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# Discrete Turbulent Spectrum Modelling for 2D Split-Step Electromagnetic Propagation Schemes

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**Abstract**—The multiple phase screen method is widely used for modelling the electromagnetic propagation in a turbulent medium. In this technique, the turbulent phase screens are classically generated from a continuous Von-Karman Kolmogorov scintillation spectrum. Recent works led to the development of an auto-coherent split-step wavelet propagation method based on a discrete formulation of the parabolic wave equation. In this configuration, the use of continuous spectra is no more suitable. In this paper, we propose an auto-coherent generation method of the turbulent phase screens. To do so, a discrete formulation of the classical Von-Karman Kolmogorov spectrum is introduced. The impact of the modelled turbulence is finally discussed through the computation of the log-amplitude variance to validate this approach.

**Index Terms**—split-step, auto-coherent, propagation, turbulence, wavelets

## I. INTRODUCTION

The tropospheric environment is characterized by unsteady temperature, pressure and humidity fields. Atmospheric turbulence is defined by a fast fluctuation of these state variables that locally modify the value of the refractive index  $n$  of the medium. The small scale fluctuations of the refractive index, called scintillation, induce additional losses during the propagation. Therefore, an accurate modelling of this turbulence is mandatory to assess quantitatively its impact and/or infer its intensity.

The 2D split-step wavelet (SSW) method [1] is developed at ENAC lab since 2015 to model long range electromagnetic (EM) propagation in the 2D parabolic wave equation (PWE) configuration [2]. This algorithm is based on the same formalism and principle as the auto-coherent discrete split-step Fourier (DSSF) [3] method which consists in solving iteratively the discrete “wide angle” 2D-PWE. Split-step methods solve iteratively this equation in two stages at each propagation step. First, the EM field is propagated in the vacuum in the spectral domain (for DSSF) or in the wavelet domain (for SSW). Then, the contribution of the atmosphere is integrated on all the propagation step in the form of a phase shift. This split-step propagation is also called multiple phase screen technique (MPS).

Classically, in the case of a turbulent atmosphere, the phase screens are randomly generated following the Von-Karman Kolmogorov theory [4][5]. However, the corresponding spectrum is expressed in a continuous domain, which does not fit with the auto-coherent formulation of the parabolic equation. In this article, a discrete Von-Karman Kolmogorov spectrum

is introduced to perform a full auto-coherent propagation in a turbulent medium.

In this paper the auto-coherent formulation of the 2D-PWE that is derived from a discretization of the Helmholtz equation is first reminded. The resulting DSSF propagator in the spectral domain is also explicated. Then, the global principle of the split-step wavelet algorithm is presented. In a second step, an auto-coherent formulation of the Von-Karman Kolmogorov spectrum that is used to generate stochastic realizations of turbulent phase screens is introduced. To validate the proposed approach, the impact of the modelled turbulence on propagation is finally discussed in Section IV with numerical simulations.

## II. AUTO-COHERENT SPLIT-STEP WAVELET PROPAGATION

In this section the derivation of the 2D auto-coherent formulation of the PWE and the associated DSSF propagator from the propagation equation are first introduced. The SSW resolution method is then presented.

In the following, the propagation is computed in free space in a 2D domain of size  $[0, x_{\max}] \times [0, z_{\max}]$ .  $u$  indicates the classical reduced scalar electromagnetic field that is here discretized along the vertical axis so that  $u_{x,p_z} = u(x, p_z \Delta z)$  for  $p_z$  in  $\llbracket 0, N_z \rrbracket$ .

### A. Auto-coherent formulation of the 2D parabolic equation

The purpose is here to develop a discrete formulation of the parabolic equation theory well-suited to a numerical implementation to avoid numerical errors. This type of procedure is called an auto-coherent method [3].

