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Mixture of noises and sampling non log-concave distributions

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1/ Aims and contributions

Real life inverse problems: **multiple noises**, **non-linear** models, need for **uncertainty** quantification.

Bayesian inference with

- Mixture of noises
- Non-log-concave posterior (multimodal)
- Non-gradient-Lipschitz log-posterior

- ★ Likelihood approximation for a mixture of noises,
- ★ New kernel to efficiently sample from posterior,
- ★ Application to astrophysical data.

2/ Observation model

→ **Forward model** $f : \Theta \in \mathbb{R}^{ND} \mapsto \mathbf{Y} \in \mathbb{R}^{NL}$, strictly positive, twice differentiable, covers multiple decades.

→ **Observation model:**

$$y_{n,\ell} = \max \left\{ \omega, \epsilon_{n,\ell}^{(m)} f_{\ell}(\theta_n) + \epsilon_{n,\ell}^{(a)} \right\}$$

with $\begin{cases} \omega > 0 \\ \epsilon_{n,\ell}^{(a)} \sim \mathcal{N}(0, \sigma_a^2) \text{ i.i.d.} \\ \epsilon_{n,\ell}^{(m)} \sim \log \mathcal{N}(0, \sigma_m^2) \text{ i.i.d.} \end{cases}$

⇒ untractable likelihood $\pi(\mathbf{Y} | \Theta)$.

→ Literature often neglects one noise [1]. But when f covers many decades: dominant noise depends on θ_n .
⇒ need to address full mixture model.

How to **approximate the likelihood** with **controlled error**? → See 4/

3/ Bayesian approach & sampling

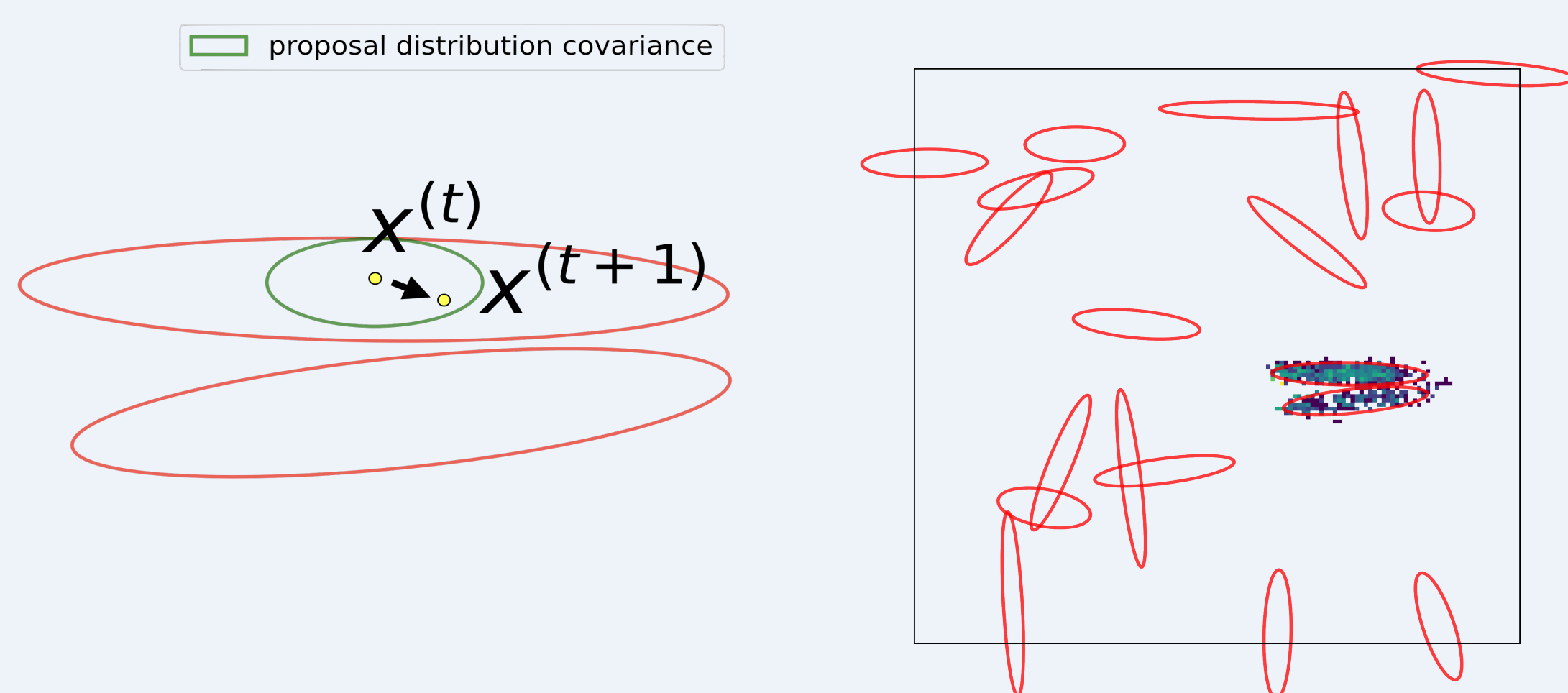
→ Combine likelihood with informative **prior** $\pi(\Theta)$: yields **Posterior** distribution:

