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Collaborative Financing and Supply Chain Coordination for Corporate Social Responsibility

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Highlights

- SCs benefit from combining a cost-sharing contract with a financing device.
- RF can help suppliers make stronger CSR investment efforts.
- RF also permits retailers to simultaneously increase the profit of all SC members.
- RF can outperform BF in terms of CSR efforts and profits.
- The best financing device to adopt depends on many factors.

Collaborative Financing and Supply Chain Coordination for Corporate Social Responsibility

Abstract

Recent research in supply chain (SC) management shows that collaborative financing and coordination can separately improve the SC's corporate social responsibility (CSR). This article examines how reverse factoring (RF) and cost-sharing (CS) contracts initiated by sizable creditworthy retailers interact and can help SCs address various challenges posed by CSR, especially for small- and medium-sized suppliers with limited working capital. RF can simultaneously lead to greater CSR effort and higher profits for all SC members compared to traditional bank financing. We highlight, nevertheless, how some factors, such as market demand uncertainty, the interest rate premia charged by the respective banks, and the supplier's or retailer's bankruptcy risks, determine the adoption and the benefits of the financing devices. Our managerial implications indicate that combining collaborative financing and coordination can simultaneously be profitable for all members of the SC and incentivize the supplier to raise the CSR efforts. Specifically, a CS contract associated with an appropriate financing mechanism can help to improve CSR and the SC's profitability.

Keywords: Corporate Social Responsibility, Collaborative Financing, Reverse Factoring, Supply Chain Coordination.

1. Introduction

Corporate social responsibility (CSR)¹ has become critical for many firms, particularly due to the mounting pressures from significant stakeholders (including governments, NGOs, shareholders, and consumers) (Xu and Lee, 2019; Barcena-Ruiz and Sagasta, 2021). Companies face increasing demands for CSR activities and show growing efforts to integrate social, environmental, and ethical concerns into their business operations. Moreover, CSR matters for individual firms and whole supply chains (SCs) (Nematollahi et al., 2017). On the one hand, the suppliers' social irresponsibility negatively affects downstream companies' image, goodwill, and sales (Huang et al., 2020). Consequently, many buying firms actively mobilize their suppliers to engage in CSR initiatives (Wilhelm et al., 2016). On the other hand, CSR may permit economic benefits for socially-responsible firms by reducing the costs of debt and equity, as stated by Boubaker et al. (2020); however, financing constraints and material CSR implementation costs can hinder suppliers' CSR initiatives (Coté et al., 2008; Becchetti et al., 2014). This is particularly true for small and medium-sized enterprises (SMEs), suppliers who tend to be cash-constrained and face problems obtaining loans from banks at favorable interest rates (Tian and Lin, 2019).

Buying companies can incentivize and support their suppliers to adopt CSR by sharing costs, for instance, via co-financing CSR initiatives². For example, Hewlett-Packard shares the CSR investment costs of its leading suppliers (Rammohan, 2008). In 2010, Walmart Stores launched a "global social compliance program" for co-financing the CSR investment of suppliers (Walmart Stores Inc., 2010). Academic research has recently integrated CSR issues and cost-sharing (CS) contracts into the SC coordination challenge (Panda, 2014; Nematollahi et al., 2017; Hosseini-Motlagh et al., 2019)³. Several benefits CS provided alone or combined with other coordination tools have been documented. For instance, CS of green initiatives can yield higher greening levels and larger profits at the firm and the SC levels (Ghosh and Shah, 2015; Xu et al., 2020). Greening-CS contracts have been combined with revenue-sharing

¹The concept of CSR, credited to Bowen (1953), states that firms should not only pursue economic benefits but also pay attention to the needs of society, the economy, the environment, and stakeholders. CSR activities may be quite diverse and contain environmental and greening efforts, health care plan development, social welfare programs, and human skill capital building, among many others (Dahlsrud, 2008).

² To avoid confusion, "sharing" will be used systematically hereafter.

³ Supply chain coordination looks for mechanisms to motivate SC members to pursue decisions based on the profitability of the entire SC while ensuring that each partner in the SC can benefit from collaborative activities (Cachon, 2003).

contracts for greening and social purposes (Raj et al., 2018). CS supplemented by retailers' marketing efforts can incentivize suppliers to make the CSR efforts (Phan et al., 2019).

Second, following an increasing number of large retailers, buying companies can recourse to collaborative financing schemes and reverse factoring programs (RF) to improve suppliers' CSR. RF is a collaborative financial agreement initiated by a large buyer (retailer hereafter) with a financial institution like a bank that benefits the supplier. Under RF, suppliers obtain, in exchange for accounts receivables, a payment upon delivery at a financing cost aligned with the retailer's credit risk (Klapper, 2006). Moreover, RF has clear advantages over traditional financing modes, such as bank financing (BF). First, RF programs improve the supplier's working capital (Lekkakos and Serrano, 2016) and result in substantial benefits for the suppliers, as they take advantage of the credit rating discrepancy between the small suppliers and the large retailers who are assumed to have better creditworthiness (Grüter and Wuttke, 2017). They can also reduce the risks and costs associated with asymmetric information (Klapper, 2006).

RF programs now integrate CSR-oriented criteria. For example, the famous apparel retailer PUMA⁴ cooperates with World Bank (WB) to pay suppliers through discounting invoices. Under the terms of this program, the discount rate set by WB notably depends on PUMA's credit standing and the supplier's adherence to the company's social and environmental standards. The apparel retailer PIMKIE⁵ partners with BNP Paribas to offer a lower financing cost to suppliers with a higher CSR performance. Other major apparel brands, including Levi Strauss & Co⁶, H&M, and Timberland, follow similar approaches. There nowadays exist, many variations of such CSR-related RF.

Most RF research focuses on an SC's operational and financial aspects. Only recently have a few papers begun to explore whether and how a financing scheme (such as trade credit, BF, and RF) can interact with the SCs' various operational decisions to impact sustainability. Sarkar et al. (2018) explore the impacts of variable carbon emission costs and multi-delay-inpayments on a global sustainable SC. They find that implementing multi-level trade credit positively affects the economic and environmental performances of a three-level SC in a single

⁴ <https://group.bnpparibas/en/press-release/bnp-paribas-puma-launch-innovative-financing-program-suppliers-reward-social-environmental-standards>

⁵ <https://www.kyriba.com/wp-content/uploads/2018/03/Strengthening-Supplier-Relationships-with-Reverse-Factoring.pdf>

⁶ <https://corporates.db.com/files/documents/Payables-Finance-A-guide-to-working-capital-optimisation.pdf>

setup multiple delivery setting. Shi et al. (2020) integrate the impacts of advance payment, cash payment, and credit payment on the carbon emissions of the SC. Their numerical results demonstrate that credit payment is the best of all three payment schemes to curb carbon emissions. Zhan et al. (2018) investigate the contribution and value of the advance payment and RF in promoting SC sustainability and efficiency. Cao et al. (2019) compare external financing (bank) with trade credit in a SC where consumers have low-carbon preferences. An et al. (2021) recently designed a green credit financing model for a SC by imposing a hard constraint on carbon emissions. Their results show that under a relatively strict carbon emission policy, the manufacturer can set an appropriate green investment range to achieve a win-win situation with the supplier.

To our knowledge, no previous research has addressed the specific role of RF, the absolute and relative merits of financing solutions, and their potential combination with CS contracts in boosting the suppliers' CSR and the financial performance of the SC. Several questions then naturally arise. Is a double incentive (with financing and cost sharing) better than a single one? How and when does the collaborative mechanism, initiated by the retailer, benefit the retailer, the supplier, and the SC? Our main goal is to explore how the SC can simultaneously use collaborative financing and CS to promote the CSR of suppliers.

To address these questions, we investigate a SC consisting of a large retailer and an SME supplier. The retailer purchases a single product from the supplier and then sells it to some consumers sensitive to the supplier's investment in CSR activities. To settle the purchase, the retailer and supplier enter an RF program in which they work with a bank to pay the supplier upon delivery of the purchasing cost of the ordered products. Notice that we also consider BF, where suppliers directly borrow from the bank to fund their businesses and then allow the retailer to delay their payments until the products are sold. BF has traditionally been used to finance the SC (Cao et al., 2019; Kouvelis and Zhao, 2018; Kouvelis and Xu, 2021); thus, it is regarded as a benchmark model in evaluating the benefit of RF.

We study in parallel the two financing modes with a CS contract, and this leads to a "CSR CS under BF" situation (CSBF) and a "CSR CS under RF" one (CSRF). Results are compared to situations without CS (BF and RF). For the CS contract design, we follow Ghosh and Shah (2015) and consider a retailer-led CS contract where the retailer sovereignly determines and fine-tunes the CS parameter.

Our work contributes to the literature in three different ways. First, we are the first to investigate the impact of the financing scheme choice on the CSR and profitability of an SC. Existing studies mainly investigated the benefit of BF and RF in improving an SC's operational and financial aspects. This paper extends past literature to characterize the conditions under which RF outperforms or under-performs BF in terms of CSR and individual profits. These conditions depend incidentally on factors like market demand uncertainty, the credit spread, and the bankruptcy risk of each SC member. We provide situations where the variables under investigation, such as the CSR effort, are possibly insensitive to some factors; hence, our study permits the SC members to choose the best financing strategies. Second, we investigate the joint impact of the financing mode selection and CS coordination on the CSR performance of the SC. Specifically, we highlight some situations where it is more beneficial for the retailer to favor RF or BF while sharing CSR costs with the supplier. In this sense, selecting the appropriate financing mode can lower the costs supported by any retailer who wants to encourage their supplier to make a CSR effort. Third, we show that no matter which financing mode of BF or RF is adopted, the CS contract always leads to a higher profit for SC members and supplier's CSR effort improvement.

The rest of this study is organized as follows. Section 2 describes the framework, model, assumptions, and notations, while Section 3 analyzes the CSBF and CSRF and derives some analytical distribution-free results for each strategy. We consider a uniformly distributed demand, explicitly demonstrate the corresponding equilibrium solutions for CSBF and CSRF in Section 4, and run numerical simulations in Section 5. Finally, Section 6 concludes, and an online Appendix provides all mathematical proofs.

2. The framework

Consider a two-echelon SC with one upstream supplier, "S," and one downstream retailer, "R." S produces and delivers a single type of finished product to "R," who sells them to final consumers during a single selling season. Neglecting financing, payment arrangement, and CSR for a while, the SC functions as follows. At some time before the selling season, say at time $t_0 = 0$, R pre-orders q units from S at a wholesale price w . Upon receiving the order, S produces R's order quantity at a unit production cost c ($0 < c < w$). The delivery to R intervenes at time $t_p = t_0 + \Delta t_p$, where Δt_p stands for the production time. At the delivery time t_p , the selling season begins, and R sells the product to consumers at a known and exogenous retail price p . The duration of the selling season is Δt_s , so that the sales season ends at time $t =$

$t_0 + \Delta t_p + \Delta t_s$. We assume, for clarity, that the demand \tilde{D} is realized at a single time t , the end of the sales period.

The random and CSR-dependent demand \tilde{D} is described by

$$\tilde{D} = \tilde{D}_0 + \beta\theta$$

where \tilde{D}_0 is a random variable capturing the level of demand in the absence of any CSR activity, θ is the CSR effort undertaken by the supplier, and β is the effort elasticity of marginal demand. This seemingly simple and linear specification (for the effect of S's CSR effort on demand) hides a real non-linear effect because the above effort made by S will cost $d\theta^2$ in terms of expected profit, where d is a positive parameter that captures the marginal CSR effort cost. Such a non-linear dependence of the demand on the CSR effort is a standard hypothesis in the literature (Liu et al., 2019; Kuiti et al., 2020; Liu et al., 2021). \tilde{D}_0 is a continuous positive random variable whose cumulative distribution function, probability density function, and complementary cumulative distribution function are respectively denoted by $F(\cdot)$, $f(\cdot)$ and $\bar{F}(\cdot) = 1 - F(\cdot)$. We assume, following the literature, that the demand distribution has the property of increasing failure rate so that its failure/hazard rate defined by $h(x) = f(x)/\bar{F}(x)$ increases in x or equivalently $h'(x) > 0$.

To encourage CSR initiatives, R signs a CS contract with S to share a proportion λ of any CSR investment costs faced by S; thus, S has only a proportion $(1 - \lambda)$ of the investment to finance. The contribution satisfies $0 \leq \lambda < 1$ and $k^R \geq \lambda d\theta^2$, because the initial capital of R must be sufficient to share the CSR investment cost. We investigate an R-led CS contract by considering the following game structure. R moves first by deciding the CS proportion, namely, λ . Then, knowing R's contribution level, S chooses the CSR effort θ . In the last stage, R sets the order quantity taking S's CSR effort θ into consideration. The CSR effort of S influences the level of demand; therefore, R can only decide the appropriate quantity to order after knowing the CSR effort undertaken by S. The parameter λ therefore influences the level of CSR effort made by S and it should consequently be denoted by $\theta(\lambda)$. The decisions to invest in CSR and to share CSR investment costs are taken *before* any ordering from R; hence, both decisions influence the quantity ordered by R, and this quantity is given by $q(\lambda, \theta(\lambda))$. Two supplementary remarks deserve attention. First, regarding SC coordination, we investigate an R-led CS contract, where R determines the level of contribution λ sovereignly and on the sole basis of the profit function π^R . Second, regarding the decision-making process, S and R internalize the steps ahead in a sequential and backward decision-making process. In particular,

when choosing the contribution λ , R maximizes the profit function $\pi^R(\lambda, \theta^*(\lambda), q^*(\lambda, \theta^*(\lambda)))$, where $\theta^*(\lambda)$ results from the optimization of the CSR effort undertaken by S and where $q^*(\lambda, \theta^*(\lambda))$ results from the optimization of the order quantity undertaken by R. This backward sequential decision-making approach allows parties to internalize the subsequent decisions and choose the optimal parameters, allowing us to derive the equilibrium strategies for players.

S needs financing to finance production and invest in CSR activities. To understand and illustrate the benefits of financing choice, we first analyze the benchmark case of BF, where S directly borrows from a bank to fund business and allows R to delay payments until the products are sold. We then model and analyze the equilibrium outcomes under the RF scheme in which R signs a financing agreement with a bank, allowing the bank to pay the purchasing cost of the ordered products to S upon delivery.

Because R may have recourse to RF without consuming cash, we assume (for comparability and to avoid artificial distortion between RF and BF), without loss of generality, that R has no cash available for other purposes after sharing CSR cost with S under CSBF and CSRF. S and R are limited-liability companies. Thus, if they default on financing obligations due to low realized demand, the creditor can only seize the realized revenue of the business. In addition, S and R may go into bankruptcy at the end of the selling season, with the probability ρ_S and ρ_R , due to exogenous default events that are independent of the random demand. In the event of bankruptcy, the bank receives zero repayments for the short-term loan issued to the debtor. Following Kouvelis and Xu (2021), we also assume that ψ_S (resp. ψ_R) is the credit spread charged by the bank based on the borrower's credit rating. For all cases, the bank offers a fairly priced loan for relevant risks; however, they charge a credit spread reflecting the firm's credit risk. Table 1 presents this study's notations.

Table 1. List of notations

Notation	Meaning	
<i>Superscript/Subscript</i>		
BF, RF	Subscripts to indicate the financing mode (BF = bank financing, RF = reverse factoring)	
S, R	Superscripts to indicate the parties (S = Supplier, R = Retailer). Sole exceptions B_S and B_R that indicate the Banks of S and R respectively.	
<i>Objective function</i>		
π	Profit or expected profit above the opportunity cost	
<i>Decision variables</i>		
θ	CSR effort level decided by S	
q	Ordered quantity of products decided by R	
r	The interest rate of the loan granted by the bank to S	
γ	The early payment rate by RF decided by the bank	
λ	The cost-sharing proportion granted by R	
v	The proportion of invoices financed by RF decided by S	
L	Loan amount borrowed by S	
<i>Problem parameters</i>		<i>Typical value</i>
p	Sale price	10
w	Wholesale price	6
c	The unit cost of production	2
ψ_S	Credit spread of S	5%
ψ_R	Credit spread of R	$\leq 5\%$
ρ_S	Probability of Bankruptcy of S	10%
ρ_R	Probability of Bankruptcy of R	$\leq 10\%$
d	CSR investment parameter	0.5
β	The marginal effect of the CSR effort on demand	5
r_f	Risk-free interest rate	1%
$\Delta t_p = t_p$	Duration of the production period	1
Δt_s	Duration of the selling season	1
t	The final date of the business ($t = \Delta t_p + \Delta t_s$)	2
k^R	The initial capital of R	

3. Financing modes and CSR CS

3.1. CSR CS under BF

Figure 1 illustrates the sequence of events and decisions under CSBF.

