A Unified Topological Approach for the Modeling: Application to a 2D Beams Structure

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Abstract In this paper, a unified topological approach for the modeling of complex systems is presented. This approach is based on topological notions such as the topological collections and the transformations. These two topological notions allow the separation between the topological structure (interconnection laws) and the physics (behavioral laws) of the studied system. In fact, regardless of the complexity of the system, interconnection laws are declared through the topological collections and behavioral laws through the transformations. Therefore, a system is considered as local elements linked by neighbor relations to which local behavioral laws are associated. In order to show that the topological modeling approach is independent of the physical nature or the number of elementary components, a 2D beams structure with topological modification is taken as an example. In fact, a beams structure is a well-structured set of beams. Classical modeling consists in considering such structure as a global system and the resolution necessities the computation of the global stiffness matrix and therefore a modification of the beams structure involves the actualization of this matrix. Contrary to the classical approach, the application of the topological collections allow to consider a beams structure as local elements with no assembling terms as it is demonstrated through this paper.

Keywords Topological collections · Transformations · Local laws · Topological structures

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1 Introduction

Technological development, consumer demand and industrial competition are leading to the emergence of more and more complex systems. These systems have a high level of integration of electronics, mechanics, automation and computing technologies, which necessitates an advanced modeling (Plateaux 2011). Since, the various domains of mechatronics can be represented by their topological structures and behavioral laws, a topological approach for modeling is taken up (Plateaux et al. 2007; Chaabane 2014). This approach permits to separate the topology and the physics of the studied system in order to have a standard model of characterization of all the physics of a complex system (Chaabane et al. 2013a, 2014a, b; Abdeljabbar Kharrat et al. 2017a, b, 2018).

The MGS (General System Model) language is applied to assure topological modeling. Indeed, this language is a research activity of he IBISC of the University of Evry allowing the study and the development of the contribution of notions of topological nature in programming languages as well as the application of these notions to the design of new data and control structures (Giavitto et al. 2002; Spicher and Michel 2007). Therefore, to assure the computation, in addition to its fundamental elements, MGS includes new types of values named topological collections, which composed, of a set of cells whose organization is captured by local relationships and affected by values. MGS applies transformations which are a series of rewriting rules to manipulate its data.

The main transformations are the case defined functions, the patches that are designed to change the cell structure and the path transformations that renew the values allied with the cells.

To ensure a topological modeling approach, collections are applied to present the system topology and transformations to identify the local behavior laws of its different components. Thus, a complex system will be described as a set of local elements.

In what follows, a short presentation of topological collections and transformations is presented. The topological approach is applied to a 2D beams structure and two particular cases of topological modification are taken to show the flexibility of the topological modeling.

2 Topological Collections and Transformations

In this section, collections and transformations are presented (Cohen 2004; Spicher 2006).

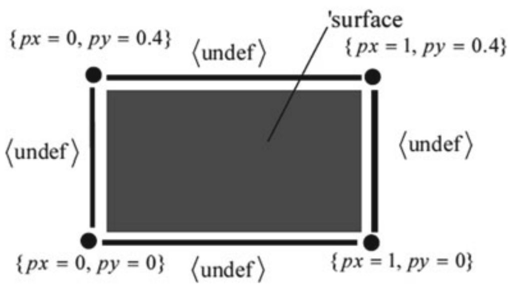
2.1 Topological Collections

There are several types of topological collections that can be classified into two categories:

- Leibnizian collection: space is defined from the elements it contains and their relative positions with respect to each other;
- Newtonian collection: Space is defined as an object having a proper existence on which other objects are placed and moved. For these type of collections, positions exist regardless of their decoration. The special value ⟨undef⟩ is associated with undecorated positions.