$$\pi(\Theta | \mathbf{Y}) \propto \pi(\mathbf{Y} | \Theta) \pi(\Theta)$$

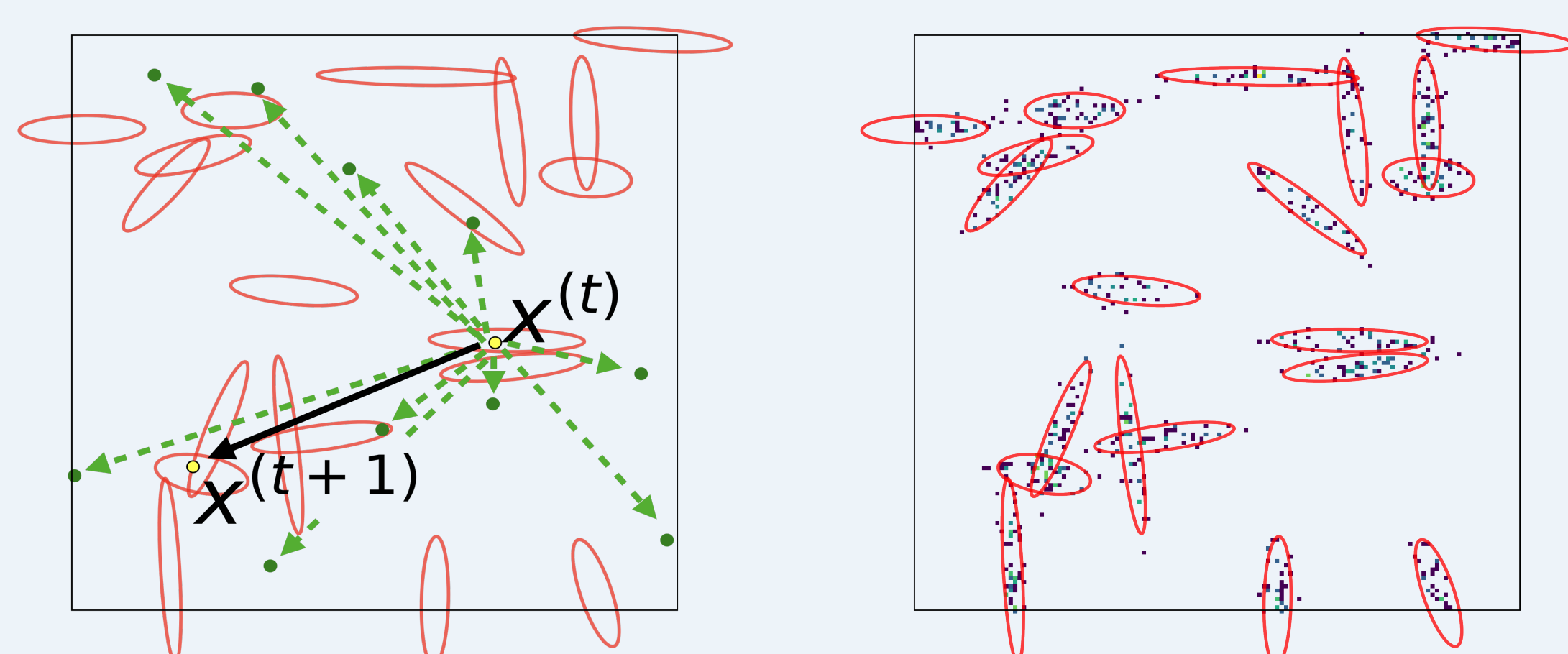
→ Sampling: MCMC
⇒ uncertainty quantification