- | | | |
|---|---|---|
| 1) R decides the sharing proportion λ_{BF} | 1) S delivers q_{BF} products to R and records accounts receivable from R which are payable at time t with face value wq_{BF} | 1) Overall demand is realized |
| 2) S decides the CSR effort level θ_{BF} | | 2) R receives the sales proceeds $p \min\{\tilde{D}, q_{BF}\}$, and pays $\min\{p\tilde{D}, wq_{BF}\}$ to S. |
| 3) R transfers $\lambda_{BF}d\theta_{BF}^2$ to S | | 3) S pays off obligation $L_{BF}e^{r_{BF}t}$ to the bank B_S to the extent possible. |
| 4) R sets the order quantity q_{BF} | | |
| 5) S borrows L_{BF} from bank B_S to finance the production and CSR investment. | | |
| 6) The bank sets the interest rate r_{BF} . | | |

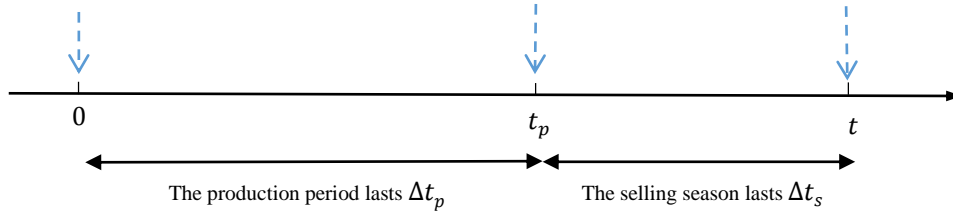


Fig. 1. The sequence of events and decisions under CSBF

At time 0, the CS proportion λ_{BF} is determined by R and, given λ_{BF} , S determines a CSR effort θ_{BF} . R then transfers an amount equal to $\lambda_{BF}d\theta_{BF}^2$ to S, and determines the ordered quantity of products q_{BF} . Upon receiving the order request, S borrows the amount $L_{BF} = cq_{BF} + (1 - \lambda_{BF})d\theta_{BF}^2$ from bank B_S . This latter sets the financing rate r_{BF} , so that S can start producing and investing in CSR activities.

At time t_p , S delivers q_{BF} products to R and records accounts receivable from R, which are payable at the end of the sales period with face value wq_{BF} .

At time t , the demand is realized, and the final payments to parties first depend on the exogenous bankruptcy risk events. When either S or R or S and R, claim bankruptcy, the bank receives zero repayments for the loan issued to S. When neither S nor R goes bankrupt, the final payments to parties depend on the realized demand. If the realized demand is above the R's obligation to S, that is $p\tilde{D} > wq_{BF}$, S receives the whole amount (wq_{BF}) from R, and the bank B_S is fully repaid. If the realized demand is between S's obligation to the bank and the R's obligation to S, namely, $L_{BF}e^{r_{BF}t} < p\tilde{D} < wq_{BF}$, then the bank B_S is fully repaid, and S receives only $p\tilde{D} - L_{BF}e^{r_{BF}t}$. If the realized demand is very low and lower than S's obligation

to the bank, that is, $p\tilde{D} < L_{BF}e^{r_{BF}t}$, then B_S obtains the whole sales revenue, and S receives nothing more.

Table 2 summarizes the revenue of each partner (at the end of the selling season) based on the demand realized and when neither S nor R goes bankrupt.

Table 2. Revenue of each partner under BF at the end of the selling season

Realized sales revenue	Revenue at the end of the selling season			
	$p\tilde{D}$	The bank B_S	Supplier	Retailer
$p\tilde{D} < L_{BF}e^{r_{BF}t} < wq_{BF}$	$p\tilde{D}$	$p\tilde{D}$	0	0
$L_{BF}e^{r_{BF}t} < p\tilde{D} < wq_{BF}$	$L_{BF}e^{r_{BF}t}$	$L_{BF}e^{r_{BF}t}$	$p\tilde{D} - L_{BF}e^{r_{BF}t}$	0
$L_{BF}e^{r_{BF}t} < wq_{BF} < p\tilde{D}$	$L_{BF}e^{r_{BF}t}$	$L_{BF}e^{r_{BF}t}$	$wq_{BF} - L_{BF}e^{r_{BF}t}$	$p \min\{\tilde{D}, q_{BF}\} - wq_{BF}$

Thus, with probability $(1 - \rho_S)(1 - \rho_R)$, the bank has only demand risks affecting the ability of S to repay the loan. The bank expects a cash flow amounting to $(1 - \rho_S)E[(1 - \rho_R) \min\{p\tilde{D}, L_{BF}e^{r_{BF}t}\}]$ at the end of the selling season and determines the competitive interest rate r_{BF} , which satisfies

$$(1 - \rho_S)(1 - \rho_R)E[\min\{p\tilde{D}, L_{BF}e^{r_{BF}t}\}] = L_{BF}e^{(r_f + \psi_S)t} \quad (1)$$

For further analysis, it is worth introducing the future value factor $\Omega_{BF} = e^{(r_f + \psi_S)t}$; Ω_{BF}^{-1} is a present value factor. At the end of the selling season, the expected profit of S is given by:

$$\pi_{BF}^S = (1 - \rho_S)(1 - \rho_R)(E[\min\{p\tilde{D}, wq_{BF}\}] - E[\min\{p\tilde{D}, L_{BF}e^{r_{BF}t}\}]) \quad (2)$$

Due to Equation (1), the expected profit of S can now be rewritten as

$$\pi_{BF}^S = (1 - \rho_S)(1 - \rho_R)E[\min\{p\tilde{D}, wq_{BF}\}] - (c_{BF} + (1 - \lambda_{BF})d\theta_{BF}^2)\Omega_{BF} \quad (3)$$

The expected profit of R is

$$\pi_{BF}^R = (1 - \rho_R)(E[p \min\{\tilde{D}, q_{BF}\}] - \min\{p\tilde{D}, wq_{BF}\}) - \lambda_{BF}d\theta_{BF}^2 \quad (4)$$

We employ a standard backward induction approach to derive the optimal decision parameters. Furthermore, to ease presentation, it is worth introducing $m_S = w(1 - \rho_S)(1 - \rho_R)\Omega_{BF}^{-1} - c$, the effective unit profit margin of S under BF, and $m_R = p - w$, the profit margin R per unit of product⁷. S's effective unit profit margin incorporates the original profit margin $w - c$ and the financing cost associated with the credit spread that the bank charged S and the bankruptcy risk of either S or R. In particular, the effective unit margin increases in S's credit spread and the bankruptcy risk of either S or R.

Proposition 1. *If, under CSBF, S and R adopt a CS contract and R selects a sharing proportion $\lambda_{BF} \in [0,1)$, then the optimal sharing proportion for R λ_{BF}^* is determined by*

$$\lambda_{BF}^* = 1 - \left(\frac{\beta}{2d\theta_{BF}^*} \right) \left(\frac{(m_S + c)p}{w} \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] + \frac{V_1 V_3}{V_2} \right), \quad (5)$$

the optimal CSR effort of S θ_{BF}^ and the optimal R's order quantity q_{BF}^* simultaneously satisfy the following set of equations:*

$$\bar{F} [q_{BF}^* - \beta\theta_{BF}^*] = \frac{w}{p} \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right], \quad (6)$$

$$\begin{aligned} 2/\beta^2 - \frac{V_1 V_3 - 2pV_2 V_4}{2d\beta\theta_{BF}^* V_2} + \frac{V_3(V_2^2 V_5 - 2V_1 V_2 V_6 + V_1^2 V_7)}{2dV_2^3} \\ = \frac{(m_S + c)}{2dw} \left(\frac{p^2}{\beta\theta_{BF}^*} F \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] \right. \\ \left. - \frac{(pV_2 - wV_1)^2 f \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right]}{V_2^2} \right) \end{aligned} \quad (7)$$

where the expressions of V_j , with $j = 1, \dots, 7$ provided in Appendix A.2 only depend on the decision parameters θ_{BF}^ and q_{BF}^* .*

This proposition informs us that, under CSBF, the optimal quantity of products ordered by R, the optimal CSR effort of S, and the R's optimal cost-sharing proportion depend on the financing conditions of S and R (via m_S); the distribution was chosen for modeling the demand uncertainty.

⁷ These profit margins are not defined at the same time. The supplier's is considered at time 0, i.e., before the production occurs, while the retailer's is considered at time t , namely, after the selling season. Clearly, m_S and m_R should not be negative; otherwise, the supplier and retailer have no incentive to conduct business.

3.2. CSR CS under RF

Figure 2 depicts the sequence of events and decisions under CSRF.

- | | | |
|--|--|--|
| 1) R decides the sharing proportion λ_{RF} | 1) S delivers q_{RF} products and invoices R | 1) Overall demand is realized |
| 2) S decides the CSR effort level θ_{RF} | 2) S decides v the proportion of invoice to finance by RF, and then the bank B_R sets γ the early payment rate. | 2) R receives the sales proceeds $p \min\{\bar{D}, q_{RF}\}$ |
| 3) R transfers $\lambda_{RF}d\theta_{RF}^2$ to S | 3) S receives an early payment $\gamma v w q_{RF}$ and pays off obligation $L_{RF} e^{r_{RF}\Delta t_p}$ to the bank B_S . | 3) R pays (to the extent possible) in priority $v w q_{RF}$ to the bank B_R then $(1 - v)w q_{RF}$ to S. |
| 4) R sets the order quantity q_{RF} | | |
| 5) S borrows L_{RF} from bank B_S at the financing rate r_{RF} to finance the production and CSR investment. | | |
| 6) B_S sets the interest rate r_{RF} . | | |

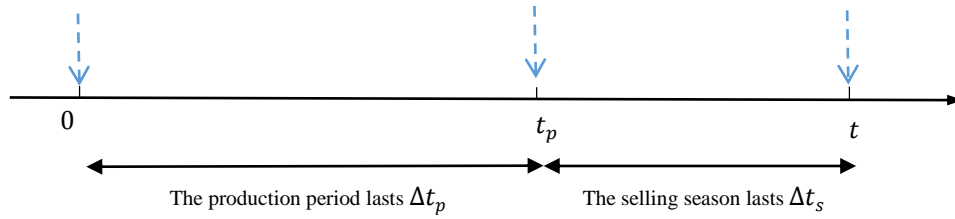


Fig. 2. The sequence of events and decisions under CSRF

At time 0, S, R, and a bank B_R enter an RF arrangement; S and R sign a CS agreement. The CS proportion λ_{RF} is determined by R; given λ_{RF} , S decides the CSR effort θ_{RF} . R then transfers $\lambda_{RF}d\theta_{RF}^2$ to S, and determines q_{RF} the quantity of products to order. S borrows an amount, $L_{RF} = cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2$, from the bank B_S . This latter sets the interest rate r_{RF} for the financing of S, who can start producing and investing in CSR.

At time t_p , S delivers products and invoices to R. Once the invoice is approved (by R), S can discount invoices (amounting to wq_{RF}) to the bank B_R to obtain an early payment amounting to $\gamma v w q_{RF}$, where v and γ are two parameters, such that $0 < v, \gamma < 1$. Here the parameter v stands for the proportion of invoices that S decides to finance by RF, and γ is the early payment rate determined by the bank B_R . Existing RF programs offer the flexibility to discount a proportion of receivables (Kramer, 2009; Vliet et al., 2015); thus, S can decide to discount only a portion⁸ $v \leq 1$ of the account receivables. Upon receiving the early payment, S pays off the obligation $L_{RF} e^{r_{RF}\Delta t_p}$ to the bank B_S .

⁸ If $v \leq 1$ stands for the proportion of invoices the supplier decides to discount. $(1 - v)$ represents what should be paid at the end of the selling season (if possible).

At time t , the demand is realized, and the final payments to parties depend on its value and whether R becomes bankrupt. If R goes bankrupt, the bank, B_R , and S get nothing; otherwise, the payment to the parties depends on the realized demand. If the realized demand is above the full payment due to B_R and S, namely, $p\tilde{D} > wq_{RF}$, then the bank receives the whole amount (wq_{RF}) from R, transfers the value of the “non-discounted invoice” ($(1 - v)wq_{RF}$) to S, and keeps the rest, vwq_{RF} . If the realized demand is sufficient to pay off the “discounted invoice” (vwq_{RF}) but insufficient to meet S’s invoice, namely, $vwq_{RF} < p\tilde{D} < wq_{RF}$, then the bank B_R is fully repaid (vwq_{RF}) and S receives only $p\tilde{D} - vwq_{RF}$. If the realized demand is very low, lower than the “discounted invoice,” that is, $p\tilde{D} < vwq_{RF}$, then B_R keeps the whole available sales revenue, and S receives nothing more. Here, we assume the credit obligation *vis-à-vis* the bank is greater than the obligation *vis-à-vis* S (Schwartz, 1997; Yang and Birge, 2018). Table 3 summarizes each partner’s revenue under RF (at the end of the selling season) based on the demand realized. Note that S received an early payment amounting to γvwq_{RF} at the start of the sales period for all demand scenarios.

Table 3. Revenue of each partner under RF at the end of the selling season

Realized sales revenue $p\tilde{D}$	Revenue at the end of the selling season		
	<i>The bank B_R</i>	<i>Supplier</i>	<i>Retailer</i>
$p\tilde{D} < vwq_{RF} < wq_{RF}$	$p\tilde{D}$	0	0
$vwq_{RF} < p\tilde{D} < wq_{RF}$	vwq_{RF}	$p\tilde{D} - vwq_{RF}$	0
$vwq_{RF} < wq_{RF} < p\tilde{D}$	vwq_{RF}	$(1 - v)wq_{RF}$	$p \min\{\tilde{D}, q_{RF}\} - wq_{RF}$

The expected profit of S at the end of the sales period can be formulated as follows

$$\pi_{RF}^S = (1 - \rho_S) \left((1 - \rho_R) \left(\underbrace{E[\min\{p\tilde{D}, wq_{RF}\} - \min\{p\tilde{D}, vwq_{RF}\}]}_X \right) + \underbrace{(\gamma vwq_{RF} - (cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2)e^{r_{RF}\Delta t_p})}_{Y} e^{r_f \Delta t_s} \right). \quad (8)$$

In Equation (8), the expression of “X” represents the S’s expected revenue from the non-discounted invoice, and the expression of “Y” is the S’s cash, which remains after paying for the loan obligation. For later analytical developments, we assume that when S requests a bank

loan at time 0, the bank B_S knows the S's discounting activities at time t_p , which guarantees the S's full repayment of the bank loan at the maturity date t_p . That is, the early payment when S decides to discount the invoices completely, $\gamma w q_{RF}$ should be not smaller than the S's loan obligation $L_{RF} e^{r_{RF} \Delta t_p}$; otherwise S has no incentive to trade with R. Hence, the bank's interest rate r_{RF} for financing the production should satisfy $L_{RF} e^{(r_f + \psi_S) \Delta t_p} = E[\min\{L_{RF} e^{r_{RF} \Delta t_p}; \gamma w q_{RF}\}] = L_{RF} e^{r_{RF} \Delta t_p}$ under the assumption of the competitive credit market; thus, we obtain $r_{RF} = r_f + \psi_S$. Regarding the financing of the selling season, once S informs the bank about the proportion v of invoice to discount, the bank communicates an early payment rate, γ , under the assumption of a competitive credit market satisfies $\gamma v w q_{RF} e^{(r_f + \psi_R) \Delta t_s} = (1 - \rho_R) E[\min\{p \tilde{D}, v w q_{RF}\}]$. This equation formally clarifies that the early payment rate announced by the bank is a non-linear function of the proportion v and that the product $\gamma v w q_{RF} = \gamma(v) v w q_{RF}$ —the effective early payment received by S—is an increasing function of the proportion v (as the right-hand side of the equation is)⁹. Thanks to the “competitive credit market” equation, the expected profit of S in Equation (8) may now be rewritten

$$\pi_{RF}^S = (1 - \rho_S) \left((1 - \rho_R) E[\min\{p \tilde{D}, w q_{RF}\}] - \gamma v w q_{RF} e^{r_f \Delta t_s} (e^{\psi_R \Delta t_s} - 1) - (c q_{RF} + (1 - \lambda_{RF}) d \theta_{RF}^2) e^{(r_f + \psi_S) \Delta t_p} \right). \quad (9)$$

This expected profit is a strictly decreasing function of the early payment effectively received (namely, $\gamma v w q_{RF}$); therefore, S is incited to choose γv as low as possible. Assuming¹⁰ S uses RF to cover (at minimum) the loan obligation due to the bank B_S at the end of the production period, one finds a new condition $(\gamma v)^* w q_{RF} = (\gamma v)_{\min} w q_{RF} = (c q_{RF} + (1 - \lambda_{RF}) d \theta_{RF}^2) e^{(r_f + \psi_S) \Delta t_p}$. By substituting this amount into Equation (9), the expected profit of S becomes

⁹ These two points are fortunately not an issue because i) only the product γv matters in our reasoning and ii) in case needed, as will be shown later, they do not impeach to determine and calculate separately the most appropriate proportion v and then the interest charged by the bank via γ .