In this paper we are interested in the application of Newtonian collections for modeling and more precisely, we are interested in the application of topological collections of type abstract chain. Figure 1 shows an example of an abstract chain and the associated MGS code. For this type of topological collection, the topology is explicitly defined by the creation of new cells. These cells are then decorated using sum and product operators. To browse the cells of an abstract string denoted c, the following functions are used:

- faces (c): provides the list of faces of c;
- cofaces (c): provides the list of cofaces of c;
- icells (c): provides the list of cells incident to c;
- pcells (c, p): provides the list of p-neighbors of c.




```
V1 := add_acell(0,seq:(),seq:());;
V2 := add_acell(0,seq:(),seq:());;
V3 := add_acell(0,seq:(),seq:());;
V4 := add_acell(0,seq:(),seq:());;
e1 := add_acell(1,(V1,V2),seq:());;
e2 := add_acell(1,(V2,V3),seq:());;
e3 := add_acell(1,(V3,V4),seq:());;
e4 := add_acell(1,(V4,V1),seq:());;
f := add_acell(2,(e1,e2,e3,e4),seq:());;
  {px = 0, py = 0}*V1
+ {px = 0, py = 0.4}*V2
+ {px = 1, py = 0.4}*V3
+ {px = 1, py = 0}*V4
+ 'surface* f;;
```

Fig. 1 Example of abstract chain and associated MGS code

2.2 Transformations

Transformations are described as a series of rewriting rules $\{m_1 \Rightarrow e_1; m_2 \Rightarrow e_2, \ldots\}$. When a rule is applied for a topological collection, sub-collections respecting that pattern m_i are substituted by the topological collection calculated according the expression e_i (e_i is the expression that substitutes the occurrences of m_i). Thus, the procedure for applying a transformation to a topological collection can be summed up in three stages. Let C be a topological collection. First, sub-collections A of C are chosen according to the pattern; second, for each chosen sub-collection A, a new sub-collection A_0 is evaluated from A. Third, the evaluated sub-collections A_0 substitute the selected sub-collections A and so on.

3 Topological Modeling of a 2D Beams Structure

The generic topological approach for the modeling of beams structures is to represent the structure by an abstract cellular complex to which the variables of interest are assigned. Once the system has been written as a topological collection, local behavior laws of the main components (beam, frame, force and node) of beams structures are specified through transformations.

The studied beams structure is presented in Fig. 2.

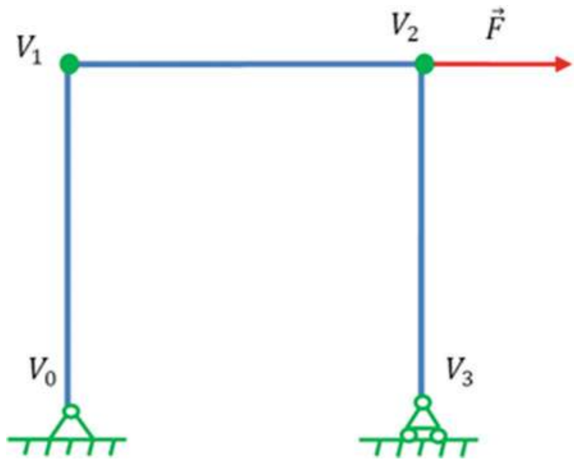


Fig. 2 Studied 2D beams structure

3.1 Topological Structure

The main components of a beams structure are beam, frame, force and node. Therefore to represent a beam structure as a cellular complex: 0-cells represent nodes, 1 cells represent beams frames or forces such as 1-cells ended with two 0-cells represent beams and 1-cells ended with one 0- cells represent forces or frames. Figure 3 represents the studied 2D beams structure as a cellular complex.

The topological structure of the 2D beams structure is connected by locals relationships: the cofaces of P_2 are $\{e_2, e_3, e_f\}$, the face of e_{b1} is $\{P_0\}$ and the faces of e_2 are $\{P_1,\ P_2\}$.

Once a beams structure is presented in the form of a cellular complex, this structure is then decorated with variables.

