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Integral Equation Modelling and Deep Learning

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Integral equation modelling

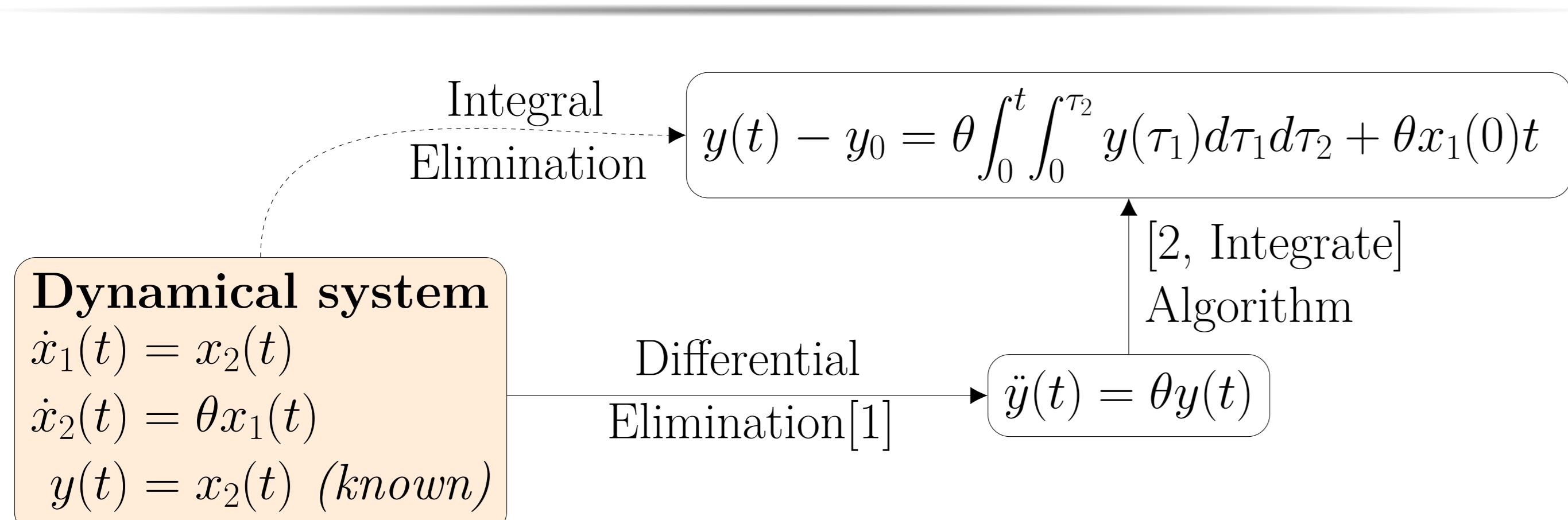


Figure 1: Modelling, elimination and [2, Integrate] algorithm.

Many models (such as the SIR epidemiology model) consist of non-linear differential equations, involving unknown numerical parameters. Recent work investigates the treatment of integro-differential equations i.e. equations involving both derivatives and integration operators. The introduction of integral equations helps to increase the expressiveness of the models, improve the estimation of parameters and reduce the size of the intermediate equations.