→ Model: Non-linear
⇒ **multimodal** distribution

✓ Preconditioned-MALA [2]: local exploration



✓ MTM [3]: jumps between minima.



4/ Mixture of noises: Likelihood approximation

1. Build 2 likelihood approximations with moment matching:

Gaussian approx $\pi^{(a)}$	lognormal approx $\pi^{(m)}$
$y_{n,\ell} \simeq f_{\ell}(\theta_n) + e_{n,\ell}^{(a)}$	$y_{n,\ell} \simeq e_{n,\ell}^{(m)} f_{\ell}(\theta_n)$
$e_{n,\ell}^{(a)} \sim \mathcal{N}(m_{a,n,\ell}, s_{a,n,\ell}^2)$	$e_{n,\ell}^{(m)} \sim \log \mathcal{N}(m_{m,n,\ell}, s_{m,n,\ell}^2)$

2. Combine approximations with weight function $\lambda_{\ell}^{a_{\ell}}$:

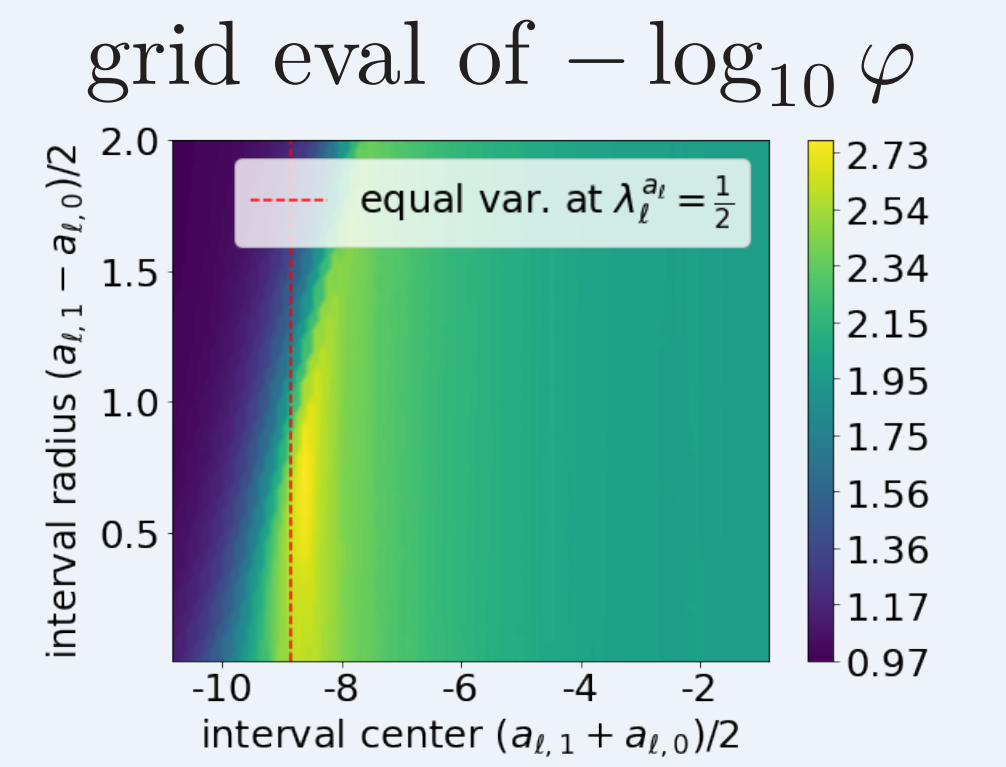
$$\tilde{\pi}_{a_{\ell}}(y_{n,\ell} | \theta_n) \propto \underbrace{\pi^{(a)}(y_{n,\ell} | \theta_n)}_{\text{Gaussian approx}}^{1 - \lambda_{\ell}^{a_{\ell}}(\theta_n)} \underbrace{\pi^{(m)}(y_{n,\ell} | \theta_n)}_{\text{lognormal approx}}^{\lambda_{\ell}^{a_{\ell}}(\theta_n)}$$

