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Application of mechano-probabilistic optimization technique to launcher model

Oussama Braydi
Phimeca

Cournon d'Auvergne, France
braydi@phimeca.com

Antoine Dumas
Phimeca

Cournon d'Auvergne, France
dumas@phimeca.com

Chahrazed Amrane
SIGMA Clermont, Institut Pascal
Université Clermont Auvergne, CNRS
Clermont-Ferrand, France
amrane@phimeca.com

Yannick Guérin

Direction des lanceurs, DLA/SDT/STM
CNES
Paris, France
yannick.guerin@cnes.fr

Résumé—Cet article présente l'application d'une méthodologie d'optimisation mécano-probabiliste pour faire face aux incertitudes sur la conception d'un démonstrateur de véhicule spatial réutilisable. La méthodologie est intégrée sous forme de module python sur l'outil de conception COLIBRI du CNES. La méthodologie est principalement composée de quatre fonctionnalités, une méthode de *screening*, l'analyse de sensibilité, l'estimation de probabilités de défaillance et l'optimisation sous incertitudes. Pour chaque fonctionnalité plusieurs méthodes sont implémentées et appliquées au modèle de lanceur. Les travaux montrent un caractère innovant dans le domaine des lanceurs, par l'application de méthodes éprouvées dans des domaines industriels fortement contraints par des aspects de fiabilité.

Abstract—This paper presents the application of a mechano-probabilistic optimization methodology to deal with the uncertainties on the design of a reusable space vehicle demonstrator. The methodology is integrated as a python module in the COLIBRI design tool of the CNES. The methodology is mainly composed with four functionalities, the screening analysis, the sensitivity analysis, the failure probability estimation and the optimization under uncertainty. For each functionality, several methods are implemented and applied to the launcher model. The work shows an innovative character in the field of launchers, through the application of proven methods in industrial fields strongly constrained by the aspects of reliability.

Keywords—Optimization, probability, sensitivity, uncertainty, space vehicle

I. INTRODUCTION

In structure and mechanical construction, there exists a lot of sources of uncertainties such as the geometrical tolerances, the material properties and others [13]. These uncertainties affect the performance, the robustness and the reliability of the products. Some types of uncertainties can be eliminated, by improving the knowledge about the model, or by investing on more accurate fabrication machines and metrology tools; while other types can not be eliminated, and their presence is inherent of the production process, as explained in [5].

In other side, ignoring the presence of uncertainties leads to un-robust and/or unreliable products. One way to limit their effects is through design under uncertainties, by taking them into consideration in the early phase of the mechanical and structure design.

Within the framework of the guarantee of the design of spacecraft structures under uncertainties and as part of the search for mass gain in these structures, while ensuring their reliability, CNES called on Phimeca to develop a mechano-probabilistic optimization module integrated into their COLIBRI design tool. The ultimate objective of this study is to perform a reliability-based design optimization of a part of a launcher.

Indeed, a global methodology based on four complementary functionalities, to deal with uncertainties, are implemented and applied on the launcher model. These functionalities are detailed in the following sections and they are enumerated below:

- Screening analysis.
- Sensitivity analysis.
- Estimation of the failure probability.
- Optimization under uncertainty.

This work shows an innovative character in the field of launchers, through the application of methods proven in industrial fields strongly constrained by reliability aspects. Results reveal how design under uncertainties could be applied for such complex model.

The paper is structured as follows: section II presents an overview of the theory, section III presents the model of the application and the results. Finally the conclusion and some perspectives are outlined in section IV

II. OVERVIEW OF THE THEORY

Several steps are described in the uncertainty methodology, and each of them uses different probabilistic techniques. Methods used in the application are developed in this section. In all this section, it is assumed that a physical model and a probabilistic model is defined. Let \mathcal{M} be the physical model, $\mathbf{X} = (X^1, \dots, X^m)$ the random vector that follows the joint distribution $f_{\mathbf{X}}$ of dimension m and $\mathbf{Y} = (Y^1, \dots, Y^n)$ the output random vector of interest where $\mathbf{Y} = \mathcal{M}(\mathbf{X})$.

A. Morris screening method

When dealing with high dimensional inputs and heavy CPU time codes, it is often needed to reduce the input dimension by selecting only the most relevant ones, meaning the most influential ones on the outputs. Screening methods are devoted to such purpose. Among them, the Morris method is one on the most known, because it requires few simulations to get an accurate estimate of the influential factors [7], [18].

The principle is to perform several one-at-a-time design of experiments and only requires the bounds of the input variables. The method acts as follows :

- 1) The input space must be discretized in a p levels grid of space δ .
- 2) A starting point is randomly chosen in this grid $\mathbf{x}^{j=1}$.
- 3) A perturbation is applied to each component but one at a time, a trajectory is then obtained containing $m + 1$ experiments, as shown in Figure 1.
- 4) The output values are computed from this trajectory.
- 5) Elementary effects ee_i associated to each input i are computed, then for one trajectory m elementary effects are available.

$$ee_i^1 = \frac{\mathcal{M}(\mathbf{x}^1, x_i^1 + \delta) - \mathcal{M}(\mathbf{x}^1)}{\delta} \quad (1)$$

- 6) Steps 2 to 5 are iterated r times, each time starting from a new point $\mathbf{x}^j, j = \{1, \dots, r\}$. Hence the total number of evaluations is equal to $r \times (m + 1)$.

