



# Teleportation of continuous variables states

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# TD7: Teleportation of continuous variables states

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## 1 Introduction

The continuous-variable teleportation protocol is a quantum communication protocol that allows for the transmission of a continuous quantum state, such as the amplitude or phase of a light beam, from one location to another. The continuous variable teleportation is deterministic, but will lead to the distortion of the state due to the finite squeezing of the EPR state, contrary to the discrete variable teleportation which is probabilistic but does not distort the output state. CV-teleportation protocol is a building block of many quantum information protocols, and can be used for instance to share information between different users [1], and in quantum repeater architecture [2].

## 2 Description of the protocol

In what follows, we describe the continuous variables teleportation protocol through the evolution of the wavefunction (Schrödinger point of view). A nice introduction on the topic, along with some codes can be found in [https://strawberryfields.ai/photonics/demos/run\\_teleportation.html](https://strawberryfields.ai/photonics/demos/run_teleportation.html).

### 2.1 Ideal protocol

(1) We start from an initial EPR state shared by Alice and Bob. The wave function can be written as:  $|\psi\rangle = \int dx |x\rangle_b |x\rangle_c$ . The continuous state to be teleported will be noted  $|\psi\rangle = \int \psi(x) dx |x\rangle_a$ . The equivalent of the Bell measurement for the continuous variables encoding is performed, consisting of a beam-splitter mixing between one mode of the EPR and the mode of the wavefunction to transmit, concluded by two homodyne detections in orthogonal directions.

- Step 1: write the tripartite state before the beam-splitter.

• Step 2: Write the quadrature operators after the output of the beam-splitter  $\hat{x}_\pm$  and  $\hat{p}_\pm$ . We remind that a beam-splitter performs the unitary operation:

$$\hat{U} |x_1, x_2\rangle_{ab} = \left| \frac{x_1 + x_2}{\sqrt{2}}, \frac{x_1 - x_2}{\sqrt{2}} \right\rangle_{+-} \quad (1)$$

Write the output wavefunction state.

• Step 3: Homodyne detections consist of projections  $|x\rangle \langle x|$  in the mode  $-$  and  $|p\rangle \langle p|$  in the mode  $+$ . Write the collapsed wavefunction and the quadratures operators  $\hat{x}_c, \hat{p}_c$ .

• Step 4: The teleported state has a correctable error. What remains to be done for Alice (classical communication), and subsequently for Bob? What is the fidelity of the teleportation protocol  $F = \langle \psi | \hat{\rho} | \psi \rangle$ ?

♣ (2) Note that the protocol can be written using the Heisenberg point of view. Using this method, we have to start from the nullifiers of the EPR state, which is the linear combination of position (or momentum) operators that applied on an entangled state give zero.

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## 2.2 Real protocol

Physical CV states do not have a infinite amount of squeezing, as it would require an infinite amount of energy to produce them. To take into account the finite quadrature width of the states, the state to be teleported and the EPR state, one can employ the Wigner phase space representation of the state, as it was done in [3]. The consequence of the finite squeezing will lead to a distorted teleported state. For instance, the fidelity  $F = \langle \psi | \hat{\rho}_{out} | \psi \rangle$  of the teleported coherent state  $|\psi\rangle = |\beta_{in}\rangle$  can be cast as [4]:

$$F = \frac{2}{\sigma_Q} \exp\left(-\frac{2}{\sigma_Q} |\beta_{out} - \beta_{in}|^2\right) \quad (2)$$

where  $\sigma_Q = \sqrt{(1 + \sigma_{in}^x)(1 + \sigma_{in}^p)}$ , where:

$$\sigma_{in}^x = \sigma_{in}^p = g^2 + \frac{1}{2}e^{2r}(1-g)^2 + \frac{1}{2}e^{-2r}(1+g)^2 \quad (3)$$

$\sigma_Q$  is the variance of the teleported state, and  $\sigma_{x,p}$  are the variance along the  $x$  and  $p$ . quadratures.  $r$  is the squeezing factor of the EPR state and  $g$  is the gain factor of the classical channels.

(1) What is the limit  $r \rightarrow \infty$ ?

(2) What other factors can limit the fidelity of the teleportation?

We now define other important bounds of the fidelity.

**[No-cloning limit  $F = 2/3$ ]** Alice sends a quantum state to Bob, but Bob wants the guarantee that Alice does not keep for her a better (less noisy) copy of the quantum state. Surpassing the no-cloning limit guarantees that the teleported state is the best remaining copy of the input state. Copying the state two times will lead to a fidelity of  $2/3$ , that we will show now. This is a security argument limit, in the perfect spirit of other attacks that were defined during the course. For a reference, see [5].

