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# Synergistic Multi-Energy CT Reconstruction with a Deep Penalty “Connecting the Energies”

Zhihan Wang, Alexandre Bousse, Noel Jeffrey Pinton, Jacques Froment, Franck Vermet, Béatrice Vedel, Jean-Pierre Tasu, Dimitris Visvikis, *Fellow, IEEE*

**Abstract**—We propose a novel penalty term for multi-channel synergistic image reconstruction with an application to multi-energy computed tomography (CT). The penalty utilizes trained convolutional neural networks (CNNs) to connect the energies to a latent image. We show on simulated data that our method has the potential to outperform reconstruction with a joint total variation (JTV) penalty.

## I. INTRODUCTION

MULTI-energy computed tomography (CT) consists of acquiring multi-energetic data, e.g., using dual energy acquisitions or photon-counting CT systems, to reconstruct multiple attenuation images at different energies. These images can be subsequently used to determine material composition. The images are traditionally obtained by model-based iterative reconstruction (MBIR) techniques, i.e., by iteratively minimizing a cost function consisting of the sum of a data fidelity term and a penalty that controls the noise while preserving basic image properties [1].

Alternatively, the images from multi-energy data can be simultaneously reconstructed using synergistic techniques in order to combine the information from all available energy channels. This can be achieved using handcrafted multi-channel penalties such as for example joint total variation (JTV) [2], or structural similarities [3]. These methods may be inappropriate when some features are visible in one channel but not in another (e.g. contrast agent, tumors).

Alternatively, the inter-channel information can be learned with patch-based multi-channel dictionaries, for example as proposed in [4]. However, training such dictionaries is computationally expensive, especially when the number of channels is high (e.g., multi-energy CT). Besides, this approach leads to spatially redundant atoms that are essentially shifts of a basic atom type to “enforce” the spatial-invariance of the representation, which is not memory-efficient.

In this work we present a novel synergistic multi-energy CT techniques that uses convolutional neural networks (CNNs) to “connect the energies”. The CNNs are trained to map the attenuation image at a reference energy to other energies. This approach is a generalization of multi-channel dictionary learning. We compared our method to joint reconstruction with a JTV penalty.

## II. METHOD

We consider a simplified multi-energy CT setting with  $K$  monochromatic X-ray sources of energies  $E_1, \dots, E_K$ . The aim is to reconstruct  $K$  linear attenuation images  $\boldsymbol{\mu}_k = [\mu_{1,k}, \dots, \mu_{J,k}]^\top \in \mathbb{R}^J$ ,  $J$  being the number of pixels or voxels, at each energy  $E_k$ . The system

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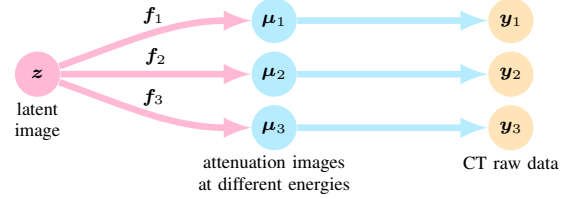


Fig. 1: Schematic representation of our generative model with  $K = 3$  energies.

comprises  $I$  detectors and the expected number of X-ray photons of energy  $E_k$  recorded at detector  $i = 1, \dots, I$ , denoted  $y_{i,k}$ , is defined by the Beer-Lambert law as

$$\bar{y}_{i,k}(\boldsymbol{\mu}_k) = I_k \cdot \exp(-[\mathbf{A}\boldsymbol{\mu}_k]_i) + r_{i,k}$$

where  $I_k$  denotes the mean photons flux at the  $k$ -th energy bin,  $\mathbf{A} \in \mathbb{R}^{I \times J}$  is a system matrix modeling line integrals along each beam and  $r_{i,k} > 0$  is a known background term. The actual number of detected photons is a Poisson random variable  $y_{i,k}$

$$y_{i,k} \sim \text{Poisson}(\bar{y}_{i,k}(\boldsymbol{\mu}_k)). \quad (1)$$

For each energy bin  $k$ , the number of detected photons are stored in a vector  $\mathbf{y}_k = [y_{1,k}, \dots, y_{I,k}]^\top \in \mathbb{R}^I$ .

Synergistic MBIR of the collection of images  $\{\boldsymbol{\mu}_k\}$  consists in iteratively solving the optimization problem

$$\min_{\{\boldsymbol{\mu}_k\}} \sum_{k=1}^K L_k(\boldsymbol{\mu}_k) + \beta R(\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K), \quad (2)$$

where  $L_k(\boldsymbol{\mu}_k)$  is a data fidelity term that evaluates the fit between the image  $\boldsymbol{\mu}_k$  and the measurement  $\mathbf{y}_k$ ,  $R(\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K)$  is a synergistic penalty term that enforces individual and joint properties on the images and  $\beta > 0$  is weight. In this work,  $L_k(\boldsymbol{\mu}_k)$  is a weighted least-squares (WLS) data fit term that approximates the negative log-likelihood defined as [1]

