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Abstract

The study takes place in the context of magneto-elastic coupling. A new magnetoelastic energy formulation is proposed at the magnetic domain scale in order to account for the non-linear and non-monotonous effect of stress on magnetization and magnetostriction. The model uses 1st order cubic invariants and a vanishing 2nd order cubic invariants.

Keywords: magnetostriction, second-order effect, invariants

1. Introduction

The main physical mechanisms involved in the magneto-mechanical coupling are on the one hand the influence of mechanical loading on the magnetic behavior and the deformation of the medium when subjected to a magnetic field on the other hand [1, 2]. They correspond to *inverse magnetostriction* and *magnetostriction* respectively. As a coupled phenomenon, their modeling generally uses a thermodynamic approach allowing for the derivation of the mechanical behavior and the magnetic behavior using a single coupled energy term. Its formulation can be done at the macroscopic scale [3]. An energy formulation at the microscopic scale is however more relevant since

stress and magnetization can be considered as homogeneous at the magnetic domains scale for example. This approach requires however homogenization rules and higher computation time. At this scale, the constitutive laws linking stress $\boldsymbol{\sigma}$ to strain $\boldsymbol{\epsilon}$, and the magnetization $\boldsymbol{m}$ to magnetic field $\boldsymbol{h}$ are obtained from the formulation of a Gibbs free energy density:

$$\Psi = \Psi^{\mu}(\boldsymbol{m}) + \Psi^{\sigma}(\boldsymbol{\sigma}) + \Psi^{\mu\sigma}(\boldsymbol{m}, \boldsymbol{\sigma}) - \mu_0 \boldsymbol{h}\boldsymbol{m} \tag{1}$$

$\Psi^{\mu}(\boldsymbol{m})$, $\Psi^{\sigma}(\boldsymbol{\sigma})$, $\Psi^{\mu\sigma}(\boldsymbol{m}, \boldsymbol{\sigma})$ and $\mu_0 \boldsymbol{h}\boldsymbol{m}$ indicate respectively magnetic, elastic, magneto-mechanical and Zeemann energy densities (μ_0: vacuum permeability $= 4\pi \times 10^{-7}$ Henry/m). The magneto-mechanical energy density is usually considered as linear in stress and quadratic in magnetization [1, 2], writing:

$$\Psi^{\mu\sigma} = -\boldsymbol{\sigma} : \boldsymbol{\mathcal{E}} : (\boldsymbol{m} \otimes \boldsymbol{m}). \tag{2}$$

$\boldsymbol{\mathcal{E}}$ is a fourth order constitutive tensor. Magnetostriction $\boldsymbol{\epsilon}^{\mu}$ is obtained by deriving the magneto-mechanical energy density with respect to stress:

$$\boldsymbol{\epsilon}^{\mu} = -\frac{\partial \Psi^{\mu\sigma}}{\partial \boldsymbol{\sigma}} = \boldsymbol{\mathcal{E}} : (\boldsymbol{m} \otimes \boldsymbol{m}),$$

Such a definition implies that magnetostriction is a stress independent free strain which can be expressed as a function of so-called magnetostriction constants λ_{100} and λ_{111} following :

$$\boldsymbol{\epsilon}^{\mu} = \frac{3}{2} \begin{pmatrix} \lambda_{100}(\gamma_1^2 - \frac{1}{3}) & \lambda_{111}\gamma_1\gamma_2 & \lambda_{111}\gamma_1\gamma_3 \\ \lambda_{111}\gamma_1\gamma_2 & \lambda_{100}(\gamma_2^2 - \frac{1}{3}) & \lambda_{111}\gamma_2\gamma_3 \\ \lambda_{111}\gamma_1\gamma_3 & \lambda_{111}\gamma_2\gamma_3 & \lambda_{100}(\gamma_3^2 - \frac{1}{3}) \end{pmatrix}.$$

where γ_i are the direction cosines of magnetization verifying $\boldsymbol{m} = M_s \boldsymbol{\gamma} = M_s \gamma_i \mathbf{e}_i$ (M_s: saturation magnetization; $\mathbf{e}_i$: canonical basis). The magne-

tostriction then respects the assumption of volume conservation during the magnetization [1, 2, 4]. However, such formulation of magneto-mechanical term is unable to describe *the morphic effect* [2, 4] occurring in materials such as iron-silicon alloys. The morphic effect results in a non-monotony of the magnetization variation with stress and a corresponding non-monotony of magnetostriction variation with magnetic field.