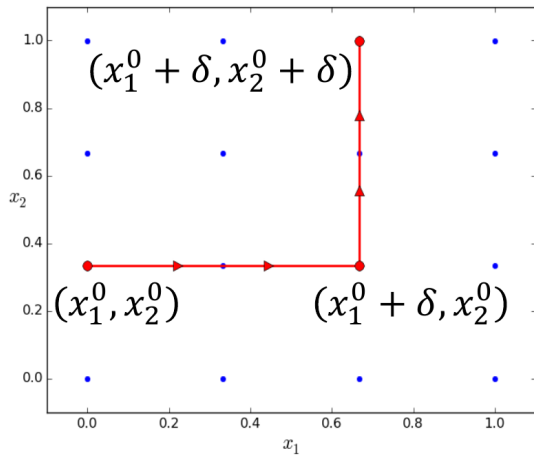


Figure 1. Example of a Morris trajectory in a 4-levels grid in dimension 2.

When the r trajectories are evaluated, a sample of elementary effects of size r and dimension m is available. Statistics from this sample can be estimated per input and according to these values, the most influential variables are determined and some useful interpretations can be derived:

- **absolute mean** $\mu_i^* = \frac{1}{r} \sum_{j=1}^r |ee_i^j|$: high values highlights the important effects, so the output is sensitive to the corresponding input.
- **standard deviation** σ_i : high values implies that it may exist interaction and/or non linearity effects. It is not possible to make the distinction between the two cases. In practice, interpretation is easier using the coefficient of variation $\delta_i = \sigma_i / \mu_i^*$. Roughly, if $\delta_i < 0.5$ the effect can be considered almost linear and without interaction.
- **mean** $\mu_i = \frac{1}{r} \sum_{j=1}^r ee_i^j$: the classic mean of elementary effects is compared to the absolute mean to determine the monotonic effect. Indeed if $|\mu_i| = \mu_i^*$, then the elementary effects have all the same sign (positive or negative) which means that the output is only increasing or decreasing. Otherwise the case $|\mu_i| < \mu_i^*$ means that the input has a non monotonic effect on the output.

In practice, dedicated graphics are used in order to help to interpret and visualize the effects. Two graphics are usually built, as it can be shown in the result section in the Figure 7: σ_i vs μ_i^* and μ_i vs μ_i^* .

B. Global sensitivity analysis using Sobol' indices

A global sensitivity analysis allows to provide a quantitative ranking of the inputs regarding their influence on the outputs. The well known Sobol' indices are dedicated to this end [23].

This method analyzes the influence of each component of an input random vector \mathbf{X} on the output random vector \mathbf{Y} by computing Sobol' indices. It computes, for every output random variable $Y^k (1 \leq k \leq n)$ the part of its variance due to each input component $X^i (1 \leq i \leq m)$ of \mathbf{X} .

Let consider only one output Y , using the Hoeffding decomposition, the variance can be written as follow :

$$\text{Var}[Y] = \sum_i^m V_i + \sum_{1 \leq i < j \leq m} V_{i,j} + \dots + V_{1,\dots,m} \quad (2)$$

where $V_i = \text{Var}_{X_i} [\mathbb{E}[Y|X_i]]$, $V_{i,j} = \text{Var}_{X_i, X_j} [\mathbb{E}[Y|X_i, X_j]] - V_i - V_j$ and $V_{i,\dots,m} = \text{Var}_{X_i, \dots, X_m} [\mathbb{E}[Y|X_i, \dots, X_m]] - \sum_i^m V_i$.

The term V_i corresponds to the part of the variance of the response only explained by the variable X_i . And $V_{i,j}$ corresponds to the part of the variance of the response explained by the interaction between X_i and X_j .

The first and second order Sobol' indices are then defined as :

$$S_i = \frac{V_i}{\text{Var}[Y]} \quad (3)$$

$$S_{i,j} = \frac{V_{i,j}}{\text{Var}[Y]} \quad (4)$$

The first order Sobol' index S_i measures the part of the variance of Y explained by X_i alone. The second order Sobol'

index $S_{i,j}$ measures the part of the variance of Y explained by the interaction of X_i and X_j . Indices varies from 0 to 1, the higher value the greater influence.

The sum of all Sobol' indices is equal to one:

$$\sum_i^m S_i + \sum_{1 \leq i < j \leq m} S_{i,j} + \dots + S_{1,\dots,m} = 1 \quad (5)$$

Instead of computing all order of Sobol' indices, total order indices S_{T_i} are computed. Let define the following conditional variance :

$$V_{-i} = \text{Var}[\mathbb{E}[Y|X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_{n_x}]] \quad (6)$$

Total order Sobol' indices are defined as follows:

$$S_{T_i} = 1 - \frac{V_{-i}}{\text{Var}[Y]} \quad (7)$$

It corresponds with the sum of all indices related to a given variable X_i . For example, if $m = 3$, the total order for the variable X_1 is $S_{T1} = S_1 + S_{1,2} + S_{1,3} + S_{1,2,3}$

In practice, it is common to compute only the first and total order indices. A difference between both indices indicates that interaction exists between some inputs. Moreover the total interaction can be easily computed from the sum of all first order indices :

$$S_{interaction} = 1 - \sum_i^m S_i \quad (8)$$

Estimation of the Sobol' indices

Indices are generally estimated from a Monte Carlo simulation whose evaluation cost is $N \times (m + 2)$, where N is the chosen simulation size. The size N is usually at least several thousands in order to get an accurate estimate of the Sobol' indices. This technique is hardly feasible if the model \mathcal{M} is computationally expensive and / or if the number of input variables is large.

Another technique consists in using a surrogate model. One can build its favorite metamodel and use the Monte Carlo simulation to compute the Sobol' indices. However, the use of polynomial chaos expansion (PCE) can be preferred, because of its structure (functional decomposition in an orthogonal basis) allows to get the Sobol' indices from a post processing of the PCE coefficients.

