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# Predicting medicine demand fluctuations through Markov chain

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**Abstract.** Nowadays, the healthcare sector is rapidly changing. Hospitals are facing limited budgets and high costs. The logistics activities of the hospitals in France (stock management, delivery, etc.) represent one of the highest cost components. The logistic costs can be reduced through an optimized inventory management system. The inventory optimization is strongly dependent on the accuracy of the demand prediction of medicines. Many factors influence this demand, such as seasonality, hospital size and location, etc. The objective of the paper is to propose a Markov chain model to estimate medicine demand fluctuations for hospital logistics in France. An analysis of the first experimental results is proposed to assess the effectiveness of the method. This preliminary result could contribute to the management of hospital inventories.

**Keywords.** Inventory management, demand forecasting, hospital pharmacy, stochastic models, Markov chain.

## 1. Introduction

The healthcare sector is rapidly changing, especially since the 90s. Organizations in the sector are promoting projects in areas such as care-related logistics, information systems and quality of care to cope with characteristics such as increased competition, patient influence and the need for a more efficient and effective service [1].

The logistics related activities can account for about 46% of a hospital's operating budget in the United States for instance [2]. The average health expenditure in France in 2019 was 11% of gross domestic products. The main logistics activities include planning, designing, implementing, and managing material [3]. These activities seek to support functions such as inventory management, procurement, and distribution [4]. One of the highest hospital costs is related to these logistical activities, as an example in OECD<sup>1</sup> it

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<sup>1</sup> The OECD's 38 members are: Austria, Australia, Belgium, Canada, Chile, Colombia, Costa

represents 30% and is the second largest cost according to Volland et al. [5]. More precisely, in the health sector, inventory costs are estimated between 10% and 18% [1].

The logistics costs in hospitals can be reduced, in some cases even in a half, by implementing efficient logistics management [5]. That is the reason why hospitals have to generate better returns in their internal service. In this case, inventory management becomes more important [1]. In order to decrease the costs associated to inventories, French hospitals are interested in the reduction of the inventory supplies [6]. Since the inventory optimization depends highly on the quality of the demand prediction, in this study, we focus on the estimation of medicines demand fluctuations for hospital logistics.

This study begins with an overview of existing methods for forecasting medicine demand in hospital pharmacies. Many factors influence this demand, such as seasonality, the size and location of hospitals, etc. In particular, demand can be stationary if its behavior does not follow an up or down trend over time. Therefore, a stochastic method may capture fluctuations in demand from period to period. Markov chain is a particular type of no-memory stochastic process with discrete or continuous time and states [22] where the prediction of the future state only depends on the current state [25]. It is commonly used in different fields including inventory management to forecast demand. Indeed, depending on the definition of the states of the model (for instance, a state can be the demand level during one season), the forecast of the demand fluctuation in the current state in next season does not depend on the previous state (previous season). Moreover, compared to machine learning methods for instance, we do not need to identify the factors impacting medicine consumption to predict the consumption fluctuation. The objective is to use monthly historical data from French hospitals to develop a Markov chain model in order to evaluate its performance to predict monthly demand fluctuations. A state of the proposed Markov chain represents an interval of consumption during one month. For this purpose, the document is structured as followed. Section 2 presents a literature review about inventory management in hospitals and stochastic models. Section 3 presents the methodology followed to develop the Markov chain model. Section 4 presents a case study in the hospital of Montpellier to exemplify the application of the proposed methodology. The results obtained from the application of the proposed method to selected medicines are presented in section 5. Section 6 discusses some of the findings in this work. The last section presents the conclusion and the perspectives.

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Rica, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovak

## **2. State of the Art**

There is considerable amount of research on inventory management and demand prediction in the literature. In this section, we first focus on papers related to factors that influence the demand in hospitals (to understand how the demand can be modeled), and secondly on existing forecasting methods.

### **2.1 Influencing factors in hospitals**

The inventory management system is an important factor that affects the performance of the operations in hospital systems. Inventory management in this context means controlling and managing a variety and quantity of items stored in a hospital pharmacy [7]. Healthcare systems are complex systems linked with several influencing factors such as the demand which is by itself related to other factors like: the size of the hospital, the type of population, etc. These factors influence the performance of inventory systems and must be considered when modeling and analyzing these systems [8].