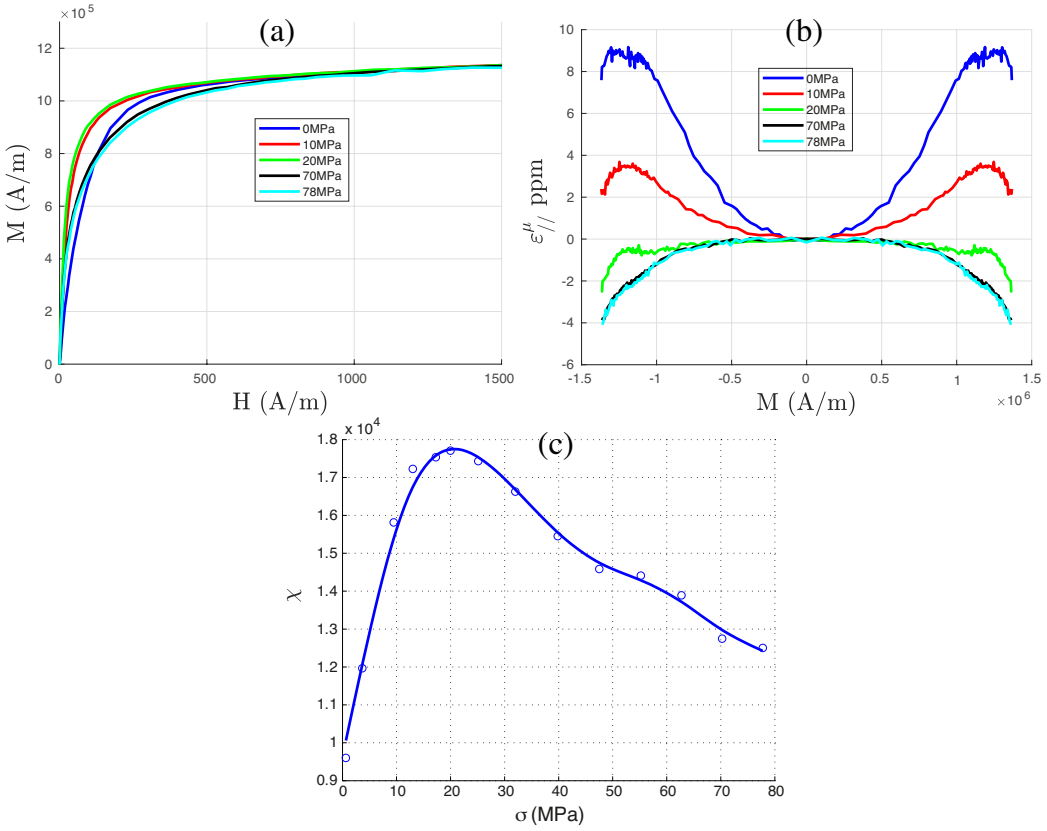

Figure 1: Anhysteretic experimental behavior of Fe-3%Si alloy: (a) magnetic behavior; (b) magnetostriction behavior; (c) initial susceptibility [5].

This point is illustrated in figure 1 plotting various anhysteretic quantities: the initial anhysteretic susceptibility increases from 0 to 20 MPa then

decreases. Conversely magnetostriction is *positive* for applied stress remaining below 20MPa and is *negative* for higher stress values. This phenomenon can not be explained by magnetization rotation. As recently proposed in [5], a higher order development of the magneto-elastic energy density allows for the modeling of these phenomena. This formulation however fails to model i) the progressive decreasing of magnetic susceptibility stress sensitivity ii) a progressive *saturation* of the magnetostriction as clearly observable at high stress. In this paper, a new formulation of the Gibbs free energy density is proposed allowing for the modeling of both non-monotony and saturation phenomena. Some simulations (analytical and multiscale model) of homogenized magnetization and magnetostriction are also presented using the different formulations.

2. Former formulations and drawbacks

To account for the stress dependency of the magnetostriction, a quadratic stress term involving a 6th order tensor $\mathbb{E}$ has been initially proposed in [2] and more recently used in [5] :

$$\Psi^{\mu\sigma} = -\boldsymbol{\sigma} : \boldsymbol{\mathcal{E}} : (\boldsymbol{m} \otimes \boldsymbol{m}) - \frac{1}{2}\boldsymbol{\sigma} : (\boldsymbol{\sigma} : \mathbb{E} : (\boldsymbol{m} \otimes \boldsymbol{m})) \tag{3}$$

The magnetostriction is simply given by:

$$\boldsymbol{\epsilon}^{\mu} = \boldsymbol{\mathcal{E}} : (\boldsymbol{m} \otimes \boldsymbol{m}) + \boldsymbol{\sigma} : \mathbb{E} : (\boldsymbol{m} \otimes \boldsymbol{m}). \tag{4}$$