To do so, the Helmholtz equation is first discretized along  $z$  such as

$$\frac{\partial^2 u_{x,p_z}}{\partial x^2} - 2jk_0 \frac{\partial u_{x,p_z}}{\partial x} + \mathbf{d}_z^2 u_{x,p_z} + k_0^2 (n_{x,p_z}^2 - 1) u_{x,p_z} = 0, \quad (1)$$

where the operator  $\mathbf{d}_z^2$  corresponds to the second-order centered finite-difference approximation. Following the same reasoning as in the classical continuous “wide angle” parabolic equation development [2], the discrete forward PWE is obtained as

$$\frac{\partial u_{x,p_z}}{\partial x} = \left[ -jk_0 (n_{x,p_z} - 1) - j \left( \sqrt{k_0^2 + \mathbf{d}_z^2} - k_0 \right) \right] u_{x,p_z}. \quad (2)$$

Then, the idea is to consider (2) in the vacuum ( $n = 1$ ) and to compute the propagator associated to the discrete split-step

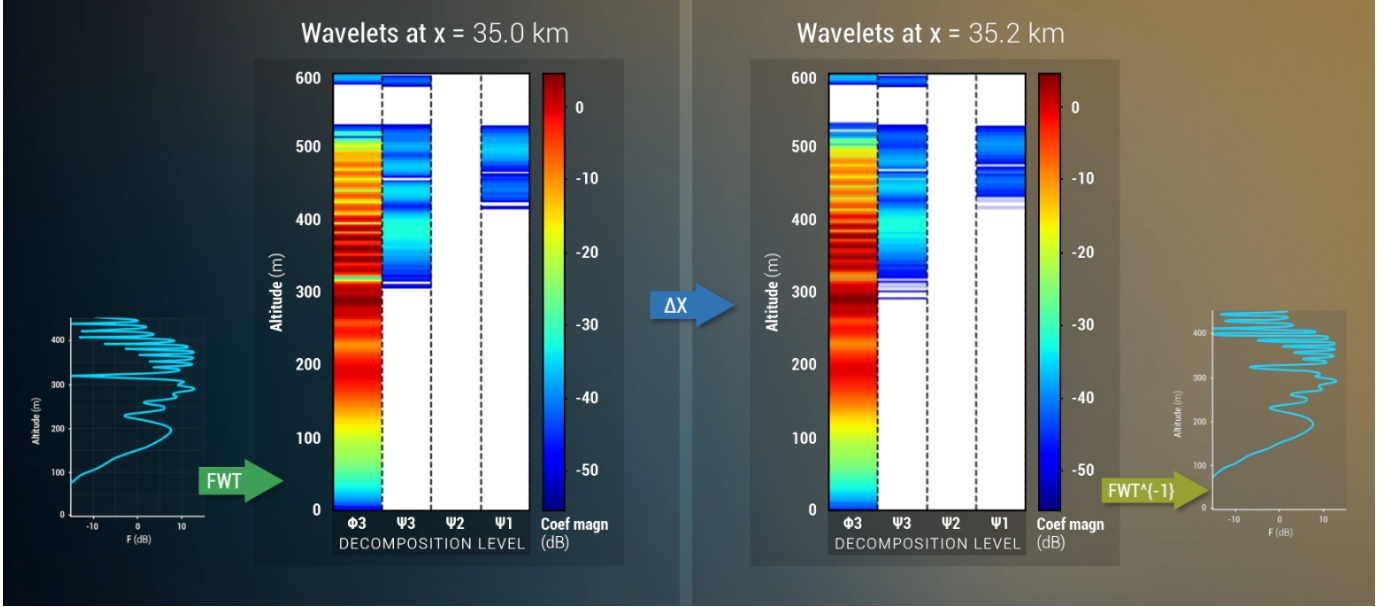


Fig. 1. Free space propagation of an electric field on one propagation step  $\Delta x = 200$  m with SSW-2D [6].

Fourier method by diagonalizing  $\mathbf{d}_z^2$  in the spectral domain. This DSSF propagator on a propagation step  $\Delta x$  is given by [3]

$$P(k_z^d) = \begin{cases} e^{-j(\sqrt{k_0^2 - (k_z^d)^2} - k_0)\Delta x} & \text{if } |k_z^d| \leq k_0, \\ e^{-j(\sqrt{(k_z^d)^2 - k_0^2} - k_0)\Delta x} & \text{if } |k_z^d| > k_0, \end{cases} \quad (3)$$

with

$$k_z^d = \frac{2}{\Delta z} \sin\left(\frac{\pi q_z}{N_z}\right), \quad q_z \in \llbracket 0, N_z - 1 \rrbracket. \quad (4)$$

The iterative DSSF solution of (2) is finally given by

$$u_{x+\Delta x, p_z} = e^{-jk_0 \int_x^{x+\Delta x} (n(x', z) - 1) dx'} \mathcal{F}^{-1} [P(k_z^d) \mathcal{F}(u_{x, p_z})], \quad (5)$$

where  $\mathcal{F}$  and  $\mathcal{F}^{-1}$  denote the direct and inverse fast Fourier transform (FFT), respectively.