An important factor in inventory management is the demand. In hospitals, the demand for health supplies is strongly linked to the doctor's recommendations based on the patient's condition [9]. The consumption of healthcare supplies is non-stationary (follows an up or down trend), the demand may depend on aspects such as the number of patients, the patient's conditions, the stage of treatment, among others. Some studies assume demand as independent and constant. However, to perform an analysis in a more real scenario, it is necessary to consider a stochastic demand [10].

The classification of inventory problems in healthcare sectors depends on the nature of the associated factors: the problems can be either deterministic or uncertain [11]. In the first case, the influencing factors are known over the period. This kind of inventory problems are not frequent in hospitals but may be applied to general medical items like syringes, gloves, intravenous fluids, and vaccines [11]. The other case is uncertainty; Healthcare faces conditions of uncertainty such as the patient's clinical conditions and response to treatment, availability of providers, demand for medicines and lead time [12]. The demand character distinguishes stochastic inventory models from deterministic models. Demand is fixed in deterministic models while in stochastic models it is a random variable with a probability distribution [13]. Several factors that fluctuate at random, such as the patient's condition, his uncertain reaction to therapy, period of stay at hospital and the transition from one type of hospital care unit to another at various phases of treatment, can all

have a substantial impact on demand [11]. These random factors change with time, which explain why demand of required medicines is uncertain [14].

## **2.2 Forecasting methods**

Traditional methods for predicting demand in a hospital exist, and several of them can also anticipate the stationary demand for healthcare elements. Several works consider time series to forecast the demand. Ramírez et al. [16] apply different traditional methods such as simple moving mean, exponential smoothing and ARIMA models on no changing demand. Causal models like simple and multiple linear regression are applied when a cause-effect relation between a dependent variable and independent one exists. A forecasting exercise is performed by Varghese et al. [17] in two hospitals using, in most cases, ARIMA models for several medicines with no significant trends or non-stationary patterns in the demand. However, healthcare item consumption is usually non-stationary and uncertain [18]. Kadri et al. [19] successfully apply the ARIMA method to forecast the daily attendances in emergency department using one year of data from the paediatric emergency department in Lille regional hospital center in France.

In recent years, machine learning approaches have gained popularity, especially when it comes to integrate parameter estimation like the demand and inventory optimization [20]. Research in pharmacology identifies accurate methods for constructing a predictive model: linear regression, random forest method, neural network, and support vector regression [20]. The demand for most medicines depends on the periods of increased risk of spreading diseases, in this case, seasonal trends need to be considered when developing the model. A study using real drug consumption data in Rwanda is conducted by Mbonyinshuti et al. [21]. The authors applied machine learning methods to predict the demand for 10 selected medicines. According to their findings, machine learning methods can be used to predict the demand for medicines and Random Forest algorithm has the best performance. Machine learning can also be used to compare and to select the best prediction model among a set of predictive models for non-stationary demand [22].

Stochastic approaches can be used to solve problems when information on the probability distribution on some parameters such as demand, patient conditions are available. For instance, Markov decision process which relies on functional stochastic dynamic programming equations is used to determine the inventory levels [8]. In this context, Saha and Ray [11] use a Markovian demand approach to incorporate external factors impacting demand. They model a patient condition-based medication demand process which allows to capture the uncertain patient conditions (in terms of treatment stages and type of care units). Hermosilla et al. [24] propose models to predict the demand for

cardiovascular drugs using Markov chain. They consider models with 3 and 4 states. Each state represents patient conditions. These methods can be used to prioritize patients with greater consumption levels in critical inventory situations. Lastly, another Markov chain model is defined by Uzunoglu Kocer [25] to model and estimate the intermittent demand in products.

The reviewed studies on methods that predict drug demand are summarized in Table 1. In this table, the objectives, the data characteristics, the performance measures, and the limitations and advantages of each method/paper are given.

**Table 1** Methods used in inventory management to predict medicines demand.

Method classification	Reference	Method(s)	Data	Performance measures	Limitations	Advantages
Traditional	Ramírez et al. (2014)	Simple moving average	Monthly	MAE <sup>2</sup> Typical deviation	-Better results in non-variable demand -Cannot capture demand variation well	-Easy to apply -Easy to understand
		Exponential smoothing				
		Simple and Multiple linear regression				
	Hyndman and Athanasopoulos (2018)	Average	Monthly	MAE, MSE <sup>3</sup> MASE <sup>4</sup> , MAPE <sup>5</sup>		
		Naïve				
		Naïve seasonal				
	Varghese et al. (2012)	Simple moving average	Weekly	MAE		
Cumulative average						
Exponential smoothing						
Naïve						
Machine learning	A G Kravets et al. (2018)	Linear regression	Monthly	MAPE MSE	-Require a large number of observations -Require good data -Not Easy to apply -Not Easy to understand	Powerful in Big data
		Random forest				
	Mbonyinshuti et al. (2021)	Linear Regression	Monthly	RMSE <sup>6</sup> R-square		
		Artificial neural network				
Stochastic processes	Hermosilla et al. (2020)	Markov chain-several numbers of states	Monthly	MAPE Inventory parameters MASE <b>R-square</b> <b>MSE</b>	-Need large observations number -Good quality	-Capture variation in demand well
	Saha and Ray (2019)	Markov chain based in stage of treatment				