(3) The cloner or duplicator has one input mode and two output modes. Starting from the two orthogonal quadratures  $X_{in}, P_{in}$ :

$$\hat{X}_a = g_{ax}\hat{X}_{in} + \hat{\beta}_{ax}, \quad \hat{P}_a = g_{ap}\hat{P}_{in} + \hat{\beta}_{ap} \quad (4)$$

$$\hat{X}_b = g_{bx}\hat{X}_{in} + \hat{\beta}_{bx}, \quad \hat{P}_b = g_{bp}\hat{P}_{in} + \hat{\beta}_{bp} \quad (5)$$

In what transformation it looks like? Can you interpret the terms  $g_{ax}$  and  $\hat{\beta}_{ax}$ ?

(4) Write the inequality verified by the product  $\Delta\hat{\beta}_{ax}\Delta\hat{\beta}_{ap}$ .

**[Classical teleportation  $F = 1/2$ ]**

The best achievable value for the fidelity without using entanglement for the coherent state input is  $F = 1/2$ . It corresponds to the case where an infinite number of copies of the state is performed, and corresponds to a classical measurement. More details can be found in [5].

The state-of-the-art CV teleportation is described in [4], and a fidelity of  $F = 0.905 \pm 0.2$  was reported. In practice, many other obstacles must be passed, such as the phase stabilization between different quantum photonic fields.

## References

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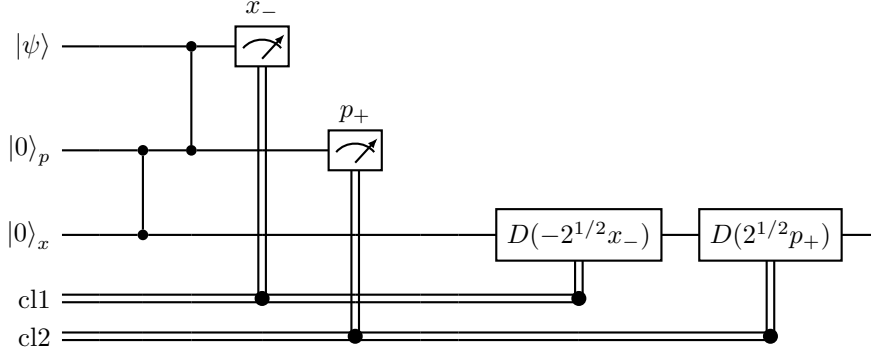


Figure 1: Quantum circuit of the continuous variable teleportation.

## A Correction

### A.1 Heisenberg picture:

The nullifiers can be written as:

$$\hat{x}_a - \hat{x}_b = \hat{p}_a + \hat{p}_b = 0 \quad (6)$$

Alice combines the input mode with one of the mode of the EPR state. The output quadratures mode can be written as:

$$\hat{x}_{\pm} = \frac{1}{\sqrt{2}}(\hat{x}_a \pm \hat{x}_{in}), \quad \hat{p}_{\pm} = \frac{1}{\sqrt{2}}(\hat{p}_a \pm \hat{p}_{in}) \quad (7)$$

Alice then performed two homodyne detections, measuring the quadrature  $x$  in the mode  $+$ , and the quadrature  $p$  in the mode  $-$ . The obtained outcome are noted  $x_-, p_-$ , the collapse is:

$$\hat{x}_a = \hat{a}_{in} + \sqrt{2}x_-, \quad \hat{p}_a = -\hat{p}_{in} + \sqrt{2}\hat{p}_+ \quad (8)$$

Due to the EPR property, the nullifier relations, Bob's quadrature are projected

$$\hat{x}_b = \hat{a}_{in} + \sqrt{2}x_-, \quad \hat{p}_b = \hat{p}_{in} - \sqrt{2}\hat{p}_+ \quad (9)$$

Classical communication: Alice communicates the outcome that she obtained...

Conditional displacement: ... so that Bob then performs the displaced operation to correct the state:

$$\hat{x}'_b = \hat{x}_b - \sqrt{2}x_- = \hat{x}_{in} \quad \hat{p}'_b = \hat{p}_b + \sqrt{2}\hat{p}_+ = \hat{p}_{in} \quad (10)$$

The fidelity of the teleportation is then one. The quantum circuit of the teleportation is represented in Fig.A.1.