Figure 4 represents the studied 2D beams structure as a topological collection. Variables are associated to each cells (length L, displacement $U,\ V,\ \Theta$, modulus young E, force F, moment M ...). For example:

$$P_1 = \{U_1, V_1, \Theta_1\};$$
$$\{e_{b1}\} = \{F_{i\_b1}, F_{j\_b1}, M_{i\_b1}, M_{j\_b1}\};$$
$$\{e_2\} = \{F_{i\_12}, F_{j\_12}, M_{i\_12}, M_{j\_12}, L_{12}\ldots\}$$

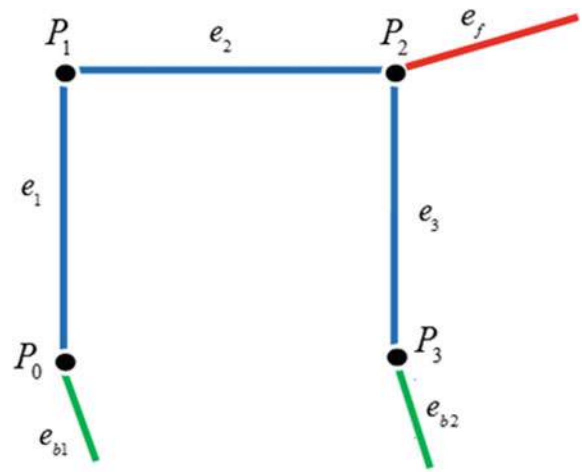


Fig. 3 Presentation as a cellular complex of the studied 2D beams structure

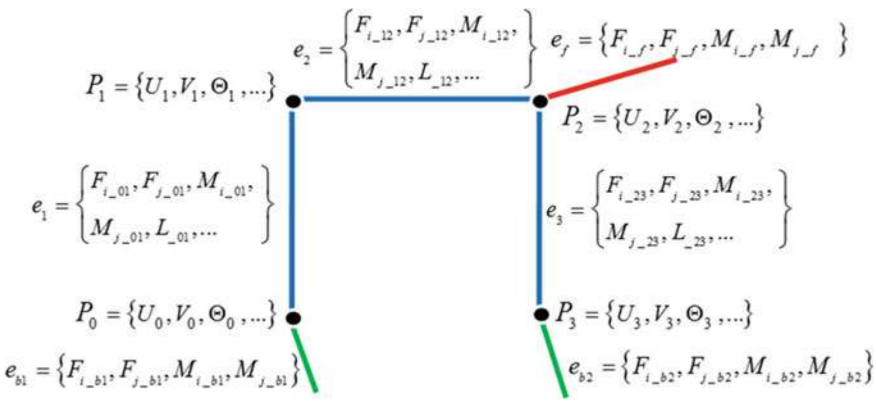


Fig. 4 Presentation as a topological collections of the studied 2D beams structure

3.2 *Local Behavior Laws*

Transformations are applied to identify local behavior laws of nodes, beams, forces and frames. The creation of system's equations is done by swiping all cells describing the beams structure.

Table 1 presents local laws for 0-cells.

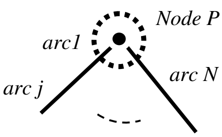
For nodes P_0, P_1, P_2 and P_3 the local equations (equilibrium equations) are respectively given by Eqs. 1, 2, 3 and 4.

$$\begin{cases} \overrightarrow{F_{i\_01}} + \overrightarrow{F_{j\_b1}} = \overrightarrow{0} \\ \overrightarrow{M_{i\_01}} + \overrightarrow{M_{j\_b1}} = \overrightarrow{0} \end{cases} \tag{1}$$

$$\begin{cases} \overrightarrow{F_{j\_01}} + \overrightarrow{F_{i\_12}} = \overrightarrow{0} \\ \overrightarrow{M_{j\_01}} + \overrightarrow{M_{i\_12}} = \overrightarrow{0} \end{cases} \tag{2}$$

$$\begin{cases} \overrightarrow{F_{j\_12}} + \overrightarrow{F_{i\_23}} + \overrightarrow{F_{i\_f}} = \overrightarrow{0} \\ \overrightarrow{M_{j\_12}} + \overrightarrow{M_{i\_23}} + \overrightarrow{M_{i\_f}} = \overrightarrow{0} \end{cases} \tag{3}$$

Table 1 Local laws for nodes (0-cells)