### B. 2D split-step wavelet algorithm

The numerical simulations of the EM propagation are performed here with the open source software ‘‘SSW-2D’’ [6] which is based on the split-step wavelet algorithm (SSW) [1]. This method relies on the same formalism as the DSSF algorithm. The main difference is that the propagation in the vacuum is performed in the wavelet domain instead of Fourier through a fast wavelet transform (FWT). This decomposition algorithm is faster than the FFT and uses wavelet bases that have remarkable properties of invariance and compression [7]. Both these two advantages motivate the choice of the SSW method.

Thus, similarly to DSSF, the iterative SSW solution is

$$u_{x+\Delta x, p_z} = e^{-jk_0 \int_x^{x+\Delta x} (n(x', z) - 1) dx'} \mathcal{W}^{-1} [\mathcal{P} \mathcal{W}(u_{x+\Delta x, p_z})], \quad (6)$$

where  $\mathcal{W}$  denotes the FWT, and  $\mathcal{P}$  is the free-space propagator in the wavelet domain. The latter is a dictionary of propagators computed and stored before the first iteration. The global numerical scheme of the SSW method is illustrated Fig. 1. The electric field at  $x$  is first decomposed on the chosen wavelet basis characterized here by a decomposition level  $L = 3$ , through a FWT [7]. It is represented in the wavelet domain by its corresponding wavelet coefficients. The wavelets are then individually propagated in the vacuum on a step  $\Delta x$  with the dictionary of propagators giving new wavelet coefficients. The field at  $x + \Delta x$  is finally retrieved in the spatial domain thanks to an inverse FWT.

### III. AUTO-COHERENT MODELLING OF THE TURBULENT REFRACTIVE INDEX

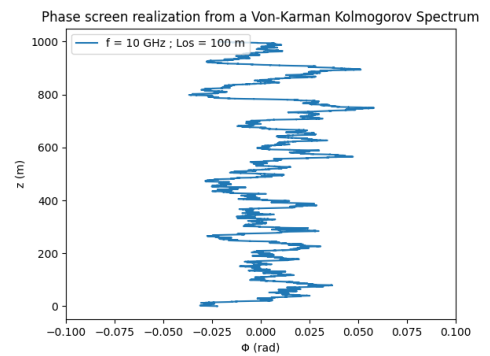


Fig. 2. Vertical phase screen realization from the auto-coherent formulation of the Von-Karman Kolmogorov spectrum.  $f = 10$  GHz;  $L_{os} = 100$  m;  $C_n^2 = 10^{-12} \text{ m}^{-2/3}$ .

The purpose of this paper is to extend the auto-coherent formulation of the PWE to the generation of the turbulent