with $\lambda_{\ell}^{a_{\ell}} \in [0, 1]$, twice differentiable, parametrized with $\mathbf{a}_{\ell} \in \mathbb{R}^2$
⇒ negative log of this approx: easy to work with.

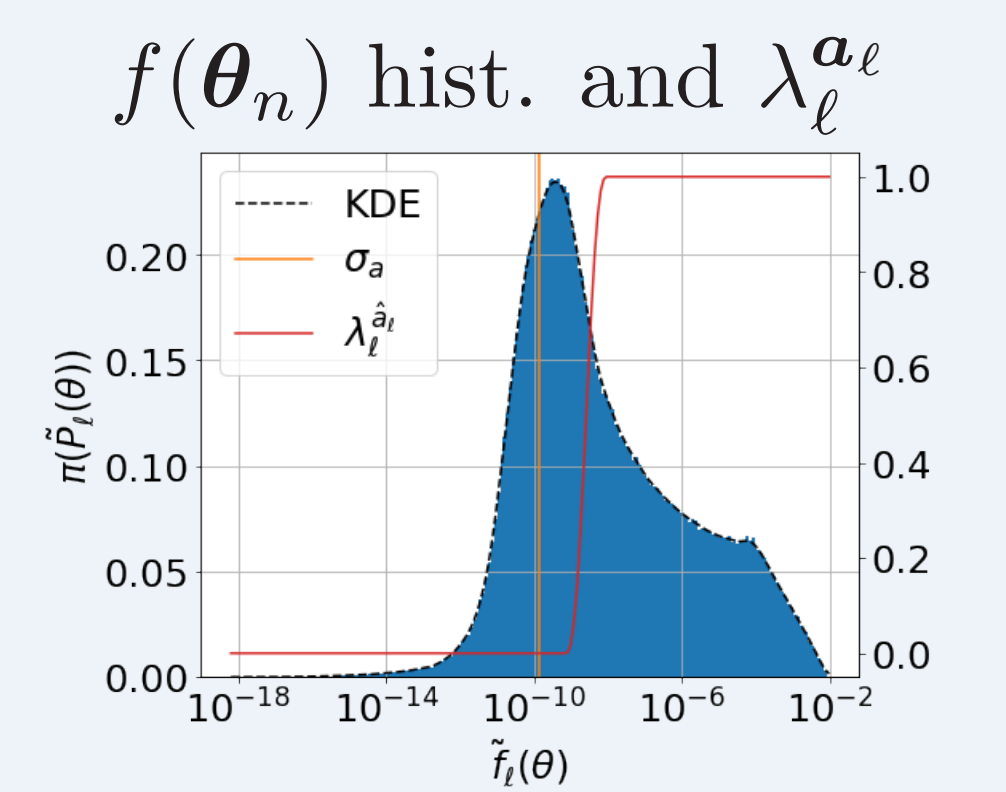
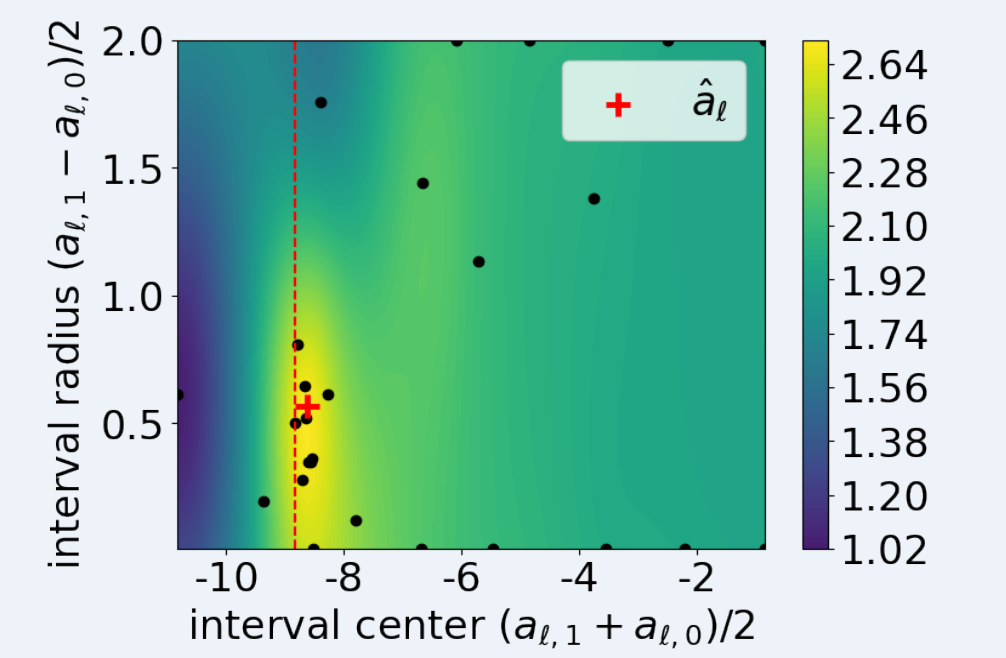
Tuning \mathbf{a}_{ℓ} :