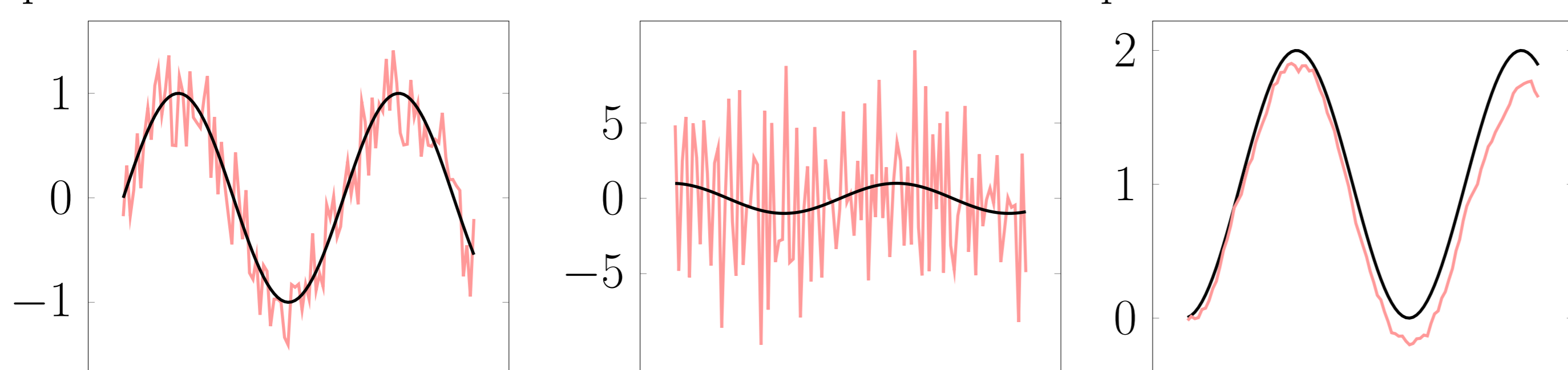


Figure 2: A signal, its derivative and its integral (with noise in red).

Integration of equations with deep learning [3]

Through learning techniques used for the translation of texts, Lample and Charton [3] succeed in calculating primitives of mathematical expressions, and solutions of differential equations.

The model used is called Transformer and is widely used in natural language processing [4]:

- It takes a list of words as input and returns the list of translated words as output,
- Lample and Charton have created a data generator to generate the train and test datasets, composed of pairs (f', f) in Polish notation.

