' \mathbf{q}^*_{\mathbf{A}}\right) \end{aligned} \tag{E.1}$$

and let us verify that this vector also satisfies Definition 1 with contract structure of $\begin{bmatrix} \mathbf{A}_{S-K}\,(\mathbf{A}_K)^{-1} \\ \mathbf{I}_K \end{bmatrix}$. With regard to point (i) of Definition 1, we observe that we obtain the new budget constraint by simply making a change of variable: replacing $\boldsymbol{\theta}$ by $\boldsymbol{\theta} = (\mathbf{A}_K)^{-1}\, \boldsymbol{\vartheta}$, i.e.,

$$\begin{cases} \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \\ \left(\mathbf{q}^*_{\mathbf{A}}\right)' \boldsymbol{\theta} + m = m_0 \end{cases} \Leftrightarrow \begin{cases} \begin{bmatrix} \mathbf{A}_{S-K}\,(\mathbf{A}_K)^{-1} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\vartheta} \geq \mathbf{0} \\ \left(\underbrace{\left(\mathbf{A}_{\mathbf{K}}^{-1}\right)' \mathbf{q}^*_A}_{=\mathbf{q}^*_{\mathbf{B}}}\right)' \boldsymbol{\vartheta} + m = m_0 \end{cases} \tag{E.2}$$

The solution of this new program $\left(\boldsymbol{\vartheta}^*, m^*\right)_{\mathbf{B}} = \left(\mathbf{A}_K \boldsymbol{\theta}^*_{c,\mathbf{A}}, m^*_{\mathbf{A}}\right)$ is therefore the same up to, of course, this change of variable. If we now move to (ii) of Definition 1, the reader knows from our early discussion (see Section 2), that the retailer maximizes his profit and that the contract markets clear if the following relation is satisfied $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^*$. Since $\mathbf{p}^*_{\mathbf{B}} = \mathbf{p}^*_{\mathbf{A}}$, it is a matter of fact to verify that:

$$\begin{aligned} &\mathbf{q}^*_{\mathbf{A}} = \mathbf{A}'\mathbf{p}^*_{\mathbf{A}} \Leftrightarrow \left(\mathbf{A}_{\mathbf{K}}^{-1}\right)' \mathbf{q}^*_{\mathbf{A}} = \left(\mathbf{A}_{\mathbf{K}}^{-1}\right)' \mathbf{A}'\mathbf{p}^*_{\mathbf{A}} \Leftrightarrow \mathbf{q}^*_{\mathbf{B}} \\ &\quad = \begin{bmatrix} \left(\mathbf{A}_{S-K}\,(\mathbf{A}_K)^{-1}\right)' & \mathbf{I}_k \end{bmatrix} \mathbf{p}^*_{\mathbf{B}} \end{aligned} \tag{E.3}$$

Moreover (iii) and (iv) of Definition 1 are not affected by the contract change, the electricity supply remains therefore unchanged. So if the retailer's demand remains unchanged, the proof is finished. From Eq. (11), this new demand at price $\mathbf{p}^*_{\mathbf{B}} = \mathbf{p}^*_{\mathbf{A}}$ is given by:

$$\mathbf{D}_{\mathbf{B}}(\mathbf{p}^*_{\mathbf{B}}) = \begin{bmatrix} \mathbf{A}_{S-K}\,(\mathbf{A}_K)^{-1} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\vartheta}^* = \mathbf{A}\boldsymbol{\theta}^* = \mathbf{D}_{\mathbf{A}}\left(\mathbf{p}^*_{\mathbf{A}}\right) \tag{E.4}$$

