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Prognostic-Based Maintenance Optimization For Complex Systems

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Abstract—This paper is concerned with prognostic information for maintenance and production optimization in complex systems. In each stage of such a system, we allow redundant components used as backup to ensure system’s availability. Remaining Useful Life (RUL, the so-called prognostic information) of components is used to evaluate each component’s redundancy. We address RUL-based maintenance and production optimization to guarantee the availability and productivity of the system such that client demands can be satisfied in a given planning horizon. We propose a mixed-integer linear programming model to minimize the total cost. Experimental results on test instances show the efficiency of the proposed approach to attain optimal solutions.

Keywords—Predictive maintenance, Production, Optimization, Complex system, Remaining useful life

I. INTRODUCTION

Industrial systems generally degrade due to different factors, which may eventually cause serious economic problems for companies. Maintenance is a widely used essential element in asset management to reduce the speed of degradation [6]. Unfortunately, traditional maintenance decisions for single-component systems are not appropriate for contemporary complex systems. Complex systems are the systems that commonly consist of multiple components with various interactions [17]. Industrial and academic researchers have thus been focusing on proposing effective maintenance optimization strategies for such complex systems.

In real-life applications, maintenance can be mainly classified as *Corrective Maintenance* (CM) and *Preventive Maintenance* (PM). CM is carried out when a component has broken down, while PM happens in advance to reduce the degradation speed and avoid a sudden failure. As a supplement to maintenance, configuring redundant components is another practical way to enhance the flexibility of industrial systems, meaning that some components are on-working while the others are

on standby [14]. Redundant components can easily take the operational responsibility whenever on-working components reach a warning condition. The managers of complex systems, therefore, need optimally arrange this redundancy to reduce system unavailability.

To make proper maintenance decisions, an interesting approach is soliciting the prognostic information of components, such as the *Remaining Useful Life* (RUL) [4]. The definition of component’s RUL is the currently remaining time of operation before it fails. As we use this kind of predictive information, the focus of our work is on using the obtained component-level RUL information to plan PM optimization in order to achieve system-level availability in complex systems.

The considered complex systems have a series of stages and each stage contains multiple redundant components. The same type of structure has been considered in multi-process industries, such as gas production [15] [14]. The overall aim consists in using prognostic information, coordinating the operations in different stages, and providing global predictive maintenance decisions such that the system can operate continuously in the planning horizon to satisfy client demands (refer to Figure 1).

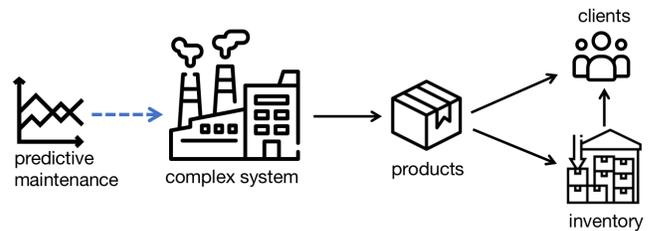


Figure 1. Introduction of the studied problem.

In summary, we address a *RUL-Based Maintenance and Production Optimization* (RMPO) problem in complex systems. The objective is to minimize the total cost over a planning optimization horizon, including maintenance cost, system-failure cost, inventory expense, and a penalty for non-met production. The contributions of our work are as follows:

- We study a predictive maintenance and production optimization problem to minimize the total cost in complex systems. Our purpose is to keep the operation of the system to satisfy client demands.
- We establish a *Mixed-Integer Linear Programming* (MILP) formulation for this problem, and we test the model by an illustrative example.

The remaining of this paper is organized as follows. In Section II, we review the previous works to highlight and position our contributions. In Section III, we mathematically formulate the considered RMPO and present the MILP model. An illustrative example is conducted and the results are reported and analyzed in Section IV. Section V is dedicated to the conclusion and future research directions.

II. RELATED WORKS

In this section, we review generally maintenance in complex systems and corresponding solution methods.

A. Maintenance within Complex Systems

Firstly, we focus on the main contributions in the maintenance literature related to RUL usage, system availability, and integrated problems.