Topological structure	Physical structure (local behavior laws and/or equilibrium equations)
	$\begin{cases} \sum_{j=1}^{N} \overrightarrow{F_j} = \overrightarrow{0} \\ \sum_{j=1}^{N} \overrightarrow{M_j} = \overrightarrow{0} \end{cases}$ N the number of concurrent arcs e to the node P

$$
\begin{cases}
\overrightarrow{F_{j\_23}} + \overrightarrow{F_{i\_b2}} = \overrightarrow{0} \\
\overrightarrow{M_{j\_23}} + \overrightarrow{M_{i\_b2}} = \overrightarrow{0}
\end{cases}
\tag{4}
$$

Table 2 presents local laws for 1-cells ended with 1 0-cell (forces or frames).

For force e_f, frame e_{b1} and frame e_{b2}, the local equations (equilibrium equation) are respectively given by Eqs. 5, 6 and 7.

$$
\begin{cases}
\overrightarrow{F_{j\_f}} + \overrightarrow{F_{i\_f}} = \overrightarrow{0} \\
\overrightarrow{M_{j\_f}} + \overrightarrow{M_{i\_f}} = \overrightarrow{0}
\end{cases}
\tag{5}
$$

$$
\begin{cases}
\overrightarrow{F_{j\_b1}} + \overrightarrow{F_{i\_b1}} = \overrightarrow{0} \\
\overrightarrow{M_{j\_b1}} + \overrightarrow{M_{i\_b1}} = \overrightarrow{0}
\end{cases}
\tag{6}
$$

$$
\begin{cases}
\overrightarrow{F_{j\_b2}} + \overrightarrow{F_{i\_b2}} = \overrightarrow{0} \\
\overrightarrow{M_{j\_b2}} + \overrightarrow{M_{i\_b2}} = \overrightarrow{0}
\end{cases}
\tag{7}
$$

Table 3 presents local laws for 1-cells ended with by two 0-cells.

Table 2 Local laws for forces or frames (1-cells bounded by 1 0-cell)

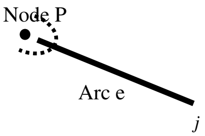
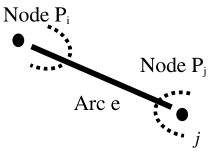
Topological structure	Physical structure (local behavior laws and/or equilibrium equations)
	$\begin{cases} \vec{F_i} + \vec{F_j} = \vec{0} \\ \vec{M_i} + \vec{M_j} = \vec{0} \end{cases}$ i and j denote the ends of the arc

Table 3 Local laws for 2D beams (1-cells bounded by 2 0-cells)

Topological structure	Physical structure (local behavior laws and/or equilibrium equations)
	$\begin{cases} \vec{F_i} + \vec{F_j} = \vec{0} \\ \vec{M_i} + \vec{M_j} + \vec{F_j} \wedge \overrightarrow{L_{ij}} = \vec{0} \\ \{\tau\} = [K]\{\Delta P\} \end{cases}$ where: i and j denote the ends of the arc $\{\vec{\tau}\} = \left\{ F_{X_i} \; F_{Y_i} \; M_i \; F_{X_j} \; F_{Y_j} \; M_j \right\}^T$ is the force vector in the global coordinate system $\left\{\overrightarrow{\Delta P}\right\} = \left\{ U_i \; V_i \; \Theta_i \; U_j \; V_j \; \Theta_j \right\}^T$ is the displacement vector in the global coordinate system $[K]$ is the local stiffness matrix of a beam element in the global coordinate system (Chaabane et al. 2014c)

For beams e_1,e_2 and e_3 the local equations (equilibrium equations and local behavior laws) are respectively given by Eqs. 8, 9 and 10.