$$L_k(\boldsymbol{\mu}_k) = \sum_{i=1}^I \frac{1}{2} w_{i,k} ([\mathbf{A}\boldsymbol{\mu}_k]_i - b_{i,k})^2 \quad (3)$$

where  $w_{i,k} = \frac{(y_{i,k} - r_{i,k})^2}{y_{i,k}}$  when  $y_{i,k} > r_{i,k}$  and  $w_{i,k} = 0$  if  $y_{i,k} \leq r_{i,k}$ , and  $b_{i,k} = \log\left(\frac{I_k}{y_{i,k} - r_{i,k}}\right)$ . The minimization problem (2) is therefore a penalized weighted least-squares (PWLS) problem. The penalty term  $R(\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K)$  is designed to promote piecewise image smoothness as well consistency between the images, for example by enforcing joint sparsity of the gradient [2] or structural similarities [3], or using multichannel dictionary learning [4].

In this work we present a novel synergistic penalty term that connects the energies through CNNs. Let  $\{\mathbf{f}_k\}$ ,  $\mathbf{f}_k: \mathbb{R}^J \rightarrow \mathbb{R}^J$ , be a collection of CNNs trained to map an attenuation image at energy

$E_1$  to the same image but at energy  $E_k$ . We assume  $\mathbf{f}_1 = \text{Id}_{\mathbb{R}^J}$ . We define our penalty term as

$$R_{\text{CNN}}(\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K) = \min_{\mathbf{z} \in \mathbb{R}^J} \sum_{k=1}^K \frac{1}{2} \gamma_k \|\mathbf{f}_k(\mathbf{z}) - \boldsymbol{\mu}_k\|_2^2 + \alpha H(\mathbf{z}) \quad (4)$$

where  $\mathbf{z} = [z_1, \dots, z_J]^\top \in \mathbb{R}^J$  is a reference image,  $\gamma_k, \alpha > 0$  are weights—typically  $\sum_k \gamma_k = 1$  with  $\gamma_k \propto \|\mathbf{y}_k\|_1$  to minimize the variance of  $\mathbf{z}$ —and  $H(\mathbf{z})$  is an image smoothness penalty term defined as

$$H(\mathbf{z}) = \sum_{j=1}^J \sum_{\ell \in \mathcal{N}_j} \omega_{j,\ell} \psi(z_j - z_\ell)$$

where  $\mathcal{N}_j$  is a neighborhood of  $j$ ,  $\omega_{j,\ell} = 1/\text{dist}(j, \ell)$  and  $\psi: \mathbb{R} \rightarrow \mathbb{R}^+$  is a potential function. In this work we used the Huber potential [5].

To summarize,  $R(\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K)$  is small if (i) each image  $\boldsymbol{\mu}_k$  is close to  $\mathbf{f}_k(\mathbf{z})$  for some latent image  $\mathbf{z}$  and (ii) the latent image  $\mathbf{z}$  is piecewise smooth. By doing so, the latent image  $\mathbf{z}$  “connects” the  $\boldsymbol{\mu}_k$ s together such that each  $\boldsymbol{\mu}_k$  is reconstructed using the entire measurement data at each energy, thus reducing the variance. The proposed generative model is summarized in Fig. 1.

This approach is a generalization of multichannel dictionary learning [4] where  $\{\mathbf{f}_k\}$  play the role of the multichannel dictionary and  $\mathbf{z}$  plays the role of the sparse code.

Solving (2) is achieved through an iterative algorithm that alternates between minimization in  $\{\boldsymbol{\mu}_k\}$  and minimization in  $\mathbf{z}$ . We used a limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm [6] for the minimization in  $\mathbf{z}$ , and separable paraboloidal surrogates (SPSS) [1] for the minimization in  $\{\boldsymbol{\mu}_k\}$ .

We compared our method with synergistic reconstruction using a JTV penalty, i.e., by solving (2) with the penalty

$$R_{\text{JTV}}(\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_K) = \sum_{j=1}^J \sum_{\ell \in \mathcal{N}_j} \omega_{j,\ell} \sqrt{\sum_{k=1}^K (\mu_{j,k} - \mu_{j,\ell})^2}.$$

The minimization was performed with a primal-dual algorithm [7].

### III. EXPERIMENTS AND RESULTS

We generated low-dose multi-energy CT data  $\{\mathbf{y}_k\}$  following (1) using  $K = 6$  ground truth (GT) images  $\boldsymbol{\mu}_k^*$  at different energies (from  $E_1 = 40$  keV to  $E_6 = 140$  keV) obtained by the Philips IQon Spectral CT scanner from the Poitiers University Hospital. We used a standard projector to model the system matrix  $\mathbf{A}$  and its transpose. The CNNs  $\{\mathbf{f}_k\}$  consist of U-Net architectures trained on a separate dataset of 100 multi-energy images obtained on the same scanner. The models were built and trained using Tensorflow in Python. The models were trained using NVIDIA GeForce GTX 1660 Ti (6 GB memory) with batch size of 10. We reconstructed the images by solving (2) using  $R = R_{\text{JTV}}$  and  $R = R_{\text{CNN}}$  with a range of values for  $\beta$ , as well as with  $R = 0$  (no penalty). The parameter  $\alpha$ , cf. (4), was set manually.