RUL is one of the essential indices that reflects the status of a component. It can be either obtained by defining the time length from the current time to the end-of-life. Or more frequently, it is defined as the time left before the health condition reaches a warning threshold [12]. More than 270 papers have studied RUL prediction [9]. We focus on RUL usage in our work and distinguish three branches in the literature. (i) RUL-based inspection: Do et al. [8] used RUL information for deciding the time point for the next coming inspection. (ii) RUL-based maintenance strategies: Chen et al. [5] proposed different maintenance actions via combinations of degradation and RUL. (iii) RUL-based constraints: Camci et al. [4] used prognostic information to formulate probability constraints related to the failure rate of a component.

For describing system-level availability by component condition, Wu and Castro [13] proposed a linear combination of the degradation processes of several components. If this value exceeded a given threshold, PM was performed. Lei and Sandborn [10] proposed a prognostic health analysis to predict the RUL of wind turbines. The authors assumed that turbines were dependent and system availability relied on the minimum RUL among them. Dong et al. [7] assumed that normal-distributed shocks occurred independently and described system reliability by conditioning on the numbers of arrived shocks.

Integrated problems combine maintenance in complex systems with other scopes to make global decisions. One of the

mainstream approaches is to simultaneously take maintenance and resource into account, such as spare part ordering. Camci [3] proposed CM and spare part inventory strategies using the given prognostic information to minimize the failure risk. A genetic algorithm was proposed to solve the problem and computational results were compared with the ones via PM strategy. Numerous papers have considered spare part ordering, see the invited review [6]. The integration of maintenance and production is also an important branch because maintenance activities eventually impact production. Bahria et al. [2] developed an integrated approach to control production, maintenance, and quality for manufacturing. Appropriate thresholds for conducting maintenance were discussed to guarantee the robustness of the system.

From the literature, we observe a lack of RUL usage and mathematical modeling via RUL-based constraints. Moreover, the influence of individual RUL information on complex systems is seldom discussed. Hence, and to the best of our knowledge, the integrated optimization of maintenance and production in series-structured systems with backup components and resource limitation has not been studied. In our work, we address maintenance optimization for complex systems considering *standby components* and *resource limitation*. The optimization aims to guarantee the continuous operation of the system. This is achieved by integrating *component-level RUL information* in the formulation.

B. Maintenance Optimization Methods

Many solution methods based on optimization for maintenance problems in complex systems have been proposed. In the following, we discuss some of the related and recent references and clearly situate our contribution compared to these approaches.

In the mentioned Camci [3], genetic algorithms were proposed to solve the problems. Rivera-Gómez et al. [11] presented a continuous production system with quality deterioration. The objective was to reduce the occurred cost with a quality constraint. A non-linear programming model was formulated for the problem. Zhou et al. [16] proposed an optimal PM policy with the purpose to get operational parameters for a production line. A non-linear model and a heuristic were designed to minimize the cost and guarantee the operating speed. Compared to the widely used non-linear formulation and (meta-) heuristics, linear formulation for maintenance optimization is very limited. In this research, we formulate a *MILP models* for the considered RMPO problems, with the purpose to provide optimal maintenance decisions.

III. PROBLEM DESCRIPTION AND FORMULATION

In this section, we first describe the problem in a mathematical way and then formulate a MILP model for the problem. Before this, for sake of clarity, we present firstly the nomenclature of parameters and variables.

Problem sets

- K : set of stages in the system;
- J_k : set of components in stage k ;

- T : set of periods.

Parameters:

- cap : production capacity of the system each period;
- $a_{k,j}$: coefficient in the RUL function of component j in stage k ;
- $b_{k,j}$: constant in the RUL function of component j in stage k ; then the RUL function can be established by $b_{k,j} - a_{k,j} \cdot t$;
- $o_{k,j}$: original RUL of component j in stage k ;
- γ_k : RUL threshold of components in stage k ;
- p_k : maintenance duration of components in stage k ;
- d^t : client demands in period t ;
- c^M : unit maintenance cost;
- c^{FL} : unit failure cost;
- c^{Inv} : unit inventory cost;
- c^{Loss} : unit production-loss cost.