C. Methods for failure probability estimation

The failure of a mechanical system occurs when requirements of its good functioning and/or its safety are not gathered. The probability of occurrence of this failure event can be assessed by taking into account the various sources of the system uncertainties that are quantified and modeled by the input random vector \mathbf{X} . In that case, the outcome $\mathbf{Y} = \{Y_0, \dots, Y_n\}$ is uncertain and the failure event is defined by:

$$\mathbf{Y} = \mathcal{M}(\mathbf{X}) \leq \mathbf{y}_t \quad (9)$$

where $\mathbf{y}_t = \{y_{t_1}, \dots, y_{t_n}\}$ denotes here the thresholds that should not be exceeded for the n outputs.

In structural reliability analysis, it is very common to define a performance function, noted g , to define the system state. This function expresses the difference between the outcome \mathbf{y} and the threshold \mathbf{y}_0 , i.e. $g = \mathbf{y} - \mathbf{y}_0$. If the performance function is unique, the reliability analysis is here called *component reliability analysis*. Otherwise, if more than one performance function is considered, the analysis is referred to *system reliability analysis*. The system might be one component that is subjected to multiple failure modes, or it might be composed of multiple components subjected to failure. Depending on the problem in hands, the interaction between the failure modes could be an intersection or a union.

In this section, the union system is to be considered. Thus, the system fails when the union of the individual failure modes is less or equal to zero, i.e. $\cup_{i=1}^n g_i(\mathbf{x}) \leq 0$, where $\mathbf{x} = (x_1, \dots, x_m)$ denotes the vector of the random input variables and g_i is the performance function of the i^{th} failure mode, i.e. $g_i = y_i - y_{t_i}$. The case where $\cup_{i=1}^n g_i(\mathbf{x}) > 0$ means that the structure is safe for the input vector \mathbf{x} . The limit between the two situations, i.e. $\cup_{i=1}^n g_i(\mathbf{x}) = 0$, is called the Limit State Function (LSF). The failure probability is defined as the integral of the joint density function (PDF) $f_{\mathbf{X}}(\mathbf{x})$ over the failure domain $D_f = \{\mathbf{x} : \cup_{i=1}^n g_i(\mathbf{x}) \leq 0\}$:

$$P_f = \int_{D_f} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (10)$$

Except for some simple academic cases, the calculation of this integral using traditional integration schemes or analytically is impossible. Indeed, the performance function is implicit since it is usually computed with computer codes and algorithms, e.g. finite element code. To approximate this integral, several methods have been proposed in the literature, see e.g. [8], [16], [17]. In this section, the methods used to estimate the failure probability are briefly recalled: Monte Carlo simulation, First Order Reliability Method (FORM), importance sampling method and subset simulation.

1) *Monte Carlo simulation*: The Monte Carlo simulation method is a very popular method for the integral calculation. Its result is considered as the reference for the failure probability estimation, if the number of simulations is enough. The multidimensional integral of Equation 10 is computed here by first generating a large sample of realizations of the input random vector \mathbf{X} according to its PDF $f_{\mathbf{X}}$. Then, the performance function is evaluated on this sample. By introducing the indicator function \mathbb{I}_{D_f} of the failure domain D_f (i.e. $\mathbb{I}_{D_f}(\mathbf{x}) = 1$ if $\mathbf{x} \in D_f$, $\mathbb{I}_{D_f}(\mathbf{x}) = 0$ otherwise), equation 10 becomes:

$$P_f = \int_{\mathbb{R}^n} \mathbb{I}_{D_f}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \mathbb{E}_f[\mathbb{I}_{D_f}(\mathbf{X})] \quad (11)$$

where $\mathbb{E}[\cdot]$ represents the mathematical expectation. For N realizations of the input random vector \mathbf{X} , the Monte Carlo estimator of P_f is expressed by:

$$\hat{P}_f^{MC} = \frac{1}{N} \sum_{i=1}^N \mathbb{I}_{D_f}(\mathbf{X}^{(i)}) = \frac{|N_f|}{N} \quad (12)$$

where $|N_f|$ indicates the cardinal of the set $N_f = \{i : g(x^{(i)}) \leq 0\}$. According to the law of large numbers, this estimator is unbiased, i.e. $E[\hat{P}_f^{MC}] = P_f$. Its coefficient of variation reads:

$$\delta_{\hat{P}_f^{MC}} = \frac{\sigma_{\hat{P}_f^{MC}}^2}{E[\hat{P}_f^{MC}]} = \sqrt{\frac{1 - P_f}{NP_f}} \quad (13)$$

According to the above equation, a precise estimator \hat{P}_f^{MC} requires a small coefficient of variation $\delta_{\hat{P}_f^{MC}}$. This is generally not an easy task when the performance function is costly to evaluate. Furthermore, the number of simulations increases drastically when small failure probabilities are to estimate. It should also be noted that the coefficient of variation $\delta_{\hat{P}_f^{MC}}$ does not depend on the dimension of the input random vector, which makes the method insensitive to problems with high dimensions.

2) *First order reliability method (FORM)*: The estimation of the failure probability with this method is performed in the standard space, where the random variables are independent standard Gaussian and are gathered in the random vector $\mathbf{U} \sim \mathcal{N}(0, 1)$. In that case, a isoprobabilistic transformation of the original space, called also physical space, to the standard space is performed. Various techniques are proposed in the literature to apply this transformation, see e.g. [19] for Nataf transformation and [21] for Rosenblatt transformation. This allows us to express the failure probability in the standard space as follows:

$$P_f = \int_{D'_f} \varphi_{\mathbf{U}}(\mathbf{u}) d\mathbf{u} \quad (14)$$

where $D'_f = \{\mathbf{u} : \cup_{i=1}^n G_i(\mathbf{u}) \leq 0\}$ and G_i is the transformed performance function of the i^{th} failure mode in the standard space.