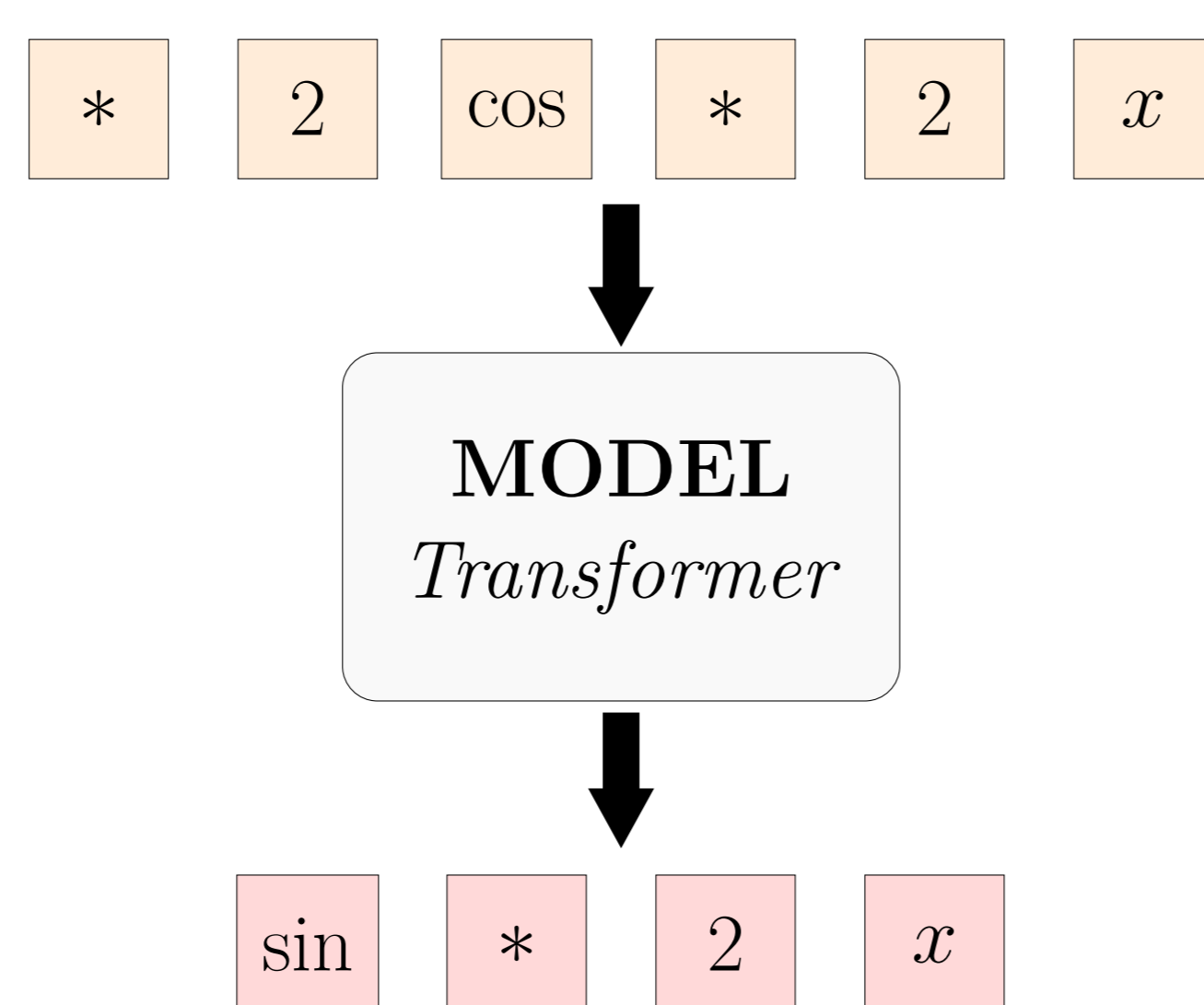


Figure 3: Integration of $2 \cos(2x)$ with Deep Learning.

Adaptation of [3] to integro-differential equations

In order to integrate integro-differential equations, we modified the implementation of [3] (based on PyTorch) to fit our needs.

Generation

The generation process has been modified to handle the ∂ and f operators. Time dependent functions have also been added. Thus, the main changes concern the unary-binary tree generator and the use of SymPy to handle these additions.

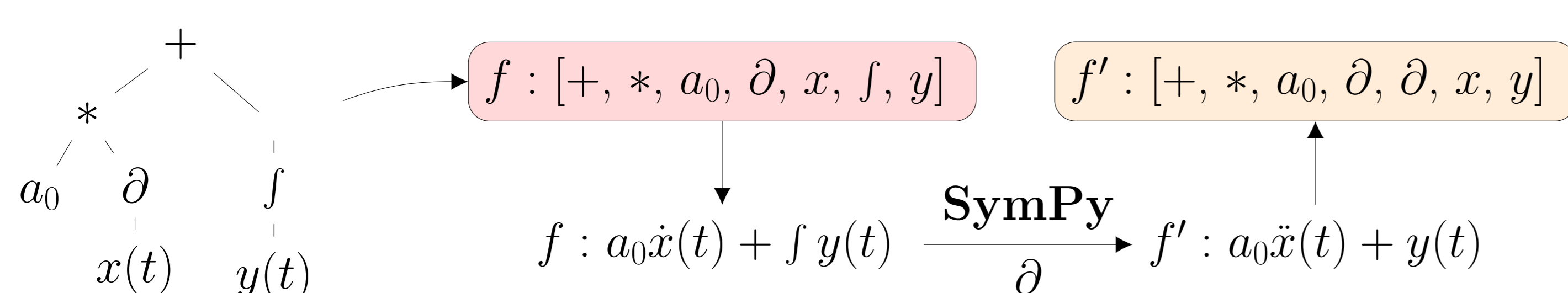


Figure 4: Generation process

Training and evaluation:

During the training and evaluation phases, we need to check whether a predicted integral is correct or not. Lample and Charton verify that a prediction \hat{f} is correct by checking that $\partial \hat{f} - f'$ is zero. Our modifications lead us to adapt this test, in particular to deal with the f operator.