¹⁰ This is not an assumption, as a rational S should seize the opportunity to use RF to cover the payment due to the bank B_S that finances the production period. RF hence permits optimal working capital management by avoiding any capital consumption at the end of the production period and helps S to reassure the bank B_S concerning the capacity within this business to meet S's loan obligation when reimbursement is due at the end of the production period. This is an important condition to not be charged more than necessary. With this setup indeed, the bank B_S that finances the production period will only charge to S the level of credit risk of S, as we show some lines earlier. Without it, the bank B_S could legitimately ask for an additional premium for the risk of not receiving the due payment.

$$\pi_{RF}^S = (1 - \rho_S) \left(\begin{array}{c} (1 - \rho_R) E[\min\{p\tilde{D}, wq_{RF}\}] \\ -(cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2) e^{(r_f + \psi_S)At_p} e^{(r_f + \psi_R)At_s} \end{array} \right) \quad (10)$$

and the profit of S is found to depend neither on the proportion v , nor the early payment rate γ . Hence, these two dimensions are instrumental and irrelevant to the following decision-making. Nevertheless, it is interesting that the two parameters v and γ can be determined separately (see Appendix A.3 for details).¹¹ In what follows, to ease presentation, it is worth introducing the future value factor

$$\Omega_{RF} = e^{(r_f + \psi_S)At_p} e^{(r_f + \psi_R)At_s}.$$

We now consider the retailer's situation. At time t at the end of the selling season, the expected profit of R is

$$\pi_{RF}^R = (1 - \rho_R) (E[p \min\{\tilde{D}, q_{RF}\} - \min\{p\tilde{D}, wq_{RF}\}] - \lambda_{RF} d\theta_{RF}^2) \quad (11)$$

We employ a standard backward induction approach to derive the optimal decision parameters. It is worth introducing $\bar{m}_S = w(1 - \rho_R)\Omega_{RF}^{-1} - c$, the effective unit profit margin of S under RF, and recall that $m_R = p - w$ is the profit margin of R per unit of product.¹² The following proposition characterizes the optimal equilibrium solution under RF.

Proposition 2. *If, under CSRF, S and R adopt a CS contract and R selects a sharing proportion $\lambda_{RF} \in [0, 1)$, then the optimal sharing proportion for R λ_{RF}^* is determined by*

$$\lambda_{RF}^* = 1 - \left(\frac{\beta}{2d\theta_{RF}^*} \right) \left(\frac{(\bar{m}_S + c)p}{w} \bar{F} \left[\frac{w}{p} q_{RF}^* - \beta\theta_{RF}^* \right] + \frac{\bar{V}_1 \bar{V}_3}{\bar{V}_2} \right), \quad (12)$$

the optimal CSR effort of S θ_{RF}^ and the optimal R's order quantity q_{RF}^* simultaneously satisfy the following set of equations:*

$$\bar{F}[q_{RF}^* - \beta\theta_{RF}^*] = \frac{w}{p} \bar{F} \left[\frac{w}{p} q_{RF}^* - \beta\theta_{RF}^* \right], \quad (13)$$

¹¹ It is also interesting to note that we can prove that the proportion v negatively impacts S's expected profit in equation (9) (i.e., $\frac{\partial \pi_{RF}^S}{\partial v} < 0$) so that, by associating this with the requirement that the early payment from RF covers, at minimum, the loan of the production period, one can find $v^* = v_{min}$. See Appendix A.3 for more details on the impact of v on S's expected profit.

¹² Paralleling the situation under BF, \bar{m}_S is related to time 0.

$$\begin{aligned}
& 2/\beta^2 - \frac{\bar{V}_1\bar{V}_3 - 2p\bar{V}_2\bar{V}_4}{2d\beta\theta_{RF}^*V_2} + \frac{\bar{V}_3(\bar{V}_2^2\bar{V}_5 - 2\bar{V}_1\bar{V}_2\bar{V}_6 + \bar{V}_1^2\bar{V}_7)}{2dV_2^3} \\
&= \frac{(\bar{m}_S + c)}{2dw} \left(\frac{p^2}{\beta\theta_{RF}^*} F \left[\frac{w}{p} q_{RF}^* - \beta\theta_{RF}^* \right] \right. \\
&\quad \left. - \frac{(p\bar{V}_2 - w\bar{V}_1)^2 f \left[\frac{w}{p} q_{RF}^* - \beta\theta_{RF}^* \right]}{\bar{V}_2^2} \right)
\end{aligned} \tag{14}$$

where the expressions of \bar{V}_j , with $j = 1, \dots, 7$ are similar to those of V_j , with $j = 1, \dots, 7$ provided in Appendix A.4, except that they involve the decision parameters θ_{RF}^* and q_{RF}^* (instead of θ_{BF}^* and q_{BF}^*) and refer to \bar{m}_S (instead of m_S).

Proposition 2 indicates that the optimal decisions $\{\lambda_{RF}^*, \theta_{RF}^*, q_{RF}^*\}$ depend critically on the financing conditions of S and R (via \bar{m}_S) and the distribution chosen for modeling the demand uncertainty; however, the solution does not generally lead to any analytical optimal sharing proportion. Therefore, we consider a uniform distribution for the random demand to develop more insight in the next section.

Remark 1: The equilibrium in RF (as shown in Proposition 2) is obtained by solving a similar system of equations as in BF (as shown in Proposition 1), but with a different effective profit margin of S. Consequently, when the effective profit margins of S are equal (i.e., when $m_S = \bar{m}_S$ or equivalent to when $\psi_R = \psi_S - \frac{\ln[1-\rho_S]}{\Delta t_S}$), RF performs the same as BF. Namely, $\theta_{RF}^* = \theta_{BF}^*$, $q_{RF}^* = q_{BF}^*$, $\pi_{RF}^{R*} = \pi_{BF}^{R*}$, and $\pi_{RF}^{S*} = \pi_{BF}^{S*}$.

Remark 2: Neither the retailer's revenue at the end of the sale season (see Column 4 in Table 3) nor the retailer's profit (see Equation 11) depends on the proportion v chosen by S (nor γ , the early payment rate charged by the bank providing RF). It is the optimal proportion, v^* , that depends on the optimal parameters listed in Proposition 2. More formally, $v^*(\lambda_{RF}^*, q_{RF}^*, \theta_{RF}^*) = v_{\min}(\lambda_{RF}^*, q_{RF}^*, \theta_{RF}^*)$, where $v_{\min}(\lambda_{RF}^*, q_{RF}^*, \theta_{RF}^*)$ is implicitly determined by $\frac{e^{(r_f + \psi_S)At_p} e^{(r_f + \psi_R)At_S}}{(1-\rho_R)} = \frac{p\beta\theta_{RF}^* + \int_0^{v^* \frac{w}{p} q_{RF}^* - \beta\theta_{RF}^*} \bar{F}(\xi) d\xi}{cq_{RF}^* + (1-\lambda_{RF}^*)d\theta_{RF}^{*2}}$ and once the resulting proportion v^* becomes available, one may compute the early payment rate charged by the bank $\gamma(v^*) = \frac{(cq_{RF}^* + (1-\lambda_{RF}^*)d\theta_{RF}^{*2})e^{(r_f + \psi_S)At_p}}{v^*wq_{RF}^*}$.

4. Analysis under uniform distribution demand

If the demand \tilde{D}_0 is uniformly distributed on the support $[A - M; A + M]$, with A and M two known parameters, such that $0 < M \leq A$, then its expected value is A , its variance is $M^2/3$, its probability density function is $f[x] = \frac{1_{\{A-M \leq x \leq A+M\}}}{2M}$, and the cumulative distribution function is $F[x] = \frac{(x-(A-M))1_{\{A-M \leq x \leq A+M\}}}{2M} + 1_{\{A+M < x\}}$. Parameter M characterizes the demand uncertainty, and the higher the value of M , the more uncertain the demand. The positivity of a uniformly distributed demand imposes $M \leq A$. Propositions 3 and 4 below explain the equilibria under CSBF and CSRF provided by Propositions 1 and 2.

Proposition 3. *Under CSBF and uniformly distributed demand:*

(a) *In the case that the condition $M < \frac{1}{1+2w/p} \left(A + \frac{\beta^2 m_S/2}{d(1-\lambda_{BF}^*)} \right)$ is met, the optimal CS proportion*

of R is $\lambda_{BF}^ = \max \left\{ \frac{m_R - m_S/2}{m_R + m_S/2}, 0 \right\}$, the CSR effort of S is $\theta_{BF}^* = \frac{\beta m_S/2}{d(1-\lambda_{BF}^*)}$, the optimal quantity*

ordered by R $q_{BF}^ = \frac{2Mm_R}{p} + A - M + \frac{\beta^2 m_S/2}{d(1-\lambda_{BF}^*)}$ and the resulting profits of S and R are,*

respectively, $\pi_{BF}^{S} = m_S \left(\frac{2Mm_R}{p} + A - M + \frac{\beta^2 m_S/4}{d(1-\lambda_{BF}^*)} \right) \Omega_{BF}$ and $\pi_{BF}^{R*} = m_R \left(\frac{Mm_R}{p} + A - M + \frac{\beta^2 m_S/2}{d(1-\lambda_{BF}^*)} \right) - \frac{\beta^2 \lambda_{BF}^* m_S^2/4}{d(1-\lambda_{BF}^*)^2}$.*

(b) *In the case that the condition $M > \frac{1}{1+2w/p} \left(A + \frac{\beta^2 m_S/2}{d(1-\lambda_{BF}^*)(1+w/p)} \right)$ is met, the optimal CS*

proportion of R is $\lambda_{BF}^ = \max \left\{ \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2) + \gamma_1 \gamma_2 \alpha_2}{\gamma_2(\alpha_1 \gamma_2 + d\gamma_1)}, 0 \right\}$ with $\gamma_1, \gamma_2, \alpha_1, \alpha_2$*

provided in Appendix A.5. The CSR effort of S is $\theta_{BF}^ = \frac{\left(\frac{p}{w}(m_S+c) + m_S \right) 2M \left(1 + \frac{w}{p} \right) - (A+M) \frac{p}{w} (m_S+c)}{\beta^2 \frac{p}{w} (m_S+c) + 4M(1-\lambda_{BF}^*) d \left(1 + \frac{w}{p} \right)^2} \beta$,*

the optimal quantity ordered by R $q^ = \frac{A+M+\beta\theta_{BF}^*}{(1+w/p)}$ and the resulting profits of S and R are,*

respectively, $\pi_{BF}^{S} = \left(m_S \frac{A+M+\beta\theta_{BF}^*}{\left(1 + \frac{w}{p} \right)} - \frac{p \left((A+M) - 2M \left(1 + \frac{w}{p} \right) + \beta\theta_{BF}^* \right)^2}{4M \left(1 + \frac{w}{p} \right)^2} \frac{(m_S+c)}{w} - (1 - \lambda_{BF}^*) d \theta_{BF}^{*2} \right) \Omega_{BF}$*

and $\pi_{BF}^{R} = (1 - \rho_R) \left(m_R \frac{(A+M+\beta\theta_{BF}^*)^2}{4M \left(1 + \frac{w}{p} \right)} - \lambda_{BF}^* d \theta_{BF}^{*2} \right)$.*

Proposition 3 highlights the circumstances under which R decides to share (or not) the costs of investing in CSR activities with S under BF. Furthermore, this proposition identifies the four regions R must examine before selecting the optimal CS proportion under BF. In the

scenarios where parameters satisfy $\left\{ m_R < m_S/2 \text{ and } M < \frac{A}{1+\frac{2w}{p}} + \frac{\beta^2 m_S/2}{d\left(1+\frac{2w}{p}\right)} \right\}$ or $\left\{ \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2) + \gamma_1 \gamma_2 \alpha_2}{\gamma_2(\alpha_1 \gamma_2 + d\gamma_1)} < 0 \text{ and } M > \frac{1}{1+2w/p} \left(A + \frac{\beta^2 m_S/2}{d(1-\lambda_{BF}^*)(1+w/p)} \right) \right\}$, R decides not to share CSR investment costs (i.e., $\lambda_{BF}^* = 0$). In scenarios where parameters satisfy $\left\{ m_R > m_S/2 \text{ and } M < \frac{A}{1+\frac{2w}{p}} + \frac{\beta^2 m_S/2}{d\left(1+\frac{2w}{p}\right)} \right\}$ or $\left\{ \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2) + \gamma_1 \gamma_2 \alpha_2}{\gamma_2(\alpha_1 \gamma_2 + d\gamma_1)} \equiv \lambda_{BF}^* > 0 \text{ and } M > \frac{A}{\left(1+\frac{2w}{p}\right)} + \frac{\beta^2 m_S/2}{d\left(1+\frac{2w}{p}\right)(1-\lambda_{BF}^*)(1+\frac{w}{p})} \right\}$, R shares CSR investment costs with S ($\lambda_{BF}^* > 0$).

The final decision of R requires the investigation of all four regions. For later use (e.g., Figure 3), we call these four scenarios Z1, Z3, Z2, and Z4, respectively.

In the same logic, Proposition 4 highlights the circumstances under which R decides to share (or not) the costs of investing in CSR activities with S under RF.

Proposition 4. *Under CSRF and uniformly distributed demand:*

(a) *In the case that the condition $M < \frac{1}{1+2w/p} \left(A + \frac{\beta^2 \bar{m}_S/2}{d(1-\lambda_{RF}^*)} \right)$ is met, the optimal CS proportion*

of R is $\lambda_{RF}^ = \max \left\{ \frac{m_R - \bar{m}_S/2}{m_R + \bar{m}_S/2}, 0 \right\}$, the CSR effort of S is $\theta_{RF}^* = \frac{\beta \bar{m}_S/2}{d(1-\lambda_{RF}^*)}$, the optimal quantity*

ordered by R $q_{RF}^ = \frac{2Mm_R}{p} + A - M + \frac{\beta^2 \bar{m}_S/2}{d(1-\lambda_{RF}^*)}$ and the resulting profits of S and R are,*

respectively, $\pi_{RF}^{S} = (1 - \rho_S) \bar{m}_S \left(\frac{2Mm_R}{p} + A - M + \frac{\beta^2 \bar{m}_S/4}{d(1-\lambda_{RF}^*)} \right) \Omega_{RF}$ and $\pi_{RF}^{R*} = (1 -$*

$\rho_R) \left(m_R \left(\frac{Mm_R}{p} + A - M + \frac{\beta^2 \bar{m}_S/2}{d(1-\lambda_{RF}^)} \right) - \frac{\beta^2 \lambda_{RF}^* \bar{m}_S^2/4}{d(1-\lambda_{RF}^*)^2} \right)$.*

(b) *In the case that the condition $M > \frac{1}{1+2w/p} \left(A + \frac{\beta^2 \bar{m}_S/2}{d(1-\lambda_{RF}^*)(1+w/p)} \right)$ is met, the optimal CS*

proportion of R is $\lambda_{RF}^ = \max \left\{ \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_R) + \gamma_2) + \gamma_1 \gamma_2 \alpha_2}{\gamma_2(\alpha_1 \gamma_2 + d\gamma_1)}, 0 \right\}$ with $\gamma_1, \gamma_2, \alpha_1, \alpha_2$ provided*

in Appendix A.6. The CSR effort of S is $\theta_{RF}^ = \frac{\left(\frac{p}{w}(\bar{m}_S + c) + \bar{m}_S \right) 2M \left(1 + \frac{w}{p} \right) - (A + M) \frac{p}{w} (\bar{m}_S + c)}{\beta^2 \frac{p}{w} (\bar{m}_S + c) + 4M(1-\lambda_{RF}^*) d \left(1 + \frac{w}{p} \right)^2} \beta$, the optimal*

quantity ordered by R $q^ = \frac{A + M + \beta \theta_{RF}^*}{(1+w/p)}$ and the resulting profits of S and R are, respectively,*

$$\pi_{RF}^{S*} = (1 - \rho_S) \left(\bar{m}_S \frac{A+M+\beta\theta_{RF}^*}{(1+\frac{w}{p})} - \frac{p \left((A+M) - 2M \left(1 + \frac{w}{p} \right) + \beta\theta_{RF}^* \right)^2}{4M \left(1 + \frac{w}{p} \right)^2} \frac{(\bar{m}_S + c)}{w} - (1 - \lambda_{RF}^*) d \theta_{RF}^{*2} \right) \Omega_{RF} \quad \text{and}$$

$$\pi_{RF}^{R*} = (1 - \rho_R) \left(m_R \frac{(A+M+\beta\theta_{RF}^*)^2}{4M \left(1 + \frac{w}{p} \right)} - \lambda_{RF}^* d \theta_{RF}^{*2} \right).$$

Proposition 3 reveals that the adoption of a CS contract under BF depends on the probability of bankruptcy of S, ρ_S , and the demand uncertainty M . In contrast, Proposition 4 shows that a CS contract is granted by R under RF, irrespectively of the bankruptcy risk of S. Figure 3 numerically illustrates the potential joint impact of S's probability of bankruptcy and the demand uncertainty on the decision to adopt a CS contract under the different financing modes. Figure 3 was generated using the base case parameters presented in Table 1 (the expected demand is 50). This figure plots zones labeled Z1, Z2, Z3, and Z4, materializing the test conditions of Proposition 3.