The main idea of this method is to approximate the LSF locally with a linear Taylor expansion. The latter is performed on a specific point of the LSF, generally named design point or most probable failure point (MPFP) and corresponds to the maximum density of probability in the standard space [16]. This point is completely defined by its coordinates which are the solution of the following constrained optimization problem:

$$\mathbf{u}^* = \underset{\mathbf{u} \in \mathbb{R}^n}{\operatorname{argmin}} \mathbf{u}^T \mathbf{u} \quad \text{s.t.} \quad G(\mathbf{u}) = 0 \quad (15)$$

To solve this optimization problem, usual algorithms such as Rackwitz-Fiessler algorithm [20] or Abdo-Rackwitz algorithm could be used [1]. The distance between the MPFP P^* and the origin of the standard space is noted β and it is named the reliability index.

Once the MPFPs P_i^* are defined for each performance function G_i , each LSF is replaced with its tangent hyperplane at P_i^* , as illustrated in Figure 2. The equation of one hyperplane is given by:

$$\tilde{G}_i(\mathbf{u}) = \beta_i - \boldsymbol{\alpha}_i^T \mathbf{u} \quad (16)$$

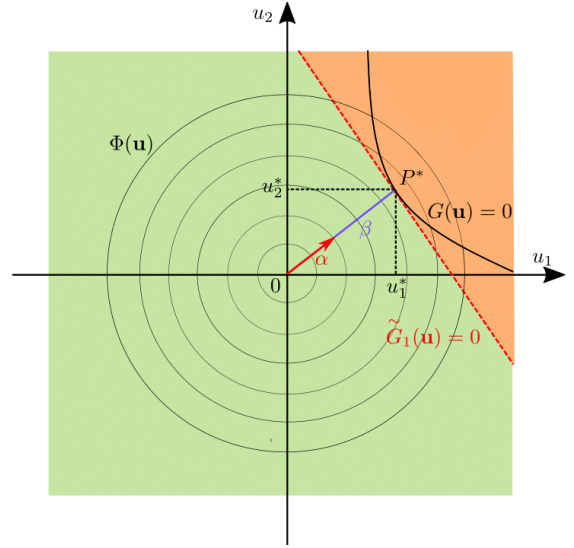


Figure 2. FORM method illustration on a 2D example. The orange region illustrates the failure domain.

where $\boldsymbol{\alpha}_i = -\nabla G_i(\mathbf{u}_i^*) / \|\nabla G_i(\mathbf{u}_i^*)\|$ is the vector unit that verifies $\boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_i = 1$ and ∇ is the gradient operator. Failure probability is calculated in that case from that approximation and Equation 14 rewrites:

$$P_f = \int_{\cup_{i=0}^n \tilde{G}_i(\mathbf{u}) \leq 0} \varphi_n(\mathbf{u}) d\mathbf{u} \quad (17)$$

The above integral can be calculated in a closed form and gives this approximation for the failure probability estimate [12]:

$$P_f^{FORM} = 1 - \Phi_m(\boldsymbol{\beta}; \boldsymbol{\rho}) \quad (18)$$

where $\boldsymbol{\beta} = \{\beta_1, \dots, \beta_n\}$ and $\boldsymbol{\rho} = \{\rho_{ij}, i, j = 1, \dots, n\}$ is the correlation matrix of LSFs, such that $\rho_{ij} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j$. For a good understanding of these notions, one can refer to Figure 2, which exhibits an illustration in a two-dimensional space the previous parameters P^* , β , $\boldsymbol{\alpha}$ and the tangent hyperplane for one performance function.

It should be noted that FORM assumes the uniqueness of the P^* and the linearity of the LSF. If these assumptions are not satisfied, the failure probability estimate could be biased w.r.t its true value.

3) *Importance sampling*: To reduce the computational burden of Monte Carlo simulation in the context of small failure probabilities, the importance sampling method is proposed in [11], [22]. Its idea is to generate more simulations in the failure domain. This is performed with the introduction of an instrumental distribution, noted $h_{\mathbf{U}}$. The latter is used as follows in the formulation of the failure probability:

$$\begin{aligned} P_f &= \int_{\mathbb{R}^n} \mathbb{I}_{D_f}(\mathbf{u}) \frac{f_{\mathbf{U}}(\mathbf{u})}{h_{\mathbf{U}}(\mathbf{u})} h_{\mathbf{U}}(\mathbf{u}) d\mathbf{u} \\ &= E_h \left[\mathbb{I}_{D_f}(\mathbf{U}) \frac{f_{\mathbf{U}}(\mathbf{U})}{h_{\mathbf{U}}(\mathbf{U})} \right] \end{aligned} \quad (19)$$

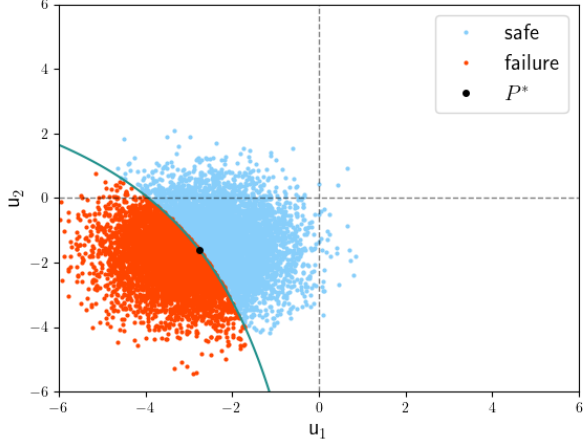


Figure 3. Illustration of the importance sampling method in the standard space.