While this formulation reflects non-monotony, it does not allow for saturation of the magnetostriction (due to linear dependency with stress) nor decreasing of magnetic susceptibility sensitivity to stress (see section 4 for

illustrations). A better formulation is required. But the search for a more appropriate energy form is made very complicated by the use of tensorial operators. A rewriting of (2) and (3) in the form of invariant polynomials has been proposed in [6]. It is based on the cubic decomposition proposed by [7], where the stress is written as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{d} + \boldsymbol{\sigma}^{\bar{d}} + \frac{1}{3}(\mathrm{tr}(\boldsymbol{\sigma}))\mathbf{1}, \tag{5}$$

with

$$\boldsymbol{\sigma}^{d} := \mathbf{P}^{d}_{\mathbb{O}} : \boldsymbol{\sigma} \quad \text{and} \quad \boldsymbol{\sigma}^{\bar{d}} := \mathbf{P}^{\bar{d}}_{\mathbb{O}} : \boldsymbol{\sigma}.$$

where projectors $\mathbf{P}^{d}_{\mathbb{O}}$ and $\mathbf{P}^{\bar{d}}_{\mathbb{O}}$ are given by [8, 9]:

$$\mathbf{P}^{\bar{d}}_{\mathbb{O}} := \frac{1}{2}\sum_{i<j} \mathbf{e}_{ij} \otimes \mathbf{e}_{ij}, \quad \text{with} \quad \mathbf{e}_{ij} := \boldsymbol{e}_i \otimes \boldsymbol{e}_j + \boldsymbol{e}_j \otimes \boldsymbol{e}_i \ (i \neq j), \tag{6}$$

and

$$\mathbf{P}^{d}_{\mathbb{O}} := \mathbf{J} - \mathbf{P}^{\bar{d}}_{\mathbb{O}}, \quad \text{with} \quad \mathbf{J} = \mathbf{I} - \frac{1}{3}\mathbf{1} \otimes \mathbf{1}. \tag{7}$$

In the cubic canonical basis $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$, stress deviators are:

$$\boldsymbol{\sigma}^{\bar{d}} = \begin{pmatrix} 0 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & 0 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & 0 \end{pmatrix}_{\boldsymbol{e}_i}, \quad \boldsymbol{\sigma}^{d} = \begin{pmatrix} \sigma'_{11} & 0 & 0 \\ 0 & \sigma'_{22} & 0 \\ 0 & 0 & \sigma'_{33} \end{pmatrix}_{\boldsymbol{e}_i}.$$

where $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \frac{1}{3}(\mathrm{tr}(\boldsymbol{\sigma}))\mathbf{1}$ is the deviatoric stress tensor. The projectors $\mathbf{P}^{d}_{\mathbb{O}}$ and $\mathbf{P}^{\bar{d}}_{\mathbb{O}}$ being applicable to any 2nd order tensor, a basis of cubic invariants can be constructed by decomposing the tensors into diagonal $(\bullet)^{d}$, out-of-diagonal $(\bullet)^{\bar{d}}$ and hydrostatic parts. Thus, a basis of invariants linear in stress and quadratic in magnetization (depending on the orientation) can be obtained [6]:

$$\boldsymbol{\sigma}^{d} : (\boldsymbol{m} \otimes \boldsymbol{m})^{d}, \quad \boldsymbol{\sigma}^{\bar{d}} : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}}. \tag{8}$$

By associating a material constant to each invariant, respectively c_{210} and c_{201}, equation (2) can be reformulated in:

$$\begin{aligned} \Psi^{\mu\sigma} &= -\boldsymbol{\sigma} : \boldsymbol{\mathcal{E}} : (\boldsymbol{m} \otimes \boldsymbol{m}) \\ &= c_{210}\boldsymbol{\sigma}^{d} : (\boldsymbol{m} \otimes \boldsymbol{m})^{d} + c_{201}\boldsymbol{\sigma}^{\bar{d}} : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}} \end{aligned} \tag{9}$$

c_{210} and c_{201} can be expressed as functions of the usual magnetostriction constants (values for Fe-3%Si NO are given in Table 1) following:

$$c_{210} = -\frac{3}{2}\frac{\lambda_{100}}{M_s^2}, \quad c_{201} = -\frac{3}{2}\frac{\lambda_{111}}{M_s^2}.$$

Now, some higher order invariants can be used. 2nd order stress terms are [6]:

$$\begin{aligned} &(\boldsymbol{\sigma}^{\bar{d}})^2 : (\boldsymbol{m} \otimes \boldsymbol{m})^{d}, \quad (\boldsymbol{\sigma}^{\bar{d}})^2 : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}}, \quad (\boldsymbol{\sigma}^{\bar{d}}\boldsymbol{\sigma}^{d}) : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}} \\ &(\boldsymbol{\sigma}^{d})^2 : (\boldsymbol{m} \otimes \boldsymbol{m})^{d}. \end{aligned} \tag{10}$$