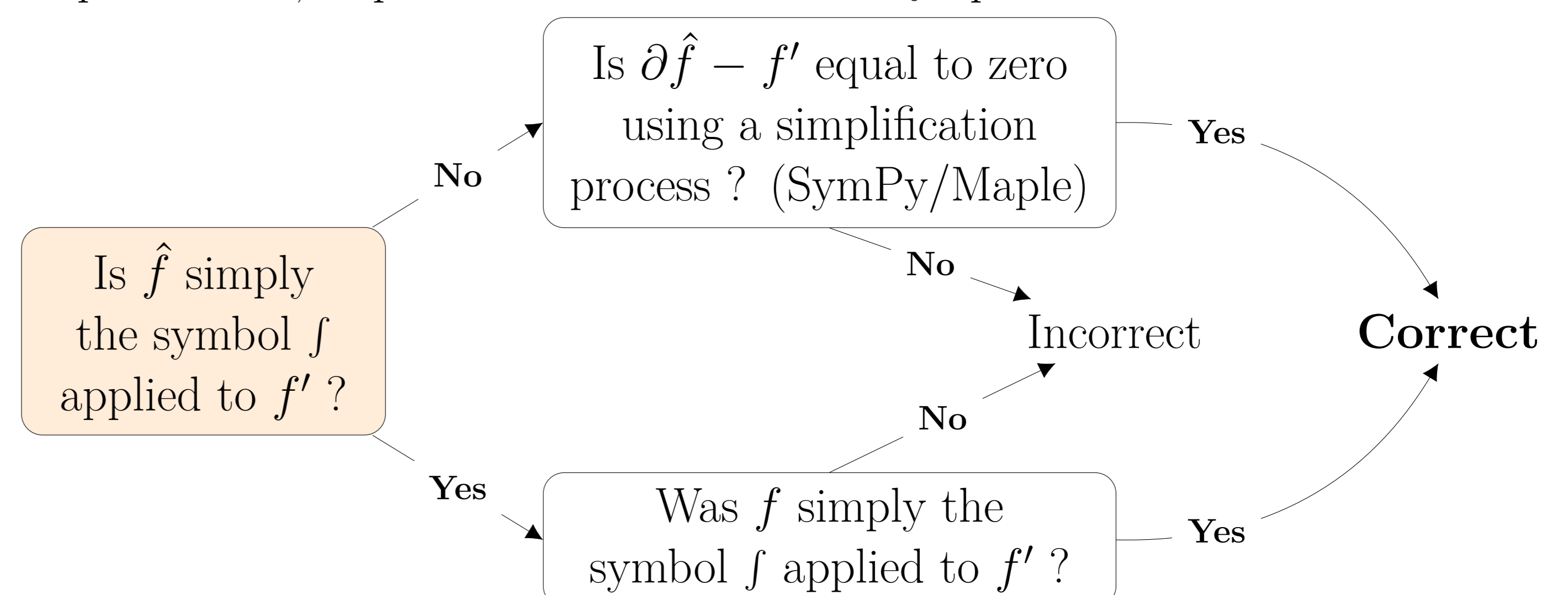


Figure 5: Test for checking if a predicted integral \hat{f} is correct.

Experiment

For our experiment, we generated functions that include the following operations/operators: $+$, $-$, $*$, \div , ∂ , f . We used two functions: $x(t)$, $y(t)$ and a parameter, a_0 . The generated functions are then differentiated in order to obtain pairs (f', f) .



$$\text{Accuracy} = \frac{\text{number of correct integrals (Figure 5)}}{\text{number of equations of the test dataset}}$$

Figure 6: Learning process and accuracy measurement

Results

	Accuracy	Comments
Maple	75.8%	We use $\text{int}(f, t)$. Could be enhanced with integration by parts, but it is difficult to automate.
Transformer	96%	Our adapted version of [3].

Table 1: Accuracy of the model versus the accuracy of Maple.

Function f'	Solution f
$\frac{1}{f x} - \frac{t x}{(f x)^2}$	$\frac{t}{f x}$
$\frac{(-a_0 x - 1) \int x y + \dot{y} (t + a_0 \int x) + a_0 x y \int x + x y t - a_0 \dot{y} x - \dot{y}}{(t + a_0 \int x)^2}$	$\frac{\dot{y} + \int x y}{t + a_0 \int x}$

Table 2: Functions that the model can integrate, but not Maple.

Conclusion

We achieve a quite good accuracy on the integration of a specific set of integro-differential equations. Different data generation methods could be investigated (such as [3, IBP and FWD methods]). Our next goal will be focused on the integral elimination of differential systems with deep learning.

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