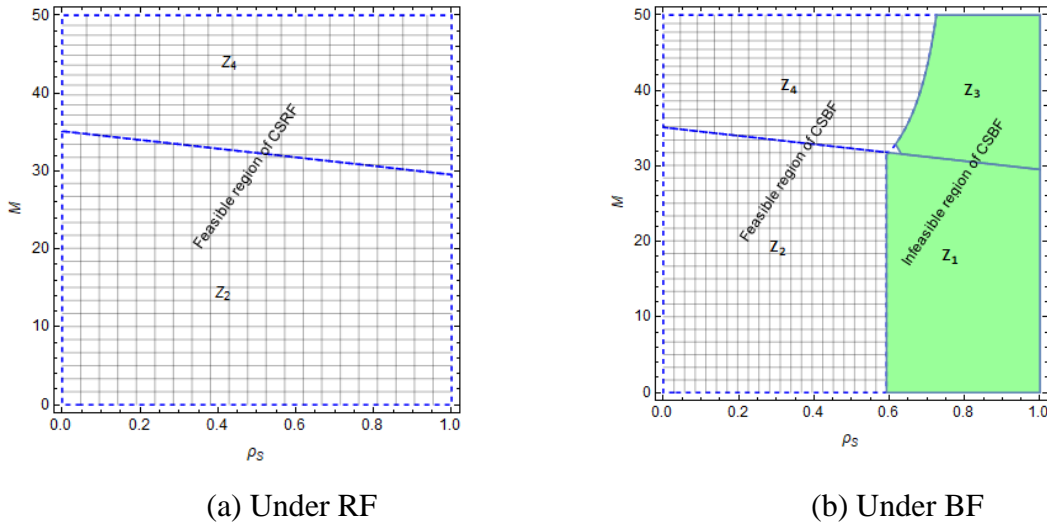


Fig. 3. The impact of supplier's bankruptcy risk and demand uncertainty on CS adoption

Figure 3 shows that adopting a CS contract under RF is always feasible, while it is not the case under BF. In particular, adopting a CS contract under BF depends on the S's bankruptcy risk and demand uncertainty (see Figure 3[b]). In addition, the feasible region under BF shrinks when the demand uncertainty and bankruptcy risk of S are high. One reason for this finding is that the bank increases the financing cost to compensate for the risks (i.e., the credit risk of S and the demand risk), which increases the overall cost for S, making adopting a CS contract more difficult. In contrast, the bankruptcy risk of S is irrelevant to adopting a CS contract under

RF (see Figure 3[a]). This result is intuitive because S's bankruptcy risk does not influence RF. Figure 3 reveals that the financing mode (BF vs. RF) matters when a CS contract is considered. In many cases, any retailer willing to engage in a CS strategy must consider RF simultaneously. As far as we know, this second result is entirely new in the literature.

Propositions 3 and 4 also offer analytical results that deserve illustration concerning the CS contract adoption. Unsurprisingly, both propositions show that a CS contract is granted by R (namely, the optimal CS ratio is positive) only if she benefits from the contract. In addition, when a CS agreement is signed with S, the CSR effort of S is higher; however, it is less straightforward to see that this does not depend on the financing scheme and that S obtains more profit for both financing schemes with the CS contract. These results can be formally and easily verified by showing that $\frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}^*}$, $\frac{\partial \pi_{BF}^*}{\partial \lambda_{BF}^*}$, $\frac{\partial \theta_{RF}^*}{\partial \lambda_{RF}^*}$ and $\frac{\partial \pi_{RF}^*}{\partial \lambda_{RF}^*}$ are positive. In other words, no matter which financing scheme is adopted, CS results in a win-win-win situation.

Further analyses of the above results lead to the following corollaries.

Corollary 1: *When $\theta_{BF}^* > 0$, (a) $\frac{\partial \theta_{BF}^*}{\partial \rho_S}$, $\frac{\partial \theta_{BF}^*}{\partial \rho_R}$, $\frac{\partial \theta_{BF}^*}{\partial \psi_S}$ are all negative, and (b) $\frac{\partial \theta_{BF}^*}{\partial \psi_R}$ is null*

Corollary 1(a) shows that under BF, S's optimal CSR effort depends on the bankruptcy risk of both parties in the SC and the interest rate premia charged by the bank to S. In particular, increasing the bankruptcy risk of either S or R would decrease the CSR effort level. The same conclusion holds for the effect of S's credit spread. Note that S's effective unit profit margin under BF incorporates S and R's bankruptcy probabilities and S's credit spread. That is, the effective unit profit margin of S m_S decreases as these parameters increase, which makes intuitive sense. Regarding the effect of R's credit spread, Corollary 1(b) shows that S's optimal CSR effort is not affected by the interest rate premia charged by the bank to R. This result is intuitive because under BF the bank focuses on the financing condition of S instead that of R.

Corollary 2: *When $\theta_{RF}^* > 0$, (a) $\frac{\partial \theta_{RF}^*}{\partial \rho_R}$, $\frac{\partial \theta_{RF}^*}{\partial \psi_R}$ are both negative, and (b) $\frac{\partial \theta_{RF}^*}{\partial \rho_S}$ and $\frac{\partial \theta_{RF}^*}{\partial \psi_S}$ are null.*

Overall, Corollary 2 shows that S's optimal CSR effort under RF depends on the financing condition of R but not on S's side. In Corollary 2 (a), S's optimal CSR effort decreases R's bankruptcy risk and the interest rate premia charged by the bank to R. The logic of Corollary 2(a) can be explained as follows; an increase in the probability of bankruptcy of R or R's credit spread would lead to higher financing cost under RF, and consequently, the financing cost of S increases, which lowers R's effective unit profit margin. As the margin profit reduces, S has

less incentive to invest in CSR. In Corollary 2 (b), S's optimal CSR effort does not depend on the bankruptcy risk of S nor the interest rate premia charged by the bank to S. This result is intuitive because, under RF, the bank only collects payment from R.

Corollary 3: *When $\psi_R < \psi_S$, then RF outperforms BF in the sense that $\theta_{RF}^* > \theta_{BF}^*$, $q_{RF}^* > q_{BF}^*$, $\pi_{RF}^{S*} > \pi_{BF}^{S*}$ and $\pi_{RF}^{R*} > \pi_{BF}^{R*}$.¹³*

Corollary 3 shows that when R's credit spread is lower than S's, RF leads to higher CSR effort, ordered quantity, and profits than BF. In other words, all SC members and the whole SC are worth considering RF. RF is suitable to incentivize SME suppliers with a low credit rating to increase CSR investment and improve the performance of the entire SC. The reason for this finding is that under RF, the bank relies on the creditworthiness of R. Therefore, when R has some financing advantages (e.g., a higher credit rating than S), RF can reduce the financial cost for the SC and thereby improve the SC performance. This result reveals that RF is especially suitable for retailers with high credit ratings.

5. Numerical experiments under a normal distribution

This section compares the performance of SC in terms of CSR effort, order quantity, and individual profit in the CSR cost sharing under RF with that in the CSR cost sharing under BF. One considers a normal distribution, namely, $\tilde{D} \sim N(\mu, \sigma)$, for modeling the market demand uncertainty without CSR investment instead of a uniform distribution in the previous sections. Note that simulations and figures confirm the robustness of our analytical findings under uniform distribution demand in the previous section and lead to some crucial insights that cannot be analytically demonstrated. We use the base case parameters, which are the same as those in Table 1 (except for μ and σ , i.e., $\mu = 50$, $\sigma = 50$). For comparison convenience, we explore the differences $\Delta\pi^k = \pi_{RF}^{k*} - \pi_{BF}^{k*}$, $\Delta\theta = \theta_{RF}^* - \theta_{BF}^*$, $\Delta q = q_{RF}^* - q_{BF}^*$ to inform on the profit difference of entity k ($k \in \{S, R\}$), CSR effort, and order quantity, respectively, between RF and BF. The following simulations reveal, for given values of parameters, whether S and R

¹³ If ever, in contrast to our setting, the retailer's credit spread is larger than the supplier's, then a couple of situations emerge. If $\psi_S < \psi_R < \psi_S - \frac{\ln(1-\rho_S)}{\Delta t_S}$, then RF still outperforms BF. If the credit spread is even larger and such that $\psi_S - \frac{\ln(1-\rho_S)}{\Delta t_S} < \psi_R$, then BF outperforms RF. This result implies that RF can improve the supply chain performance even if the retailer has no financing advantage over the supplier. This finding somewhat contrasts with the conventional wisdom that large creditworthy retailers mainly initiate RF.

benefit from choosing the same financing mode ($\Delta\pi^S$ and $\Delta\pi^R$ have the same sign) or not ($\Delta\pi^S$ and $\Delta\pi^R$ have opposite signs).

5.1. SC Performance without a CS Contract

The RF's performance without a CS contract is compared to the BF's performance without a CS contract to determine the potential benefits of the SC members' financing choice. We examine the impacts of four exogenous factors (S's probability of bankruptcy, credit spread of either S or R, and demand variance) on the SC's performance under each financing choice and investigate four scenarios. In scenario 1, S's probability of bankruptcy lies in the range of $[0, 0.5]$ and other parameters remain unchanged. In scenario 2, S's credit spread lies in the range of $[0.05, 0.15]$. In scenario 3, R's credit spread lies in the range of $[0, 0.05]$. In scenario 4, the standard deviation of demand (which represents demand variability) lies in the range of $[10, 50]$. The equilibrium results for under-financing modes BF and RF are in Tables B1.1–B1.4 in Appendix B.1 for scenarios 1, 2, 3, and 4, respectively. Based on the computational results, we have the following observations. (1) The equilibrium results (CSR effort, order quantity, and R's and S's profit) under BF decrease with the probability of bankruptcy and S's credit spread. They increase with the demand variability for S's profit but decrease for R's profit and do not change with R's credit spread. (2) The equilibrium results under RF decrease with R's credit spread, increase with the demand variability, and remain unchanged with S's credit spread. Regarding the impact of bankruptcy probability of S on the outcomes under RF, we find that the CSR effort, R's order, and R's profit are unaffected, but S's profit decreases as the bankruptcy probability increases; hence, these observations are consistent with Corollaries 2 and 3 in the case of uniform distribution. Next, panels (a), (b), (c), and (d) in Figure 4 show the impact of S's probability of bankruptcy, credit spread of either S and R, and demand variance on differences, indicating that in all situations, the outcomes under RF are higher than those under BF (because all Δ are positive). Panels (a) and (b) also indicate that the values of Δ increase with the probability of bankruptcy and S's credit spread, confirming our conclusion in Corollary 3. Thus, RF can lead to a higher SC performance when R has a high credit rating, or S is an SME with a low credit rating. Regarding the impact of demand variance, panel (d) of Figure 4 shows that S benefits more from RF as demand variance increases; however, the reverse holds for R. The reason for this finding lies in the following aspects. From S's perspective, S can protect itself by the early payment received from the bank under RF. When the demand risk is high, S can ask the bank to consider the entire invoice and eliminate the risk. From R's perspective, R is responsible for the loan repayment under RF, even when the market

demand is low, which can hurt R's profit. Therefore, a profit-maximizing retailer should not favor RF when demand uncertainty is high.

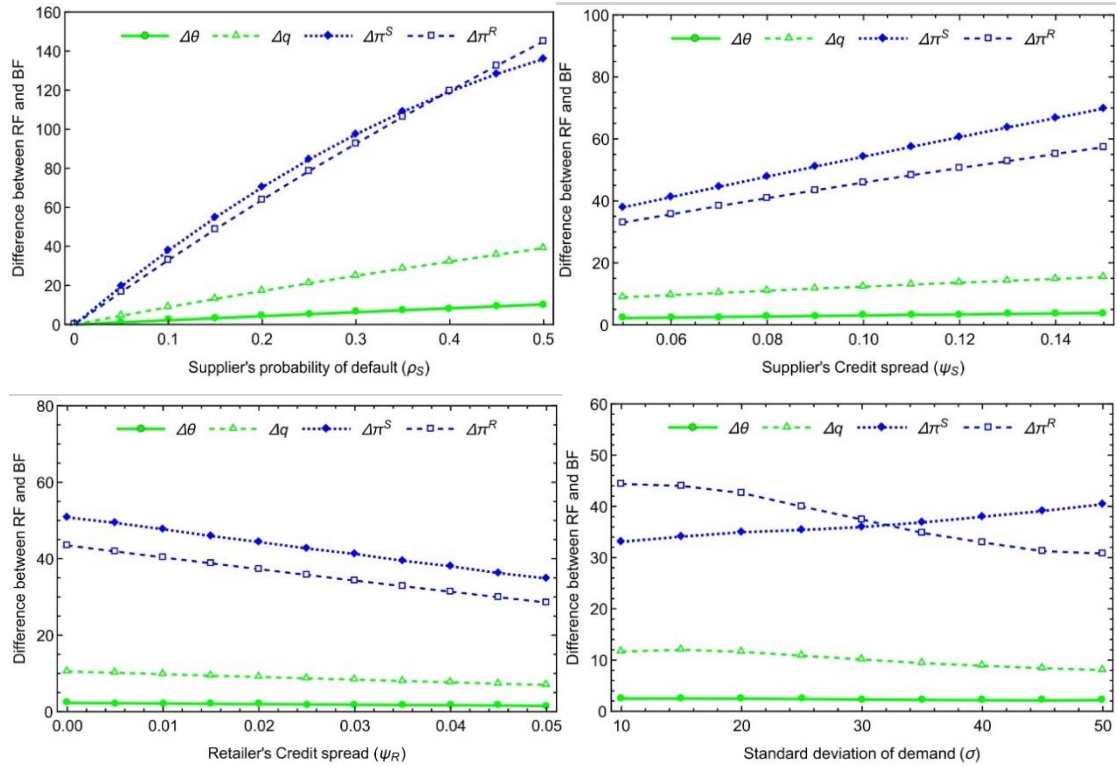


Fig. 4. Impact of parameters on the difference of outcomes between RF and BF

5.2. SC Performance with CS contract

5.2.1. Impact of CS proportion

The effects of the cost-sharing proportion on the equilibrium CSR effort, order quantity, and the profits of S and R are shown in Figure 5. Panels (a) and (b) consider BF and RF separately and respectively, while panel (c) illustrates the difference between RF and BF. Under BF and RF, the CSR effort and the profit of S increase with sharing ratio, while the profit of R is a concave function of the sharing ratio; thus, S always benefits from the CS contract under different financing modes. An interesting result can be found in panel (c) in Figure 5, where the profit of R under RF can be higher or lower than that under BF (i.e., $\Delta\pi^R$ can be higher or lower than zero) depending on the CS level; however, the CSR effort, order quantity, and S's profit under RF always exceeds BF ($\Delta\theta$, Δq , and $\Delta\pi^S$ are always higher than zero). These observations have two implications. First, from the CSR perspective, RF is the best choice to enhance S's CSR effort. Second, R's contribution level within the CS contract strongly affects the financing preference of R; R tends to favor RF, except when the contribution is too high.