Equation 3 can consequently be rewritten as:

$$\begin{aligned} \Psi^{\mu\sigma} &= -\boldsymbol{\sigma} : \boldsymbol{\mathcal{E}} : (\boldsymbol{m} \otimes \boldsymbol{m}) - \frac{1}{2}\boldsymbol{\sigma} : (\boldsymbol{\sigma} : \mathbb{E} : (\boldsymbol{m} \otimes \boldsymbol{m})) \\ &= c_{210}\boldsymbol{\sigma}^{d} : (\boldsymbol{m} \otimes \boldsymbol{m})^{d} + c_{201}\boldsymbol{\sigma}^{\bar{d}} : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}} \\ &\quad + c^{a}_{202}(\boldsymbol{\sigma}^{\bar{d}})^2 : (\boldsymbol{m} \otimes \boldsymbol{m})^{d} + c^{b}_{202}(\boldsymbol{\sigma}^{\bar{d}})^2 : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}} \\ &\quad + c_{211}(\boldsymbol{\sigma}^{\bar{d}}\boldsymbol{\sigma}^{d}) : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}} + c_{220}(\boldsymbol{\sigma}^{d})^2 : (\boldsymbol{m} \otimes \boldsymbol{m})^{d} \\ &\quad + c_{210,010}\mathrm{tr}(\boldsymbol{\sigma})\boldsymbol{\sigma}^{d} : (\boldsymbol{m} \otimes \boldsymbol{m})^{d} + c_{201,010}\boldsymbol{\sigma}^{\bar{d}} : (\boldsymbol{m} \otimes \boldsymbol{m})^{\bar{d}}. \end{aligned} \tag{11}$$

where c^{a}_{202}, c^{b}_{202}, c_{211}, c_{220}, $c_{210,010}$ and $c_{201,010}$ are material constants. The assumption of incompressibility of the magnetostriction results in $c_{210,010} =$

$c_{210,010} = 0$. A fully isotropic formulation considers $c_{210} = c_{201}$, $A = c_{220} = c^a_{202} = c^b_{202} = \frac{1}{2}c_{211}$ et $B = c_{210,010} = c_{201,010}$. These 2nd order constants are related to isotropic second-order constants λ'_s and λ''_s introduced by Kraus [10] :

$$A = \frac{3}{2}\frac{\lambda''_s}{M_s^2}, \quad B = -\frac{1}{2}\frac{3\lambda'_s + \lambda''_s}{M_s^2}.$$

Whether isotropic or cubic, these energy density forms allow for the modeling of a second order effect in stress. They do not account for the progressive saturation of the magnetostriction at high stress or the progressive reduction in the susceptibility sensitivity to stress.

3. New cubic vanishing 2nd order energy

A new formulation of Gibbs energy density must involve consequently: i) a second-order stress effect at low stress; ii) a progressive transition to first order form at higher stresses; iii) a cubic symmetry description for both situations; iv) remaining in a multiaxial framework; v) volume conservation. Following these requirements, we propose the following formulation:

$$\begin{aligned}\Psi^{\mu\sigma} = {} & c_{210}\boldsymbol{\sigma}^d : (\boldsymbol{m}\otimes\boldsymbol{m})^d + c_{201}\boldsymbol{\sigma}^{\bar{d}} : (\boldsymbol{m}\otimes\boldsymbol{m})^{\bar{d}} + \\ & \frac{c_{220}(\boldsymbol{\sigma}^d)^2 : (\boldsymbol{m}\otimes\boldsymbol{m})^d + c^b_{202}(\boldsymbol{\sigma}^{\bar{d}})^2 : (\boldsymbol{m}\otimes\boldsymbol{m})^{\bar{d}}}{1 + \alpha\sqrt{\frac{3}{2}(\boldsymbol{\sigma}^d + \beta\boldsymbol{\sigma}^{\bar{d}}) : (\boldsymbol{\sigma}^d + \beta\boldsymbol{\sigma}^{\bar{d}})}} + \\ & \frac{c^a_{202}(\boldsymbol{\sigma}^{\bar{d}})^2 : (\boldsymbol{m}\otimes\boldsymbol{m})^d + c_{211}(\boldsymbol{\sigma}^{\bar{d}}\boldsymbol{\sigma}^d) : (\boldsymbol{m}\otimes\boldsymbol{m})^{\bar{d}}}{1 + \alpha\sqrt{\frac{3}{2}(\boldsymbol{\sigma}^d + \beta\boldsymbol{\sigma}^{\bar{d}}) : (\boldsymbol{\sigma}^d + \beta\boldsymbol{\sigma}^{\bar{d}})}}\end{aligned} \tag{12}$$

