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Classical charged particle models derived from complex shift methods

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Abstract

Extended charged objects embedded in complex space-time are proposed using the double-copy or complex shift method. Most of the objects studied are 3D strings in different shapes. The most detailed case studied is a static-charged open string. It is found that this can be interpreted purely electromagnetically. It exhibits the same relation between charge, mass, angular momentum, and magnetic moment as the Dirac equation and the Kerr-Newman metric. The spin of the particle is purely electromagnetic, as is the mass. A gyromagnetic ratio of 2 is obtained. The fields in this case are multi-valued, and the singular part of the fields can be arranged to be on an unphysical Riemann sheet with a judicious selection of Riemann cut surfaces. The calculations of mass and angular momentum are done numerically using multi-precision algorithms included as a Python script. The mass calculation agrees with the measured electron mass. Particles for knotted or linked strings in 3 space dimensions are also proposed, and these inherit the topological invariants of the knot or link they are based on. A liquid drop model with complex shift is also discussed. The multi-valued behavior of the solutions, related to that of the Kerr-Newman metric, can be thought of as the origin of the Einstein-Rosen bridge, and the conjectures that this is the origin of quantum entanglement, ER=EPR, is therefore supported in this theory. So we have here a classical theory that has some properties of quantum mechanics. Hopefully it can offer a new phenomenological application of string theory as a semiclassical model for elementary particles, nuclei, and solitons in condensed matter, fluids, and gases.

KEYWORDS electron model, Kerr-Newman, Kerr-Schild, complex space-time, C^4 , double copy, complex offset, emergent quantum mechanics, string theory, ER=EPR, quantum gravity

1 Introduction

Constructing purely electromagnetic particles is an ongoing effort in physics. Electrons and other fundamental charged particles tend to have both spin and magnetic dipole moments. Here we consider only classical models which have both of these. Point charges, of course, have divergent electrostatic energy. Line charges also have infinite energy, but the divergence is weaker than for the point charge. It is known that if classical electromagnetism is analytically continued into complex space-time then the point charge is still divergent [1, 2, 3], but including gravity yields the Kerr-Newman metric which has finite mass and angular momentum. This technique is sometimes called the complex-shift or double-copy method and it yields many exact solutions to general relativity that have a Kerr-Schild form of metric [4, 5, 6]. The idea of this paper is to apply the complex shift method to charged strings and other extended structures in complex space-time. Open and closed strings are considered, as is a liquid drop model. We study only static solutions. Most of these are multi-valued and they can have finite electromagnetic energy on at least one Riemann sheet. Although the fields can be finite, they have branch point singularities making them multi-valued. Because the stress-energy tensor is finite, we ignore gravitational effects and study only the electromagnetic fields in a flat spacetime.

There are several modifications to electromagnetism that allow for finite electromagnetic energy, the Born-Infeld theory [7] which is nonlinear, and the Bopp-Podolsky theory [8, 9] which adds higher-order derivative couplings to the field equations. Both of these theories modify the Maxwell equations and are interesting, but they are not considered in this paper.

Classical charged particle models have historically been constructed for 2D surfaces, like charged spherical shells, or for 3D blobs of charge [10, 11, 12, 13, 14, 15]. The complex-shifted extended-charge models considered here tend to have magnetic moments as well as electric charge and hidden angular momentum. The limiting case of a zero-length open string yields the Kerr-Newman solution, which has a g factor of 2 when the gravitational field is included, the same as an ideal Dirac electron [16]. The nonzero length strings that are considered here also have a g factor of 2 to high precision, but they don't require the gravitational field to be included. They can also have anomalous magnetic moments. Thus it seems they may be a better classical model of the electron and other particles than the spherical shell models or other classical extended charge models.

In general, the solutions we consider can have finite total electromagnetic energy as well as finite electromagnetic angular momentum calculated from the Poynting vector. Since the underlying theory is relativistically covariant, the usual "4/3 problem" can be understood and tolerated if the boost to moving frames is done properly [14, 15, 17, 13]. The stability of these particles is still an issue and may require some form of binding tension or negative pressure to oppose the electromagnetic repulsion. As they have electromagnetic charge, mass, angular momentum, and magnetic moment, there is the opportunity to search for an explanation for the fine structure constant which is proportional to the charge squared divided by the angular momentum of an electron in this

framework, but the present author has not found an explanation yet.

It is expected by most physicists that the electric quadrupole moment of an electron must be zero. This is because the Dirac equation does not allow the construction of a symmetric rank 2 or higher tensor using the Dirac matrices alone. There is a lingering possibility that the “dressed” electron might nevertheless have some perhaps state-dependent electric quadrupole that must be phenomenologically added to the wave equation due to its interactions. The Kerr-Newman particle has a large electric quadrupole [18]. In several of the simple string models studied below, it is found that the electric quadrupole moment can be zero while the magnetic dipole is non-zero.

We also consider several models which are not strings. One is a complex-shift version of the interesting and complete particle model in [13] which includes an elegant stability mechanism caused by a negative pressure force. We find for this that we can have a finite mass, angular momentum, and dipole moment, but that the electric quadrupole moment cannot be zero if there is a dipole moment. This could perhaps be a model for nuclei, but not for Dirac Fermions with no quadrupole. We also consider a simple spherical shell model in which all unwanted multipole moments are exactly zero. This model does not involve a complex shift, and the fields are single-valued, but its stability is problematical because the forces on the shell are not in the radial direction.

In this paper, natural units for particle and atomic physics shall be employed so that $c = 1$, $\epsilon_0 = 1$, $\mu_0 = 1$, $\hbar = 1$, $k_e = 1/4\pi$, electron mass =1, and $e = \sqrt{4\pi\alpha}$. We deal with the microscopic fields \mathbf{E} and \mathbf{B} here exclusively. In the Appendix, a Python software script is listed which can run on a desktop computer and reproduce all of the numerical calculations presented in this paper for the open string.

2 A discussion of some complex manifold methods in physics

Complex manifold techniques in general relativity have a long history. The majority of these works utilize a Hermitian metric. Einstein himself was one of the first [19, 20, 21]. Many others have followed this path [22, 23, 24, 25, 26, 27, 28, 29, 30]. A second method for dealing with complex space-time utilizes analytic continuation of fields from the real axes to complex coordinate axes. This technique was pioneered in general relativity too [31, 32, 3], but it has been applied to other fields as well. The idea that elementary particles might be related to the Kerr-Newman metric solution and string theory has been championed by Burinskii [16, 33]. Newman has also speculated about this [6]. Complex space methods have been proposed and applied in Bohmian and stochastic mechanics [34]. The wormholes of general relativity are related to complex space-time by the Kerr-Schild metric construction. The wormhole’s origin is the Riemann cut in the Maxwell potentials in complex space-time. These have been proposed as sources of quantum entanglement “ER=EPR”

[35].