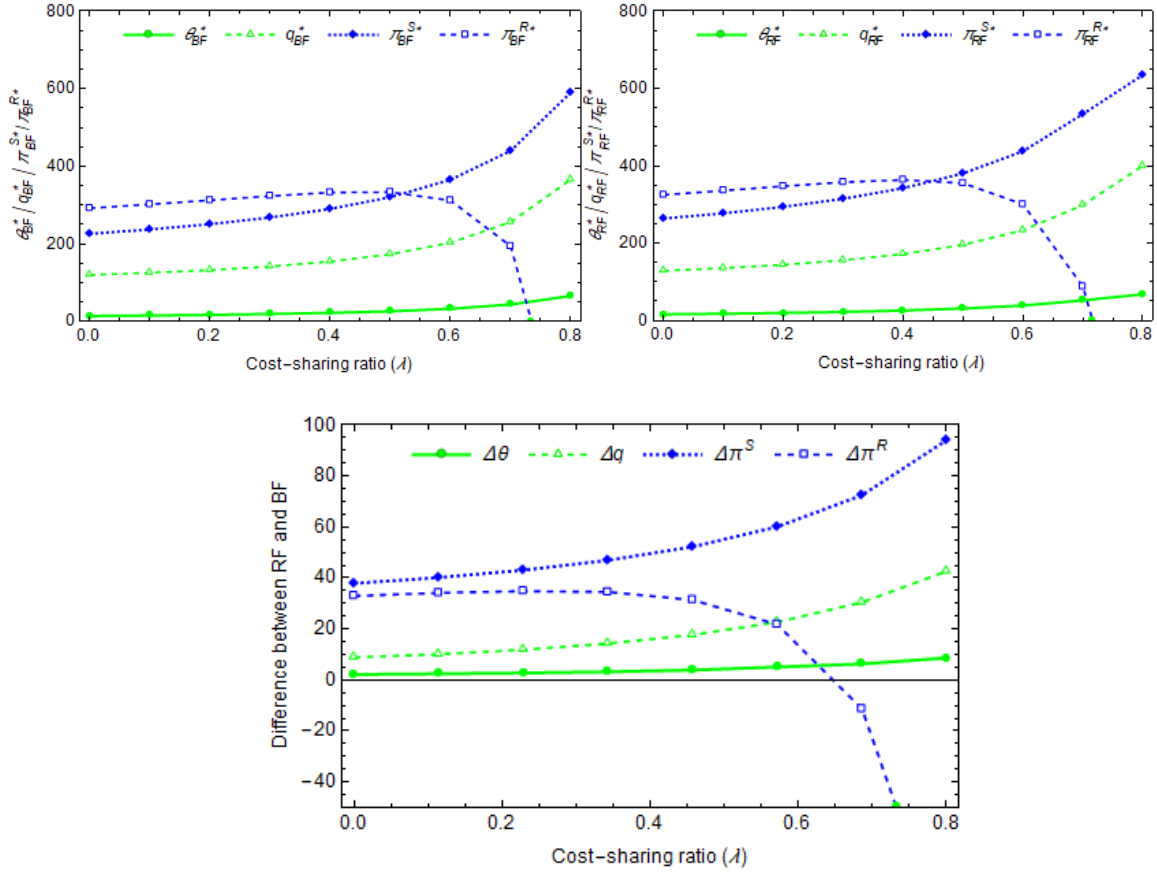


Fig. 5. Impact of the cost sharing on S's and R's profits and the CSR effort

Note: Panel (a) is related to BF, panel (b) to RF, and panel (c) highlights the difference.

5.2.2. Impact of CS contract

Figure 6 shows the impact of our four structural parameters on the CS proportion optimally chosen by R, indicating that in all situations, the optimal CS proportion is higher under BF than under RF. Hence, R is more willing to share CSR costs with S when S recurses to BF to fund the business. In other words, R provides more subsidies to S when S recurses to S's financing vehicle, and the adoption of RF can automatically lower the costs supported by R. Nonetheless, as R's financing condition deteriorates as the credit spread increases, the adoption of RF may lead R to share more CSR costs with S. For example, under RF, the optimal CS proportion increases with R's credit spread, illustrated in panel (c) of Figure 6. In the last panel of Figure 6, the optimal cost-sharing proportion under RF and BF decreases with demand variance, implying that the higher the demand uncertainty, the harder it is to adopt a CS contract, regardless of the chosen financing mode.

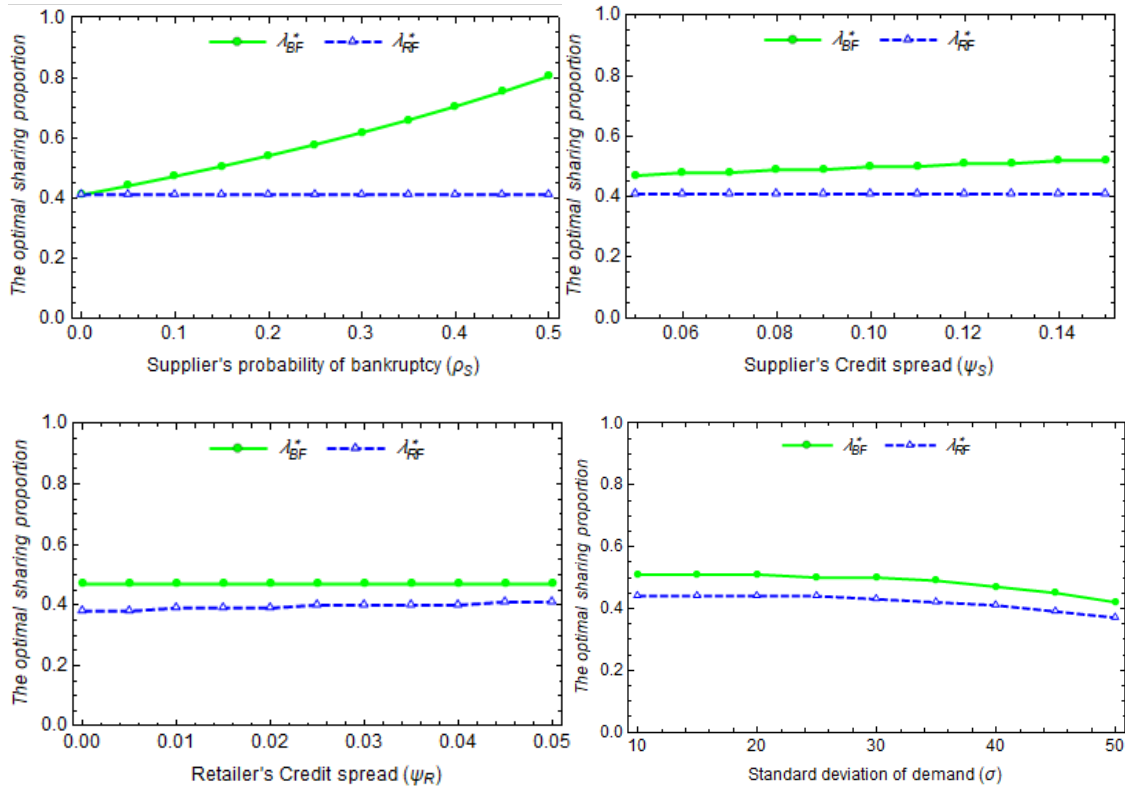


Fig. 6. Impact of parameters on the Retailer's optimal sharing proportion

In Tables B2.1–B2.4 in Appendix B2, we present some equilibrium results under the strategies of BF and RF with an R-led CS contract for scenarios 1, 2, 3, and 4, respectively. Similarly, Figures 4 and 7 (in Appendix C) show the impact of S's probability of bankruptcy, credit spread of S and R, and demand variance on the difference of outcomes between RF and BF when the CS contract is adopted. We observe that this figure appears to be very similar to Figure 4; hence, our observations for the comparison between RF and BF (as conducted in Section 5.1) remain valid for the case of a CS contract. Therefore, we can conclude that, when compared to BF, RF is more effective when combined with a CS contract if R has a high credit rating or S has a low credit rating.

6. Concluding Remarks

In this study, we investigate whether and to what extent combining RF, collaborative financing, and cost-sharing contracts can impact the supplier's CSR effort and the profitability of SC members. In an SC without CS, RF can encourage the supplier to increase the CSR efforts and generate more profits for the supplier and the retailer; however, we show that these benefits depend critically on some important factors: the market demand uncertainty, the interest rate premia charged by the bank, and the supplier's bankruptcy risk. Our model predicts that demand

uncertainty drives the supplier's adoption of RF, and the relative credit spread of the supplier drives that of the retailer. This finding corroborates some empirical observations on the supplier's adoption of RF (Wuttke et al., 2019).

In an SC with CS, we find, in line with intuition, that the sharing of CSR cost, granted by the retailer, can incentivize the supplier to increase the CSR efforts; however, we also find evidence that it can lead to higher profits for both SC members under the two financing schemes we investigate. These findings complement the early results of the literature (Ghosh and Shah, 2015, Raj et al., 2018). Furthermore, our results show that the adoption and the design of the CS contract depend on the financing condition of each SC member and the financing mode (BF vs. RF). Adopting RF can be the best choice because this automatically lowers the retailer's costs; however, in some situations, BF can be more effective in stimulating suppliers to enhance the CSR effort. As a managerial implication, this finding supports the use of various and alternative resources to co-finance suppliers' CSR activities, and this also calls for contingency in the analysis of SC management practices. SC members may have convergent or divergent opinions about adopting a specific financing mechanism. Our study also reports interesting managerial insights regarding some potential "double incentive" (cost sharing plus collaborative financing). By combining a collaborative CSR strategy with a collaborative financing device, we contribute to the present-day research efforts to understand the interaction between operational and financial decisions. Our findings help explain why many large companies (PUMA, Hewlett-Packard Company, etc.) favor RF to support their suppliers and achieve CSR goals.

Our model can be extended in other directions. First, one may combine the RF collaborative financing with other sharing contracts, such as revenue-sharing or buyback contracts. Second, one may compare RF with other collaborative financing practices like early or delayed payment mechanisms. These alternative combinations of collaborative financing devices with coordination modes could help conclude whether the benefits from the financing and operational collaboration always exist (and in which situations). Finally, as the effectiveness of RF depends critically on demand uncertainty, there is room for additional initiatives. The retailer can invest in a CSR marketing effort to improve consumer perception and increase market demand. The final impact of such retailers' CSR marketing and suppliers' CSR investment efforts deserves specific and detailed examination. Our work may be viewed as a first attempt to combine the financing and sharing contracts that can improve the CSR and

the profitability of the SC. Undoubtedly, much remains to be done, and integrating SC finance and coordination will be a fertile direction for future research.

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Appendices

Appendix A. Proofs and further theoretical explanations

Appendix A.1.

Demonstrations of the main propositions of our work require an important result we find interesting to promote *per se* with a lemma. As far as we know, it has never been formally demonstrated earlier that the solution put forward in this lemma is *global* solution of the optimization problem. This lemma will be of repeated use in the proofs that follow.

Lemma A: *Denote*

$$\Pi(q) = \int_{\alpha q - \gamma}^{q - \gamma} \bar{F}[x] dx - \beta$$

With $\alpha < 1$, β and γ three positive constants and \bar{F} the complementary cumulative distribution function of an IFR distribution, then the optimization problem $\max_q \Pi(q)$ admits a global solution which is unique. The solution defined by $q^* = \arg \max_q \Pi(q)$ satisfies

$$\bar{F}[q^* - \gamma] - \alpha \bar{F}[\alpha q^* - \gamma] = 0.$$

Proof of lemma¹: The first-order derivative of Π with respect to q is

$$\frac{\partial \Pi}{\partial q} = p \bar{F}[q - \gamma] - \alpha \bar{F}[\alpha q - \gamma]$$

and leads to the obvious FOC condition satisfied by q^* . To go further in this demonstration and shed some lights on this first-order derivative, it is worth investigating the properties of $G[q]$, via $\ln(G[q])$ where $G[q] = \frac{p \bar{F}[q - \gamma]}{\alpha \bar{F}[\alpha q - \gamma]}$.² There is indeed an obvious one-to-one correspondence between the sign of the derivative, whether the function G is above or below the value 1 and the sign of the function $\ln(G[q])$. Hence, one has

$$\ln(G[q]) = \ln \frac{p}{\alpha} + \ln(\bar{F}[q - \gamma]) - \ln(\bar{F}[\alpha q - \gamma]),$$

and $\frac{\partial \ln(G[q])}{\partial q} = \alpha \frac{f[\alpha q - \gamma]}{\bar{F}[\alpha q - \gamma]} - \frac{f[q - \gamma]}{\bar{F}[q - \gamma]} = \alpha h[\alpha q - \gamma] - h[q - \gamma].$

¹ Finding the last expression for a possible solution is not a big challenge, because one just needs to apply the FOC to the function $\Pi(q)$. But existence and uniqueness are not-that-straightforward to prove and, as far as we know, it has been formally demonstrated earlier that q^* is the global solution of the optimization problem. The aim of our demonstration is to prove that the solution is effectively global. Our proof relies on two tricks.

² This is our first trick.

Since $\alpha < 1$ and $h[x]$ is a strictly increasing function of x , one has $\alpha h[\alpha q - \gamma] - h[q - \gamma] < h[\alpha q - \gamma] - h[q - \gamma] < 0$ and the derivative $\frac{\partial \ln(G[q])}{\partial q}$ is found to be strictly negative. The function $\ln G[q]$ is thus strictly decreasing for all q , so is the function $G[q]$. Of course, their respective maximum is located at $q = 0$, which is the smallest possible value for the quantity and one has $G[0] = \frac{p}{w} > 1$ and $\ln G[0] = \ln \frac{p}{w} > 0$. The decreasing feature of the function $\ln G[q]$ is a necessary condition but not a sufficient one to formally prove the existence of the solution. Let's apply the mean value theorem to the continuous function $g[q] = \ln(\bar{F}[q - \gamma])$ and the closed interval $\left[\frac{w}{p}q; q\right]^3$. The mean value theorem tells us that there exists a constant $c \in \left[\frac{w}{p}q; q\right]$ such that

$$\begin{aligned}
\ln(G[q]) &= \ln \frac{p}{\alpha} + \ln(\bar{F}[q - \gamma]) - \ln(\bar{F}[\alpha q - \gamma]) \\
&= \ln \frac{p}{w} + g'[c] \left(q - \frac{w}{p}q \right) \\
&= \ln \frac{p}{w} - \frac{f[c - \gamma] p - w}{\bar{F}[c - \gamma] p} q \\
&= \ln \frac{p}{w} - qh[c - \gamma] \frac{p - w}{p} \\
&< \ln \frac{p}{w} - qh \left[\frac{w}{p}q - \gamma \right] \frac{p - w}{p} \\
&= \ln \frac{p}{w} - \frac{w}{p}qh \left[\frac{w}{p}q - \gamma \right] \frac{p - w}{w}
\end{aligned}$$

The inequality comes from the decreasing property of “ $-h$ ”, which is minus the failure rate function and the use of the lower interval bound. The two last expressions make it clear that the upper boundary for $\ln(G[q])$ may be arbitrarily and limitlessly negative for large q . Observe that $qh[q - \gamma]$ is indeed strictly increasing in q because 1) the product of two increasing positive functions (q and $h[q - \gamma]$) is increasing and 2) the composite of two increasing functions (namely h and ‘ $x - \gamma$ ’) is increasing. This implies that $\ln G[q]$ tends to minus infinity as q becomes large or equivalently $G[q]$ tends to zero as q becomes large. Hence, summing up, the strictly decreasing function G starts at $\frac{p}{w} > 1$ and tends to zero, so that the critical value q_{RF}^* (such that $G[q_{RF}^*] = 1$ and $\left. \frac{\partial \Pi}{\partial q} \right|_{q=q_{RF}^*} = 0$) exists and is unique. q_{RF}^* is the unique root of equation $p\bar{F}[q_{RF}^* - \gamma] = w\bar{F}\left[\frac{w}{p}q_{RF}^* - \gamma\right]$. In passing, it is worth mentioning that the convergence of

³ This is our second trick.

$\ln(G[q])$ as q increases is strictly bounded. By applying the mean value theorem and considering the upper bound of the closed interval too, one may show that the function $\ln(G[q])$ tends to minus infinity but within a certain interval:

$$\ln \frac{p}{w} - qh[q - \gamma] \frac{p - w}{p} < \ln(G[q]) < \ln \frac{p}{w} - \frac{w}{p}qh \left[\frac{w}{p}q - \gamma \right] \frac{p - w}{w}.$$

Appendix A.2. Proof of Proposition 1.