Decision variables:

- $x_{k,j}^t$: binary variable, equals to 1 if component j in stage k needs to be maintained in period t , 0 otherwise;
- $y_{k,j}^t$: binary variable, equals to 1 if component j in stage k is working in period t , 0 otherwise;
- $R_{k,j}^t$: RUL of component j in stage k in period t ;
- ST_k^t : binary variable, equals to 1 if stage k is working in period t , 0 otherwise;
- SY^t : binary variable, equals to 1 if the system is working in period t , 0 otherwise;
- $quat^t$: production quantity in period t ;
- inv^t : inventory in period t . Note that there is no inventory at the beginning of the horizon; - z^t : binary variable, equals to 1 if the inventory is positive in period t , 0 otherwise.

A. RMPO Problem Description

For describing the studied RMPO problem, we first present the system structure in Figure 2. To be specific, a generic complex system consists of a series of $|K|$ processing stages where K denotes the stage set. One stage k has a redundancy of $|J_k|$ components where J_k represents the component set in stage k . Each component is under one of the following three conditions: working (in green), standby (in white), and maintenance (in grey), respectively. We suppose that each component is repairable, and that maintaining it does not affect the operation of a stage if there exist any available standby component. The length of the planning horizon is $|T|$ periods (weeks for example), where T is the period set. Our fundamental purpose is to satisfy client demands in this horizon. In the following, we describe main constrain sets for proposing the MILP model.

Evolution of RUL. As we have component redundancy in each process, therefore, we need to keep track of components' RUL over the optimization horizon. It is assumed that the prognostic RUL of component j in stage k follows a linear function $b_{k,j} - a_{k,j} \cdot t$, where $a_{k,j}$ and $b_{k,j}$ respectively denote the coefficient and constant. Note that RUL can be alternatively described using values, quantiles, or probabilities,

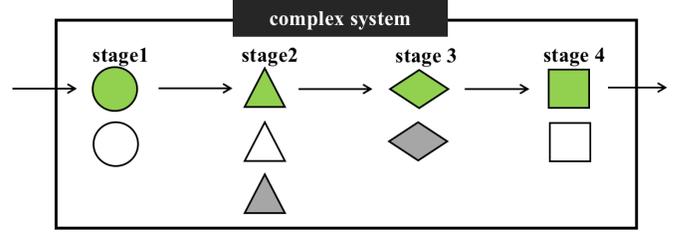


Figure 2. Structure of a generic complex system.

and we choose the first option in this research. As illustrated in Figure 3, for component 1 in stage 1, if it is operating in period 1, its RUL value (in the circles) decreases $a_{k,j} = 1$, from 8 to 7. If it is in standby, its RUL will not change. If it is under maintenance, its RUL will stay at the threshold $\gamma_k = 4$ until maintenance is carried out. Note that any component reaching the corresponding threshold can no longer operate and needs to be maintained. After maintenance, its RUL is restored to $b_{k,j} = 20$. The initial RUL of each component is given as $o_{k,j}$. We assume that threshold γ_k is provided by experts.

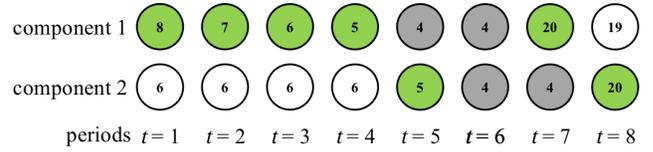


Figure 3. Evolution of RUL of redundant components.

System availability. The operation of a stage requires that at least one component (at the same stage) is working. If not, it eventually results in the unavailability of the system because it operates if and only if all stages are working.

Production and inventory. The system has a production of $quat^t$ in period t to satisfy demands d^t . However, its production capacity for each period is limited by Q . If the system cannot work during a period, there is no production and it may cause some production loss. To avoid demand unmet, we allow some possible stocks in preparation (if needed) to serve for subsequent client demands.