(ii) Conversely, let $\left(\boldsymbol{\vartheta}^*_c, m^*, \boldsymbol{\vartheta}^*_r, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*\right)_{\mathbf{B}}$ be an equilibrium with the contract structure $\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}$ satisfying Definition 1 and let $\mathbf{C}$ be a K dimensional invertible matrix. Define:

$$\left(\boldsymbol{\theta}^*, m^*, \boldsymbol{\theta}^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*\right)_{\mathbf{A}} = \left(\mathbf{C}^{-1}\boldsymbol{\vartheta}^*_B, m^*_{\mathbf{B}}, \mathbf{C}^{-1}\boldsymbol{\vartheta}^*_B, \mathbf{y}^*_{\mathbf{B}}, \kappa^*_{\mathbf{B}}, \mathbf{p}^*_{\mathbf{B}}, \mathbf{C}'\mathbf{q}^*_{\mathbf{B}}\right) \tag{E.5}$$

and let us verify that this vector satisfies Definition 1 with $\mathbf{A} = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix}$. First notice that by replacing $\boldsymbol{\vartheta}$ by $\boldsymbol{\vartheta} = \mathbf{C}\boldsymbol{\theta}$, the budget constraints become

$$\begin{cases} \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\vartheta} \geq \mathbf{0} \\ \left(\mathbf{q}^*_{\mathbf{B}}\right)' \boldsymbol{\vartheta} + m = m_0 \end{cases} \Leftrightarrow \begin{cases} \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \\ \left(\underbrace{\mathbf{C}'\mathbf{q}^*_{\mathbf{B}}}_{\mathbf{q}^*_{\mathbf{A}}}\right)' \boldsymbol{\theta} + m = m_0 \end{cases} \tag{E.6}$$

It follows that, at price $\mathbf{q}^*_{\mathbf{A}}$, the new solution of (i) of Definition 1 is given by $\left(\boldsymbol{\theta}^*_{\mathbf{A}}, m^*_{\mathbf{A}}\right) = \left(\mathbf{C}^{-1}\boldsymbol{\vartheta}^*_B, m^*_{\mathbf{B}}\right)$. It is also immediate to verify that the relation between the contract and the state-contingent prices are maintained since:

$$\mathbf{q}^*_{\mathbf{B}} = \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix} \mathbf{p}^*_{\mathbf{B}} \Leftrightarrow \mathbf{C}'\mathbf{q}^*_{\mathbf{B}} = \mathbf{A}'\mathbf{p}^*_{\mathbf{B}} \Leftrightarrow \mathbf{q}^*_{\mathbf{A}} = \mathbf{A}'\mathbf{p}^*_{\mathbf{A}} \tag{E.7}$$

Finally we also notice that the electricity demand at price $\mathbf{p}_{\mathbf{A}}^* = \mathbf{p}_{\mathbf{B}}^*$ remains the same since:

$$\mathbf{D}_{\mathbf{A}}\left(\mathbf{p}_{\mathbf{A}}^*\right) = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix} \mathbf{C}^{-1}\vartheta_B^* = \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \vartheta_{\mathbf{B}}^* = \mathbf{D}_{\mathbf{B}}(\mathbf{p}_{\mathbf{B}}^*) \tag{E.8}$$

We can therefore conclude that $\left(\boldsymbol{\theta}^*, m^*, \boldsymbol{\theta}^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*\right)_{\mathbf{A}}$ is an equilibrium solution with $\mathbf{A} = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix}$ for all choice of $\mathbf{C}$, a K dimensional invertible matrix.

Point (iii) directly follows from the computation given by Eq. (21).

Appendix F. Proof of Proposition 6

This result follows directly from our identification of the first-order conditions.

Appendix G. Proof of Proposition 7

From our discussion in Section 5, we only need to verify that Eq. (26) holds. So let us consider $SW(\mathbf{B})$ introduced in Definition 2 and denote by $(\boldsymbol{\lambda}(\mathbf{B}), \boldsymbol{\mu}(\mathbf{B}))$, the Lagrangian multipliers associated to the constraints. In order to compute the derivative of $SW(\mathbf{B})$, we adopt the following convention: we differentiate per contracts (per column of $\mathbf{B}$) and do this for each contract. $\partial_B SW(\mathbf{B}, \mathbf{g})$ is therefore a vector of dimension $(1, (S-K)K)$ which satisfies the FOC (see Eq. (24)):