The linear terms in the equation (9) are duplicated. The other four terms are fractions formed by a 2nd order stress invariants which do not depend on tr($\boldsymbol{\sigma}$) at the numerator (assumption of incompressibility of the magnetostriction removes two other terms) and purely mechanical invariants

at the denominator (see equation (10)). Each term is related to the diagonal or out–of-diagonal parts of the stress deviator. New constant α allows for the second order effect vanishing to be adjusted by giving more or less weight to the purely mechanical part of energy. As shown in figure 5 where several α values are tested, it is possible to adjust the monotony of the magnetic behavior using this parameter. Constant β is introduced to take into account the cubic symmetry : the stress invariants at the denominator correspond to a cubic form of the Von Mises equivalent stress (remark: the von Mises equivalent stress is defined by $\sigma_{vm} = \sqrt{\frac{3}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}'}$ where $\boldsymbol{\sigma}'$ is the deviatoric part of $\boldsymbol{\sigma}$). For low stress amplitude, the magneto-elastic energy density exhibits a 2nd order character. At high stress amplitude, a 1rst order stress magneto-elastic energy density is obtained. As for the full second order situation, isotropy is obtained for $K = c_{220} = c^a_{202} = c^b_{202} = \frac{1}{2}c_{211}$ and $\beta = 1$ (since $\boldsymbol{\sigma}^d + \boldsymbol{\sigma}^{\bar{d}} = \boldsymbol{\sigma}'$). Therefore, 6 constants are used to model the morphic effect. We reach the same constants number as Kraus [10]. But unlike his proposition, our formulation allows for the magnetostriction saturation at high stress and respects the volume conservation.

4. Illustrations using analytical modeling

We propose in this section to illustrate the various energy density forms using a simple analytical model [5] derived from a more complete multiscale model [11] using homogeneous stress and magnetic field conditions. The simplified modeling argues that since an isotropic polycrystal is an aggregate of single crystals with random orientation, and that magnetic domains are a substructure of each single crystal, an isotropic polycrystal can be considered as an aggregate of magnetic domains with random orientation and whose

magnetization is aligned along one of the easy axes of the crystal. If we consider on the other hand this direction as fixed in space and following a Boltzmann-like distribution, the volume fraction of each domain family α is given by:

$$f_\alpha = \frac{\exp\left(-A_s\Psi\right)}{\int_\alpha \exp\left(-A_s\Psi\right)\mathrm{d}\alpha}, \quad \text{with} \quad A_s = \frac{3\chi_0}{\mu_0 M_s^2} \tag{13}$$

where χ_0 is the stress-free initial anhysteretic susceptibility. The Gibbs free energy density Ψ is only the sum of Zeeman and magneto-elastic energy densities. This form consequently ignores any magnetization rotation mechanism on the one hand. Considering on the other hand the six $< 100 >$ directions as easy axes leads to get a magnetostriction tensor defined as an equivalent isotropic magnetostriction tensor with magnitude $\lambda_m = 2/5\lambda_{100}$ following homogenization rules [11, 5] (λ_{100} is the magnetostriction amplitude along $< 100 >$ axis that can be stress-dependent for higher-than-one order formulations). The following definitions of the magnetoelastic energy density and of the magnetostriction tensors are obtained:

Isotropic first order

$$\Psi_o^{\mu\sigma} = \frac{2}{5}c_{210}\boldsymbol{\sigma}' : (\boldsymbol{m}\otimes\boldsymbol{m})' \quad , \quad \boldsymbol{\epsilon}_o^{\mu} = -\frac{2}{5}c_{210}(\boldsymbol{m}\otimes\boldsymbol{m})' \tag{14}$$

Isotropic second order

$$\begin{aligned} \Psi_o^{\mu\sigma} &= \frac{2}{5}c_{210}\boldsymbol{\sigma}' : (\boldsymbol{m}\otimes\boldsymbol{m})' + \frac{2}{5}A\boldsymbol{\sigma}'^2 : (\boldsymbol{m}\otimes\boldsymbol{m})' \\ \boldsymbol{\epsilon}_o^{\mu} &= -\frac{2}{5}c_{210}(\boldsymbol{m}\otimes\boldsymbol{m})' - \frac{2}{5}A\left(\boldsymbol{\sigma}'(\boldsymbol{m}\otimes\boldsymbol{m})' + (\boldsymbol{m}\otimes\boldsymbol{m})'\boldsymbol{\sigma}'\right) \end{aligned} \tag{15}$$

Isotropic vanishing second order

$$\Psi_o^{\mu\sigma} = \frac{2}{5}c_{210}\boldsymbol{\sigma}' : (\boldsymbol{m}\otimes\boldsymbol{m})' + \frac{2}{5}K\frac{\boldsymbol{\sigma}'^2 : (\boldsymbol{m}\otimes\boldsymbol{m})'}{1+\alpha\sigma_{vm}}$$
$$\boldsymbol{\epsilon}_o^{\mu} = -\frac{2}{5}c_{210}(\boldsymbol{m}\otimes\boldsymbol{m})' - \frac{2}{5}K\left(\frac{(\boldsymbol{\sigma}'(\boldsymbol{m}\otimes\boldsymbol{m})' + (\boldsymbol{m}\otimes\boldsymbol{m})'\boldsymbol{\sigma}')'}{1+\alpha\sigma_{vm}} - \frac{\frac{3\boldsymbol{\sigma}'}{2\sigma_{vm}}\alpha(\boldsymbol{\sigma}')^2 : (\boldsymbol{m}\otimes\boldsymbol{m})'}{(1+\alpha\sigma_{vm})^2}\right) \tag{16}$$