<sup>2</sup> MAE: Mean Absolute Error,

<sup>3</sup> MSE: Mean Squared Error

<sup>4</sup> MASE: Mean Absolute Scaled Error

<sup>5</sup> MAPE: Mean Absolute Percentage Error

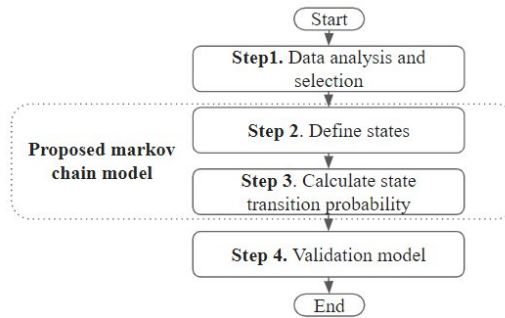
<sup>6</sup> RMSE: Root Mean Square Error

	Uzunoglu Kocer (2013)	Modified Markov chain model		<b>MAE</b>	data	
	<b>Present study</b>	<b>Markov chain based on consumption intervals</b>				

In the present study, our objective is to capture medicine demand fluctuations as the demand is variable. In this preliminary study, we will focus on Markov chain using monthly data model to confirm the fluctuations of demand and predict them. In such model, we do not need to include factors such as patients conditions, hospital size, etc.

### 3. Methodology

The Markov chain technique is used to capture consumption fluctuations from one period (month) to another. The objective of this section is to propose a discrete-time (representing the months) Markov chain model that can predict the medicine demand fluctuations in future periods. The proposed steps of our Markov-Chain-based methodology are summarized in the following:



**Fig. 1** Proposed methodology for forecasting consumption - Markov chain model.

#### Step 1. Data analysis and selection

We consider the historical data related to consumption of medicines aggregated by months in Montpellier hospital for 2016, 2017 and 2018. The data set has 65450 observations (consumption of a medicine during one month), and 2396 medicines. For one observation, the quantity, the name, the year, and the month of the medicine consumed is available. In the following, for each year  $y$ , for each medicine  $m$ , we denote by  $M_m^y$  the set of observations. Note that an observation can be any type of medicine.

Preliminary results show that medicines which are not consumed frequently or constant are not interesting for our study. The following criteria were used to select the relevant medicines:

1. High level of consumption at each month (greater than 500)

2. Large variation of consumption value. The ratio between the minimum and the maximum value was calculated and medicines with a ratio less than 0.5 were taken.

After applying these criteria, the resulting data set has 156 medicines. For each of these medicines, steps 2, 3 and 4 are then applied.

## Step 2. Define states of Markov Chain model

We define the states as intervals of consumption. The number of states is denoted by  $n$ . Four methods are proposed to define the intervals.

*Method 1.* For each year  $y$ , for each medicine  $m$ , the states are determined dividing the demand data in quantiles. It is divided into equal sized groups. It means that the number of points in each state/interval is the same. The intervals for  $n$  states are represented as follow:

$$[0, Q_{i1}^y], [Q_{i1}^y, Q_{i2}^y], \dots, [Q_{i, n-1}^y, Q_{inn}^y], [Q_{inn}^y, \infty[ \quad (1)$$

where  $Q_{inn}^y$  is the  $n$ -th quartile of medicine  $m$  at year  $y$ .