Given a certain cost sharing parameter λ_{BF} , the supplier determines his CSR effort θ_{BF} and then the retailer establishes her order quantity q_{BF} according to this particular supplier's CSR effort. Such a sequence of strategic decisions to be made by the retailer and supplier naturally calls for a backward approach. We therefore proceed backward to find the best-response order quantity of the retailer and then the best-response CSR effort of the supplier.

The expected profit of the retailer (see Equation (4)) can be written

$$\pi_{BF}^R = (1 - \rho_R) \left(p \int_{\frac{w}{p}q_{BF} - \beta\theta_{BF}}^{q_{BF} - \beta\theta_{BF}} \bar{F}[x] dx - \lambda_{BF} d\theta_{BF}^2 \right) \quad (\text{A.1})$$

And, of course, this expression conforms to lemma A by identifying α , β and γ to the appropriate values. We can conclude that the optimal quantity q_{BF}^* the retailer has to order solves the first order condition (FOC hereafter), and hence $\left. \frac{\partial \pi_{BF}^R}{\partial q_{BF}} \right|_{q_{BF}=q_{BF}^*} = 0$ and more explicitly

$$\bar{F}[q_{BF}^* - \beta\theta_{BF}] = \frac{w}{p} \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF} \right]. \quad (\text{A.2})$$

This is where Equation (6) in the core text comes from. This equation (A.2) implies the optimal quantity is a function of θ_{BF} only. And this has a couple of important implications. First, the Implicit Function Theorem then tells that

$$\frac{\partial q_{BF}^*}{\partial \theta_{BF}} = \beta \frac{f[q_{BF}^* - \beta\theta_{BF}] - \frac{w}{p} f \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF} \right]}{f[q_{BF}^* - \beta\theta_{BF}] - \frac{w^2}{p^2} f \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF} \right]} = \beta \frac{v_1}{v_2},$$

by introducing

$$v_1 = \frac{w}{p} f \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF} \right] - f[q_{BF}^* - \beta\theta_{BF}]$$

and

$$v_2 = \frac{w^2}{p^2} f \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF} \right] - f[q_{BF}^* - \beta\theta_{BF}].$$

Second, the equation (A.2) shows that the optimal quantity does not depend directly on the sharing cost parameter λ_{BF} . Consequently, the optimal quantity will depend on λ_{BF} only indirectly, via the influence of λ_{BF} on the optimal CSR effort if any. And one will get

$$\frac{\partial q_{BF}^*}{\partial \lambda_{BF}} = \frac{\partial q_{BF}^*}{\partial \theta_{BF}} \Big|_{\theta_{BF}=\theta_{BF}^*} \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} = \beta \frac{V_1}{V_2} \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}}, \quad (\text{A.2}')$$

where the terms

$$V_1 = \frac{w}{p} f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] - f[q_{BF}^* - \beta \theta_{BF}^*]$$

and

$$V_2 = \frac{w^2}{p^2} f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] - f[q_{BF}^* - \beta \theta_{BF}^*]$$

Emphasize (with their capital letters) the fact that both involve the optimal CSR effort θ_{BF}^* made the supplier. Now, the optimal quantity ordered by the retailer is a known function of the CSR effort made by the supplier, one can investigate the optimal level of effort made by the supplier.

The expected profit of the supplier (see Equation (3)) can be written:

$$\begin{aligned} \pi_{BF}^S &= (1 - \rho_S)(1 - \rho_R) E[\min\{p\tilde{D}, wq_{RF}\}] - (cq_{BF} + (1 - \lambda_{BF})d\theta_{BF}^2)\Omega_{BF} \\ &= (1 - \rho_S)(1 - \rho_R)p \left(\beta\theta_{BF} + \int_0^{\frac{w}{p}q_{BF} - \beta\theta_{BF}} \bar{F}(\xi) d\xi \right) \\ &\quad - (cq_{BF} + (1 - \lambda_{BF})d\theta_{BF}^2)\Omega_{BF} \\ &= \left(\frac{p}{w}(m_S + c) \left(\beta\theta_{BF} + \int_0^{\frac{w}{p}q_{BF} - \beta\theta_{BF}} \bar{F}(\xi) d\xi \right) \right. \\ &\quad \left. - (cq_{BF} + (1 - \lambda_{BF})d\theta_{BF}^2) \right) \Omega_{BF} \end{aligned} \quad (\text{A.3})$$

The third equality made use of $(1 - \rho_S)(1 - \rho_R) = \frac{(m_S + c)}{w} \Omega_{BF}$. Neglecting the integral and the dependence of the quantity to the effort for a short while, one observes that the expected profit of the supplier is a simple concave quadratic function of the CSR effort. The intricate dependence of the quantity to the effort (cf. equation A.2) and the presence of the integral obscure this simple picture however. And, for instance, a possible concavity of the supplier's profit with respect to θ_{BF} , which is a sufficient condition to show the existence and unicity of the supplier's profit, cannot be proved formally in full generality and for any demand distribution. Consequently, from now on, we consider only some demand distributions with

suitable properties and that permit the use of the FOC $\left(\frac{d\pi_{BF}^S}{d\theta_{BF}}\right)\Big|_{\theta_{BF}=\theta_{BF}^*} = 0$) to determine the supplier's optimal CSR effort θ_{BF}^* . The first order derivative of the supplier's profit with respect to θ_{BF} is

$$\begin{aligned}
\frac{d\pi_{BF}^S}{d\theta_{BF}} &= \frac{\partial \pi_{BF}^S}{\partial q_{BF}^*} \frac{\partial q_{BF}^*}{\partial \theta_{BF}} + \frac{\partial \pi_{BF}^S}{\partial \theta_{BF}} \\
&= \left(\left((m_S + c) \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF} \right] - c \right) \beta \frac{v_1}{v_2} - 2(1 - \lambda_{BF}) d\theta_{BF} \right. \\
&\quad \left. + \frac{p\beta}{w} (m_S + c) F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF} \right] \right) \Omega_{BF} \\
&= \left((m_S + c) \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF} \right] - c \right) \left(\beta \frac{v_1}{v_2} \right. \\
&\quad \left. - \frac{2(1 - \lambda_{BF}) d\theta_{BF} - \beta \frac{p}{w} (m_S + c) F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF} \right]}{(m_S + c) \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF} \right] - c} \right) \Omega_{BF}
\end{aligned} \tag{A.4}$$

Now, it appears that θ_{BF}^* can nullify only the second term with brackets in (A.4). One therefore finds

$$\beta \frac{V_1}{V_2} = \frac{2(1 - \lambda_{BF}) d\theta_{BF}^* - \beta \frac{p}{w} (m_S + c) F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right]}{V_3} \tag{A.5}$$

Where we use

$$V_3 = (m_S + c) \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] - c.$$

The equation (A.5) now deserves two important remarks. First, it shows that (and how) the θ_{BF}^* is a function of λ_{BF} , as our backward reasoning implies. Second, it implies that the optimal CSR effort made by the supplier θ_{BF}^* and the resulting optimal quantity ordered by the retailer $q_{BF}^*(\theta_{BF}^*)$ will adjust, so as to satisfy

$$1 - \lambda_{BF} = \frac{\beta}{2d\theta_{BF}^*} \left(\frac{(m_S + c)p}{w} F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] + \frac{V_1 V_3}{V_2} \right)$$

Whatever the value of cost sharing parameter λ_{BF} is. This is in particular true at equilibrium where the optimal parameters will satisfy

$$1 - \lambda_{BF}^* = \frac{\beta}{2d\theta_{BF}^*} \left(\frac{(m_S + c)p}{w} F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] + \frac{V_1 V_3}{V_2} \right) \tag{A.6}$$

The equation (5) exposed in the core text is then demonstrated.

In principle, in the Retailer-led CS contract under study, the equilibrium solution $\{\lambda_{BF}^*, \theta_{BF}^*(\lambda_{BF}^*), q_{BF}^*(\theta_{BF}^*(\lambda_{BF}^*))\}$ is obtained by solving: $\max_{\lambda_{BF}} \pi_{BF}^R$ subject to the equations (A.2) and (A.5). And solving this optimization problem provides simultaneously the resulting retailer's expected profit in equilibrium under CSBF. Assuming the FOC applies, it is sufficient to consider the first-order derivative of π_{BF}^R with respect to λ_{BF} . This first-order derivative is given by

$$\begin{aligned} \frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}} = & -d\theta_{BF}^{*2} - 2d\lambda_{BF}\theta_{BF}^* \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \\ & + p \left(\bar{F}[q_{BF}^* - \beta\theta_{BF}^*] \left(\frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) \right. \\ & \left. - \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] \left(\frac{w}{p} \frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) \right) \end{aligned} \quad (A.7)$$

Since $\frac{\partial q_{BF}^*}{\partial \lambda_{BF}} = \beta \frac{V_1}{V_2} \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}}$ by virtue of equation (A.2'), we get

$$\begin{aligned} \frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}} = & -d\theta_{BF}^{*2} \\ & + \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \left(-2d\theta_{BF}^* \lambda_{BF} \right. \\ & \left. + p\beta \left(\bar{F}[q_{BF}^* - \beta\theta_{BF}^*] \left(\frac{V_1}{V_2} - 1 \right) - \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] \left(\frac{wV_1}{pV_2} - 1 \right) \right) \right) \end{aligned}$$

After arranging, we have

$$\begin{aligned} \frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}} = & -d\theta_{BF}^{*2} \quad (A.7') \\ & + \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \left(-2d\theta_{BF}^* \lambda_{BF} - p\beta \left(\bar{F}[q_{BF}^* - \beta\theta_{BF}^*] - \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] \right) \right. \\ & \left. + \frac{p\beta \left(\bar{F}[q_{BF}^* - \beta\theta_{BF}^*] - \frac{w}{p} \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] \right) V_1}{V_2} \right). \end{aligned}$$

Now, by reconsidering (A.2) (i.e. $\bar{F}[q_{BF}^* - \beta\theta_{BF}^*] - \frac{w}{p} \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] = 0$) and introducing

$$V_4 = \bar{F}[q_{BF}^* - \beta\theta_{BF}^*] - \bar{F} \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right],$$

we have

$$\frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}} = -d\theta_{BF}^{*2} - (2d\theta_{BF}^* \lambda_{BF} + p\beta V_4) \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \quad (A.7'')$$

One therefore needs an expression for $\frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}}$. By differentiating the equation (A.5) viewed as a function of λ_{BF} , we get in full length

$$\begin{aligned} & \frac{\beta \left(-p \left(\frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) f' [q_{BF}^* - \beta \theta_{BF}^*] + w \left(\frac{w}{p} \frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right)}{pV_2} \\ & - \frac{-\beta V_1 \left(p^2 \left(\frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) f' [q_{BF}^* - \beta \theta_{BF}^*] + w^2 \left(\frac{w}{p} \frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right)}{p^2 V_2^2} \\ & = \frac{(c + m_S) \left(\frac{w}{p} \frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) \left(2(1 - \lambda_{BF}) d\theta_{BF}^* - \beta \frac{p}{w} (m_S + c) F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right) f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right]}{V_3^2} \\ & + \frac{-2d\theta_{BF}^* + 2d(1 - \lambda_{BF}) \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} - \frac{(m_S + c)p\beta \left(\frac{w}{p} \frac{\partial q_{BF}^*}{\partial \lambda_{BF}} - \beta \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} \right) f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right]}{V_3} \end{aligned}$$

By replacing $\frac{\partial q_{BF}^*}{\partial \lambda_{BF}} = \beta \frac{V_1}{V_2} \frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}}$, and reminding that $2(1 - \lambda_{BF}) d\theta_{BF}^* - \beta \frac{p}{w} (m_S + c) F \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] = \frac{\beta V_1 V_3}{V_2}$ (i.e. A.5), one finds

$$\frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}} = - \frac{2d\theta_{BF}^* p^3 w V_2^3}{G}$$

Where

$$\begin{aligned} G = & p^3 V_2^3 \left(2dw(-1 + \lambda_{BF}) - (m_S + c)p\beta^2 f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right) \\ & + w\beta^2 V_1^2 V_3 \left(p^3 f' [q_{BF}^* - \beta \theta_{BF}^*] - w^3 f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right) \\ & - p^2 w\beta^2 V_2^2 \left(-2(m_S + c)pV_1 f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right. \\ & \left. + V_3 \left(pf' [q_{BF}^* - \beta \theta_{BF}^*] - wf' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right) \right) \\ & + pw\beta^2 V_1 V_2 \left(-(m_S + c)pwV_1 V_1 f \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right. \\ & \left. + 2V_3 \left(-p^2 f' [q_{BF}^* - \beta \theta_{BF}^*] + w^2 f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right) \right) \end{aligned}$$

Now, introducing

$$V_5 = \frac{w}{p} f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] - f' [q_{BF}^* - \beta \theta_{BF}^*],$$

$$V_6 = \frac{w^2}{p^2} f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] - f' [q_{BF}^* - \beta \theta_{BF}^*]$$

and

$$V_7 = \frac{w^3}{p^3} f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] - f' [q_{BF}^* - \beta \theta_{BF}^*],$$

the expression of G reduces to

$$G = p^2 \left(-pw(2d(1 - \lambda_{BF})V_2^3 + \beta^2 V_2^2 V_3 V_5 - 2\beta^2 V_1 V_2 V_3 V_6 + \beta^2 V_1^2 V_3 V_7) \right. \\ \left. - (m_S + c)\beta^2 V_2 (pV_2 - wV_1)^2 f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right] \right)$$

Substituting $\frac{\partial \theta_{BF}^*}{\partial \lambda_{BF}}$ into the expression of $\frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}}$ in (A.7''), then setting $\frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}} = 0$ give

$$1 - \lambda_{BF}^* = \beta^2 \left(2/\beta^2 + \frac{pV_4}{d\beta\theta_{BF}^*} + \frac{V_3(V_2^2 V_5 - 2V_1 V_2 V_6 + V_1^2 V_7)}{2dV_2^3} \right. \\ \left. + \frac{(m_S + c)(pV_2 - wV_1)^2 f' \left[\frac{w}{p} q_{BF}^* - \beta \theta_{BF}^* \right]}{2dpwV_2^2} \right) \quad (A.8)$$

The imbricated equilibrium solution $\{\lambda_{BF}^*, \theta_{BF}^*, q_{BF}^*\}$ provided by the system of equations (A.2), (A. 6) and (A.8) is in line with the principle of our backward approach. It is nevertheless immediate to see that the system can be further simplified. Equating the equation (A.6) with equation (A.8) provides a new equation (A.9) satisfied by $\{\theta_{BF}^*, q_{BF}^*\}$

$$2/\beta^2 - \frac{V_1 V_3 - 2pV_2 V_4}{2d\beta\theta_{BF}^* V_2} + \frac{V_3(V_2^2 V_5 - 2V_1 V_2 V_6 + V_1^2 V_7)}{2dV_2^3} \\ = \frac{(m_S + c)}{2dw} \left(\frac{p^2}{\beta\theta_{BF}^*} F \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right] \right. \\ \left. - \frac{(pV_2 - wV_1)^2 f' \left[\frac{w}{p} q_{BF}^* - \beta\theta_{BF}^* \right]}{V_2^2} \right). \quad (A.9)$$

This is the equation (7) provided in the core text. Now, the system of (A.2) and (A.9) does not depend on the cost sharing the optimal cost-sharing effort λ_{BF}^* . Solving it provides $\{\theta_{BF}^*, q_{BF}^*\}$. Once $\{\theta_{BF}^*, q_{BF}^*\}$ available, the optimal cost-sharing effort is determined by the Equation (A.6), and one finds Equation (5) in the core text. Proposition 1 is proved. Of course, one must check that all solutions do satisfy some natural respective conditions. For instance, $\lambda_{BF}^* \in [0,1]$. \square

Appendix A.3: On the expected profit function of S

This appendix provides detailed explanations about two claims we make in the core text about the expected profit function of S. The first claim affirms that the two parameters ν and γ can

be determined separately. The second claim in footnote 11 affirms that the expected profit function is a negative function of the sole parameter v .