It has been proposed that complex space-time can account for quantum non-locality [29], a concept championed early on by the radical physicists of the Fundamental Fysiks Group [36, 37]. It has also recently been proposed that complex space-time can provide a novel explanation for observed galaxy rotation without dark matter [38, 39].

Stability is another issue that needs to be addressed in these models. The Kerr-Newman solution is stable obviously because of general relativity and gravity. The string shapes we consider, which can be knotted and linked as well as open, may not be stable unless some tension is added along the string. We partially address this in a later section. Most of the cases we consider here are examples of what Wheeler called “Charge without charge” [40]. They deserve this moniker because the charge is located at coordinates that have non-zero imaginary parts so that there will be no charge on the real subspace. Developing an equation of motion for these objects in the presence of an external electromagnetic field is a non-trivial task.

3 Riemann-Silberstein Vector

It is convenient to work with the Riemann-Silberstein vector given by

$$\mathbf{F}(\mathbf{x}) = \mathbf{E}(\mathbf{x}) + i\mathbf{B}(\mathbf{x}) \quad (1)$$

If the fields are time-dependent, the Maxwell equations in the absence of charges on the real subspace become simply [41]

$$i \frac{\partial \mathbf{F}}{\partial t} = \nabla \times \mathbf{F} \quad (2)$$

In the static case, we find $\nabla \times \mathbf{F} = 0$ and consequently we can write \mathbf{F} as a gradient. In most of the models we consider here, there is no charge on the real subspace, and it follows that at all real points \mathbf{x} where the gradient exists we have

$$\nabla \cdot \mathbf{F}(\mathbf{x}) = 0 \quad (3)$$

and consequently

$$\nabla \cdot \mathbf{E}(\mathbf{x}) = \nabla \cdot \mathbf{B}(\mathbf{x}) = 0 \quad (4)$$

The electromagnetic energy density is given by

$$\mathcal{E} = |\mathbf{F}|^2 / 2 = (\mathbf{E}^2 + \mathbf{B}^2) / 2 \quad (5)$$

the momentum density we assume is equal to the Poynting vector given by

$$\mathbf{P} = (\mathbf{F}^* \times \mathbf{F}) / 2i = \mathbf{E} \times \mathbf{B} \quad (6)$$

and the angular momentum of the electromagnetic fields by the integral

$$\mathbf{J} = \int \mathbf{r} \times \mathbf{P} dVol \quad (7)$$

4 A static line charge in complex space

A straight-line charge with uniform charge per unit length is a good starting point. In the complex-shift method, the arena for physics is complex Minkowski space or C^4 . Electromagnetism in complex coordinates can be described by two equivalent methods. The first utilizes the Riemann-Silberstein (RS) complex vector [41], and the second utilizes a 4D complex Faraday tensor and is manifestly covariant. For simplicity, we start with the RS approach. We introduce a complex potential function $\psi = \phi + i\chi$ where ϕ is the electrostatic potential and χ the magnetostatic potential in a vacuum away from charges. We consider a static line charge oriented along the z -axis. We also choose the imaginary shift to be a constant in the z -direction. We write

$$\psi_L(\mathbf{x}) = \frac{1}{4\pi} \int_{-L/2}^{L/2} \frac{\rho(s)}{\sqrt{(\mathbf{x} - s\hat{\mathbf{z}} - ia\hat{\mathbf{z}})^2}} ds \quad (8)$$

where L is the length of the line charge, $\rho(s)$ the charge per unit length, and $ia\hat{\mathbf{z}}$ is an imaginary vector representing how far off the real axis the charge is shifted, and in which direction so that a has units of length. So long as $\rho(s)$ is real, there will be no apparent magnetic monopole charge far from this source. The observation or field point \mathbf{x} is real-valued. The Riemann-Silberstein vector is given by

$$\mathbf{F}(\mathbf{x}) = -\nabla\psi_L(\mathbf{x}) \quad (9)$$

If we take the limit of $\mathbf{a} \rightarrow 0$ we get the ordinary line charge which has divergent electrostatic energy. We shall search for systems for which the total energy and angular momentum are finite. The magnitude of the angular momentum will be proportional to the electric charge squared. The proportionality factor can be compared to the fine structure constant, and we dare to hope that some special class of these systems will give an explanation for its value. This model has two parameters, L and a , with dimension length. Dimensional considerations require that the total angular momentum be a function of the ratio L/a and not of L and a independently.

4.1 The Kerr-Newman limit

In the limit $L \rightarrow 0$ we get the familiar point particle with complex shift which yields the Kerr-Newman solution in general relativity [1, 6, 2, 3]. The complex potential, in this case, is simply given by

$$\psi_0(\mathbf{x}) = \frac{q}{4\pi\sqrt{(\mathbf{x} - ia\hat{\mathbf{z}})^2}} \quad (10)$$

For the Kerr-Newman metric, the angular momentum J_{KN} and the mass m_{KN} are determined by solving the Einstein field equations and examining the metric behavior far from the source. It is found that $a = J_{KN}/m_{KN}c$. For an electron

$a = \hbar/(2m_e c) = \lambda_e/(4\pi)$ where λ_e is the Compton wavelength of the electron. In our unit system, $a = 1/2$. This well-studied potential function has a ring singularity perpendicular to the z-axis with a radius a centered at $z = 0$. Because of the square root, it is a two-sheeted function. The cut can be chosen to be across the face of the ring singularity, and so long as we don't pass through this ring in analytic continuation, the function remains single-valued. This choice of the cut surface is not unique, and any deformation of it which is bounded by the ring is also a possible cut surface. If we pass through the cut though, the function exhibits its double-valued nature and we enter another Riemann sheet. This multi-valued behavior is the origin of the wormhole in general relativity which is popular in science fiction as a stargate. It is also the origin of the Einstein-Rosen bridge [42], and it has been proposed that this multi-connectedness of space-time is the origin of quantum entanglement [35]. The integral of the electrostatic energy for the potential function in (10) is infinite, but when the metric tensor is calculated using general relativity, the resulting mass of the solution as measured in the far field multipole expansion is finite. It has also been proposed that elementary particles might be small Kerr-Newman solutions modified by interaction with quantum fields [43, 44, 45]. The possibility that quantum mechanics itself might be derivable in the context of complex space-time and classical field theory has been explored [46, 47], and this paper is an extension of that exploration.