*Method 2.* The grouping of the data is performed taking the average value of consumption  $\bar{C}_m^y = \sum_{c \in M_m^y} c / |M_m^y|$  and applying a percentage tolerance  $\alpha$  above and below this value. The method can only be used for odd number of states. For three states for instance, we have the following intervals:

$$[0, (1 - \alpha) \bar{C}], [(1 - \alpha) \bar{C}, (1 + \alpha) \bar{C}], [(1 + \alpha) \bar{C}, \infty[ \quad (2)$$

*Method 3.* In this method, the extreme of the intervals for each year  $y$  and each medicine  $m$  are calculated taking the maximum  $\bar{l}_{ym}$  and minimum  $\underline{l}_{ym}$  values of consumption in demand data set  $M_m^y$  and dividing by the number of groups desired. We denote by  $L_m^y = \left\lceil \frac{(\bar{l}_{ym} - \underline{l}_{ym})}{n} \right\rceil$  the size of the intervals for medicine  $m$  in year  $y$ . In contrary to method 1, the number of values in each interval/state is not necessarily the same. The intervals are given by:

$$[0, vL_m^y], [vL_m^y, (v + 1)L_m^y] \text{ for } v = 1, \dots, n - 1 \quad (3)$$

*Method 4.* According to Du et al. [26] a common machine learning technique can be used which is K-means. K-means clustering takes the data sequence and classifies each data point into a specific cluster. To define the Markov chain states, we use K-means to compute  $n$  clusters. Each cluster will be equal to a state. Now that the four methods are defined. The states are generated and the probability transition between each pair of states is defined in step3.

The number of states  $n$  depends on the method used. In the following, we set  $n$  to 3 and compare the performance of the four proposed methods.

### Step 3. Calculate the transition probability matrix

The probability transition  $P_{ij}$  between states  $S_i$  and  $S_j$  can be determined using the frequencies of transitions following expression [26]:

$$P_{ij} = \frac{T_{ij}}{\sum_{j=1}^K T_{ij}} = P_{ij}\{X = S_j | X = S_i\} \quad (4)$$

where  $T_{ij}$  is the number of transitions between the state  $i$  ( $S_i$ ) and the state  $j$  ( $S_j$ ) in one period, and  $K$  is the total number of states. In the present study the state transition matrix is composed of historical data of consumption by months. The probability transition from one state to another describes the probability of changing the state (demand interval) in the next month.

### Step 4. Validation of the proposed model

To validate our model, we choose to create it based only on historical consumption data between 2016 and 2017. We call it the prediction model. Ideally, this model should allow to predict the consumptions for every year such as 2018. Therefore, we apply it on 2018 data using the same consumption intervals/states (the probability transitions are updated). Then, we compare the prediction error/gap: the real probabilities of 2018 are compared with the probabilities of the prediction model.

In the literature, there are a variety of ways for evaluating predicting error metrics. For this purpose, coefficient of determination ( $R^2$ ), Mean Square Error ( $MSE$ ), and Mean Absolute Error ( $MAE$ ) were selected for evaluating the predicted model with the real model constructed using historical data. The closer the  $R^2$  is to 1, the greater the prediction value's fit to the observation value. The average squared difference (resp. mean of absolute errors) between the estimated values and the actual values are measured by the  $MSE$  (resp.  $MAE$ ). A  $MSE/MAE$  value closer to zero reflects a smaller difference between the original values and the estimated values.

## 4. Case study

A case study is presented to illustrate the proposed Markov chain model. Within the selected medicines in step 1, the consumption of a particular medicine MED1 at Montpellier hospital is selected and presented as an example of illustration. We assume that the Markov chain has three states for this preliminary work implying that we have three consumption intervals. For sake of clarify, we call the three intervals L “Low consumption”, M “Medium consumption” and, H: “High consumption”. This simple assumption comes

from the fact that a demand is usually normal, however, depending on multiple parameters, consumption can either be higher or less than usual.

**Step 2. Define states of Markov Chain model :**

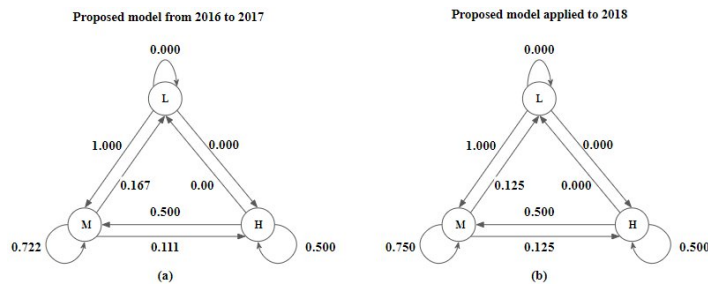
Table 2 presents the results to define the states after applying the four methods on MED1 using the historical data between 2016 and 2017.