- **Parameters v and γ can be determined separately**

To see v and γ can be determined separately, it is sufficient to reconcile the ‘competitive credit market’ equation and the requirement that the early payment from RF must at minimum cover the loan of the production period. One indeed gets a couple of non-linear equations, namely

$$\gamma(v_{min})v_{min} = \frac{(cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2)e^{(r_f + \psi_S)\Delta t_p}}{wq_{RF}} \quad \text{and} \quad \gamma(v_{min})v_{min} =$$

$$\frac{(1 - \rho_R)}{wq_{RF}} e^{-(r_f + \psi_R)\Delta t_s} E[\min\{p\tilde{D}, v_{min}wq_{RF}\}].$$

Reconciling them together implies that v_{min} necessarily satisfies $(cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2)e^{(r_f + \psi_S)\Delta t_p} = (1 - \rho_R)e^{-(r_f + \psi_R)\Delta t_s} E[\min\{p\tilde{D}, v_{min}wq_{RF}\}]$. And, once v_{min} available, it is a simple exercise to calculate $\gamma(v_{min})$ by the ‘competitive credit market’ equation or the requirement that the early payment from RF must at minimum cover the loan of the production period.

- **Proportion v has a negative impact on the supplier’s expected profit**

To see this, consider simultaneously the expected profit function of S given by the Equation (9)

$$\begin{aligned} \pi_{RF}^S = & (1 - \rho_S) \left((1 - \rho_R) E[\min\{p\tilde{D}, wq_{RF}\}] - \gamma v w q_{RF} e^{r_f \Delta t_s} (e^{\psi_R \Delta t_s} - 1) \right. \\ & \left. - (cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2)e^{(r_f + \psi_S)\Delta t_p} \right) \end{aligned}$$

and the ‘competitive credit market’ condition

$$\gamma v w q_{RF} e^{(r_f + \psi_R)\Delta t_s} = (1 - \rho_R) E[\min\{p\tilde{D}, v w q_{RF}\}].$$

Note this latter leads to an expression $\gamma v w q_{RF} e^{r_f \Delta t_s} = (1 - \rho_R) \frac{E[\min\{p\tilde{D}, v w q_{RF}\}]}{e^{\psi_R \Delta t_s}}$ we can plug into the former. We get

$$\begin{aligned} \pi_{RF}^S = & (1 - \rho_S) \left((1 - \rho_R) E[\min\{p\tilde{D}, wq_{RF}\}] - (1 - \rho_R) \frac{(e^{\psi_R \Delta t_s} - 1)}{e^{\psi_R \Delta t_s}} E[\min\{p\tilde{D}, v w q_{RF}\}] \right. \\ & \left. - (cq_{RF} + (1 - \lambda_{RF})d\theta_{RF}^2)e^{(r_f + \psi_S)\Delta t_p} \right) \end{aligned}$$

And we can observe the supplier’s profit does not depend on γ , and only depends directly on v . Now, because

$$E[\min\{p\tilde{D}, v w q_{RF}\}] = p E\left[\min\left\{\tilde{D}, v \frac{w}{p} q_{RF}\right\}\right] = p\beta\theta_{BF} + \int_0^{v \frac{w}{p} q_{BF} - \beta\theta_{BF}} \bar{F}(\xi) d\xi,$$

We find that $E[\min\{p\tilde{D}, v w q_{RF}\}]$ is increasing function of v and consequently the expected profit function π_{RF}^S decreases in v (i.e., $\frac{\partial \pi_{RF}^S}{\partial v} < 0$).

Appendix A.4. Proof of Proposition 2.

By similar arguments as in the proof of Proposition 1, we obtain the results as presented in Proposition 2. We omit the details for brevity.

Appendix A.5. Proof of Proposition 3.

In the Proof of Proposition 3, to simplify notations, we omit the subscript ‘BF’ for the cost sharing parameter, the retailer’s order quantity and the supplier’s CSR effort. When the demand D_0 is uniformly distributed on the support $[A - M; A + M]$ with A and M two known parameters such that $0 < M \leq A$, then its expected value is A , its variance is $M^2/3$, its probability density function is $f[x] = \frac{1_{\{A-M \leq x \leq A+M\}}}{2M}$, the cumulative distribution function is $F[x] = \frac{(x-(A-M))1_{\{A-M \leq x \leq A+M\}}}{2M} + 1_{\{A+M < x\}}$, the inverse cumulative distribution function is $F^{-1}[y] = 2My + (A - M)$, for $y \in (0,1)$ the complementary cumulative distribution function or tail function is $\bar{F}[x] = 1 - F[x] = 1_{\{x < A-M\}} + \frac{[2M-(x-(A-M))]1_{\{A-M \leq x \leq A+M\}}}{2M}$ and the inverse complementary cumulative distribution function is $\bar{F}^{-1}[y] = 2M + (A - M) - 2My = A + M - 2My$, for $y \in (0,1)$. Therefore, the demand with CSR \tilde{D} has the lowest value $A - M + \beta\theta \equiv D_{\min}$. And, the expression for the profit of the retailer under CSBF (see Equation (4)) shows it is worth considering a couple of situations depending on whether wq is above or below the lowest possible revenue pD_{\min} .

Scenario 1 : When $pD_{\min} > wq$ is possible, then $E[\min\{p\tilde{D}, wq\}] = wq$. In this situation, the profit of the supplier (see Equation (4)) can be written

$$\begin{aligned} \pi_{BF}^S &= (1 - \rho_S)(1 - \rho_R)wq - (cq + (1 - \lambda)d\theta^2)\Omega_{BF} \\ &= ((1 - \rho_S)(1 - \rho_R)w - c\Omega_{BF})q - (1 - \lambda)d\theta^2\Omega_{BF} \\ &= (m_S q - (1 - \lambda)d\theta^2)\Omega_{BF} \end{aligned} \tag{A4.1}$$

Where, recall that $m_S = w(1 - \rho_S)(1 - \rho_R)\Omega_{BF}^{-1} - c$.

The profit function of the retailer can be written as

$$\begin{aligned} \pi_{BF}^R &= (1 - \rho_R)(E[p \min\{\tilde{D}, q\}] - wq - \lambda d\theta^2) \\ &= (1 - \rho_R)(p(q + \int_0^q Df_{\tilde{D}}[D] dD - qF_{\tilde{D}}[q]) - wq - \lambda d\theta^2) \end{aligned} \tag{A4.2}$$

$$= (1 - \rho_R) \left(m_R q - \frac{p(q - A + M - \beta\theta)^2}{4M} - \lambda d\theta^2 \right) \quad (\text{A4.3})$$

Where, $m_R = p - w$.

Taking the first and second order derivatives of π_{RF}^R with respect to q then gives $\frac{\partial \pi_{RF}^R}{\partial q} = (1 - \rho_R) \left(m_R - \frac{p(q - A + M - \beta\theta)}{2M} \right)$ and $\frac{\partial^2 \pi_{RF}^R}{\partial q^2} = -\frac{p}{2M} (1 - \rho_R) < 0$. The second order derivative being negative, the profit function of the retailer is concave w.r.t. q and the optimal quantity is

$$q^*(\theta) = 2M \left(\frac{m_R}{p} \right) + A - M + \beta\theta = \left(1 - 2\frac{w}{p} \right) M + A + \beta\theta. \quad (\text{A4.4})$$

Observe for later use that $q^*(\theta) - (A - M + \beta\theta) = 2\frac{m_R}{p}M$. Now, substituting this optimal quantity into the retailer's profit (A5.3) gives

$$\begin{aligned} \pi_{BF}^R &= (1 - \rho_R) \left(m_R q^*(\theta) - \frac{p \left(2\frac{m_R}{p}M \right)^2}{4M} - \lambda d\theta^2 \right) \\ &= (1 - \rho_R) \left(m_R \left(q^*(\theta) - M\frac{m_R}{p} \right) - \lambda d\theta^2 \right) \end{aligned} \quad (\text{A4.5})$$

Now, substituting this linear quantity $q^*(\theta)$ into the profit function of the supplier (A4.1) gives

$$\pi_{BF}^S = \left(m_S \left(2M \left(\frac{m_R}{p} \right) + A - M + \beta\theta \right) - (1 - \lambda)d\theta^2 \right) \Omega_{BF}$$

a concave quadratic function of θ , whose first and second derivatives are $\frac{\partial \pi_{BF}^S}{\partial \theta} = (m_S\beta - 2(1 - \lambda)d\theta)\Omega_{BF}$ and $\frac{\partial^2 \pi_{BF}^S}{\partial \theta^2} = -2(1 - \lambda)d\Omega_{BF}$. The second-order derivative is negative, so that the profit function of the supplier is concave and the associated optimal effort provided by the FOC is

$$\theta^*(\lambda) = \frac{m_S}{2d(1 - \lambda)}\beta.$$

Hence, the condition $\{pD_{\min} > wq\}$ translates into

$$p(A - M + \beta\theta^*(\lambda)) > w \left((1 - 2w/p)M + (A + \beta\theta^*(\lambda)) \right)$$

And then

$$M < \frac{A + \beta\theta^*(\lambda)}{1 + 2w/p} = \frac{A}{1 + 2w/p} + \frac{m_S\beta^2}{2d(1 + 2w/p)(1 - \lambda)}$$

Now, by plugging $\theta^*(\lambda)$ and $q^*(\theta)$ into the profits of the retailer and supplier, one finds for the retailer's profit function

$$\pi_{BF}^R = (1 - \rho_R) \left(m_R \left(\frac{Mm_R}{p} + A - M + \frac{m_S\beta^2}{2d(1 - \lambda)} \right) - \frac{m_S^2\beta^2\lambda}{4d(1 - \lambda)^2} \right)$$

And for the profit function of the supplier $\pi_{RF}^S = m_S \left(\frac{2Mm_R}{p} + A - M + \frac{m_S\beta^2}{4d(1-\lambda)} \right) \Omega_{BF}$. By taking the first and second-order derivatives of π_{BF}^R with respect to λ one gets $\frac{\partial \pi_{BF}^R}{\partial \lambda} = -\frac{m_S\beta^2(m_S(1+\lambda)-2m_R(1-\lambda))}{4d(1-\lambda)^3}$ and $\frac{\partial^2 \pi_{BF}^R}{\partial \lambda^2} = -\frac{m_S\beta^2(m_S(2+\lambda)-2m_R(1-\lambda))}{2d(1-\lambda)^4}$. Hence, the second-order derivative can be positive, negative or null depending on, say, the contribution level λ . Consequently, the profit function of the retailer is subtler. Actually, if λ is smaller (resp. larger) than $\check{\lambda} = \frac{m_R - m_S}{m_R + m_S/2}$ then the profit function is convex (resp. concave). Any optimal cost-sharing parameter relevant for the retailer is a non-negative λ^* , lower than one satisfying $\left. \frac{\partial \pi_{BF}^R}{\partial \lambda_{BF}} \right|_{\lambda=\lambda^*} = 0$ and one gets $\lambda^* = \frac{m_R - m_S/2}{m_R + m_S/2}$. Clearly, this optimal parameter is smaller than one (all involved parameters are positive) and it is strictly positive if the condition $m_R > \frac{m_S}{2}$ is met. The optimal cost sharing proportion is also larger than the level where the convexity changes (i.e., $\lambda^* > \check{\lambda}$). We thus summarize the above results in the proposition 3(a).

Scenario 2: When $pD_{\min} > wq$ is not possible, the profits of the supplier and the retailer in Equations (3) and (4) can be written

$$\begin{aligned}
\pi_{BF}^S &= (1 - \rho_S)(1 - \rho_R)E[\min\{p\check{D}, wq\}] - (cq + (1 - \lambda)d\theta^2)\Omega_{BF} \\
&= (1 - \rho_S)(1 - \rho_R)p \left(\frac{w}{p}q + \int_0^{\frac{w}{p}q} Df_{\check{D}}[D] dD - \frac{w}{p}qF_{\check{D}}\left[\frac{w}{p}q\right] \right) \\
&\quad - (cq + (1 - \lambda)d\theta^2)\Omega_{BF} \\
&= m_Sq\Omega_{BF} - \frac{(qw - p(A - M + \beta\theta))^2}{4Mp} (1 - \rho_S)(1 - \rho_R) - d\theta^2(1 - \lambda)\Omega_{BF}. \\
&= \left(m_Sq - \frac{(qw - p(A - M + \beta\theta))^2}{4Mp} - \frac{(m_S + c)}{w} - d\theta^2(1 - \lambda) \right) \Omega_{BF}
\end{aligned} \tag{A4.6}$$

And, by the same lines of reasoning

$$\begin{aligned}
\pi_{BF}^R &= (1 - \rho_R)(E[p \min\{\check{D}, q\}] - E[\min\{p\check{D}, wq\}] - \lambda d\theta^2) \\
&= (1 - \rho_R) \left(m_Rq + p \left(\frac{1}{2M} \int_{A-M+\beta\theta}^q D dD - qF_{\check{D}}[q] \right) \right. \\
&\quad \left. - \frac{1}{2M} \int_{A-M+\beta\theta}^{\frac{w}{p}q} D dD + \frac{w}{p}qF_{\check{D}}\left[\frac{w}{p}q\right] \right) - \lambda d\theta^2 \\
&= (1 - \rho_R) \left(m_Rq \left(\frac{A + M + \beta\theta}{2M} - \frac{q \left(1 + \frac{w}{p} \right)}{4M} \right) - \lambda d\theta^2 \right)
\end{aligned} \tag{A4.7}$$

The first and the second-order derivatives of the retailer's profit π_{BF}^R (A4.7) with respect to q are $\frac{\partial \pi_{BF}^R}{\partial q} = (1 - \rho_R) m_R \left(\frac{A+M+\beta\theta}{2M} - \frac{q(1+w/p)}{2M} \right)$ and $\frac{\partial^2 \pi_{BF}^R}{\partial q^2} = -\frac{m_R(1+w/p)}{2M} (1 - \rho_R)$ respectively. The second-order derivative is negative, the profit function π_{BF}^R is concave in q and the FOC leads to an optimal quantity

$$q^*(\theta) = \frac{A + M + \beta\theta}{(1 + w/p)}.$$

Substituting $q^*(\theta)$ into (A4.6) and then taking the first and second-order derivatives of π_{BF}^S with respect to θ lead to respectively $\frac{\partial \pi_{BF}^S}{\partial \theta} = \Omega_{BF} \left(\frac{m_S \beta}{(1+w/p)} - \frac{p^{(A+M)-2M(1+w/p)+\beta\theta} \beta (m_S+c)}{2M(1+w/p)^2 w} - 2(1-\lambda)d\theta \right)$ and $\frac{\partial^2 \pi_{BF}^S}{\partial \theta^2} = -\left(\frac{p\beta^2}{2M(1+w/p)^2} \frac{(m_S+c)}{w} + 2(1-\lambda)d \right) \Omega_{BF}$. This second order derivative is negative, the profit function π_{BF}^S is concave in θ and the FOC leads to

$$\begin{aligned} \theta^*(\lambda) &= \frac{(p(1-\rho_S)(1-\rho_R) + m_S \Omega_{BF}) 2M(1 + \frac{w}{p}) - p(A+M)(1-\rho_S)(1-\rho_R)}{p\beta^2(1-\rho_S)(1-\rho_R) + 4M(1-\lambda)d\Omega_{BF} \left(1 + \frac{w}{p}\right)^2} \beta \\ &= \frac{\left(\frac{p}{w}(m_S+c) + m_S\right) 2M \left(1 + \frac{w}{p}\right) - (A+M) \frac{p}{w} (m_S+c)}{\beta^2 \frac{p}{w} (m_S+c) + 4M(1-\lambda)d \left(1 + \frac{w}{p}\right)^2} \beta \end{aligned} \quad (A4.8)$$

The condition $p(A - M + \beta\theta^*(\lambda)) < w \left(\frac{A+M+\beta\theta^*(\lambda)}{(1+w/p)} \right)$ is converted to

$$M > \frac{1}{1 + 2w/p} \left(A + \frac{\beta^2 m_S/2}{d(1-\lambda_{RF}^*)(1+w/p)} \right)$$

To find the appropriate cost-sharing parameter, it is worth reconsidering the profit function of the retailer in (A4.7) given by

$$\pi_{BF}^R = (1 - \rho_R) \left(m_R \frac{(A + M + \beta\theta^*(\lambda))^2}{4M \left(1 + \frac{w}{p}\right)} - \lambda d \theta^*(\lambda)^2 \right)$$

and rewriting

$$\theta^*(\lambda) = \frac{\gamma_1}{p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda)}$$

with $\gamma_1 = \beta \left[(p(1-\rho_S)(1-\rho_R) + m_S \Omega_{BF}) 2M(1 + \frac{w}{p}) - p(A+M)(1-\rho_S)(1-\rho_R) \right]$