### A.2 Schrodinger picture

Starting by mixing the EPR state  $|\psi\rangle = \int dx_2 |x_2\rangle_b |-x_2\rangle_c$  with the state of interest  $|\psi\rangle_{in} = \int \psi(x_1) |x_1\rangle dx_1$ , into a balanced beam-splitter:

$$|\psi\rangle = \iint dx_1 dx_2 \psi(x_1) \left| \frac{x_1 + x_2}{\sqrt{2}} \right\rangle_a \left| \frac{x_1 - x_2}{\sqrt{2}} \right\rangle_b |-x_2\rangle_c \quad (11)$$

Two homodyne detection are performed in the two spatial ports, the final state is

$$|\psi'\rangle_c = {}_a \langle x_- |_b \langle p_+ | \psi \rangle = e^{ip_+ x_-} \int dx_1 \psi(x_1) e^{ip_+ \sqrt{2}x_1} |x_1 - \sqrt{2}x_-\rangle_c \quad (12)$$

The displacement operators are:

$$\hat{D}(x) = e^{i\hat{p}x}, \quad \hat{D}(x) |x'\rangle = |x' - x\rangle \quad (13)$$

$$\hat{D}(p) = e^{-i\hat{x}p}, \quad \hat{D}(p) |x'\rangle = e^{-ipx'} |x'\rangle \quad (14)$$

Then, by applying the displacement operators, we obtain the initial state:

$$\hat{D}(\sqrt{2}p_+) \hat{D}(-\sqrt{2}x_-) |\psi'\rangle_c = |\psi\rangle_{in} \quad (15)$$

with a non-relevant global phase. The fidelity of the state is one because the EPR state are infinitively squeezed.

### A.3 Finite squeezing case

The calculation of the output wavefunction can be dealt with the wavefunction or the Wigner representation [3]. The physical EPR state can be written as

$$|\psi\rangle = \iint \psi_{\text{EPR}}(x_2, x_3) dx_2 dx_3 |x_2\rangle_b |x_3\rangle_c \quad (16)$$

where:

$$\psi_{\text{EPR}}(x_2, x_3) = e^{-(x_2+x_3)^2/4\sigma_+^2} e^{-(x_2-x_3)^2/4\sigma_-^2}. \quad (17)$$

where  $\sigma_+ \ll \sigma_-$  for an anti-correlated state. After the homodyne detection, the state can be written as:

$$|\psi'\rangle_c = \iint dx_1 dx_3 \psi(x_1) \psi_{\text{EPR}}(\sqrt{2}x_- - x_1, x_3) |x_1\rangle_a. \quad (18)$$

One has to perform the integration over  $x_3$ , which will lead to a supplementary function in  $x_1$  that will distort the state. With Gaussian states, the calculation is straightforward. The fidelity can be written as:

$$F = |\langle\psi|\psi'\rangle|^2 = \left| \iint dx_1 dx_3 |\psi(x_1)|^2 \psi_{\text{EPR}}(\sqrt{2}x_- - x_1, x_3) \right|^2. \quad (19)$$

## B Non-cloning limit

From [5, 4]. For the relation Eq.(4) takes inspiration from a non-conserving energy beam-splitter (the copying is costly), with nothing (the vacuum) at one of the input port. Since the creation and annihilation between different modes commute, we then have  $[\hat{X}_a, \hat{P}_b] = 0$ , the commutators between the noise operators are:

$$[\hat{\beta}_{ax}, \hat{\beta}_{bp}] = -g_{ax}g_{bp}[\hat{X}_{in}, \hat{P}_{in}] \quad (20)$$

The Robertson inequality can be cast as

$$\Delta\hat{\beta}_{ax}\Delta\hat{\beta}_{bp} \geq \frac{1}{2} \left| \langle [\hat{\beta}_{ax}, \hat{\beta}_{bp}] \rangle \right| \quad (21)$$

Thus, we have

$$\Delta\hat{\beta}_{ax}\Delta\hat{\beta}_{bp} \geq \frac{1}{4} |g_{ax}g_{bp}| \quad (22)$$

1/4 is the vacuum's noise variance. We can define the variances of the equivalent input noises are:

$$N_{ix} = \left( \frac{\Delta\hat{\beta}_{ix}}{|g_{ix}|} \right)^2, \quad N_{ip} = \left( \frac{\Delta\hat{\beta}_{ip}}{|g_{ip}|} \right)^2 \quad (23)$$

And the symmetric inequalities can be obtained:

$$N_{ax}N_{bp} \geq \frac{1}{8}, \quad N_{bx}N_{ap} \geq \frac{1}{8} \quad (24)$$

If a coherent state is perfectly copied  $\beta_{out} = \beta_{in}$  (used of an ideal EPR state) and the gains are unity, then the fidelity of the teleportation is:

$$F = \frac{2}{\sqrt{(1+4N_x)(1+4N_p)}} \quad (25)$$

The fidelity is maximized when  $N_{ax} = N_{bp} = N_{bx} = N_{ap} = 1/4$  and takes the value is 2/3 which is an effect of the fluctuation of the vacuum.