4.2 What about the mean value theorem and the maximum principle for harmonic functions?

The Kerr-Newman metric and the solutions that we consider here are derived from complex potential functions that satisfy Laplace's equation, that are multi-valued, and that have branch-cut singularities near the center of the particle. In order to use the mean value theorem in this case one must choose a boundary for the domain of the theorem which does not include any branch point. Therefore, the boundary can never enclose the entire particle or soliton, and so the various fields can fall off at large distances on the physical Riemann sheet without violating the maximum principle. This is clearly true for the Kerr-Newman case, and it is also true for the cases we consider here as well. A non-constant single-valued static potential satisfying Laplace's equation everywhere must have infinite energy because of the maximum principle. But, when the potential is multi-valued, finite energy is possible. So multi-valuedness is critical to most of the solutions we obtain here. This complicates the identification of the physical domain for volume integration over field-derived quantities like energy, momentum, and angular momentum.

4.3 Evaluation of the complex potential function

Let us consider the simplest case of constant $\varrho(s)$ in (8). The indefinite integral is of the form (up to a constant multiplier of $q/4\pi L$):

$$I(s) = \int \frac{ds}{\sqrt{\alpha + \beta s + \gamma s^2}} \quad (11)$$

This integral can be done analytically. The result is a multi-valued function whose Riemann-cut surfaces are quite complicated. A better way to proceed is to complete the square of the quadratic under the square root first:

$$\alpha + \beta s + \gamma s^2 = \gamma \left(s + \frac{\beta}{2\gamma} \right)^2 + \left(\alpha - \frac{\beta^2}{4\gamma} \right) \quad (12)$$

If we change integration variables to $u(s) = s + \beta/(2\gamma)$ then $du = ds$ and the integral becomes a contour integral in the complex u plane

$$I(s) = \frac{1}{\sqrt{\gamma}} \int_{\kappa} \frac{du}{\sqrt{u^2 + \left(\frac{\alpha - \beta^2/(4\gamma)}{\gamma} \right)}} \quad (13)$$

where κ is a smooth curve joining the two endpoints $u(-L/2)$ and $u(L/2)$. Define

$$D = \left(\frac{\alpha - \beta^2/(4\gamma)}{\gamma} \right) \quad (14)$$

The parameters $\{\alpha, \beta, \gamma\}$ are determined by $\alpha + \beta s + \gamma s^2 = (\mathbf{x} - s\hat{\mathbf{z}} - ia\hat{\mathbf{z}})^2$, and so

$$\begin{aligned} (\mathbf{x} - s\hat{\mathbf{z}} - ia\hat{\mathbf{z}})^2 &= r_{\perp}^2 + (z - s - ia)^2, \quad r_{\perp}^2 = x^2 + y^2 \\ \alpha + \beta s + \gamma s^2 &= r_{\perp}^2 + z^2 - a^2 - i2za - 2zs + i2as + s^2 \end{aligned} \quad (15)$$

$$\begin{aligned} \alpha &= r_{\perp}^2 + z^2 - a^2 - i2za \\ \beta &= -2z + i2a \\ \gamma &= 1 \end{aligned} \quad (16)$$

$$\alpha = r_{\perp}^2 + \frac{1}{4}\beta^2 \quad (17)$$

From this formula, we see that

$$D = r_{\perp}^2 \quad (18)$$

$$u(s) = (s + \beta/2) = (s - z + ia) \quad (19)$$

This integral $I(s)$ becomes

$$I(s) = \frac{1}{\sqrt{\gamma}} \operatorname{asinh}(u(s)/\sqrt{D}) = \pm \operatorname{asinh}(u(s)/r_{\perp}) \quad (20)$$

for all values of the field position vector \mathbf{x} . Note the ambiguity in the sign that comes from the $\sqrt{\gamma}$ term.

The inverse hyperbolic sine function $\operatorname{asinh}(\omega)$ has branch points in the complex ω plane at $\omega = \pm i$. It is customary to draw the two Riemann cuts along

the segments from $+i$ to $+i\infty$ and from $-i$ to $-i\infty$. Therefore, $I(s)$ has branch points whenever $u(s) = \pm ir_{\perp}$. If $a > 0$ then the $+i$ branch point maps into a ring in space defined by $z = s$ and $r_{\perp} = a$. The Riemann cut from this point maps into the disk in space bounded by this ring. Let $Asinh$ denote the standard principal value function for $asinh$. It is discontinuous across this planar cut, and $Asinh(u(s)/r_{\perp})$ will be discontinuous across the cut disk. The Riemann cut curve is not at all unique, but for any other choice the cut surface would be curved and with a larger area than the minimum planar cut across the disk. Choosing a different cut surface could change the mass and angular momentum calculation. The complex potential function is then given by

$$\psi_L(\mathbf{x}) = \frac{1}{4\pi} \frac{q}{L} S(x) (asinh(u(L/2)/r_{\perp}) - asinh(u(-L/2)/r_{\perp})) \quad (21)$$

S is a sign function that has values ± 1 . The sign function is determined by the physical constraint that at large radius the potential must approach a monopole electric charge. It turns out that this requires a for large r_{\perp} that we must choose the principal value form

$$\psi_L(\mathbf{x}) = -\frac{1}{4\pi} \frac{q}{L} (Asinh(u(L/2)/r_{\perp}) - Asinh(u(-L/2)/r_{\perp})), \quad r_{\perp} > a \text{ or } |z| > L/2 \quad (22)$$

If we use this formula for all values of r_{\perp} and z , which is equivalent to choosing the cut planes across the two singular rings at $z = \pm L/2$, then the electrostatic energy is divergent at the z -axis for $-L/2 < z < L/2$, and as $r_{\perp} \rightarrow 0$. The resulting electrostatic energy is infinite, and so is the angular momentum. We have learned that to avoid this divergence one can choose the cut surface differently, as shown in Figure 1 on page 9. This deformation of the Riemann cuts does not affect at all the fields at large values of r , but it greatly affects the near field and makes the total electromagnetic energy and angular momentum finite and calculable. We choose the cylindrical shape to facilitate the integration, but we discuss the non-uniqueness of the cut surface further on in this paper.

It is useful to calculate the value of ψ_L on the positive z -axis. We find

$$\psi_L(z\hat{\mathbf{z}}) = -\frac{q}{4\pi L} \ln \left(\frac{L/2 - z + ia}{-L/2 - z + ia} \right), \quad z > L/2 \quad (23)$$

The large z expansion of (23) is

$$\psi_L(z\hat{\mathbf{z}}) = \frac{q}{4\pi z} + \frac{iaq}{4\pi z^2} + \frac{(L^2/12 - a^2)q}{4\pi z^3} + O(1/r^4) \quad (24)$$

From this expression, we can calculate the full multipole expansion of the fields.