$$\begin{cases} \overrightarrow{F_{i\_01}} + \overrightarrow{F_{j\_01}} = \overrightarrow{0} \\ \overrightarrow{M_{i\_01}} + \overrightarrow{M_{j\_01}} + \overrightarrow{F_{j\_01}} \wedge \overrightarrow{L\_01} = 0 \\ \begin{Bmatrix} \overrightarrow{F_{i\_01}} \\ \overrightarrow{M_{i\_01}} \end{Bmatrix} = \left[K_{\_01}\right]\left\{ U_0 \ V_0 \ \Theta_0 \ U_1 \ V_1 \ \Theta_1 \right\}^t \end{cases} \tag{8}$$

$$\begin{cases} \overrightarrow{F_{i\_12}} + \overrightarrow{F_{j\_12}} = \overrightarrow{0} \\ \overrightarrow{M_{i\_12}} + \overrightarrow{M_{j\_12}} + \overrightarrow{F_{j\_12}} \wedge \overrightarrow{L\_12} = 0 \\ \begin{Bmatrix} \overrightarrow{F_{i\_12}} \\ \overrightarrow{M_{i\_12}} \end{Bmatrix} = \left[K_{\_12}\right]\left\{ U_1 \ V_1 \ \Theta_1 \ U_2 \ V_2 \ \Theta_2 \right\}^t \end{cases} \tag{9}$$

$$\begin{cases} \overrightarrow{F_{i\_23}} + \overrightarrow{F_{j\_23}} = \overrightarrow{0} \\ \overrightarrow{M_{i\_23}} + \overrightarrow{M_{j\_23}} + \overrightarrow{F_{j\_23}} \wedge \overrightarrow{L\_23} = 0 \\ \begin{Bmatrix} \overrightarrow{F_{i\_23}} \\ \overrightarrow{M_{i\_23}} \end{Bmatrix} = \left[K_{\_23}\right]\left\{ U_2 \ V_2 \ \Theta_2 \ U_3 \ V_3 \ \Theta_3 \right\}^t \end{cases} \tag{10}$$

For beam 3 the local rigidity matrix is given by Eq. 11.

$$[K_{23}] = \begin{bmatrix} k_{2,23} & 0 & k_{4,23} & -k_{2,23} & 0 & k_{4,23} \\ 0 & k_{1,23} & 0 & 0 & -k_{1,23} & 0 \\ k_{4,23} & 0 & k_{3,23} & -k_{4,23} & 0 & \frac{k_{3,23}}{2} \end{bmatrix} \tag{11}$$

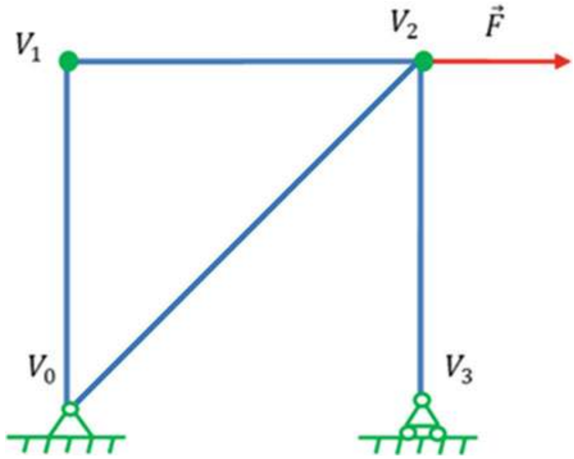
where $k_1 = \frac{EA}{L}$; $k_2 = \frac{12EI}{L^3}$; $k_3 = \frac{4EI}{L}$ and $k_4 = \frac{6EI}{L^2}$ (E, A, L and I respectively represent young's modulus, cross sectional area, length and inertia of the beam).

We can notice from the equations generated by the MGS language for the different cells representing the system (Eqs. 1–11) that local laws of elementary components are considered instead of the global behavior law of the structure. The assembly is indirectly done from neighborhood relationships.

3.3 Advantages of Topological Modeling

Contrary to the classical approach based on the displacement method(Liu and Quesk 2003), the advantage of the topological modeling for beams structure (Chaabane et al. 2012, 2013b) is that local behavior laws of beams are independently declared of their number and the way in which they are connected. Therefore the topological approach allows the simplification of the modeling of the beams structures: only topological structure is required. Then all the modifications (such as the addition/deletion of one or more beams, modification at the level of connections or efforts) are only realized through the topological structure. Local laws are the same (Tables 1, 2 and 3).