where $(\bullet)'$ is the deviatoric part of $(\bullet)$. Macroscopic magnetization and magnetostriction strain are given by the following averaging operations:

$$\boldsymbol{M} = \int_\alpha f_\alpha \boldsymbol{m}\mathrm{d}\alpha \quad , \quad \boldsymbol{\epsilon}_m^{\mu} = \int_\alpha f_\alpha \boldsymbol{\epsilon}_o^{\mu}(\boldsymbol{m},\boldsymbol{\sigma})\mathrm{d}\alpha \tag{17}$$

Figures 2, 3 and 4 show respectively the results of models using the three previous formulations of magneto-mechanical free energy densities ((14), (15) and (16): on the one hand magnetic behavior (a) and magnetostriction (b) measured along the applied field (uniaxial stress condition), initial susceptibility vs. stress (c) and initial susceptibility in biaxial stress condition (d) (with magnetic field applied along axis 1) on the other hand. Paramaters have been improved to fit properly the uniaxial experimental results since no relevant result in multiaxial stress condition is available in literature for iron-silicon). They are gathered in Table 2. The monotonous character of the linear formulation is clearly highlighted in figure 2 associated with a magnetostriction remaining positive. The introduction of second order allows for an efficient modeling of non-monotony effect but magnetostriction exhibits a divergence at high stress (figure 3). The vanishing second-order formulation in figure 4 allows for a modeling of both the non-monotony of magnetization (maximum of susceptibility at about 18MPa - transition from 2nd order to 1st order after 40 MPa) and the saturation of magnetostriction passing from positive to negative values.

Simulations of biaxial stress situation lead on the other hand to results in accordance to previous simulations using equivalent stress formulation [12] for the first order expression. The second order vanishing simulations simulate properly the non-monotony close to uniaxial condition (axis 1). Some strong susceptibility spots occur with or without vanishing term that was not observed with the first order simulation. This seems due to components σ'^2_{11} that is null along $\sigma_2 = 2\sigma_1$ axis, leading to a dominant first order term in energy along this axis. These results that can not be compared to experimental data must however be considered with caution. They are outputs of a simplified model considering isotropic material with parameters adjusted for uniaxial stress condition.

M_s	λ_{100}	λ_{111}
1.61×10^{-6}	23.5	-4.5
$A.m^{-1}$	ppm	ppm

Table 1: Material constants of Fe-3%Si NO.

χ_0	A	K	α
9600	0.51×10^{-18}	2.83×10^{-18}	4.69×10^{-2}
-	$MPa^{-1}.A^{-2}.m^2$	$MPa^{-1}.A^{-2}.m^2$	MPa^{-1}

Table 2: New parameters used for simulations.