4.4 Multipole expansion for complex line charge

The complex potential function satisfies Laplace's equation, therefore it can be expanded in a spherical harmonic series in analytic domains. Using azimuthal symmetry, we have the following complex multipole expansion for large r :

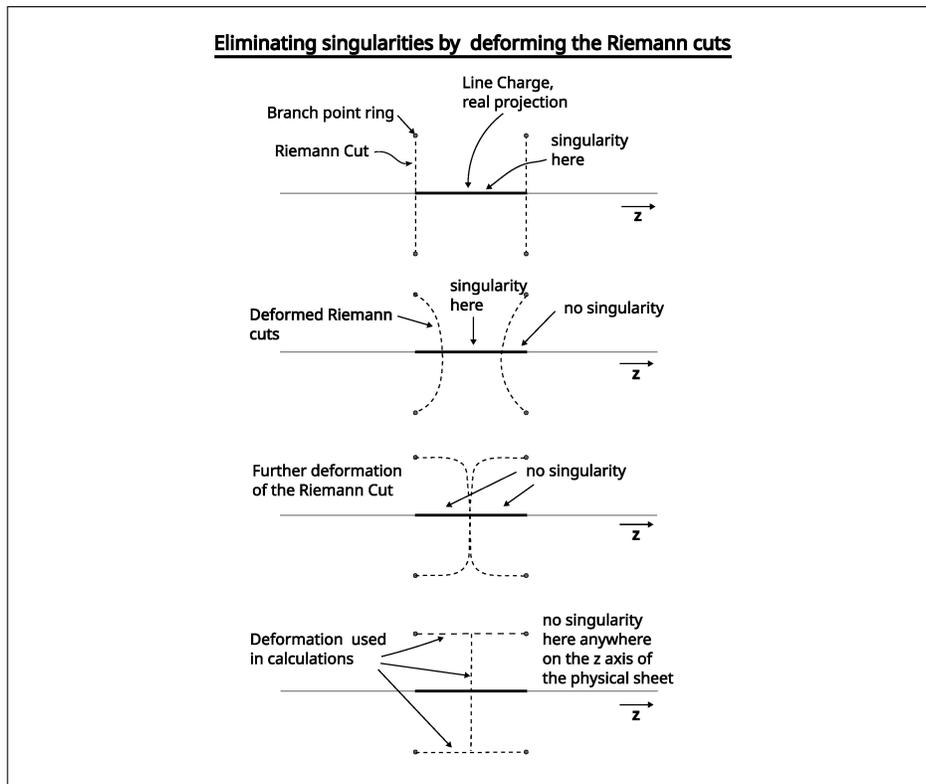


Figure 1:

$$\psi_L(\mathbf{x}) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos(\theta)) \quad (25)$$

Along the positive z-axis, $\theta = 0$, and $P_l(\cos(\theta)) = 1$, where P_l is the Legendre polynomial, and where θ is the usual polar angle in spherical coordinates. The B_l are the multipole moments, real values are electric moments and imaginary values are magnetic moments. If the potential has azimuthal symmetry as it does for the line charge, then the harmonic expansion coefficients B_L are determined by the values on the positive z-axis. The asymptotic power series for large r for the complex line charge is found from (24) to be:

$$\psi_L(\mathbf{x}) = \frac{q}{4\pi r} + \frac{iaqP_1(\cos(\theta))}{4\pi r^2} + \frac{(L^2/12 - a^2)qP_2(\cos(\theta))}{4\pi r^3} + O(1/r^4) \quad (26)$$

All of the even Legendre-order terms in this series are real and therefore electric, and all the odd terms are imaginary and magnetic, and in the limit $a \rightarrow 0$ all the odd terms vanish.

The first term is the usual Coulomb term. The next term is a magnetic dipole term and it is exactly the same as in the Kerr-Newman solution. The third term is an electric quadrupole term. It can be seen that in the limit $L \rightarrow 0$ it approaches the Kerr-Newman result. But, the interesting fact is that if $L = \sqrt{12}a$ then the electric quadrupole moment vanishes. This would be good for modeling a real electron because it is strongly believed that the electron has zero electric quadrupole moment, but unfortunately this value gives the wrong number for the spin and mass of the electron calculated numerically. The higher-order multipole terms are not zero with this value of L either. We find in the limit $L \rightarrow 0$, the usual result for the Kerr-Newman electromagnetic field [1] which has a non-zero electric quadrupole is obtained

$$\psi_0(\mathbf{x}) = \frac{1}{4\pi} \frac{q}{r} \sum_{n=0}^{\infty} \left(\frac{ia}{r}\right)^n P_n(\cos(\theta)) \quad (27)$$

4.5 Electromagnetic fields for complex line charge

We can now derive the electromagnetic fields starting with the Riemann-Silberstein vector. We use the principal value function outside the cylinder region defined by the inequalities $r_{\perp} > a$ or $|z| > L/2$

$$\mathbf{F}(\mathbf{x}) = -\nabla\psi_L(\mathbf{x}) = \nabla \frac{1}{4\pi} \frac{q}{L} (A \sinh(u(L/2)/r_{\perp}) - A \sinh(u(-L/2)/r_{\perp})) \quad (28)$$

Inside the cylindrical region, we use analytic continuation utilizing the cut surfaces in Figure 1 on page 9. The $r_{\perp} \rightarrow 0$ singularity from the left term now cancels the singularity of the right term for all $z \neq 0$. The way this works is that as we continue into the interior half of the cylinder through the left ring, the left term changes sign, but the right term does not. This results in the cancellation

of their singularities. The right half cylinder is just the mirror image of this, and cancellation occurs there too.