$$\begin{aligned} \partial SW(\mathbf{B}) &= (\partial\mathcal{U}(\mathbf{x}(\mathbf{B})))' \partial\mathbf{x}(\mathbf{B}) - (\partial\mathcal{C}(\mathbf{x}(\mathbf{B})))' \partial\mathbf{y}(\mathbf{B}) \\ &\quad - \tfrac{d\mathcal{K}(\kappa(\mathbf{B}))}{d\kappa}\partial\kappa(\mathbf{B}) \\ &= (\boldsymbol{\lambda}(\mathbf{B}))'(\partial\mathbf{x}(\mathbf{B}) - \partial\mathbf{y}(\mathbf{B}) - \mathbf{g}\partial\kappa(\mathbf{B})) \\ &\quad + (\boldsymbol{\mu}(\mathbf{B}))' \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial\mathbf{x}(\mathbf{B}) \end{aligned} \tag{G.1}$$

Using the first set of constraints of Eq. (23) as an identity, we know that:

$$\partial\mathbf{x}(\mathbf{B}) - \partial\mathbf{y}(\mathbf{B}) - \mathbf{g}\cdot\partial\kappa(\mathbf{B}) = \mathbf{0} \tag{G.2}$$

It follows that:

$$\partial SW(\mathbf{B}) = (\boldsymbol{\mu}(\mathbf{B}))' \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial\mathbf{x}(\mathbf{B}) \tag{G.3}$$

Now, by using the second set of constraints of Eq. (23) as an identity and by computation, we get that:

$$\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial\mathbf{x}(\mathbf{B}) - \left((x_{S-K+k}(\mathbf{B}))_{k=1}^K\right)' \otimes \mathbf{I}_{S-K} = \mathbf{0} \tag{G.4}$$

where $\otimes$ denotes the tensor product. We can therefore say that:

$$\begin{aligned} \partial SW(\mathbf{B}) &= (\boldsymbol{\mu}(\mathbf{B}))' \left(\left((x_{S-K+k}(\mathbf{B}))_{k=1}^K\right)' \otimes \mathbf{I}_{S-K}\right) \\ &= \left(x_{S-K+k}(\mathbf{B})(\boldsymbol{\mu}(\mathbf{B}))'\right)_{k=1}^K \end{aligned} \tag{G.5}$$

Appendix H. Proof of Lemma 1

(i) Computation of $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$

Let us apply the Implicit Function Theorem to $f(\mathbf{p}, \mathbf{B}) = \mathbf{0}$ given by Eq. (28). To apply this theorem, we first need to verify that $\partial_{\mathbf{p}} f(\mathbf{p}, \mathbf{B})$ is a square matrix of full rank, here S. This is done by showing that $\ker\left(\partial_{\mathbf{p}} f(\mathbf{p}, \mathbf{B})\right) = \{\mathbf{0}\}$. By computation:

$$\partial_{\mathbf{p}} f(\mathbf{p}, \mathbf{B})\,\mathbf{h} = \mathbf{0} \Leftrightarrow \begin{cases} \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix} \left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p}))\,\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}) - \mathbf{I}_S\right)\mathbf{h} = \mathbf{0} \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial_{\mathbf{p}}\mathbf{S}(\mathbf{p})\mathbf{h} = \mathbf{0} \end{cases} \tag{H.1}$$

Since $\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p})$ is invertible, in fact even positive definite (see (ii) of Proposition 3), we obtain by setting $\mathbf{h}_1 = \partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}, \mathbf{g})\mathbf{h}$:

$$\begin{cases} \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix} \left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p}, \mathbf{g})) - \left(\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}, \mathbf{g})\right)^{-1}\right)\mathbf{h}_1 = \mathbf{0} \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \mathbf{h}_{\mathbf{1}=0} \end{cases} \tag{H.2}$$