Approx error = Kolmogorov-Smirnov-based metric $\varphi(\mathbf{a}_{\ell})$.
 φ minimized with Bayesian Optimization (BO).

→ From approx error: $\tilde{\pi}_{a_{\ell}}$ **better than** $\pi^{(a)}$ **or** $\pi^{(m)}$ **alone.**



BO, 25 points



5/ Astrophysics synthetic dataset

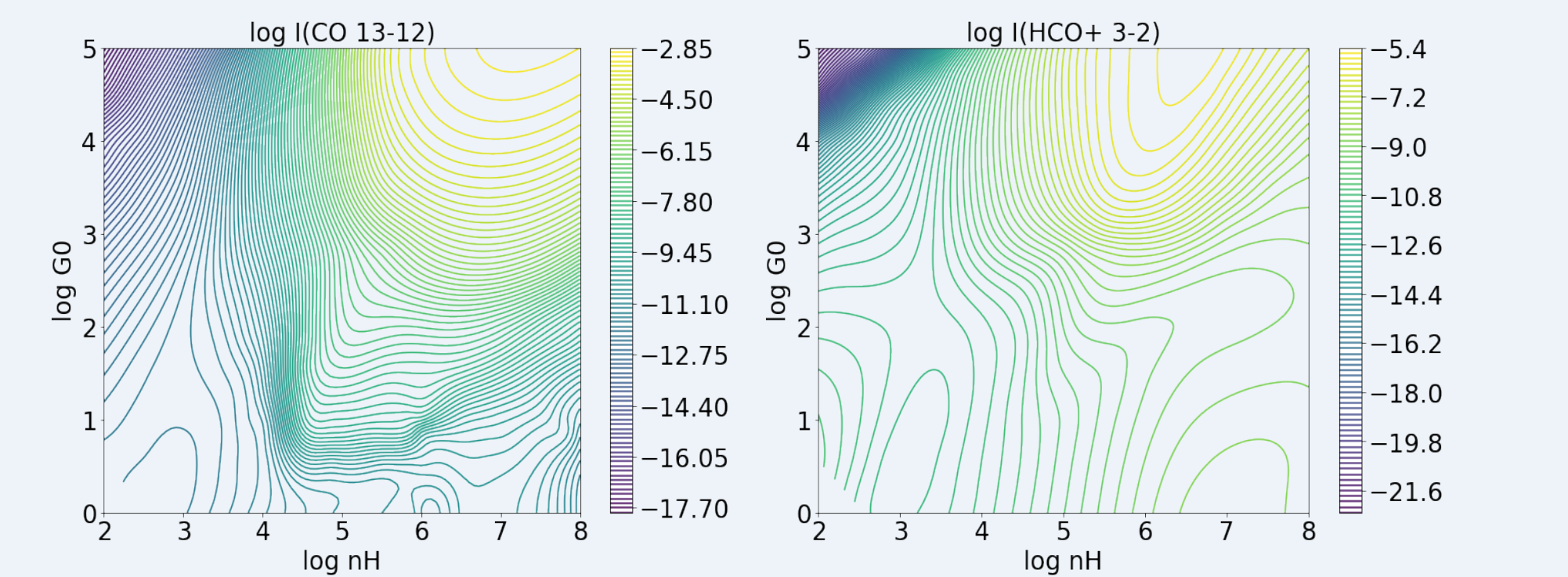
→ $\Theta \in \mathbb{R}^{900 \times 4}$: high dimensional