B. RMPO Formulation

The MILP model for the RMPO problem is formulated. The objective function (1) is a sum of the following four terms:

- C^M : total maintenance cost calculated by unit maintenance cost times the number of maintenance activities.
- C^{FL} : total failure cost computed by unit failure cost times the number of failures.
- C^{Inv} : total inventory cost computed by unit inventory cost times the amount of inventory.
- C^{Loss} : total production-loss cost computed by unit production-loss cost times negative inventory. Here negative inventory represents the quantity of unsatisfied demands in a period.

$$\begin{aligned}
(\text{M1}) &: \min C^M + C^{FL} + C^{Inv} + C^{Loss} \quad (1) \\
C^M &= c^M \cdot \sum_{k \in K} \sum_{j \in J_k} \sum_{t \in T} x_{k,j}^t \\
C^{FL} &= c^{FL} \cdot \sum_{t \in T} (1 - SY^t) \\
C^{Inv} &= c^{Inv} \cdot \sum_{t \in T} z^t \cdot inv^t \\
C^{Loss} &= c^{Loss} \cdot \sum_{t \in T} (1 - z^t) \cdot inv^t
\end{aligned}$$

Note that the non-linear term $z^t \cdot inv^t$ can be linearized by introducing an auxiliary variable $\mu^t = z^t \cdot inv^t$ with two additional constraints $inv^t \leq \mu^t + \bar{M} \cdot (1 - z^t)$ and $0 \leq \mu^t + \bar{M} \cdot z^t$. However, we chose to let CPLEX handle these terms automatically [1]. The optimization of formula (1) is under the following sets of constraints:

Evolution of RUL constraints. This constraint group is formulated to track the RUL of components. To be more specific, a maintenance is required when the RUL of a component is no bigger than the corresponding threshold, which is guaranteed by constraints (2) and (3). If a component is under maintenance, its unavailable time respects the maintenance duration, which is calculated by constraints (4). Constraints (5) restricts that only one component can be working in each stage if there is a need. Evolution of RUL contains three cases: (i) if a component is maintained, its RUL will be restored to a given value, that is, $R_{k,j}^{t+1} = b_{k,j}$ if $x_{k,j}^t = 1$. Please refer to constraints (6) to (7); (ii) If a component is being used in a period, its RUL will decrease respecting the given RUL function, that is, $R_{k,j}^{t+1} = R_{k,j}^t - a_{k,j}$ if $x_{k,j}^t = 0$ and $y_{k,j}^t = 1$, which is calculated by constraints (8) to (9); (iii) The RUL of standby components will not change, that is, $R_{k,j}^{t+1} = R_{k,j}^t$ if $x_{k,j}^t = 0$ and $y_{k,j}^t = 0$, which is established by (10) to (11).

$$R_{k,j}^t > \gamma_k - \bar{M} \cdot x_{k,j}^t, \forall k \in K, j \in J_k, t \in T \quad (2)$$

$$R_{k,j}^t \leq \gamma_k + \bar{M} \cdot (1 - x_{k,j}^t), \forall k \in K, j \in J_k, t \in T \quad (3)$$

$$\begin{aligned}
&\sum_{t' \in [t, t+p_k-1]} y_{k,j}^{t'} \leq 1 - x_{k,j}^t, \\
&\forall k \in K, j \in J_k, t \in [1, |T| - p_k + 1] \quad (4)
\end{aligned}$$

$$\sum_{j \in J_k} y_{k,j}^t \leq 1, \forall k \in K, t \in T \quad (5)$$

$$R_{k,j}^{t+1} \geq b_{k,j} - \bar{M} \cdot (1 - x_{k,j}^t), \forall k \in K, j \in J_k, t \in [1, |T| - 1] \quad (6)$$

$$R_{k,j}^{t+1} \leq b_{k,j} + \bar{M} \cdot (1 - x_{k,j}^t), \forall k \in K, j \in J_k, t \in [1, |T| - 1] \quad (7)$$

$$R_{k,j}^{t+1} \geq R_{k,j}^t - a_{k,j} - \bar{M} \cdot (1 - y_{k,j}^t), \forall k \in K, j \in J_k, t \in [1, |T| - 1] \quad (8)$$

$$R_{k,j}^{t+1} \leq R_{k,j}^t - a_{k,j} + \bar{M} \cdot (1 - y_{k,j}^t), \forall k \in K, j \in J_k, t \in [1, |T| - 1] \quad (9)$$

$$R_{k,j}^{t+1} \geq R_{k,j}^t - \bar{M} \cdot (x_{k,j}^t + y_{k,j}^t), \forall k \in K, j \in J_k, t \in [1, |T| - 1] \quad (10)$$

$$R_{k,j}^{t+1} \leq R_{k,j}^t + \bar{M} \cdot (x_{k,j}^t + y_{k,j}^t), \forall k \in K, j \in J_k, t \in [1, |T| - 1] \quad (11)$$