(which can be positive or negative) and $\gamma_2 = 4M(1 + w/p)^2 d \Omega_{BF}$ (which is positive whatever

the parameters). Of course, one has $\frac{\partial \theta^*(\lambda)}{\partial \lambda} = \frac{\gamma_1 \gamma_2}{(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda))^2}$. The (total) first-order derivative of the retailer's profit function to the cost-sharing parameter is then

$$\frac{d\pi_{BF}^R}{d\lambda} = \frac{\partial \pi_{BF}^R}{\partial \lambda} + \frac{\partial \pi_{BF}^R}{\partial \theta^*(\lambda)} \frac{\partial \theta^*(\lambda)}{\partial \lambda}$$

$$\text{With } \frac{\partial \pi_{BF}^R}{\partial \lambda} = -(1-\rho_R)d\theta^*(\lambda)^2, \quad \frac{\partial \pi_{BF}^R}{\partial \theta^*(\lambda)} = (1-\rho_R) \left(\frac{m_R(A+M+\beta\theta^*(\lambda))\beta}{2M(1+\frac{w}{p})} - 2\lambda d\theta^*(\lambda) \right) =$$

$$(\alpha_1 + \alpha_2\theta^*(\lambda) - 2\lambda d\theta^*(\lambda)), \text{ with } \alpha_1 = \frac{m_R(A+M)\beta}{2M(1+w/p)} \text{ and } \alpha_2 = \frac{m_R\beta^2}{2M(1+w/p)}.$$

One finds more explicitly

$$\begin{aligned} & \frac{d\pi_{BF}^R}{d\lambda} \frac{1}{(1-\rho_R)} \\ &= -d \left(\frac{\gamma_1}{p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda)} \right)^2 \\ &+ \left(\alpha_1 + \alpha_2 \frac{\gamma_1}{p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda)} \right. \\ &\left. - 2\lambda d \frac{\gamma_1}{p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda)} \right) \frac{\gamma_1 \gamma_2}{(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda))^2} \\ &= \gamma_1 \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda)) + \gamma_1 \gamma_2 \alpha_2 - 2\lambda \gamma_1 \gamma_2 d}{(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda))^3} \\ &= \gamma_1 \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2) + \gamma_1 \gamma_2 \alpha_2 - \lambda \gamma_2 (\alpha_1 \gamma_2 + \gamma_1 d)}{(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda))^3} \end{aligned}$$

Considering the second equality just above, the second derivative is found to be

$$\begin{aligned} \frac{\partial^2 \pi_{BF}^R}{\partial \lambda^2} &= \gamma_1 \gamma_2 \frac{3(\alpha_1 \gamma_2 - d\gamma_1)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2) + 3\gamma_1 \gamma_2 \alpha_2}{(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda))^4} \\ &\quad - \gamma_1 \gamma_2 (\alpha_1 \gamma_2 + \gamma_1 d) \frac{p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1+2\lambda)}{(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2(1-\lambda))^4} \end{aligned}$$

And, clearly, it is not possible in full generality to conclude on the concavity of π_{BF}^R with respect to λ . However, from the various numerical experiments we have conducted, we observe that the profit function of the retailer is concave (in λ) for many sets of parameters and most of those of interest for our setting. Under such a concavity, the FOC (applied to the third equality of the first-order derivative) leads to one candidate for the cost-sharing parameter that is equal to

$$\hat{\lambda} = \frac{(\alpha_1 \gamma_2 - \gamma_1 d)(p\beta^2(1-\rho_S)(1-\rho_R) + \gamma_2) + \gamma_1 \gamma_2 \alpha_2}{\gamma_2(\alpha_1 \gamma_2 + d\gamma_1)}.$$

Of course, the solution cost-sharing parameter must be non-negative and lower than 1, otherwise it is simply irrelevant for decision-making. If the above candidate $\hat{\lambda}$ is lower than 0, then the optimal cost-sharing to choose is $\lambda_{BF}^* = 0$. Otherwise and assuming that $\hat{\lambda} < 1$, the optimal cost sharing is $\lambda_{BF}^* = \hat{\lambda}$. Once, λ_{BF}^* is determined, all previous expressions can be determined.

Appendix A.6. Proof of Proposition 4.

By similar arguments as in the proof of Proposition 3, we obtain the results as presented in Proposition 4. We omit the details for brevity.

Appendix B: Additional Numerical Simulations

Appendix B.1. Equilibrium under RF and BF for different scenarios (without cost-sharing)

Table B1.1. Results for scenario 1

ρ_S	BF				RF			
	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
0	16.11	128.80	293.22	324.97	16.11	128.81	293.22	325.02
0.05	15.02	124.30	258.92	308.29	16.11	128.81	278.55	325.02
0.1	13.94	119.90	226.00	292.02	16.11	128.81	263.89	325.02
0.15	12.88	115.70	194.46	276.31	16.11	128.81	249.23	325.02
0.2	11.84	111.60	164.26	261.18	16.11	128.81	234.57	325.02
0.25	10.81	107.70	135.39	246.48	16.11	128.81	219.91	325.02
0.3	9.80	103.90	107.85	232.42	16.11	128.81	205.25	325.02
0.35	8.80	100.20	81.60	218.69	16.11	128.81	190.59	325.02
0.4	7.80	96.61	56.66	205.39	16.11	128.81	175.93	325.02
0.45	6.82	93.12	33.01	192.51	16.11	128.81	161.27	325.02
0.5	5.83	89.72	10.67	180.01	16.11	128.81	146.61	325.02

Table B1.2. Results for scenario 2

ψ_S	BF				RF			
	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
0.05	13.94	119.90	226.00	292.02	16.11	128.81	263.89	325.02
0.06	13.76	119.20	222.65	289.33	16.11	128.81	263.89	325.02
0.07	13.58	118.50	219.34	286.65	16.11	128.81	263.89	325.02
0.08	13.41	117.80	216.06	284.13	16.11	128.81	263.89	325.02
0.09	13.24	117.10	212.82	281.62	16.11	128.81	263.89	325.02
0.1	13.07	116.50	209.62	279.11	16.11	128.81	263.89	325.02
0.11	12.91	115.80	206.45	276.75	16.11	128.81	263.89	325.02
0.12	12.75	115.20	203.32	274.41	16.11	128.81	263.89	325.02
0.13	12.60	114.60	200.22	272.21	16.11	128.81	263.89	325.02
0.14	12.44	114.00	197.15	269.88	16.11	128.81	263.89	325.02
0.15	12.29	113.40	194.11	267.69	16.11	128.81	263.89	325.02

Table B1.3. Results for scenario 3

ψ_R	BF				RF			
	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
0	13.95	119.00	226.00	292.09	17.22	133.40	281.94	342.21
0.005	13.95	119.00	226.00	292.09	17.10	132.90	280.07	340.34
0.01	13.95	119.00	226.00	292.09	16.99	132.40	278.23	338.62
0.015	13.95	119.00	226.00	292.09	16.87	131.90	276.39	336.75
0.02	13.95	119.00	226.00	292.09	16.76	131.50	274.57	335.04
0.025	13.95	119.00	226.00	292.09	16.65	131.00	272.76	333.33
0.03	13.95	119.00	226.00	292.09	16.54	130.50	270.96	331.62
0.035	13.95	119.00	226.00	292.09	16.43	130.10	269.18	329.92
0.04	13.95	119.00	226.00	292.09	16.32	129.60	267.40	328.21
0.045	13.95	119.00	226.00	292.09	16.21	129.20	265.64	326.52
0.05	13.95	119.00	226.00	292.09	16.11	128.80	263.89	324.97

Table B1.4. Results for scenario 4

σ	BF				RF			
	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
10	13.05	108.76	213.66	317.83	15.54	120.40	246.75	362.20
15	13.05	108.80	213.66	317.21	15.55	120.80	247.75	361.20
20	13.08	110.10	213.79	315.46	15.56	121.70	248.75	358.07
25	13.20	111.40	215.46	306.42	15.59	122.26	250.85	346.39
30	13.39	113.60	218.35	299.31	15.70	123.71	254.32	336.74
35	13.65	116.50	221.89	294.81	15.87	125.90	258.76	329.64
40	13.94	119.90	225.93	292.01	16.11	128.80	263.90	324.97
45	14.26	123.73	230.26	290.68	16.38	132.13	269.36	321.94
50	14.50	127.75	234.58	289.22	16.67	135.80	274.99	320.00

Appendix B.2. Equilibrium under RF and BF under the cost sharing contract for different scenarios

Table B2.1. Results for scenario 1

ρ_S	BF					RF				
	λ_{BF}^*	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	λ_{RF}^*	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
0	0.41	26.40	173.90	383.63	363.85	0.41	26.40	173.97	383.84	363.85
0.05	0.44	25.63	170.40	346.65	349.36	0.41	26.40	173.97	364.65	363.85
0.1	0.47	24.86	166.90	310.51	335.32	0.41	26.40	173.97	345.45	363.85
0.15	0.50	24.09	163.40	275.20	321.73	0.41	26.40	173.97	326.26	363.85
0.2	0.54	23.32	160.00	240.98	308.58	0.41	26.40	173.97	307.07	363.85
0.25	0.58	22.55	156.50	207.41	295.88	0.41	26.40	173.97	287.88	363.85
0.3	0.62	21.78	153.10	174.90	283.62	0.41	26.40	173.97	268.69	363.85
0.35	0.66	21.01	149.70	143.26	271.80	0.41	26.40	173.97	249.49	363.85
0.4	0.70	20.23	146.30	112.55	260.43	0.41	26.40	173.97	230.30	363.85
0.45	0.75	19.47	143.00	82.73	249.49	0.41	26.40	173.97	211.11	363.85
0.5	0.80	18.70	139.70	53.85	238.99	0.41	26.40	173.97	191.92	363.85

Table B2.2. Results for scenario 2

ψ_S	BF					RF				
	λ_{BF}^*	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	λ_{RF}^*	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
0.05	0.47	24.86	166.90	310.51	335.32	0.41	26.40	173.97	345.45	363.85
0.06	0.48	24.73	166.30	307.32	332.98	0.41	26.40	173.97	345.45	363.85
0.07	0.48	24.60	165.70	304.18	330.70	0.41	26.40	173.97	345.45	363.85
0.08	0.49	24.48	165.20	301.17	328.47	0.41	26.40	173.97	345.45	363.85
0.09	0.49	24.35	164.60	298.07	326.29	0.41	26.40	173.97	345.45	363.85
0.1	0.50	24.23	164.10	295.11	324.16	0.41	26.40	173.97	345.45	363.85
0.11	0.50	24.11	163.50	291.95	322.09	0.41	26.40	173.97	345.45	363.85
0.12	0.51	24.00	163.00	288.93	320.06	0.41	26.40	173.97	345.45	363.85
0.13	0.51	23.88	162.50	286.04	318.07	0.41	26.40	173.97	345.45	363.85
0.14	0.52	23.77	162.00	283.05	316.14	0.41	26.40	173.97	345.45	363.85
0.15	0.52	23.66	161.50	280.11	314.24	0.41	26.40	173.97	345.45	363.85

Table B2.3. Results for scenario 3

ψ_R	BF					RF				
	λ_{BF}^*	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	λ_{RF}^*	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
0	0.47	24.86	166.90	310.51	335.32	0.38	27.16	177.40	361.31	378.79
0.005	0.47	24.86	166.90	310.51	335.32	0.38	27.08	177.10	359.88	377.21
0.01	0.47	24.86	166.90	310.51	335.32	0.39	26.99	176.70	358.21	375.65
0.015	0.47	24.86	166.90	310.51	335.32	0.39	26.92	176.30	356.44	374.10
0.02	0.47	24.86	166.90	310.51	335.32	0.39	26.85	176.00	354.88	372.58
0.025	0.47	24.86	166.90	310.51	335.32	0.40	26.77	175.60	353.19	371.08
0.03	0.47	24.86	166.90	310.51	335.32	0.40	26.69	175.30	351.75	369.60
0.035	0.47	24.86	166.90	310.51	335.32	0.40	26.62	174.90	349.96	368.13
0.04	0.47	24.86	166.90	310.51	335.32	0.40	26.54	174.60	348.55	366.69
0.045	0.47	24.86	166.90	310.51	335.32	0.41	26.47	174.20	346.75	365.26
0.05	0.47	24.86	166.90	310.51	335.32	0.41	26.39	173.90	345.35	363.85

Table B2.4. Results for scenario 4

σ	BF					RF				
	λ_{BF}^*	θ_{BF}^*	q_{BF}^*	π_{BF}^{S*}	π_{BF}^{R*}	λ_{RF}^*	θ_{RF}^*	q_{RF}^*	π_{RF}^{S*}	π_{RF}^{R*}
10	0.51	26.57	177.80	309.44	428.17	0.44	27.86	184.20	340.55	459.66
15	0.51	26.58	176.60	307.33	410.71	0.44	27.87	183.00	338.45	442.17
20	0.51	26.52	175.20	307.07	393.33	0.44	27.85	181.80	338.30	424.64
25	0.50	26.33	173.50	308.31	376.56	0.44	27.71	180.30	339.72	407.47
30	0.50	25.99	171.40	308.81	360.99	0.43	27.43	178.40	341.61	391.28
35	0.49	25.49	169.20	310.62	347.14	0.42	26.98	176.20	343.71	376.61
40	0.47	24.86	166.90	310.09	335.32	0.41	26.39	173.90	345.35	363.85
45	0.45	24.16	164.90	310.58	325.64	0.39	25.71	171.80	346.40	353.20
50	0.42	23.43	163.30	308.60	318.04	0.37	24.99	170.00	346.41	344.67

Appendix C: Additional Figure

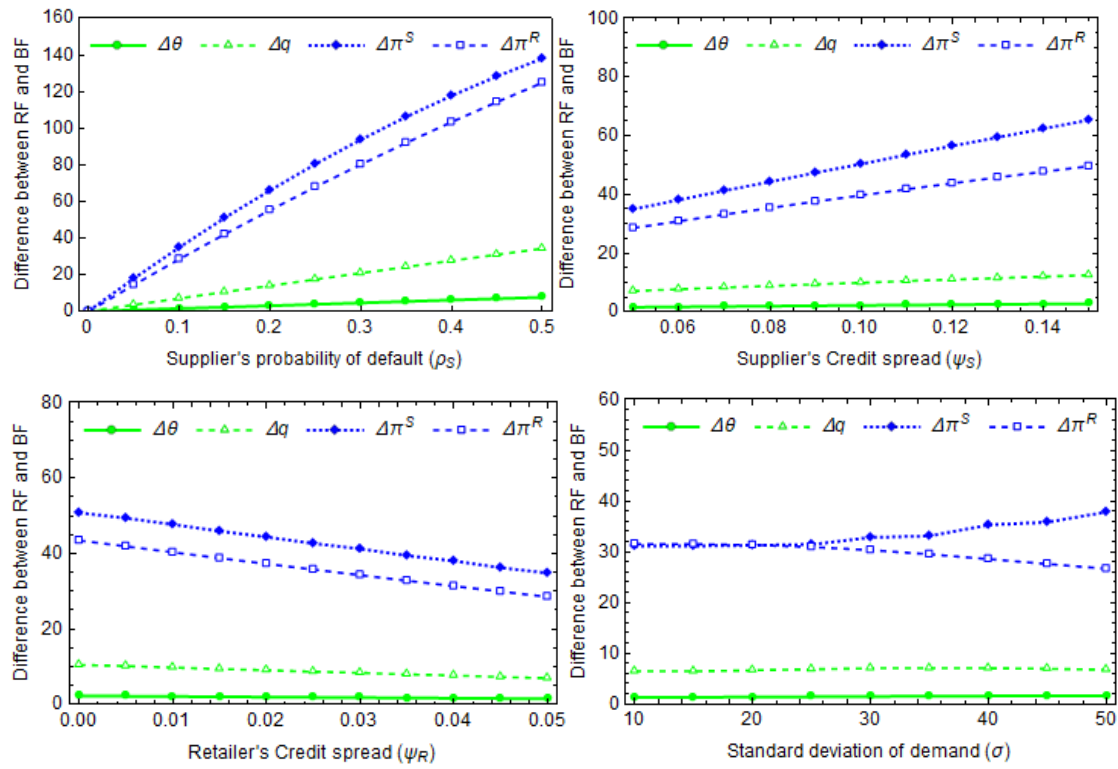


Fig. 7. Impact of parameters on the difference of outcomes between RF and BF when the cost sharing contract is adopted