Now, let us observe that $\ker\left(\begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix}\right) \perp \ker\left(\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix}\right)$. More precisely:

$$\begin{aligned} &\begin{cases} \mathbf{v}_1 \in \ker\left(\begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix}\right) \\ \mathbf{v}_2 \in \ker\left(\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix}\right) \end{cases} \\ &\Leftrightarrow \begin{cases} \mathbf{v}_1 = \begin{bmatrix} \mathbf{I}_{S-K} \\ -\mathbf{B}' \end{bmatrix} \mathbf{x}_1 \text{ with } \mathbf{x}_1 \in \mathbb{R}^{S-K} \\ \mathbf{v}_2 = \begin{bmatrix} \mathbf{B} \\ I_K \end{bmatrix} \mathbf{x}_2 \text{ with} \mathbf{x}_2 \in \mathbb{R}^K \end{cases} \Rightarrow \mathbf{v}_1'\mathbf{v}_2 = 0 \end{aligned} \tag{H.3}$$

From Eq. (H.2), we deduce that:

$$\mathbf{h}_1'\left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p}, \mathbf{g})) - \left(\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}, \mathbf{g})\right)^{-1}\right)\mathbf{h}_1 = 0 \tag{H.4}$$

Now remember that $\partial^2\mathcal{U}$ is negative definite by assumption; the same being true for $-\left(\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}, \mathbf{g})\right)^{-1}$ since $\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}, \mathbf{g})$ is positive definite (see (ii) of Proposition 3). It follows that $\mathbf{h}_1 = \mathbf{0}$ and therefore, $\ker\left(\partial_{\mathbf{p}} f(\mathbf{p}, \mathbf{g}, \mathbf{B})\right) = \{\mathbf{0}\}$.

We can now move to the construction of $\partial_{\mathbf{B}} f(\mathbf{p}, \mathbf{B})$. To compute this derivative with respect to the coefficient of $\mathbf{B}$, we keep the same convention as in the proof of Proposition 7. Let us start with the first part of this function f given by $\begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix} \cdot (\partial\mathcal{U}(\mathbf{S}(\mathbf{p})) - \mathbf{p})$ whose generic term is:

$$\left(\sum_{j=1}^{S-K} b_{j,i}\left(\partial_{x_j}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_j\right) + \left(\partial_{x_{S-K+i}}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_{S-K+i}\right)\right)_{i=1}^K \tag{H.5}$$

If we denote by $\mathbf{e}_k$ a vector of $\mathbb{R}^K$ containing 1 at rank k and 0 elsewhere, the derivative of Eq. (H.5) with respect to $b_{s,k}$ is $\left(\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_s\right)\mathbf{e}_k$. It follows that the derivative with respect to $\left(b_{s,k}\right)_{s=1}^{S-K}$ will be $\mathbf{e}_k \otimes \left(\left(\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}\right)'$ and finally the $(K, (S-K)K)$ matrix of the derivatives with respect to $\mathbf{B}$ is:

$$\mathbf{I}_K \otimes \left(\left(\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}\right)' \tag{H.6}$$

Let us now move to the second part of f given by $\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \cdot \mathbf{S}(\mathbf{p})$ whose generic term is:

$$\left(\mathbf{S}_i(\mathbf{p}) - \sum_{j=1}^{K} b_{i,j}\mathbf{S}_{S-K+j}(\mathbf{p})\right)_{i=1}^{S-K} \tag{H.7}$$

If $\boldsymbol{\epsilon}_s$ now denotes a vector of $\mathbb{R}^{S-K}$ containing 1 at rank s and 0 elsewhere, the derivative of Eq. (H.7) with respect to $b_{s,k}$ is $(-\mathbf{S}_{S-K+k}(\mathbf{p}, \mathbf{g}))\,\boldsymbol{\epsilon}_s$ and, with a same argument, the $(K, (S-K)K)$ matrix of the derivatives with respect to $\mathbf{B}$ becomes:

$$\left(-\left(\mathbf{S}_{S-K+k}(\mathbf{p})\right)_{k=1}^K\right)' \otimes \mathbf{I}_{S-K} \tag{H.8}$$

