

Supplementary material

Toward thermal autarky for large-scale biogas plants: Dynamic energy modeling for energy efficiency in anaerobic digesters with enhanced multimembrane gasholders

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2 M. Avila-Lopez^a, C. Robles-Rodriguez^a, L. Tiruta-Barna^a, A. Ahmadi^{a,*}

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4 ^aTBI, Université de Toulouse, CNRS, INRAE, INSA 135 Avenue de Rangueil, 31077, France

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7 1. Radiative heat exchange in confined spaces

8 1.1 Formulation of the infrared radiative network

9 The net rate at which radiation leaves a surface i can be expressed as

$$q_i = A_i(J_i - G_i). \quad (1)$$

10

11 Using the following definition of the radiosity,

$$J_i = E_i + (1 - \alpha_i)G_i, \quad (2)$$

12

13 and knowing that, for a gray surface, $E_i = \varepsilon_i \sigma T_i^4$, G_i can be solved for using Eq. (2) and substituted into

14 Eq. (1), resulting in

$$q_i = \frac{\varepsilon_i \sigma T_i^4 - \alpha_i J_i}{(1 - \alpha_i)/A_i}. \quad (3)$$

* Corresponding author at: Toulouse Biotechnology Institute, Bio & Chemical Engineering, 135 avenue de Rangueil, 31077 Toulouse CEDEX 04, France.
E-mail address: ahmadi@insa-toulouse.fr (A. Ahmadi).

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16 To use Eq. (3), the surface radiosity J_i must be known. To determine this quantity, it is necessary to
17 consider the radiation exchange between the surfaces of the enclosure.

18 The irradiation of surface i can be evaluated from the radiosities of all the surfaces in
19 the enclosure. The total rate at which radiation reaches the surface i from all surfaces, including i , can
20 be expressed as

$$A_i G_i = \sum_{j=1}^n F_{ji} A_j J_j, \quad (4)$$

21

22 where F_{ji} is the view factor from surface j to surface i . Applying the reciprocity relation ($A_i F_{ij} = A_j F_{ji}$),

$$A_i G_i = \sum_{j=1}^n A_i F_{ij} J_j. \quad (5)$$

23

24 Canceling A_i and substituting Eq. (5) into Eq. (1) for G_i , we find

$$q_i = A_i \left(J_j - \sum_{j=1}^n F_{ij} J_j \right). \quad (6)$$

25

26 From the summation rule ($\sum_{j=1}^n F_{ij} = 1$), which follows the conservation requirement that all radiation
27 leaving surface i must be intercepted by the enclosure surfaces, we find

$$q_i = A_i \left(\sum_{j=1}^n F_{ij} J_i - \sum_{j=1}^n F_{ij} J_j \right). \quad (7)$$

28

29 Therefore,

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{(A_i F_{ij})^{-1}}. \quad (8)$$

30

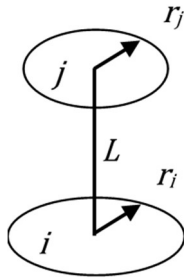
31 Combining Eqs. (3) and (8), the net radiative flow leaving any surface in a confined space can be
 32 obtained:

$$q_i = \frac{\varepsilon_i \sigma T_i^4 - \alpha_i J_i}{(1 - \alpha_i)/A_i} = \sum_{j=1}^n \frac{J_i - J_j}{(A_i F_{ij})^{-1}}. \quad (9)$$

33

34 1.2 Geometric considerations and determination of the view factors

35 The view factor between the digestate and the inner membrane, $F_{d,im,inf}$, is determined by assuming
 36 two parallel coaxial disks, as entailed in Eqs. (10)–(13).



$$R_i = \frac{r_i}{L} \quad (10)$$

$$R_j = \frac{r_j}{L} \quad (11)$$

$$S = 1 + \frac{1 + R_i^2}{R_j^2} \quad (12)$$

$$F_{ij} = \frac{1}{2} \left\{ S - \left[S^2 - 4 \left(\frac{r_j}{r_i} \right)^2 \right]^{1/2} \right\} \quad (13)$$

37

38 Applying the summation rule ($\sum_{j=i}^N F_{ij} = 1$), with $F_{i,i} = 0$ for a plane or convex surface, the view factor
 39 between the digestate and the inner walls ($F_{d,w}$) equals $1 - F_{d,im,inf}$. The space created between the
 40 inner membrane and the cover can be treated as a two-surface enclosure, and the view factor between
 41 the inner membrane and the cover ($F_{im,sup,c}$) is set to 1 because the total net radiation heat from the
 42 inner membrane goes to the cover. The rest of the view factors are obtained using the reciprocity relation
 43 ($A_i F_{ij} = A_j F_{ji}$), which can be used to determine one view factor from another.

44

45 1.3 Instantaneous global solar irradiation on the ground

46 The method for calculating the solar heat flux gain from the solar radiation is presented in the following
 47 section [46]. Solar time is the time used in all of the Sun-angle relationships and is obtained such that

$$ST = SLT + \frac{E}{60} + \frac{l_{st} - l_{loc}}{15} - 1 \quad \text{for daylight saving time,} \quad (14)$$

$$ST = SLT + \frac{E}{60} + \frac{l_{st} - l_{loc}}{15} \quad \text{for non-daylight saving time,} \quad (15)$$

48

49 where ST is the solar time expressed in hours, SLT is the standard local time in hours, l_{st} is the standard
50 meridian for the local time zone (15° in France), l_{loc} is the local longitude of the location, and E is a
51 correction term in minutes:

$$E = 229.2(0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos 2B - 0.04089 \sin B), \quad (16)$$

$$B = (n - 1) \frac{2\pi}{365}, \quad (17)$$

52

53 where n is the day of the year.