System availability constraints. This set of valid inequalities describes stage availability and further system availability for each period via the components' RUL information. The premise that a stage operates normally is that at least one component in the stage is working, i.e., $\exists y_{k,j}^t = 1$. To this end, the availability of a stage can be described by constraints (12). For system availability, it strictly requires that all the stages are available, i.e., $\forall ST_k^t = 1$, which is expressed by constraints (13) and (14).

$$ST_k^t \leq \sum_{j \in J_k} y_{k,j}^t, \quad \forall k \in K, t \in T \quad (12)$$

$$\frac{1}{|K|} \sum_{k \in K} ST_k^t \leq SY^t + \sum_{k \in K} (1 - ST_k^t), \quad \forall t \in T \quad (13)$$

$$ST_k^t \geq SY^t, \quad \forall k \in K, t \in T \quad (14)$$

Production integration constraints. These constraints describe how system availability impacts production and inventory. Constraints (15) provide the upper bound of production quantity in each period respecting system production capacity, while constraints (16) give the lower bound because a system cannot produce if any one of the stages is not active. The inventory is calculated by the sum of production quantity and the inventory in the last period minus the currently total demands (constraints (17)). Note that the inventory can be negative if the maintenance leads to a non-working state for the system or production capacity is surpassed. Then, constraints (18) and (19) record that the on-hand inventory is non-negative and negative, respectively, meaning that the inventory is enough or not for satisfying client demands.

$$qua^t \leq cap \cdot ST_k^t, \quad \forall k \in K, t \in T \quad (15)$$

$$qua^t \geq \sum_{j \in J_k} y_{k,j}^t, \quad \forall k \in K, t \in T \quad (16)$$

$$inv^t = qua^t + inv^{t-1} - d^t, \quad \forall t \in T \quad (17)$$

$$inv^t \geq -\bar{M} \cdot (1 - z^t), \quad \forall t \in T \quad (18)$$

$$inv^t \leq \bar{M} \cdot z^t, \quad \forall t \in T \quad (19)$$

Note that the initial RUL of a component is equal to $o_{k,j}$ and there is no inventory at the beginning of the optimization planning horizon. The ranges of decision variables in this model are detailed as follows:

$$x_{k,j}^t, y_{k,j}^t, ST_k^t, SY^t, z^t \in \{0, 1\}, \forall k \in K, j \in J_k, t \in T \quad (20)$$

$$R_{k,j}^t, inv^t, qua^t \in \mathbb{Z}, \forall k \in K, j \in J_k, t \in T \quad (21)$$

IV. COMPUTATIONAL RESULTS

In this section, an illustrative example is presented to show the effectiveness of the proposed mathematical model. The tests are conducted on a computer with Core I7 and 8GB RAM system. The MILP models are solved using CPLEX 12.8 and coded in Python 3.7.8.

For this illustrative example, the computational results obtained by our proposed MILP model are concluded as follows and in Figures 4 and 5, respectively. The objective value, i.e., the total cost, is equal to 641, including maintenance cost of 150, inventory cost of 296, and production-loss cost 195. Notice that there is no system failure penalty as our model can appropriately arrange component redundancy and coordinate different processing stages, such that the system can operate