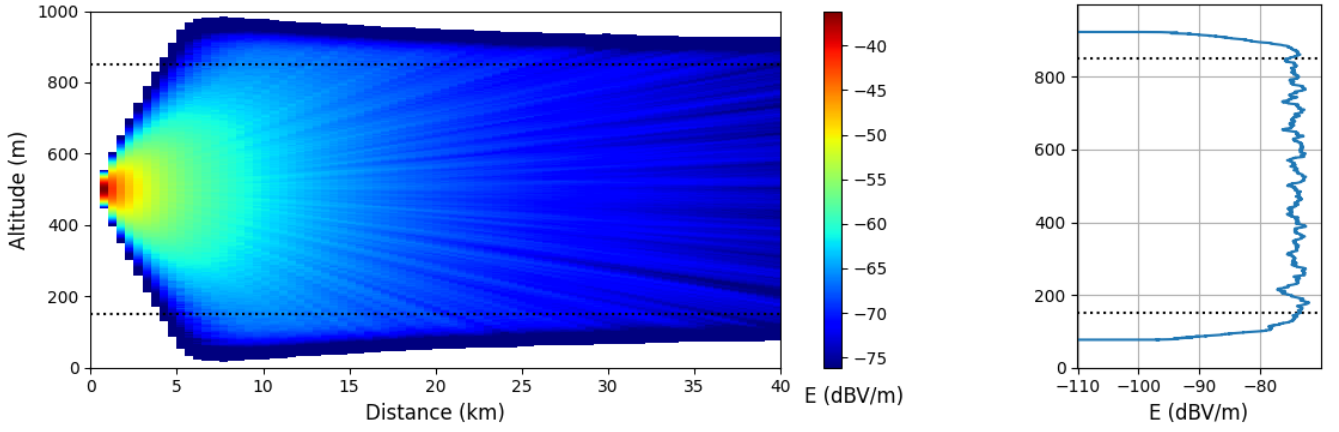


Fig. 3. Propagation of an electric field in a turbulent atmosphere with SSW-2D at  $f = 10$  GHz,  $C_n^2 = 10^{-12} \text{ m}^{-2/3}$  and  $L_{os} = 100$  m.

propagation medium. In this section some generalities on both the modelling of the turbulent refractive index and the phase screens generation are reminded. Then, a discrete version of the classical Von-Karman Kolmogorov spectrum is introduced. This spectrum is consistent with the auto-coherent formulation of the propagation method.

The tropospheric refractive index depends on thermodynamic fields that chaotically evolve in the presence of turbulence. Therefore, the refractive index is classically modelled by a mean term  $\langle n \rangle$  and a fluctuating contribution  $\Delta n$  such that

$$n = \langle n \rangle + \Delta n. \quad (7)$$

$\langle n \rangle$  is deterministic and is taken equal to 1 in this paper. On the contrary  $\Delta n$  is random and requires a stochastic modelling that relies on the K41 theory of Kolmogorov on isotropic and homogeneous turbulent processes [8].

Turbulence is a multiscale phenomenon: multiple turbulent structures (eddies) of various scales coexist within a turbulent flow. In particular, in the inertial range scales delimited by the outer scale length  $L_{os}$  and the inner scale length  $L_{is}$ , the turbulent energy is universally transmitted from the larger scales to the smaller ones following a constant rate. In this inertial range, the tropospheric scintillation  $\Delta n$  can be modelled as a stochastic process by well-chosen analytical spectra [9]. In this paper the classical Von-Karman Kolmogorov spectrum is used, defined in 2D as

$$S_{\Delta n}(\kappa_x, \kappa_z) = 0.055 C_n^2 \left( \kappa_x^2 + \kappa_z^2 + \left( \frac{2\pi}{L_{os}} \right)^2 \right)^{-4/3}, \quad (8)$$

where  $C_n^2$  is the turbulent structure constant that varies from  $10^{-18} \text{ m}^{-2/3}$  for weak scattering to  $10^{-12} \text{ m}^{-2/3}$  for strong scattering.  $\kappa_x$  and  $\kappa_z$  correspond to the horizontal and vertical wavenumbers of the turbulent flow, respectively.

In SSW, as explicited in (6), the contribution of the refractive index is applied at each propagation step to the electric field through a phase shift given by  $e^{-j\Phi_{x+\Delta x}(z)}$  with

$\Phi_{x+\Delta x}(z)$  the vertical phase screen located at  $x + \Delta x$  and defined as

$$\Phi_{x+\Delta x}(z) = k_0 \int_x^{x+\Delta x} \Delta n(x', z) dx' \quad (9)$$

considering  $\langle n \rangle = 1$ .