54 The declination angle of the sun, δ (in $^\circ$), is given by

$$\delta = 0.006918 - 0.399912 \cos B + 0.070257 \sin B - 0.006758 \cos 2B + 0.000907 \sin 2B - 0.002697 \cos 3B + 0.00148 \sin 3B. \quad (18)$$

55

56 The sunset hour angle, ω_s (in $^\circ$), is given by

$$\omega_s = \cos^{-1}(-\tan \Phi + \tan \delta), \quad (19)$$

57

58 where Φ is the local latitude in degrees.

59 The sunrise and sunset standard local times (denoted with subscripts sr and ss , respectively) are
60 calculated as follows:

$$SLT_{sr} = 12 - \frac{\omega_s}{15}, \quad (20)$$

$$SLT_{ss} = 12 + \frac{\omega_s}{15}. \quad (21)$$

61

62 The sunrise and sunset solar times (ST_{sr} and ST_{ss} , respectively) are obtained by substituting Eqs. (20)
 63 and (21) into Eqs. (14) and (15).

64 The daily extraterrestrial radiation on a horizontal surface at the upper limit of the Earth's atmosphere,
 65 H_0 , is calculated as follows:

$$H_0 = \frac{24 \times 3600}{\pi} G_0, \quad (22)$$

$$G_0 = G_{on} \cos \theta_z, \quad (23)$$

$$G_{on} = G_{sc}(1.000110 + 0.034221 \cos B + 0.001280 \sin B + 0.00719 \cos 2B \\ + 0.000077 \sin 2B), \quad (24)$$

$$\cos \theta_z = \cos \Phi \cos \delta \sin \omega_s + \omega_s \sin \Phi \sin \delta, \quad (25)$$

66
 67 where G_0 is the extraterrestrial irradiance on the horizontal plane, G_{on} is the extraterrestrial radiation
 68 incident on the plane normal to the radiation on the n th day of the year, θ_z is the zenith angle, and G_{sc}
 69 is the solar constant, equal to 1367 W m^{-2} .

70 To estimate the daily global solar irradiation on a horizontal surface at the Earth's surface, the following
 71 equation is used:

$$G_H = K_T H_0, \quad (26)$$

72
 73 where K_T is the daily insolation clearness index. An empirical correlation for the calculation of K_T was
 74 established. Monthly K_T data were obtained from the Nasa Data Access Viewer [39] and correlated with
 75 the monthly sunshine fraction (equal to the difference between the sunset and sunrise solar times) of the
 76 location. The following equation was obtained for the specific location of this study:

$$K_T = 0.6163ss + 0.2025, \quad (27)$$

77
 78 where ss is the monthly sunshine fraction for the n th day:

$$ss = \frac{ST_{ss} - ST_{sr}}{24}. \quad (28)$$

79

80 Depending on the value of K_T , the daily diffuse solar irradiation on a horizontal surface at the Earth's
 81 surface, D_H , is given by Eqs. (29)–(32).

$$D_H = 0.99G_H \quad K_T \leq 0.17 \quad (29)$$

$$D_H = (1.188 - 2.272K_T + 9.473K_T^2 - 21.865K_T^3 + 14.648K_T^4)G_H \quad 0.17 < K_T \leq 0.75 \quad (30)$$

$$D_H = (-0.54K_T + 0.632)G_H \quad 0.75 < K_T \leq 0.85 \quad (31)$$

$$D_H = 0.2G_H \quad K_T > 0.85 \quad (32)$$

82

83 The daily beam irradiation on a horizontal surface at the Earth's surface (in $\text{J m}^{-2} \text{d}^{-1}$) is

$$B_H = G_H - D_H. \quad (33)$$

84

85 The instantaneous diffuse (D_H^i) and beam (B_H^i) solar irradiance values on a horizontal surface at the
 86 Earth's surface depend on the time of day. During the daytime ($ST_{sr} \leq ST \leq ST_{ss}$), D_H^i and B_H^i are
 87 calculated such that

$$D_H^i = \frac{\pi}{24 \times 3600} (a + b \cos \omega) \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \omega_s \cos \omega_s} D_H, \quad (34)$$

$$B_H^i = \frac{\pi}{24 \times 3600} (a + b \cos \omega) \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \omega_s \cos \omega_s} B_H, \quad (35)$$

$$\omega = (15ST - 180), \quad (36)$$

$$a = 0.409 + 0.5016 \sin(\omega_s - 60), \quad (37)$$

$$b = 0.6609 - 0.4767 \sin(\omega_s - 60), \quad (38)$$

88

89 where ω is the solar hour angle at the time in question. The instantaneous global solar irradiation on a
 90 horizontal surface at the Earth's surface under cloudy sky conditions (in W m^{-2}) is given by

$$G_H^i = B_H^i + D_H^i. \quad (39)$$

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