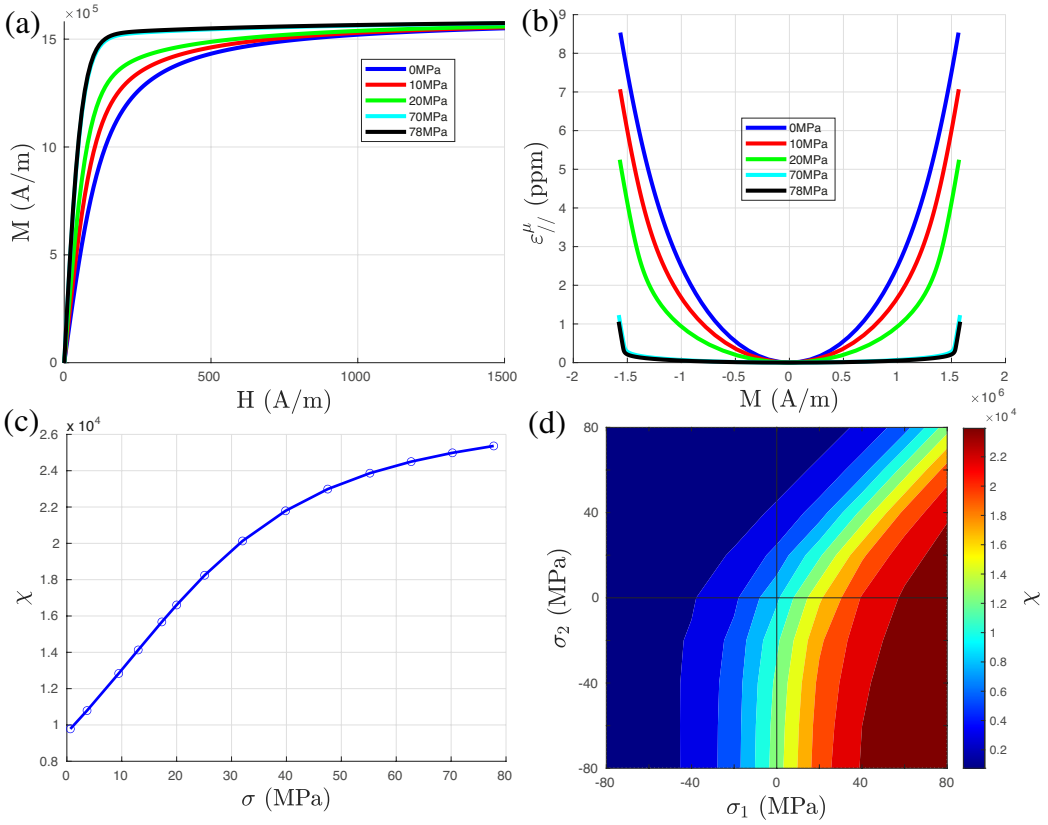

Figure 2: Modeling result for 1rst order formulation: (a) magnetic behavior; (b) magnetostriction behavior; (c) initial susceptibility; (d) initial susceptibility in biaxial stress condition with magnetic field applied along axis 1.

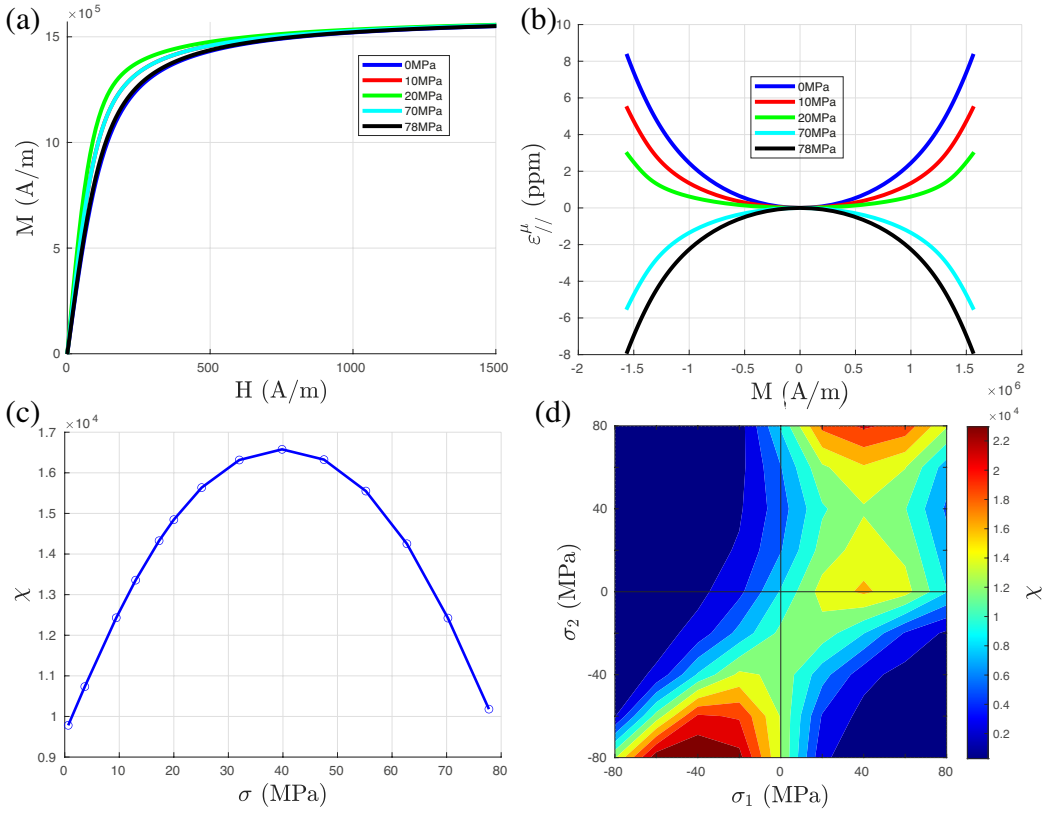

Figure 3: Modeling result for 2nd order formulation: (a) magnetic behavior; (b) magnetostriction behavior; (c) initial susceptibility; (d) initial susceptibility in biaxial stress condition with magnetic field applied along axis 1.