The widely used MPS technique [4][5] consists in considering directly the phase  $\Phi$  as a stochastic process. An analytical expression of the vertical phase spectrum  $S_\Phi$  is obtained from the scintillation spectrum  $S_{\Delta n}$ . Following Markov approximation, the phase screens need to be decorrelated, which requires that  $\Delta x > 5L_{os}$ . Under this assumption,  $S_\Phi$  is given by

$$S_\Phi(\kappa_z) = 2\pi k_0^2 \Delta x S_{\Delta n}(0, \kappa_z). \quad (10)$$

Usually, the spectral variable  $\kappa_z$  of the Von-Karman spectrum is discretized *a posteriori* as  $\kappa_z = \frac{2\pi q_z}{N_z \Delta z}$ ,  $q_z \in \llbracket 0, N_z - 1 \rrbracket$ . Here, we propose an *a priori* discretization that is consistent with the auto-coherence of the SSW method. Thus, the discretized formulation of the Von-Karman Kolmogorov used in this paper can be written as

$$S_{\Delta n}(\kappa_x, \kappa_z^d) = 0.055 C_n^2 \left( \kappa_x^2 + \kappa_z^{d2} + \left( \frac{2\pi}{L_{os}} \right)^2 \right)^{-4/3}, \quad (11)$$

with

$$\kappa_z^d = k_z^d = \frac{2}{\Delta z} \sin \left( \frac{\pi q_z}{N_z} \right), \quad q_z \in \llbracket 0, N_z - 1 \rrbracket. \quad (12)$$

Finally, the turbulent phase screens realizations are randomly generated by filtering a Gaussian white noise with  $\sqrt{S_\Phi(\kappa_z^d)}$ . An example of vertical turbulent phase screen generated from the *a priori* discretization of the Von-Karman Kolmogorov spectrum defined in (11) is given in Fig. 2.

#### IV. COMPUTATION OF THE PROPAGATION IN AN AUTO-COHERENT TURBULENT ATMOSPHERE

The aim of this section is to simulate the propagation of an electromagnetic wave in a turbulent atmosphere modelled

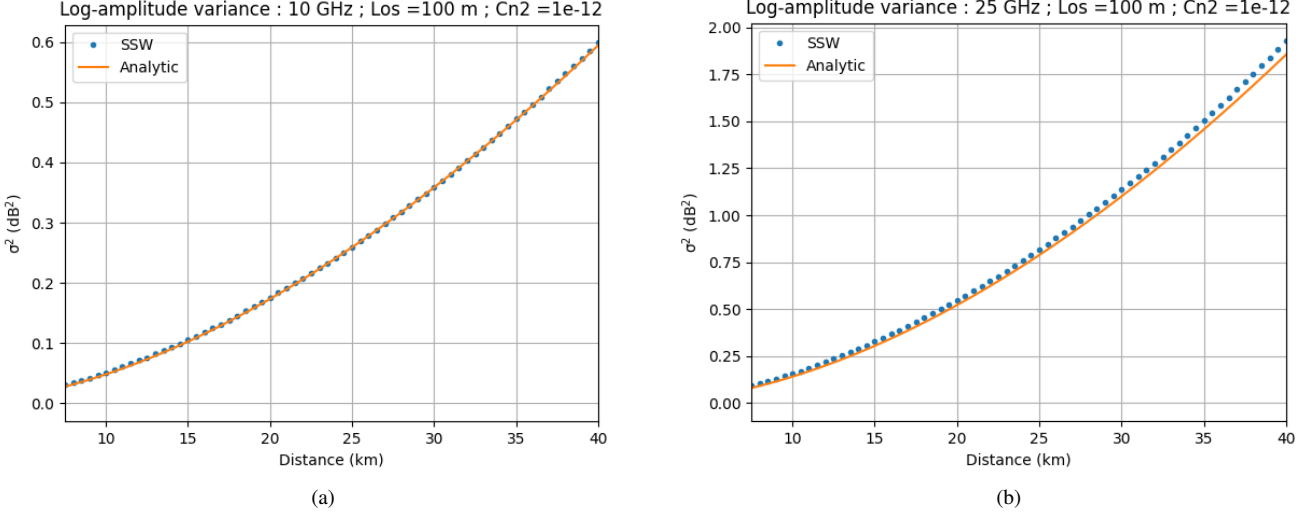


Fig. 4. Numerical and analytical log-amplitude variance at  $f = 10$  GHz (a) and  $f = 25$  GHz (b),  $C_n^2 = 10^{-12} \text{ m}^{-2/3}$  and  $L_{os} = 100$  m.

by successive phase screens randomly generated from the discrete Von-Karman Kolmogorov spectrum defined in (11). The impact of the scintillation is then discussed and compared to the analytical formulation of the log-amplitude variance of the field [10].