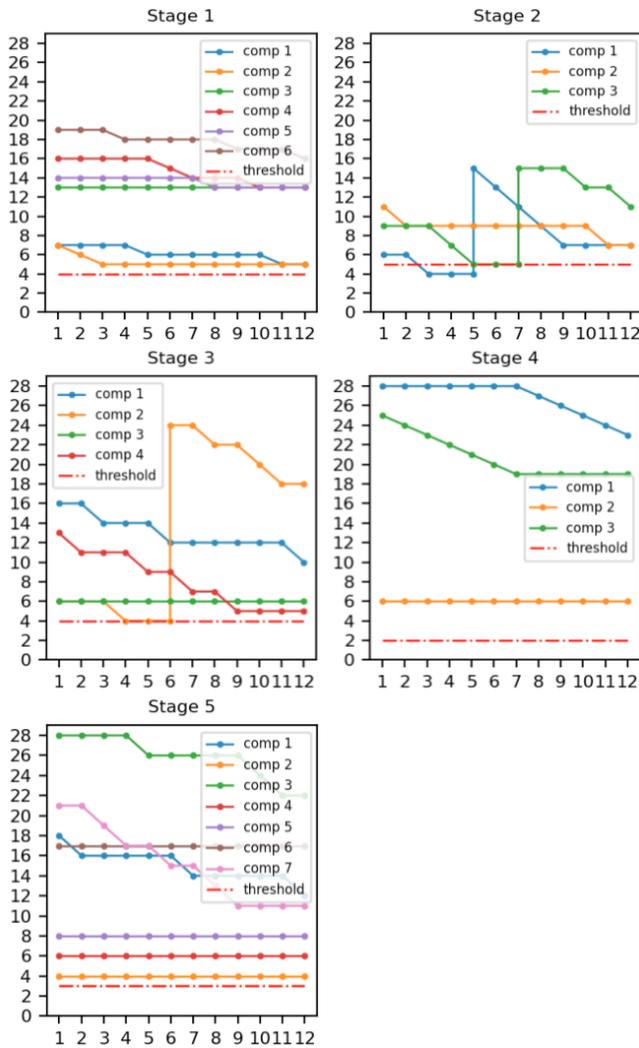


Figure 4. RUL track of this example.

continuously in the planning horizon. The computational time is less than one minute for this instance.

As for RUL track of components and maintenance decisions, we can directly observe in Figure 4. To be specific, we can see that there are 5 processing stages in this instance, and we know exactly how many redundant component in each stages. For example, there are 6 components belonging to stage 1. The RUL of a component will decrease if it is used in a period, at the same time, other components are in standby. The maintenance is conducted when component's RUL reaches the given threshold. Examples can be found for component 1 in stage 2 in period 3, component 3 in stage 2 in period 5, and component 2 in stage 3 in period 4. Once the maintenance is done (maintenance duration is 2 periods in this instance), component's RUL recovers to a given level.

We can also clearly see the production and inventory decisions over time through Figure 5. An interesting point is that the inventory in period 10 and period 12 are negative, this

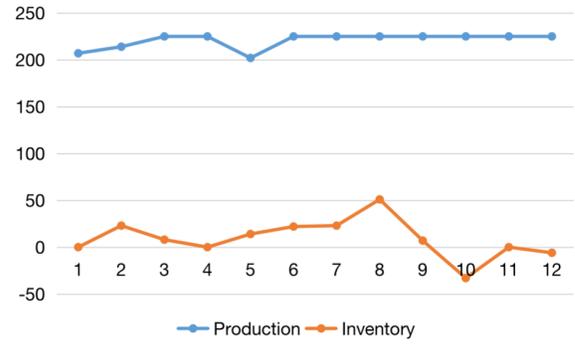


Figure 5. Production and inventory of this example.

provides a convince that production-loss cost appears in the objective. The reason is that even we can have some inventory in previous periods to satisfy demands, we prefer not to pay too much for inventory than the production-loss penalty.

V. CONCLUSIONS AND PERSPECTIVES

In this paper, we addressed RUL-based maintenance optimization in generic complex production systems. Component-level RUL information was used to arrange redundancy in each stage to guarantee the availability of the system. Besides, resource limitation constraints were integrated with respect to real-life applications and scenarios. The purpose is to satisfy client demands with minimum overall cost during the maintenance planning horizon. We provided a mixed-integer linear programming approach to cope with problem instances. Through different test instances, we showed the efficiency of our approach to reach the optimal solutions of the addressed problems in different complex systems.

Our future work will focus on the integrated maintenance of different complex systems located in different places.

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