Now we use the chain rule in cylindrical coordinates (r_\perp, z, ϕ) . We can start with the principal value function outside the cylinder and continue inside after differentiating.

$$\nabla A \sinh(u/r_\perp) = \frac{1}{\sqrt{(u/r_\perp)^2 + 1}} \nabla \left(\frac{u}{r_\perp} \right) = \frac{1}{\sqrt{(u/r_\perp)^2 + 1}} \left(\frac{\nabla u}{r_\perp} - \frac{u \hat{r}_\perp}{r_\perp^2} \right) \quad (29)$$

Using $u(s) = (s - z + ia)$ we have $\nabla u(s) = -\hat{z}$ and the field expression becomes (for points \mathbf{x} outside the cylindrical region)

$$\mathbf{F}(\mathbf{x}) = \frac{1}{4\pi} \frac{q}{L} \left[\frac{-\hat{z} - u(L/2)\hat{r}_\perp/r_\perp}{\sqrt{u(L/2)^2 + r_\perp^2}} - \frac{-\hat{z} - u(-L/2)\hat{r}_\perp/r_\perp}{\sqrt{u(-L/2)^2 + r_\perp^2}} \right] \quad (30)$$

The real part of $\mathbf{F}(\mathbf{x})$ gives the electric field, and the imaginary part gives the magnetic field. It is convenient to define $\mathbf{F}_+(\mathbf{x})$ and $\mathbf{F}_-(\mathbf{x})$ by

$$\mathbf{F}_\pm(\mathbf{x}) = \pm \frac{1}{4\pi} \frac{q}{L} \left[\frac{-\hat{z} - u(\pm L/2)\hat{r}_\perp/r_\perp}{\sqrt{u(\pm L/2)^2 + r_\perp^2}} \right] \quad (31)$$

$$\mathbf{F} = \mathbf{F}_+ + \mathbf{F}_- \quad (32)$$

Assume that a is positive without loss of generality. For $z = \pm L/2$, and $r_\perp < a$ we find that \mathbf{F}_\pm is discontinuous across the disk of radius a centered on the z -axis if we analytically continue from one side of the disk to the other without going through it by continuing around its rim and back again to the other side. We can determine the fields in the vicinity of the charged string by analytic continuation. This reveals two singularity rings of radius $|a|$ centered on the z -axis at the points $z = \pm L/2$.

The square root function of a complex variable $\sqrt{\omega}$ has a branch point at $\omega = 0$. It is customary to draw the Riemann cut from along the negative real axis $\{\omega \in \mathbb{R}, \omega < 0\}$. The convention for defining the principal value $\sqrt[\ell]{\omega}$ of $\sqrt{\omega}$ is that whenever the real part of ω is positive, then the real part of $\sqrt[\ell]{\omega}$ is also positive. The discontinuity across the Riemann cut is then $2i \sqrt[\ell]{|\omega|}$ since the square root must change sign when crossing the cut to land on the other Riemann sheet. In order to obtain finite values for the electromagnetic mass, it is necessary to deform the cut as shown in Figure 1 on page 9. After doing this deformation, if we approach the z -axis anywhere along it, the limit of the potential is finite so long as we haven't crossed any part of the cut surface.

4.6 Four Riemann sheets

In (31) we see that \mathbf{F} has four Riemann sheets since \mathbf{F}_+ and \mathbf{F}_- each have two. Now consider that the physical Riemann sheet at spatial infinity might be any one of these four. Two of them have field lines that describe particles of opposite charge. These could be interpreted as particles and antiparticles.

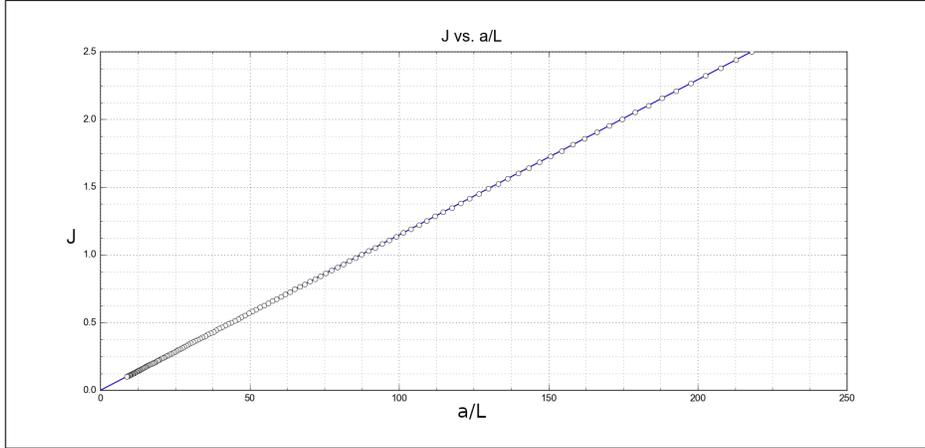


Figure 2: Angular momentum plot for total charge e as a function of the dimensionless ratio a/L in units of \hbar

The existence of antiparticles is normally considered a prediction of relativistic quantum mechanics. On the other two sheets, the fields give infinite energy.

4.7 Calculation of (hidden) angular momentum and electromagnetic energy of complex line charge

We now calculate the electromagnetic angular momentum of the fields about this charged string using (7). This integral must be a function of a/L based solely on dimensional grounds. A plot of spin J vs L/a is shown in Figure 2 on page 12. The graph is surprisingly linear considering the complexity of the integral for electromagnetic angular momentum.

We find the following linear curve fit to the data in 2

$$J(a/L) = A + B(a/L) \quad (33)$$

$$A = 3.400917E - 05; B = 1.146253E - 02 \quad (34)$$

This is equivalent to the following extremely simple formula with an accuracy of about 6 decimal places:

$$J(a/L) = \frac{\alpha\pi a}{2L} = \frac{e^2 a}{8L} \quad (35)$$

since in our units $e^2 = 4\pi\alpha$.

The $L=0$ limit gives the fields of the Kerr-Newman metric. Since the angular momentum and the mass of that solution is finite in general relativity, it means that the effects of gravity prevent a divergence of the present string solution as L tends to zero. The electric quadrupole moment is still non-zero and in fact quite large for the Kerr-Newman case [18].

The electron's spin is $|\mathbf{J}| = 1/2$ in our units. We note that the magnetic dipole moment does not depend on L as can be seen from (26). The volume integral of the angular momentum was calculated using the multi-precision capabilities of Python (see Appendix A). It was found that the "double-exponential" or "tanh-sinh" quadrature [48, 49] was critical in obtaining good convergence. This algorithm performs extremely well on integrands that are singular at the boundary of the integration domain. In the present instance, the singularities occur at $r_{\perp} = 0$ or $|a|$, and $z = \pm L/2$. One must break up the integration domains into sub-domains that have these singular points on the boundaries only. The precision value for L/a that was found to give the correct electron spin is:

$$L/a = 0.0229254974781 \rightarrow |\mathbf{J}| = 1/2 \quad (36)$$

In order to have zero electric quadrupole moment, we would need to have $L/a = \sqrt{12}$ which makes the spin angular momentum much too small to describe an electron or any other particle.