**Table 2** Intervals of consumption for the four proposed methods.

	States	Method 1	Method 2	Method 3	Method 4
Intervals	Low	[0 - 773]	[0 - 717]	[0 - 901]	[(0 - 825]
	Medium	]773 - 860]	]717 - 971]	]901 - 1139]	]825 - 1051]
	High	> 860	> 971	> 1139	> 1051

After applying each of the methods to group the data, it can be observed that the size of the intervals varies depending on the method implemented.

**Step3. Calculate the transition probability matrix:** For each method, the frequencies of transitions are computed using equation (4) on historical data of year 2016 and 2017. For validation purposes, model of 2018 is generated using the same states calculated for 2016 and 2017. Figure 2 (a) and (b) show these two models using method 2.



**Fig. 2** Prediction Markov chain model from 2016 to 2017 (a) applied to 2018 (b) using method 2.

**Step4. Validating the proposed model**

Table 3 shows the error metrics comparing the predicted model for 2018 using the historical data of 2016-2017 and the Markov chain model using the real consumption of 2018 for the 4 methods.

**Table 3** Error metrics for the proposed model applying the 4 methods with 3 states

Methods	R <sup>2</sup>	MSE	MAE
Method 1	-11.095	0.382	0.350
Method 2	<b>0.997</b>	<b>0.017</b>	<b>0.009</b>
Method 3	0.683	0.205	0.123
Method 4	0.784	0.159	0.115

The results show that the proposed model using method 2 to define states has the best value which is 0.997 for  $R^2$ . Method 2 also has the best accuracy from the values of MSE and MAE that are 0.017 and 0.009, respectively. Method 1 has the worst results for  $R^2$ , MSE, MAE.

## 5. General results

We propose to evaluate the approach using 3 states model for the 156 selected medicines. We assume that a predicted Markov chain model is valid if the corresponding  $R^2$  (resp. MSE/MAE) is higher (resp. smaller) than 0.9 (resp. 0.1). Table 4 shows the medicines for which the predicted Markov chain is valid for each method.

**Table 4** Error metrics for the proposed model applying the 4 methods with 3 states

Method	$R^2$ (>0.9)	MSE (<0.1)	MAE (<0.1)
Method 1	/	/	/
Method 2	MED1	MED1, MED3	MED1, MED3, MED4, MED6
Method 3	MED2	/	MED2, MED4, MED5, MED7
Method 4	/	/	MED4, MED5, MED8

The results show that methods1 did not predict the consumption of any medicine. However, method 2 was able to perform better. MED1 presented in the previous sections has been predicted only by method2 confirming a good  $R^2$ , MSE and MAE. MED2 only by method 3. Other medicines such as MED4 were predicted by the three methods but only using MAE. This shows that the metric evaluation affect the results. However, with this metrics we are comparing values between 0 and 1 and it is easy to have an error less then 10% (MAE<0,1); This can be observed in figure 2 where method2 is applied. A low difference between the actual probability value and the predicted value makes the MAE always generating good result. The same is true for  $R^2$  and MAE. Hence, these metrics do not allow to validate precisely the models. In such case, it is worth to mention that our prediction is not accurate because it predicts a range of values and not the exact value. Other metrics need to be analyzed and tested in future research to confirm the results.

## 6. Conclusion

Uncertainty about medicine demand in hospitals decrease the performance of their inventory management. Predicting medicine demand fluctuations can provide an important input for inventory decision making. In this study, we propose Markov chain models where a state represents an interval of consumption during one month to predict medicine demand fluctuations. Four methods are proposed to define the states using historical data from Montpellier hospital. The experimental study shows that the proposed Markov chain models can be used as a forecasting tool for some of the medicines.

Some limitations of this study are: (1) The present study used monthly data available for two years to propose the model. This means that the model was proposed with 24 observations per medicine. More data could better capture fluctuations in drug consumption. Cloud systems can be used to store and treat more data. (2) The proposed model can only model stationary consumptions and not trendy ones. However, if consumption behavior changes significantly in the future (the minimum or maximum values changes for instance), the model will have to be adjusted to the new conditions. Furthermore, seasonality behavior can be studied to better understand the fluctuations in demand. Moreover, evaluation metrics can be adapted to confirm the accuracy of the proposed models.

For future research, new methods to group the data can be implemented. Even the variation in the parameters used to calculate the intervals in each method can generate interesting results. In addition, the application of the proposed model in the other hospitals can generate interesting results to validate the model with more data. Models that include more variables about influencing factors such as patient condition and number of care units can be built to improve the accuracy. For example, the model developed by Saha and Ray [1] can be explored in depth to capture better the demand fluctuations. Furthermore, in future work the proposed model can be compared with the ones presented in table 1.

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