Since Eqs. (H.6) and (H.8) are the two parts of $\partial_{\mathbf{B}} f(\mathbf{p}, \mathbf{B})$ we can say that:

$$\partial_{\mathbf{B}} f(\mathbf{p}, \mathbf{B}) = \begin{bmatrix} \mathbf{I}_K \otimes \left(\left(\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}\right)' \\ \left(-\left(\mathbf{S}_{S-K+k}(\mathbf{p})\right)_{k=1}^K\right)' \otimes \mathbf{I}_{S-K} \end{bmatrix} \tag{H.9}$$

From Eqs. (H.1) and (H.9), we finally conclude that:

$$\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B}) = \left[\begin{array}{c} \left[\begin{array}{cc} \mathbf{B}' & \mathbf{I}_K \end{array} \right] \left(\partial^2\mathcal{U}\left(\mathbf{S}(\mathbf{p})\right)\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}) - \mathbf{I}_S\right) \\ \left[\begin{array}{cc} \mathbf{I}_{S-K} & -\mathbf{B} \end{array} \right] \partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}) \end{array} \right]^{-1}$$
$$\times \left[\begin{array}{c} \mathbf{I}_K \otimes \left(\left(\partial_{x_s}\mathcal{U}\left(\mathbf{S}(\mathbf{p})\right) - p_s\right)_{s=1}^{S-K}\right)' \\ \left(-\left(\mathbf{S}_{S-K+k}(\mathbf{p})\right)_{k=1}^{K}\right)' \otimes \mathbf{I}_{S-K} \end{array} \right] \tag{H.10}$$

(ii) Rank of $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$

Let us first consider the two particular cases given by $K = S-1$ and $K = 1$. If $K = S-1$, $\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right)$ writes:

$$\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right) = \left[\begin{array}{c} \left(\partial_{x_1}U\left(\mathbf{S}(\mathbf{p})\right) - p_1\right)\mathbf{I}_{S-1} \\ \left(-\left(\mathbf{S}_s(\mathbf{p})\right)_{s=2}^{S}\right)' \end{array} \right] \tag{H.11}$$

If $\left(\partial_{x_1}U\left(\mathbf{S}(\mathbf{p})\right) - p_1\right) = 0$, $rank\left(\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right)\right) = 1$ and therefore $rank\left(\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})\right) = 1$. Otherwise $rank\left(\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right)\right) = S-1$ and $rank\left(\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})\right) = S-1$. If $K = 1$, $\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right)$ becomes

$$\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right) = \left[\begin{array}{c} \left(\left(\partial_{x_s}U\left(\mathbf{S}(\mathbf{p})\right) - p_s\right)_{s=1}^{S-1}\right)' \\ -S_S(\mathbf{p})\mathbf{I}_{S-1} \end{array} \right] \tag{H.12}$$

which is obviously a matrix of rank $S-1$.

Let us now move to the case $1 < K < S-1$ and let us observe that the derivative of $f\left(\mathbf{p},\mathbf{B}\right)$ with respect to $(b_{s,k})_{s=1}^{S-K}$, i.e., the kth vertical block of $(S-K)$ columns of $\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right)$, is given by:

$$\partial_{(b_{s,k})_{s=1}^{S-K}}f\left(\mathbf{p},\mathbf{B}\right) = \left[\begin{array}{c} \mathbf{e}_k \otimes \left(\left(\partial_{x_s}U\left(\mathbf{S}(\mathbf{p})\right) - p_s\right)_{s=1}^{S-K}\right)' \\ -\left(\mathbf{S}_{S-K+k}(\mathbf{p})\right)\mathbf{I}_{S-K} \end{array} \right] \tag{H.13}$$