The electromagnetic mass density is given by (5). This value will depend on both the a and L parameters in our linear string model.

$$\mathcal{M} = \frac{1}{2} \int_V (\mathbf{E}^2 + \mathbf{B}^2) dVol \quad (37)$$

A numerical calculation of this value for the L/a which gave the correct value of $\hbar/2$ for the angular momentum (36) gives a correct mass of the electron to about 4 millielectronvolts accuracy for the case $a = 1/2$. This value of a corresponds to a magnetic moment which is the same as a Dirac electron, i.e. $a = J_e/m_e$ since in our units $J_e = 1/2$ and $m_e = 1$. The multiple-precision calculated values are

$$J = 0.5000000 \quad (38)$$

$$\mathcal{M} = 0.99999993093 \quad (39)$$

$$\mathcal{M}_E = \frac{1}{2} \int_V \mathbf{E}^2 dVol = 0.502865 \quad (40)$$

$$\mathcal{M}_B = \frac{1}{2} \int_V \mathbf{B}^2 dVol = 0.497134 \quad (41)$$

This is a stunning result. The electrostatic mass is equal to the true electron mass with tiny error. Moreover, it appears that in the high precision limit, this error gets smaller and smaller. Let's focus on what this means. The Dirac equation relates the mass, the spin magnitude, and the magnetic dipole moment, and it is in the framework of quantum mechanics. If you know any two of these parameters, then you also know the third. The Kerr-Newman metric solution gives exactly the same relation between mass, spin, and magnetic dipole

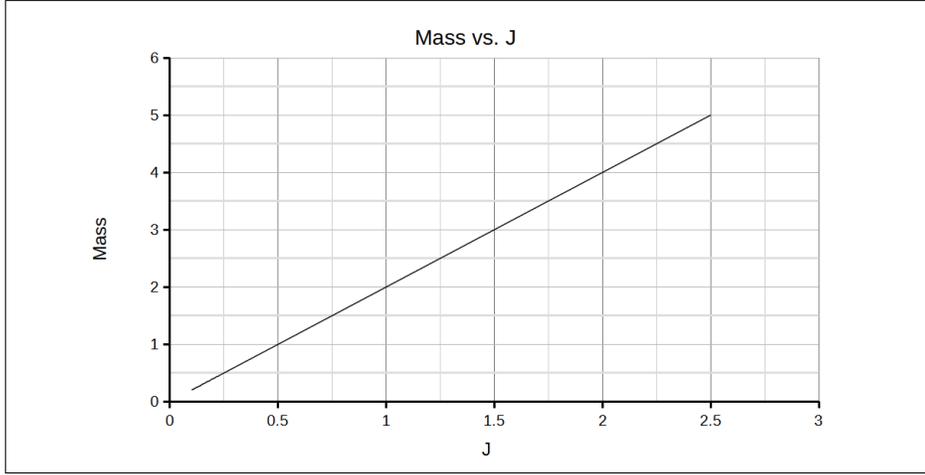


Figure 3: Total electromagnetic energy plot for $a=1/2$

moment, but it doesn't require quantum mechanics, but it does require classical general relativity together with Maxwell electrodynamics. Our charged string model gives exactly the same relation as these other two methods, but it does not require quantum mechanics or general relativity to obtain this relation, but rather only requires Maxwell electrodynamics with complex space-time. Moreover, our model gives a specific description of both spin and mass as being entirely due to electromagnetic fields. Why? What is going on here? A graph of calculated energies vs. J for fixed $a = 1/2$ is shown in Figure 3 on page 14

Once again we have an extremely linear fit to the equation (in our units)

$$Mass = 2 * J \quad (42)$$

Next we consider the electromagnetic Lagrangian which is the volume integral of the Lagrangian density $E^2 - B^2$. For this graph, the value of a was held fixed and L was varied. This shows slight nonlinearity as can be seen in Figure 4 on page 15

Note the scale on the vertical axis. The result is basically quite constant. Running a curve-fitting algorithm reveals the following functional relationship

$$\int (\mathbf{E}^2 - \mathbf{B}^2) dvol = A_1 + B_1 * J + C_1/J^2 \quad (43)$$

$$A_1 = 1.146244E - 02; B_1 = 5.515069E - 08; C_1 = -4.587288E - 08 \quad (44)$$

This can be approximated by the simple constant result independent of J that appears to be limiting behavior of the integral as the precision tends to infinity (α is the fine structure constant)

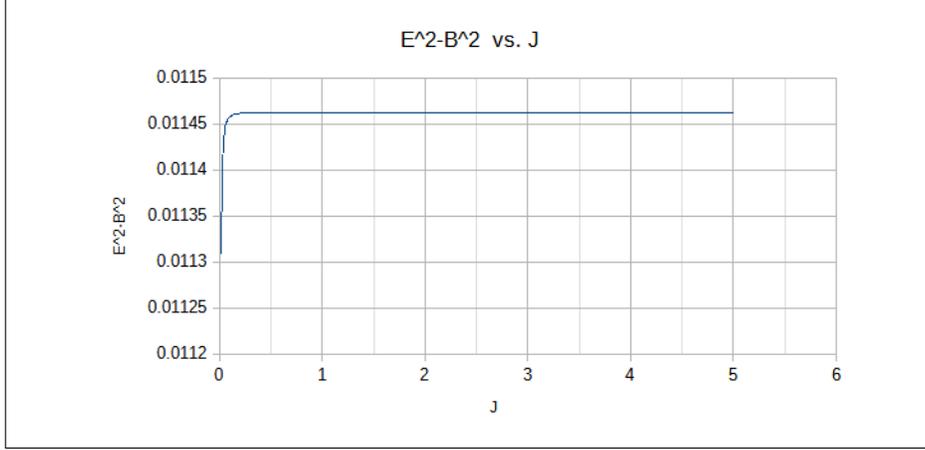


Figure 4: Volume integral of $\mathbf{E}^2 - \mathbf{B}^2$

$$\int (\mathbf{E}^2 - \mathbf{B}^2) dvol \approx \frac{\pi}{2} \alpha = \frac{e^2}{8} \quad (45)$$

The extreme simplicity of these results has caused this author to worry if perhaps the numerical integrations were somehow giving wrong results, but persistent checking has not revealed an error.