Fig. 5 Topological modification: addition of a beam



For example, if a fourth beam is added (Fig. 5), it suffices to change the topological structure by adding the cells related to the addition of the fourth beam (e_4).

The neighborhood relationships change for the nodes P_0 and P_2 (cofaces of P_0 are $\{e_1, e_4, e_{b1}\}$, cofaces of P_2 are $\{e_2, e_3, e_4, e_f\}$). Also, the relationship of neighborhood associated to the bar 4 is added (the faces of e_4 are $\{P_0,\ P_2\}$).

The equations generated by the MGS languages automatically change according to the topological structure. Therefore equilibrium equations of node P_0 and P_2 are changed and the local equation of beam 4 is added.

For P_0, P_2 and e_4 the local equations are respectively given by Eqs. 12, 13 and 14.

$$\begin{cases} \overrightarrow{F_{i\_01}} + \overrightarrow{F_{i\_04}} + \overrightarrow{F_{j\_b1}} = \overrightarrow{0} \\ \overrightarrow{M_{i\_01}} + \overrightarrow{M_{i\_04}} + \overrightarrow{M_{j\_b1}} = \overrightarrow{0} \end{cases} \tag{12}$$

$$\begin{cases} \overrightarrow{F_{j\_12}} + \overrightarrow{F_{i\_23}} + \overrightarrow{F_{j\_04}} + \overrightarrow{F_{i\_f}} = \overrightarrow{0} \\ \overrightarrow{M_{j\_12}} + \overrightarrow{M_{i\_23}} + \overrightarrow{M_{j\_04}} + \overrightarrow{M_{i\_f}} = \overrightarrow{0} \end{cases} \tag{13}$$

$$\begin{cases} \overrightarrow{F_{i\_04}} + \overrightarrow{F_{j\_04}} = \overrightarrow{0} \\ \overrightarrow{M_{i\_04}} + \overrightarrow{M_{j\_04}} + \overrightarrow{F_{j\_04}} \wedge \overrightarrow{L\_04} = 0 \\ \begin{Bmatrix} \overrightarrow{F_{i\_04}} \\ \overrightarrow{M_{i\_04}} \end{Bmatrix} = \left[K_{\_04}\right]\left\{ U_0\ V_0\ \Theta_0\ U_4\ V_4\ \Theta_4 \right\}^T \end{cases} \tag{14}$$

Also if we consider the case where the force is applied at the node P_1 instead of P_2 compared with the structure presented in Fig. 5. The equations generated by the MGS languages automatically change according to the new topological structure. Therefore only the equilibrium equations of the node P_1 and P_2 changed.

For P_1 and P_2 the local equations are respectively given by Eqs. 15 and 16.

$$\begin{cases} \overrightarrow{F_{j\_01}} + \overrightarrow{F_{i\_12}} + \overrightarrow{F_{i\_f}} = \overrightarrow{0} \\ \overrightarrow{M_{j\_01}} + \overrightarrow{M_{i\_12}} + \overrightarrow{M_{i\_f}} = \overrightarrow{0} \end{cases} \tag{15}$$

$$\begin{cases} \overrightarrow{F_{j\_12}} + \overrightarrow{F_{i\_23}} + \overrightarrow{F_{j\_04}} = \overrightarrow{0} \\ \overrightarrow{M_{j\_12}} + \overrightarrow{M_{i\_23}} + \overrightarrow{M_{j\_04}} = \overrightarrow{0} \end{cases} \tag{16}$$

4 Conclusion

In this paper, a topological approach based on topological collections and transformations is used as a unified approach for the modeling. This approach makes it possible to distinguish between structure and behavior. Systems are considered as a set of local elements linked by neighborhood relationships that facilitate the modeling of complex systems. The example of a 2D beams structure is studied. Two cases of topological modification are considered to approve the generic approach of topological modeling. Indeed, only the topological structure is modified and the local behavior and equilibrium equations of the different elements of a beams structure are independently declared of the studied beam structure.

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