#### A. Analytical log-amplitude variance

The log-amplitude variance is classically used to estimate the impact of the scintillation on the propagation. The idea here is to compute its value as a function of the range. The objective is to validate the phase screen generation method introduced in this paper by comparing the numerical results to the analytical model developed by Rytov in a similar configuration as in [5].

Under the Rytov assumption, also called the method of smooth perturbations [10], the impact of turbulence on the EM propagation can be estimated by analysing the log-amplitude fluctuations. It is defined as  $\chi(x, z) = \ln(|E(x, z)/E_0(x, z)|)$ . The analytical log-amplitude variance at a distance  $R$  from the source is obtained as

$$\sigma_\chi^2(R) = 2\pi k_0^2 R \int_{-\infty}^{+\infty} S_{\Delta n}(0, \kappa_z) F(\kappa_z) d\kappa_z, \quad (13)$$

where  $S_{\Delta n}$  is the Von-Karman Kolmogorov spectrum defined by equation (8) and  $F$  corresponds to a filtering function given by

$$F(\kappa_z) = 0.5 \int_0^1 \left( 1 - \cos \left( R u \frac{\kappa_z^2 (1-u)}{k_0^2} \right) \right) du. \quad (14)$$

#### B. Numerical results

In the following, numerical simulations are performed with the SSW-2D software. We study here the free-space propagation of a wave front at  $f = 10$  GHz and  $f = 25$  GHz on a 40 km range. The vertical size of the computational domain is  $z_{\max} = 1$  km with a discretization step  $\Delta z = \lambda$ .  $C_n^2$  is

set at  $10^{-12} \text{ m}^{-2/3}$  which corresponds to a strong turbulence. The tropospheric outer scale length ( $L_{os}$ ), that describes the largest eddy of the turbulent flow, varies from few meters to more than 100 m depending on the altitude in the troposphere. Here,  $L_{os}$  is set to 100 m. The propagation step  $\Delta x$  that indicates the distance separating each turbulent phase screen is equal to  $500 \text{ m} = 5L_{os}$  to satisfy the Markov approximation. To simulate a Gaussian aperture, the source is a complex source point of size  $6\lambda$  set at 10 m from the first screen at  $z = z_{\max}/2 = 500$  m. An absorbing condition is imposed to the electric field at  $z = 150$  m and 850 m through a Hanning window to avoid numerical spurious reflexions at the top and the bottom of the computational domain.

The result of a propagation at  $f = 10$  GHz is given Fig. 3. The complete electric field is plotted on the left of the figure while the field computed after the last phase screen is plotted on the right. The apodisation window is represented by two dotted lines at  $z = 150$  m and 850 m. As expected, the crossing of the successive turbulent phase screens generates interferences that induce additional losses.

Fig. 4 compares the simulated log-amplitude variance in the proposed approach to the analytical formulation defined in (13) from 7.5 km to 40 km at  $f = 10$  GHz (Fig. 4a) and  $f = 25$  GHz (Fig. 4b). 500 Monte-Carlo runs are performed to obtain these results. Fig. 4a shows excellent agreements between the numerical and the analytical  $\sigma_\chi^2$  which is consistent with previous works using the continuous MPS method [5]. In Fig. 4b, the numerical model also gives results close to the analytical model. However, the log-amplitude variance exceeds  $1 \text{ dB}^2$  which means that this particular case is beyond the validity domain of the first Rytov approximation ( $\sigma_\chi^2 < 1 \text{ dB}^2$ ) [10]. This explains the gap between the theoretical and simulated values that starts increasing from 30 km.

## V. CONCLUSION

In this paper, a discrete formulation of the classical Von-Karman Kolmogorov has been introduced. The resulting turbulent phase screen generation method is consistent with the auto-coherent 2D SSW propagation technique. The proposed approach has been validated by computing the log-amplitude variance in strongly turbulent atmospheres at 10 and 25 GHz. It gives similar results as the classical continuous MPS method used in the literature for the presented cases. It leads to future studies of the auto-coherent propagation in a turbulent atmosphere.

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