4.8 Anomalous Magnetic moment correction

The anomalous magnetic moment of the electron is currently reported to have the experimental value[50]

$$a_e = \frac{g-2}{2} = 0.001159652181643 \quad (46)$$

In order to incorporate this into our model, we must increase the complex offset a from $1/2$ to $(1 + a_e)/2$. We must simultaneously increase L by this same factor in order to maintain the ratio L/a in order that J doesn't change. Dimensional analysis mandates in this situation that the electrostatic mass varies in proportion to $1/a$, and since a is increased by this anomalous factor, the mass will be decreased by it

$$\mathcal{M}_{\text{anomalous}} = \frac{m_e}{1 + a_e} = 0.998841691053m_e \quad (47)$$

or

$$m_e - \mathcal{M}_{\text{anomalous}} = 591.894651eV \quad (48)$$

And so the agreement is worse than without the anomalous g factor correction where the electrostatic mass was essentially exactly the electron mass. This

has a positive implication though. The mass deficit introduced by the anomalous g factor can be made up by a non-electromagnetic term in the particle's Lagrangian, such as the string term to improve the stability of the model. Alternatively, the anomalous g factor may be due to other effects which are not included in this simple model.

4.9 Straightforward application for the muon and tau leptons

It is trivial to apply this model to the muon and tau leptons. All we have to do is adjust the value of a for the mass of each lepton

$$a_{muon} = \frac{m_e}{m_\mu} a_e; \text{ and } a_{tau} = \frac{m_e}{m_\tau} a_e \quad (49)$$

where $a_e = 1/2$ in our units, and m_μ is the mass of the muon, and m_τ the mass of the tau. The L values would scale by these same factors to maintain the ratio a/L and thus J . This would automatically give the Dirac relations between mass, angular momentum, and magnetic moment for each lepton. Their anomalous magnetic moments would then allow for a non-electromagnetic term in the Lagrangian to help to stabilize the particle, as for the electron.

4.10 Stability of string models

For the detailed calculations of the line charge, we found that the electromagnetic Lagrangian was approximately constant if a was held fixed (45), and this suggests that in this purely electromagnetic model, there is no tendency for the string to stretch in this case, but this is not a stiff equilibrium point. If the length L were to change, there would be no restoring force to bring it back from a change. This reminds us of geons [51], but gravity is not playing a role in stability here.

To stabilize the string models, we might choose to add an internal tension force to the Lagrangian borrowed from string theory. As a candidate for complex space-time embedding, the most logical choice is the Polyakov action [52, 30, 23]. It might also be of interest in the future to consider the much less well-known Stueckelberg-Horwitz-Piron (SHP) action [53] since this theory generally has fewer problems with relativistic covariance than more conventional approaches [54]. We only consider bosonic strings here, but we keep in mind that fermionic properties, such as the g factor of 2, can arise from the complex shift technique as in the Kerr-Newman electron results. There is no requirement (yet) for 26 space-time dimensions in this theory, as there is in standard bosonic string theory because we are not quantizing the strings, and in fact, it is the author's hope that more features usually attributed to quantum mechanics may emerge from this classical theory.

4.11 An action principle

The complex form of the Polyakov action for a string was treated in [30, 23]. The Hermitian metric approach in these theories can be applied to a purely bosonic string as in the case here. For the electromagnetic field, we could choose to work either with the Riemann-Silberstein vector or the manifestly covariant Faraday tensor. Presumably, they are equivalent. The action for the combined system is the sum of the string action plus the field action plus an interaction term. The interaction term between the electromagnetic field and the charged string is the tricky part here because the coordinates of the string are complex. Since there is no actual charge on the real subspace for the string models, we plausibly can write for the real-valued action [23]

$$I = \int d\sigma d\bar{\sigma} g_{\mu\bar{\nu}} \partial_{\bar{\sigma}} X(\sigma, \bar{\sigma}) \partial_{\sigma} \bar{X}(\sigma, \bar{\sigma}) - \int d^4x \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} \quad (50)$$

$F^{\alpha\beta}$ here is the real Faraday tensor calculated by the complex-shift method for currents that are off the real subspace using the complexified forms for the Liénard-Wiechert potentials [47]. The background metric g here is derived from the Minkowski metric in 4D. $\bar{\sigma}$ is the complex conjugate of σ . In the usual bosonic string theory, quantization leads to a violation of relativistic covariance except in 26 space-time dimensions. Here we are not quantizing this Lagrangian, but borrowing it for use in our classical charged particle models. So there is no problem with it in ordinary 4D Minkowski space. If you want to add more dimensions though go right ahead. The Hermitian metric g is effectively 8×8 and given by

$$g_{\mu\bar{\nu}} = g_{\bar{\nu}\mu} = \eta_{\mu\nu} \quad (51)$$

$$g_{\mu\nu} = g_{\bar{\mu}\bar{\nu}} = 0 \quad (52)$$

where η is the real Minkowski metric and the Greek subscripts $\{\mu, \bar{\mu}, \nu, \bar{\nu}\}$ are Lorentz indices.

In this action, the Faraday tensor F must be expressed as a functional of the string's curve $X(\sigma, \bar{\sigma})$ utilizing the complexified Liénard-Wiechert potentials plus an additional free-field solution. The functions to vary would be the string's world-sheet and the part of the vector potential contribution to F which is not due to the string current. In this way, we can avoid the problem of an interaction term of the form $j_{\mu} A^{\mu}$ which would need to be evaluated at a complex space-time point. This requires a nonlocal action [55, 56, 57]. So basically, it's pretty difficult to work with.

The problem with introducing a $j_{\mu} A^{\mu}$ term in the action is that there is not a perfect generalization of the Dirac delta function to complex spaces which preserves analyticity. There is, however, an imperfect one due to Lindell [58, 59, 60, 61], and perhaps a local action could be developed using this. The complex-shift method does not use a Hermitian metric. Consider the point particle. Its potential is $q/\sqrt{(\mathbf{x} - \mathbf{o})^2}$ and when \mathbf{o} is complex the result is no longer

real. So you can't use a Hermitian metric to evaluate $(\mathbf{x} - \mathbf{o})^2$. Rather you must use analytic continuation. The fields generated by the string this way are generally multi-valued. This situation, for point particles, was considered in [47] where a new mathematical technique called generalized analytic continuation was applied to the Liénard-Wiechert potentials in complex Minkowski space. The result of this is that one can imagine wave-particle duality emerging from it, although the concept is a bit of a metaphysical leap.

4.12 Dependence on the choice of the cut surface

For the cut surface found in Figure 1 on page 9, the fields are discontinuous across the curved cylindrical cut surface. Because of this, if we make an infinitesimal change in the surface, it will change the result of calculations for total mass and angular momentum. However, it is found numerically that the changes in these are in the same ratio as the linear slope relating mass and angular momentum so that the mass versus angular momentum curve won't change, and of course, deforming the cut won't affect the large distance moments like the magnetic moment either. Infinitesimal changes in the shape of the planar cuts at $z=0$ do not affect the mass or angular momentum either because the two densities are continuous in z there for all values of $r > 0$. So the main results that we obtain should allow some variation in the shape of the cut surface before it leads to noticeable effects. The arbitrariness of the cut surface, at least for small deviations acts a bit like a gauge invariance of this theory.