→ $\pi(\Theta)$: spatial (Laplacian L_2 -norm)

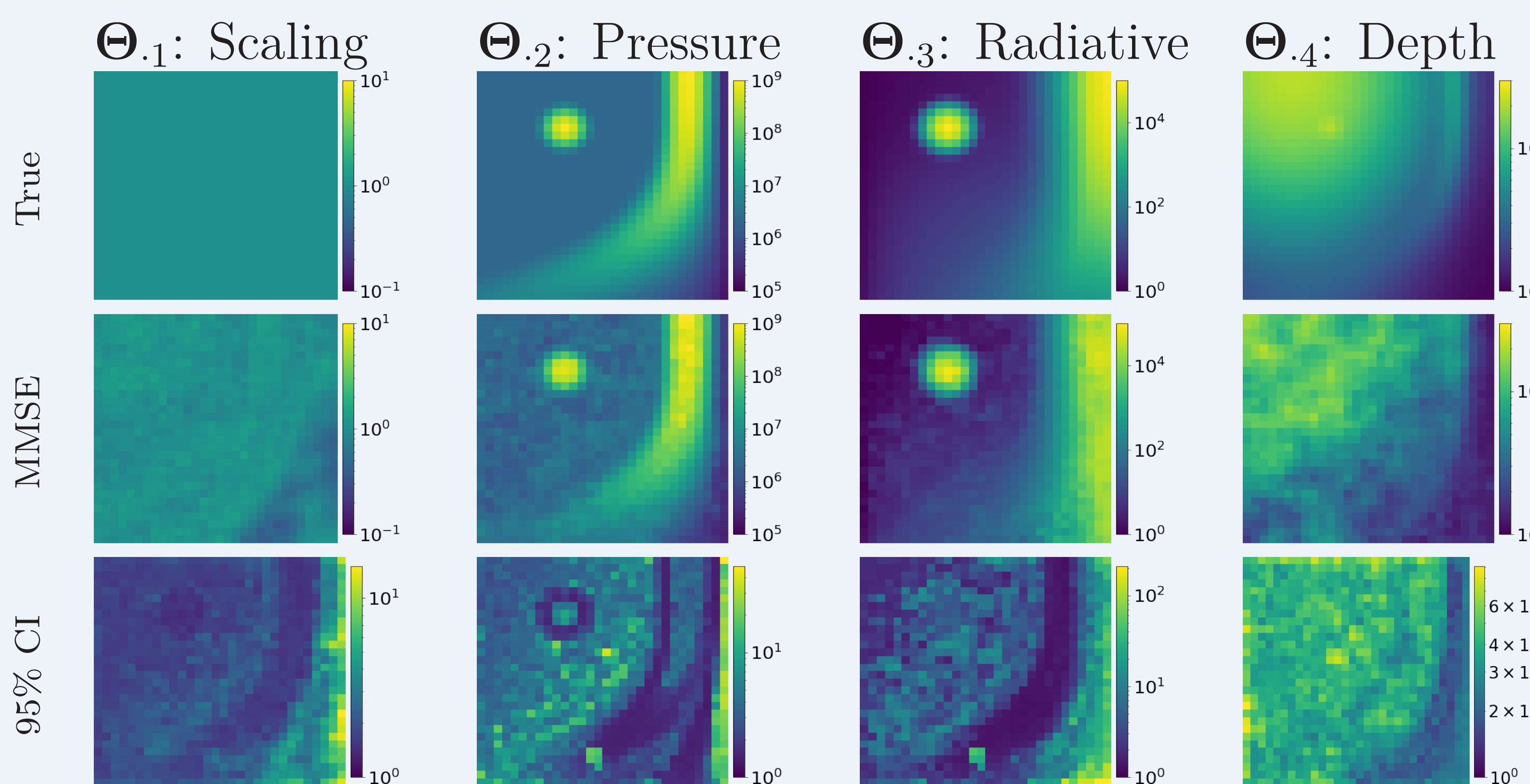
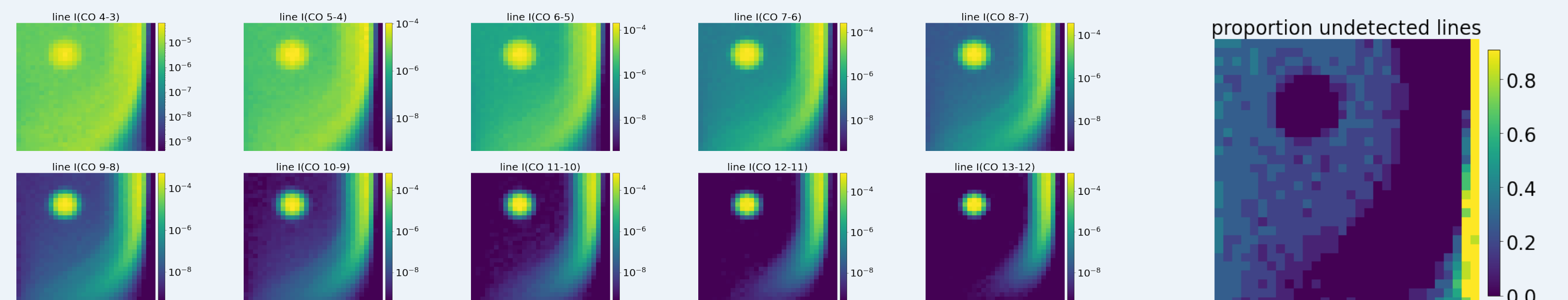
Forward model: Meudon PDR code [4]