Fig. 2 shows the reconstructed images at 40, 60 and 80 keV with optimal  $\beta$  according to the peak signal-to-noise ratio (PSNR) (see below). We observe that the reconstructed images using  $R = R_{\text{CNN}}$  are less noisy and less blurry than the images reconstructed using  $R = R_{\text{JTV}}$ .

Fig. 3 shows the PSNR values for reconstructed 40 keV images using  $R = R_{\text{JTV}}$  and  $R = R_{\text{CNN}}$  for a range of values for  $\beta$ . We observe that  $R_{\text{CNN}}$  outperforms  $R_{\text{JTV}}$  for all  $\beta$ .

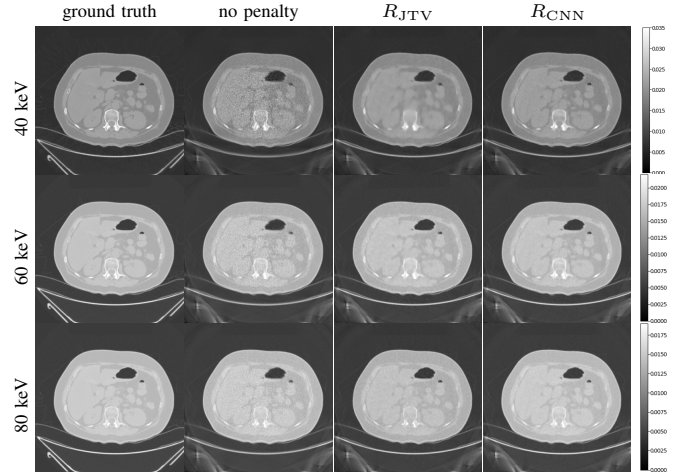


Fig. 2: GT and reconstructed images at  $E_1 = 40$ ,  $E_2 = 60$  and  $E_3 = 80$  keV.

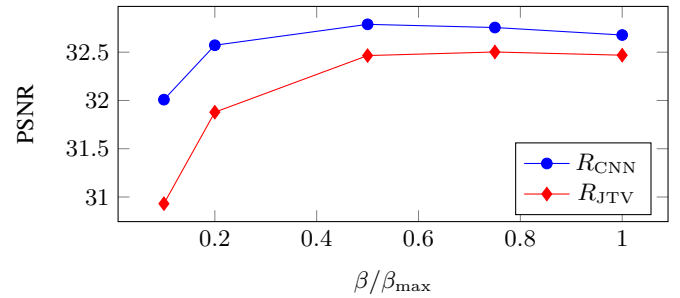


Fig. 3: PSNR values of the reconstructed 40 keV images using  $R = R_{\text{JTV}}$  and  $R = R_{\text{CNN}}$  versus  $\beta$  (normalized by  $\beta_{\max}$ ).

### IV. DISCUSSION AND CONCLUSION

We proposed a novel penalty for PWLS synergistic multi-energy CT reconstruction. The penalty “connects the energies” using CNNs. Our method may also be applicable for other modalities such as multi-parametric magnetic resonance imaging (MRI). We demonstrated that our penalty outperforms JTV in terms of PSNR. Additional improvements can be foreseen by optimally tuning  $\alpha$  and using a different  $H$  penalty in (4) such as total variation (TV). Further work includes bias/variance quantitative analysis from multiple noise replicates.

### REFERENCES

- [1] I. A. Elbakri and J. A. Fessler, “Statistical image reconstruction for polyenergetic x-ray computed tomography,” *IEEE transactions on medical imaging*, vol. 21, no. 2, pp. 89–99, 2002.
- [2] E. Haber and M. Holtzman Gazit, “Model fusion and joint inversion,” *Surveys in Geophysics*, vol. 34, no. 5, pp. 675–695, 2013.
- [3] M. J. Ehrhardt, K. Thielemans, L. Pizarro, D. Atkinson, S. Ourselin, B. F. Hutton, and S. R. Arridge, “Joint reconstruction of pet-mri by exploiting structural similarity,” *Inverse Problems*, vol. 31, no. 1, p. 015001, 2014.
- [4] V. P. Sudarshan, G. F. Egan, Z. Chen, and S. P. Awate, “Joint pet-mri image reconstruction using a patch-based joint-dictionary prior,” *Medical Image Analysis*, vol. 62, p. 101669, 2020.
- [5] P. J. Huber, *Robust statistics*. John Wiley & Sons, 2004, vol. 523.
- [6] C. Zhu, R. H. Byrd, P. Lu, and J. Nocedal, “Algorithm 778: L-bfgs-b: Fortran subroutines for large-scale bound-constrained optimization,” *ACM Transactions on mathematical software (TOMS)*, vol. 23, no. 4, pp. 550–560, 1997.
- [7] A. Chambolle and T. Pock, “A first-order primal-dual algorithm for convex problems with applications to imaging,” *Journal of mathematical imaging and vision*, vol. 40, no. 1, pp. 120–145, 2011.