For N simulations drawn according to the instrumental density $h_{\mathbf{U}}$, the failure probability estimator can be expressed by:

$$\hat{P}_{fIS} = \frac{1}{N} \sum_{i=1}^N \mathbb{I}_{D_f}(\mathbf{U}^{(i)}) \frac{f_{\mathbf{U}}(\mathbf{U}^{(i)})}{h_{\mathbf{U}}(\mathbf{U}^{(i)})} \quad (20)$$

Using an instrumental density defined in the standard space is a common choice. The aim is to generate simulations in the vicinity of the MPFP P^* , previously introduced in section II-C2. Thus, the instrumental density is a multivariate standard normal PDF φ_m centered on P^* and it writes:

$$h_{\mathbf{U}} = \varphi_m(\mathbf{u} - \mathbf{u}^*) = \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{(\mathbf{u} - \mathbf{u}^*)^t(\mathbf{u} - \mathbf{u}^*)}{2}\right) \quad (21)$$

where \mathbf{u}^* is the coordinate vector of P^* . The failure probability estimator, given in 20, rewrites:

$$\hat{P}_{fIS} = \frac{1}{N} \sum_{i=1}^N \mathbb{I}_{D_f}(\mathbf{U}^{(i)}) \frac{\varphi_m(\mathbf{U}^{(i)})}{\varphi_m(\mathbf{U}^{(i)} - \mathbf{u}^*)} \quad (22)$$

The convergence of this method is relatively better since the rate of failing simulations is almost 50%, see Figure 3. If more than one MPFP has to be considered, one can use a mixture of multivariate standard normal PDFs centered on the different MPFPs, see e.g. [3].

D. Optimization under uncertainties

The ultimate phase of the design under uncertainties is the model optimization under it. Deterministic optimization (DO) type is the classical optimization, its formulation is given in equation 23 where X is the vector of control variables, having \bar{X} as nominal values, $P^{(k)}$ is the vector of the environmental parameters with dimension k . The objective and constraint functions are given by $f(\bar{X}, P^{(k)})$ and $g(\bar{X}, P^{(k)})$ respectively. The optimal and admissible design configuration is obtained by \bar{X}_{OptAdm} .

Find \bar{X}_{OptAdm} such that :

$$\bar{X}_{OptAdm} = \arg \max_{\bar{X}} f(\bar{X}, P^{(k)}) \quad (23)$$

Subject to : $g(\bar{X}, P^{(k)}) \geq 0$

As explained in [6], there exists three types of optimization under uncertainties:

- Robust design optimization design (RDO).
- Reliability based design optimization (RBDO).
- Reliability-based robust design optimization (RBRDO)

The main difference between these types is where the uncertainties are taken into consideration:

- In RDO: the uncertainties are mainly included in the objective functions.
- In RBDO: the uncertainties are mainly included in the constraint functions.
- In RBRDO: the uncertainties are mainly included in both objective and constraint functions.

In this work, we are interested in RBDO type, which aims to found reliable and optimal design. It consists in optimizing deterministic objective function under probabilistic constraints. Many papers have studied RBDO like [2], [9], [14] and [24]. As given in [15], RBDO problem can be formulated by equation 24, where the associated uncertainties on X and P are given with the vector of random variables w . The probabilistic constraint functions are given by $g(X(\bar{X}, w), P(w))$. The allowed failure probability threshold is given by Pf_{target} and the optimal and reliable design configuration is obtained by \bar{X}_{OptRel} . Alternative formulations for RBDO problem can be found in the literature in [6] and [25].

Find \bar{X}_{OptRel} such that :

$$\bar{X}_{OptRel} = \arg \max_{\bar{X}} f(\bar{X}, P^{(k)}) \quad (24)$$

Subject to : $\text{Prob } g(X(\bar{X}, w), P(w)) \leq 0 \leq Pf_{target}$

The main difference between the results of deterministic optimization and RBDO is that the first one, leads to an optimal point which is located on the boundary of the deterministic feasible domain which is defined by: $[X \in \mathbb{R}^n \mid g(X, P) \geq 0]$. Such as optimal point produces unreliable design, where its failure probability is usually about 50%. At this optimal point, slightest uncertainty is enough to put the model on the failure domain. However the optimal point resulting from RBDO problem is far from the boundary of the deterministic feasible domain, and the corresponding distance is controlled by the admissible failure probability threshold. This difference between \bar{X}_{OptRel} and \bar{X}_{OptAdm} is shown in Figure 4 taken from [15], where an example of optimization problem is illustrated, the problem has two control variables (R and h). The optimal point \bar{X}_{OptAdm} is found on the constraint (*i.e.* the limit state function) as expected but this configuration is likely to lead to a probability of failure about 50%. On the contrary, the optimal point \bar{X}_{OptRel} is shifted from the constraint, it has been specially found such as the probability of failure is equal to $Pf_{target} = 0.1$.

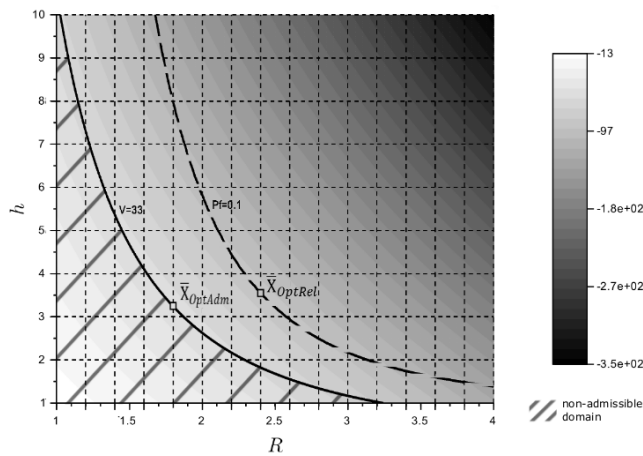


Figure 4. Example of difference between DO and RBDO results [15].