We can now make a first observation. Since the supply $\mathbf{S}(\mathbf{p})$ is strictly positive at equilibrium, the lower part of the previous matrix is the identity of $(S-K)$ up to multiplication by non-zero constant. So if we select the first block of $(S-K)$ columns of $\partial_{\mathbf{B}}f\left(\mathbf{p},\mathbf{B}\right)$, we have $(S-K)$ linearly independent vectors. Now assume that $\left(\partial_{x_s}U\left(\mathbf{S}(\mathbf{p})\right) - p_s\right)_{s=1}^{S-K} \neq \mathbf{0}$. Thus, at least one term is non-zero, say the $s_0 - th$. Now observe that the upper part of $\partial_{(b_{s,k})_{s=1}^{S-K}}f\left(\mathbf{p},\mathbf{B}\right)$ is mostly composed of zeros except in line k and this line changes with the order number of the block. So if we select from block 2 to K the $s_0 - th$ column, we obtain again $(K-1)$ linear independent vectors. This shows point (ii) of the Lemma. Moreover, if at least one other component of $\left(\partial_{x_s}U\left(\mathbf{S}(\mathbf{p})\right) - p_s\right)_{s=1}^{S-K}$ is non-zero, say the $s_1 - th$, we can add to the previously selected columns, the $s_1 - th$ column of the second block and therefore conclude that $rank\left(\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})\right) = S$.

Appendix I. Proof of Proposition 8

The proof directly follows from our discussion.

Appendix J. Proof of Proposition 9

Remember that the Gordan's form of Farkas' Lemma states that either (i) $\exists\mathbf{x}\ \mathbf{A}\mathbf{x} \gg \mathbf{0}$ or (ii) $\exists\mathbf{y} \geq \mathbf{0}$, and $\mathbf{y} \neq \mathbf{0}$, $\mathbf{y}'\mathbf{A} = \mathbf{0}$. It follows:

(i) if we set $\mathbf{A} = \left[\begin{array}{c} \partial_{\mathbf{B}}\kappa^*\left(\mathbf{p}^*(\mathbf{B})\right) \\ \partial_{\mathbf{B}}SW(\mathbf{B}) \end{array} \right]$ and assume that $\nexists\mathbf{y} \geq \mathbf{0}$, and $\mathbf{y} \neq \mathbf{0}$, $y_1\partial_B\kappa^*\left(p^*(B)\right) + y_2\partial_B SW(B) = 0$ or, in other words, that the two vectors are collinear with a positive coefficient, we can say that $\exists d\mathbf{B}$ with the property that:

$$\left\{ \begin{array}{l} \partial_{\mathbf{B}}\kappa^*\left(\mathbf{p}^*(\mathbf{B})\right)d\mathbf{B} > 0 \\ \partial_{\mathbf{B}}SW(\mathbf{B})d\mathbf{B} > 0 \end{array} \right. \tag{J.1}$$

In other words, there exists changes in $\mathbf{B}$ which improve both investment in renewables and social welfare.

(ii) since $\mathbf{g} \gg \mathbf{0}$ and $\partial_p\mathbf{Y}(\mathbf{p}) = \mathcal{D}$, a diagonal matrix of generic term $\frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)} > 0$ (see (i) of Proposition 3), we can say that $\mathbf{g}' = \alpha'\ \partial_p\mathbf{Y}(\mathbf{p})$ with $\alpha_s = g_S\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)$ for $s = 1,\ldots,S$, or, in other words, $\exists\mathbf{y} = \left(\begin{array}{c} 1 \\ \alpha \end{array}\right) \geq \mathbf{0}$ and $\mathbf{y} \neq \mathbf{0}$, $\mathbf{y}'\left[\begin{array}{c} \mathbf{g}' \\ -\partial_p\mathbf{Y}(\mathbf{p}) \end{array}\right] = 0$. It implies $\nexists\mathbf{x}$, $\left[\begin{array}{c} \mathbf{g}' \\ -\partial_p\mathbf{Y}(\mathbf{p}) \end{array}\right]\mathbf{x} \gg \mathbf{0}$. So, if we replace $\mathbf{x}$ by $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})d\mathbf{B}$, we get:

$$\nexists d\mathbf{B} \text{ which satisfies } \left\{ \begin{array}{l} \mathbf{g}'\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})d\mathbf{B} > 0 \\ \partial_p\mathbf{Y}(\mathbf{p})d\mathbf{B} < 0 \end{array} \right. \tag{J.2}$$

In other words, it is impossible by a contract change to improve investment in renewables and to decrease conventional electricity production in each state of nature.

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