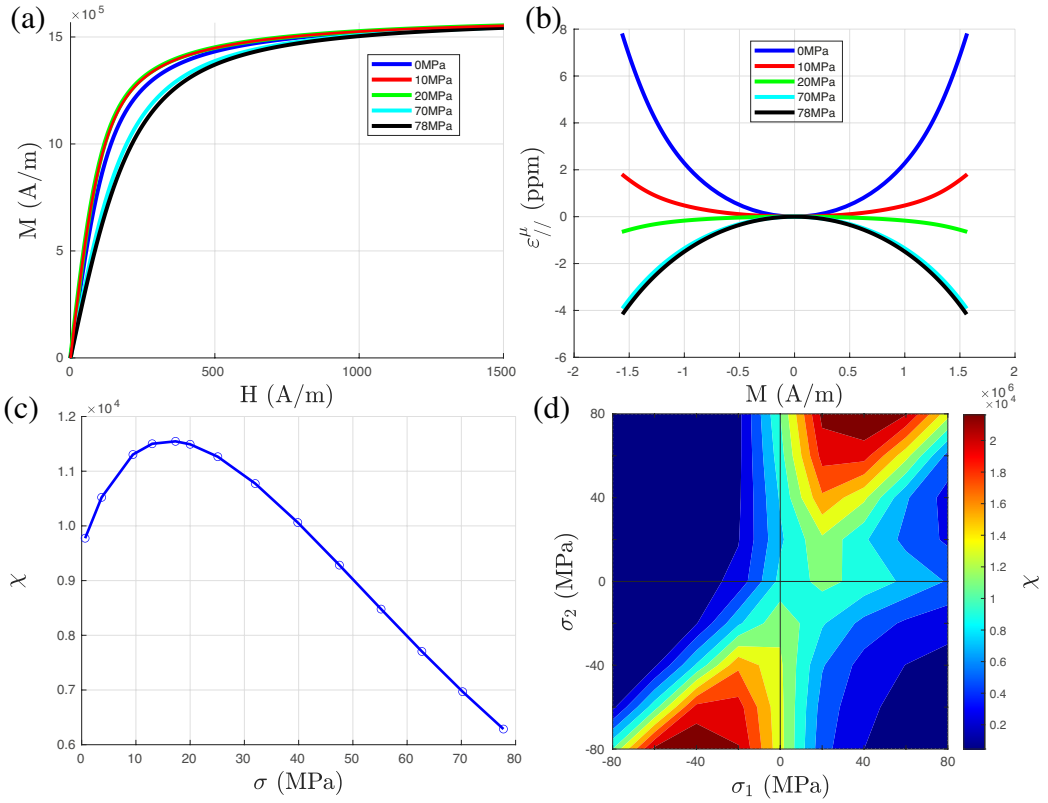

Figure 4: Modeling result for 2nd order vanishing formulation: (a) magnetic behavior; (b) magnetostriction behavior; (c) initial susceptibility; (d) initial susceptibility in biaxial stress condition with magnetic field applied along axis 1.

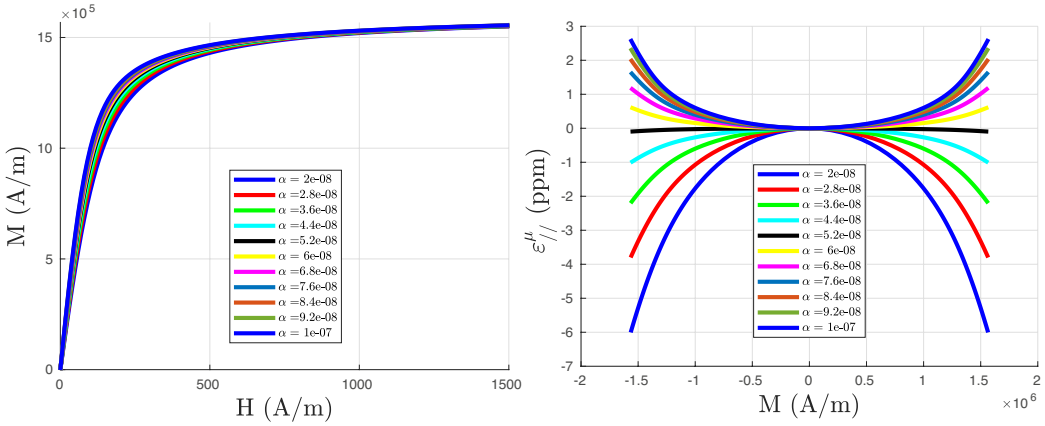

Figure 5: Modeling of magnetic behavior (left) and magnetostriction (right) for $K = 2.83 \times 10^{-18}$ $\mathrm{MPa}^{-1}.\mathrm{A}^{-2}.\mathrm{m}^2$ and $\sigma = 20$ MPa.

5. Conclusion

Various formulations of magneto-mechanical Gibbs free energy density built from a basis of cubic invariants are proposed in this study. The mentioned limitations of the pre-existing models (first order in stress and second order in stress energy densities) have been corrected by the introduction of a vanishing second order term in the framework of cubic symmetry. Illustrations have been proposed for isotropic and incompressibility conditions. Indeed the identification of the 6 cubic parameters remains a strong issue. It could be made possible by the introduction of the cubic vanishing model inside the so-called multiscale model [11] and the implementation of an optimization process based on measurements under stress (including multiaxial stress) of the magnetic behavior and the magnetostriction of single and polycrystals.

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