III. APPLICATION ON THE VEB

The uncertainty methodology is applied as part of the CALLISTO project, which is a reusable vehicle demonstrator in which CNES is the prime contractor in partnership with the DLR and the JAXA. An illustration of the vehicle is shown in Figure 5. The study focuses on the vehicle equipment bay (VEB) which is located on the top. The final objective is to reduce the mass of the VEB while ensuring its reliability.



Figure 5. Illustration of the CALLISTO project.

This section starts presenting the physical model and the probabilistic model of the inputs. Then a section shows the results of the probabilistic analyses.

A. Presentation of the model

The VEB is modeled by a finite element model, using Nastran, where a static and a buckling analysis is performed. Some characteristics of this model are provided below:

- Unit evaluation: about 5 minutes.
- Number of random variables: 47 divided into 3 categories:

- 20 thickness variables,
- 1 Young Modulus,
- 26 loading variables : general forces, aerodynamic forces and QSL.

Inputs are assumed to follow a Gaussian distribution with a coefficient of variation of 3% for the thicknesses and 5% for the other variables.

- Output of interest : 2 maximum Von Mises (VM) stress ($MAX_Stress_VM_Z1$ and $MAX_Stress_VM_Z2$), one maximum flux (MAX_IF_Flux).

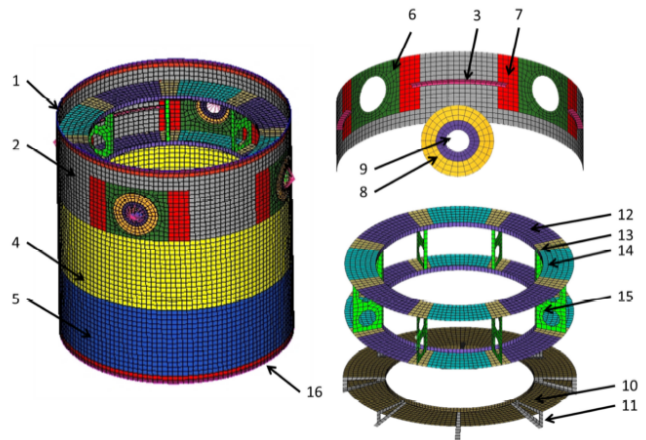


Figure 6. Finite element model of the vehicle equipment bay.

The main difficulty of this study is to consider several outputs which implies to use system reliability techniques. It was initially planned to study more outputs but due to some delay this parts is not ready for the current paper. All the probabilistic analyses were performed using the OpenTURNS library [4]. These methodologies are currently integrated into the Colibri design tool of CNES.

B. Result

The uncertainty methodology has been applied in the following order :

- 1) Morris analysis : the goal is to reduce the number of inputs to take into account in the probabilistic analysis. Due the required time for a unit evaluation (parallel evaluations were not possible), it will be required to use a surrogate model. It is then easier to build the metamodel with the lowest possible number of inputs.
- 2) Sobol analysis : it will provide an accurate ranking of the inputs.
- 3) Probability of failure estimation : this allows to get a first probability of failure estimation before the optimization step and to adjust parameters method.
- 4) Optimization under uncertainty : this is the main goal which is optimizing the mass subjected to the respect of a target probability of failure.

1) *Screening analysis:* The Morris analysis has been performed with the following parameters:

- grid discretization number: 6
- number of trajectories $r = 15$
- total number of model evaluations: 720

Among the 47 random inputs, only 7 appears (4 thicknesses and 3 forces) to be influential on the 3 outputs (all taken together). The Figure 7 shows the Morris result for one output, only the 7 selected inputs are represented. It can easily be seen that 3 inputs are mainly influential (μ^* is high, they are on the right side of each figure). The 4 other influential inputs are close to the defined threshold that splits the influential and non influential area. Moreover, based on the figure, it is possible to conclude that the inputs have a monotonous effect but as σ is high, it may exist non linearity or interaction.

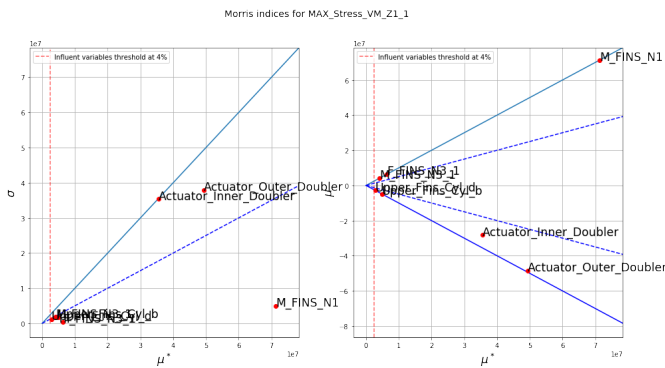


Figure 7. Morris graphics for the first Von Mises stress output.

2) *Sensitivity analysis:* The Sobol indices are computed using the Polynomial Chaos Expansion (PCE) surrogate model. In order to build this metamodel a design of experiments (optimized Latin Hypercube) of size 850 has been computed. In order to validate the metamodel, the leave-one-out cross validation criterion Q^2 is computed as well as a R^2 using a test sample of 720 points, also built with an optimized Latin Hypercube DOE. All these validation values are greater than 0.92 for the three outputs so the metamodel can be used with confidence in further analysis.