✗ non-linear

✗ non-gradient-Lipschitz

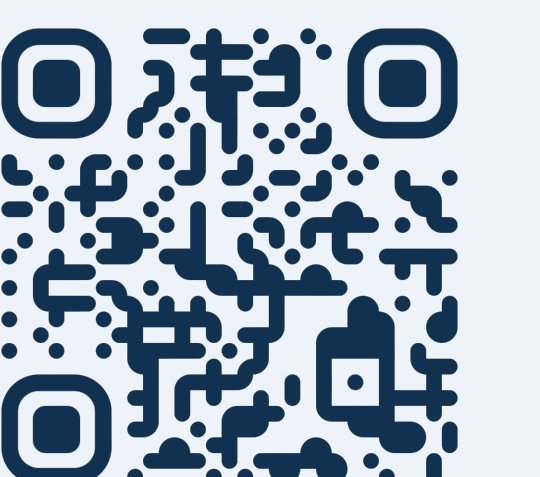


Synthetic observations $\mathbf{Y} \in \mathbb{R}^{900 \times 10}$: integrated intensities of excited lines of CO



6/ Conclusion

- ✓ Mixture of noises ⇒ likelihood approx.
- ✓ Non-log-concave posterior ⇒ MTM kernel.
- ✓ Non-gradient-Lipschitz log-posterior ⇒ P-MALA kernel.
- ✓ Application on astrophysical inverse problem.
- Application to Orion-B data, James Webb Spatial Telescope, etc.



References

- [1] Nicholson and Kaipio, *An additive Approximation to Multiplicative Noise*, 2018
- [2] Xifara et al., *Langevin diffusions and the Metropolis-adjusted Langevin algorithm*, 2014
- [3] Martino, *A review of multiple try MCMC algorithms for signal processing*, 2018
- [4] Le Petit et al., *A Model for Atomic and Molecular Interstellar Gas: The Meudon PDR Code*, 2006