N.B. : in order to get an accurate PCE, for each output, only its own influential input variables were considered.

From the PCE, Sobol indices are computed for the 3 outputs. Results are shown in Figure 8, the main influential variable for the 3 outputs is the force M_FINS_N1 , the value of the first order index is at least 0.5. The first order and total order indices are almost equals, which means that no interaction of some inputs has influence on the output variability.

It also shows that the selected thicknesses have different influences according to the chosen output. The input *Actuator_Outer_Doubler* is important for the first VM stress and the flux whereas it is not for the second VM stress. *Actuator_Inner_Doubler* is only influential for the first VM stress, and *Upper_Fins_Cyl_b* and *Upper_Fins_Cyl_d* are important for the second VM stress. To finish the last forces

($F_FINS_N3_1$ and $M_FINS_N3_1$) may not be considered as influential, they were selected during the screening analysis (it only uses bounds) but taking into account their input distribution in Sobol' analysis reduces their influence to almost nothing.

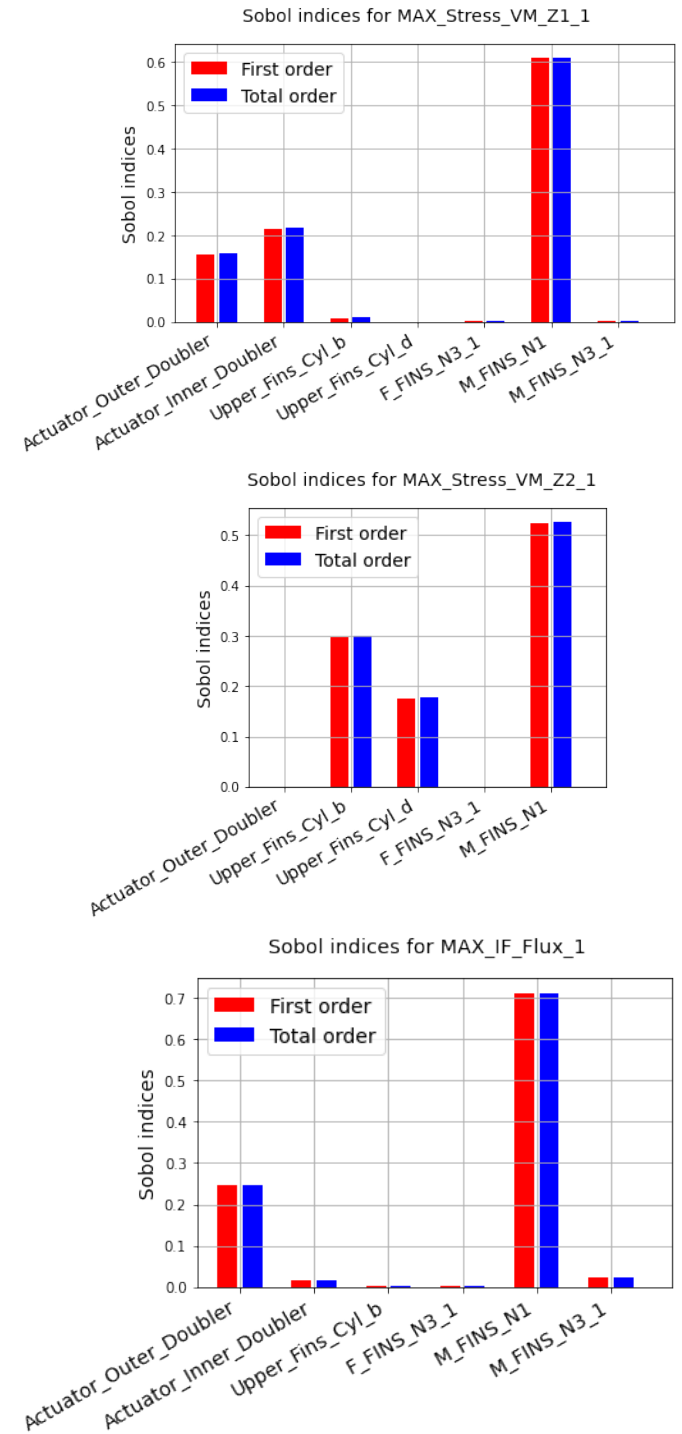


Figure 8. Sobol indices of the 3 outputs (2 Von Mises stress and one flux).

3) *Probability evaluation:* The current problem needs to take into account 3 failure events, each associated to one

outputs. The probability of failure that must be computed is defined in Equation 26. This is a union event of the 3 individual events.

$$\begin{aligned}
P_f &= \text{Prob}(MAX_Stress_VM_Z1 > 4.2 \times 10^8 \\
&\cup MAX_Stress_VM_Z2 > 4.56 \times 10^8 \quad (25) \\
&\cup MAX_IF_Flux > 8.3 \times 10^5) \\
&= \text{Prob}(E_1 \cup E_2 \cup E_3)
\end{aligned}$$

Here the events E_1 , E_2 and E_3 are introduced to simplify the notation in the optimization section.

This probability can be estimated using several reliability technique : Monte Carlo simulation, FORM + Importance Sampling, Subset simulation or metamodel-based technique. In the current study, due to a lack of time, the PCE metamodel built for the Sobol' analysis has been used to compute the probability of failure. In order to avoid to do prediction outside the domain of definition, the design of experiments used to build the metamodel was built from an extended distribution. The coefficients of variation of the input random variables were 4 times greater than the original definition.

Results of the probability failure are given in table I. The three estimation techniques provide equivalent results.

Table I
PROBABILITY OF FAILURE RESULT VALUES

Method	Probability value	95% confidence length
Monte Carlo simulation	6.38×10^{-3}	1.25×10^{-3}
FORM System	5.70×10^{-3}	None
Importance sampling	6.54×10^{-3}	1.28×10^{-3}

Importance factors can be derived from the simulations, it corresponds with a reliability sensitivity. This is the weight of each inputs regarding the failure of the system. These factors are shown in Figure 9, it can be seen that the force M_FINS_N1 is the most important, which is consistent with the Sobol indices. Secondly the thickness $Actuator_Outer_Doubler$ appears also as an important variable.

4) *RBDO*: The optimization performed consists in minimizing the sum of the thickness variables, with the constraint that the probability of failure remains lower than a probability threshold. The current used optimization algorithm does not allow to optimize the random variables, only deterministic parameters can be optimized. So all thicknesses will be now considered as deterministic parameters. However to keep a conservative estimation of the probability of failure, the chosen constant value will not be the mean but the quantile at 1% of the original distribution (Gaussian with 3% of coefficient of variation)

Let be \mathcal{P} the set of the variables to be optimized $\mathcal{P} = \{Actuator_Outer_Doubler, Actuator_Inner_Doubler, Upper_Fins_Cyl_b, Upper_Fins_Cyl_d\}$ and \mathcal{X} the set of the random variables $\mathcal{X} = \{F_FINS_N3_1, M_FINS_N1, M_FINS_N3_1\}$.

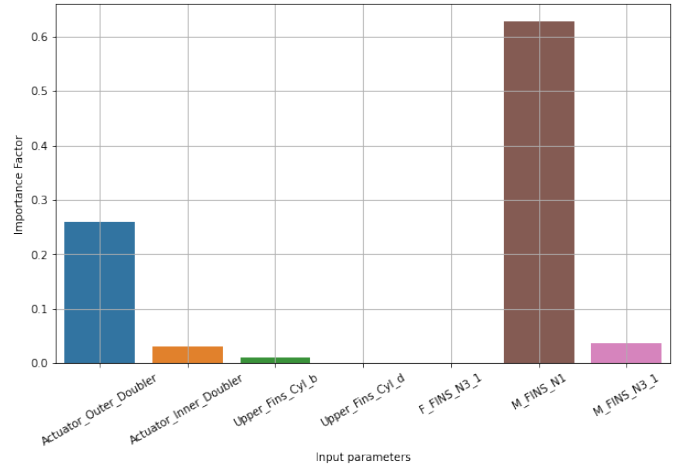


Figure 9. Reliability importance factors.

The optimization problems writes :

$$\min_{\mathcal{P}} \sum_{p \in \mathcal{P}} p \quad (26)$$

$$s.t. \quad \text{Prob}(E_1 \cup E_2 \cup E_3) < P_{f,threshold} \quad (27)$$

$$\mathcal{P}_{min} \leq \mathcal{P} \leq \mathcal{P}_{max} \quad (28)$$

with \mathcal{P}_{min} and \mathcal{P}_{max} are the boundaries of the optimized parameters.

This optimization problem has been solved using a global optimization algorithm from the NLOpt library (Steven G. Johnson, <http://github.com/stevengj/nlopt>), named *GN_ORIG_DIRECT_L* [10]. The probability threshold is set to $P_{f,threshold} = 10^{-3}$.

The table II sums up the main value of the optimization results. It shows that the objective has been slightly minimized (loss of 6×10^{-4}). However in the same time the probability of failure has been improved, as it respects the given threshold. This means that some critical thicknesses have been increased in order to improve the reliability and in the same time less important thicknesses decreased making the total thickness lower than the value at the initial start.

Table II
OPTIMIZATION UNDER UNCERTAINTY RESULTS

Initial objective (at the quantile values)	0.0474
Initial probability of failure	3.4×10^{-3}
Final objective (at the quantile values)	0.0468
Final probability of failure	9.9×10^{-4}

Eventually it is possible to update the probability of failure computed in the section III-B3 by considering once again the thicknesses as random variables. First it requires to compute the mean of these thicknesses assuming the optimized values are the quantiles at 1% of Gaussian distribution with 3% of coefficient of variation.

The probability of failures before and after the optimization procedure are given in table III.

Table III
PROBABILITY OF FAILURE AFTER THE OPTIMIZATION STEPS

Thickness mean	Probability estimation (importance sampling)	95% confidence length
Original mean	6.54×10^{-3}	1.25×10^{-3}
Modified mean	3.73×10^{-5}	7.30×10^{-6}

IV. CONCLUSION

This paper presents a reliability-based design optimization (RBDO) of a part of a launcher using a mechano-probabilistic optimization module, which is developed by Phimeca based on the OpenTURNS library. The objective is to apply the probabilistic methods to the field of launchers by optimizing the design of the vehicle equipment bay and considering the inherent uncertainties of the system.

A screening analysis is first performed in order to reduce the number of uncertain inputs. The analysis shows that among the 47 random inputs, only seven of them must be considered in the probabilistic design. Then, for the failure probability estimation, three failure events were considered, each one of them is associated to one output. The problem is treated as a system problem and three reliability methods are used: Monte Carlo simulation, FORM system and importance sampling. The results of the three methods converge roughly to the same estimate for the failure probability.

The final step of this study is the RBDO. The aim is to reduce the sum of the thickness variables under the constraint that the failure probability should not exceed a given threshold. In the present study, it is shown that the critical thicknesses have to be increased, unlike the less important ones which can be reduced. Thus, the reliability is improved and the total thickness decreases compared to the initial one.

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