4.13 Compatibility with the Kerr-Schild metric

A Kerr-Schild metric in general relativity can be expressed in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu \quad (53)$$

where $\eta_{\mu\nu}$ is the background metric, which in our case is the Minkowski metric. ϕ is a scalar function and k_μ a null congruence. The Einstein field equations take the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{4\pi} \left(F_{\mu\alpha} F^\alpha{}_\nu + \frac{1}{4}g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) \quad (54)$$

A concise necessary condition that must be satisfied by the electromagnetic fields to allow a Kerr-Schild metric is presented in [4]. The condition is that the fields in the background metric must satisfy the following constraint.

$$F_\mu{}^\nu A_\nu - \chi A_\mu = 0, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (55)$$

where χ is a scalar function. If $\chi \neq 0$ then it follows that A is a null vector. In general, this is a difficult condition to test for. Our charged line model provides us with \mathbf{E} and \mathbf{B} fields from which we can construct the Maxwell-Faraday tensor $F^{\mu\nu}$ and see if it satisfies (55). I've tested the condition numerically. It is found to be satisfied in the limit $L \rightarrow 0$ as it should because this is the Kerr-Newman

limit, but for nonzero values of L , it does not allow an exact Kerr-Schild metric solution. The numerical method used the following approach. First, pick a point x in 3 space. Next, calculate $F^{\mu\nu}$ there for the charged line. Next, find the eigenvectors which satisfy the equation

$$F^{\mu}_{\nu} l^{\nu} = \lambda l^{\mu} \quad (56)$$

These produce a null tetrad with two degenerate real and null eigenvectors and two complex conjugate null ones. Choose one of the real eigenvectors and do the eigenvector calculation in a neighborhood of x making sure that the resulting function $l^{\mu}(x)$ varies smoothly in this neighborhood. This smoothness condition is critical because as there are two real null eigenvectors if the computer algorithm selects the wrong one for a neighboring point, then there will be a discontinuity which will prevent taking derivatives of A^{μ} . We write

$$A^{\mu}(x) = \chi l^{\mu}(x) \quad (57)$$

We determine χ by the condition

$$\chi = A^0(x)/l^0(x) \quad (58)$$

where $A^0(x)$ is the known electric potential. Then the 3-vector potential to test is

$$\mathbf{A}_{\text{test}}(\mathbf{x}) = \chi \mathbf{l}(\mathbf{x}) \quad (59)$$

and so $\mathbf{B}_{\text{test}} = \nabla \times \mathbf{A}_{\text{test}}$, and if this agrees with the magnetic field that we started with, then we have met the condition for a Kerr-Schild metric. This was all done numerically, and it was found that there is a small difference between the magnetic fields, so a Kerr-Schild metric solution cannot be exact, but it is approximate, and the deviation could in principle be added as a small perturbation to the Kerr-Schild metric. It is noteworthy that there are two null fields to choose from, and they produce vector potentials that have a handedness or chiral property that distinguishes them, although the magnetic and electric fields are identical. These two solutions must therefore differ by a gauge transformation. This handedness feature could perhaps be related to the two chiral states of the Dirac equation.

5 Other models

5.1 A curve in complex space with uniform charge density

Let's now consider a curved string parameterized by arclength (of its real part) s . Let \mathbf{x}_s denote the real part of the curve, and let the total length be L with charge per unit length constant and given by q/L . We shall use the Frenet-Serret apparatus to describe this curve in terms of tangent, normal, and binormal vectors. In the static case, we can write ψ as

$$\psi(x) = \int_0^L \frac{\rho(s)}{\sqrt{(\mathbf{x} - \mathbf{x}_0(s))^2}} ds \quad (60)$$

$$\mathbf{x}_0(s) = \mathbf{X}_{re}(s) + it(s)\hat{\mathbf{t}}(s) + in(s)\hat{\mathbf{n}}(s) + ib(s)\hat{\mathbf{b}}(s) \quad (61)$$

$$\int_0^L \rho(s) ds = q \quad (62)$$

where $X_{re}(s)$ is a real curve in space, $\hat{\mathbf{t}}(s)$, $\hat{\mathbf{n}}(s)$, and $\hat{\mathbf{b}}(s)$ are the tangent, normal, and binormal unit vectors for this curve which is parametrized by arclength s . The complex shift is characterized by the functions t , n , and b , and where $\rho(s)$ is the charge per unit length along the curve. Closed as well as open strings are possible. All knots and links are possible. In general, we would expect such a structure to be time varying. This would require a complex Minkowski space to be completely general. The fields would need to be calculated by using Liénard-Wiechert potentials suitably generalized, and this is highly non-trivial.

Alternatively, We can write the coordinates of the line charge in of the form

$$\mathbf{x}(s) = \mathbf{X}_{re}(s) + i\mathbf{X}_{im}(s) \quad (63)$$

where \mathbf{X}_{re} and \mathbf{X}_{im} are independent functions of s . In the case of a closed loop, we could obviously generalize this further by including a line-current contribution to the electromagnetic potential. This model is not generally invariant under parity, and so it can have a chirality property.

5.2 A simple case of a ring of charge with a complex shift along the axis of the ring

The simplest case is the following source geometry

$$\mathbf{x}_0(s) = R\cos(\theta(s))\hat{x} + R\sin(\theta(s))\hat{y} + ia\hat{z} \quad (64)$$

$$\theta(s) = \frac{s}{R}, \quad \rho(s) = \frac{q}{2\pi R} \quad (65)$$

$$\psi(\mathbf{x}) = \frac{1}{4\pi} \int_0^{2\pi R} \frac{q/2\pi R}{\sqrt{(\mathbf{x} - \mathbf{x}_0(s))^2}} ds \quad (66)$$

When we shrink the ring diameter to zero we obtain the Kerr-Newman source. The integral can be done analytically in terms of Appel hypergeometric functions. Along the z -axis, the result is trivial.

$$\psi(\mathbf{x})|_{x=y=0} = \frac{1}{4\pi} \frac{q}{\sqrt{R^2 + (z - ia)^2}} \quad (67)$$

Doing a large- z Taylor expansion we find

$$4\pi\psi(\mathbf{x})| = q/z + qia/z^2 - q(2a^2 + R^2)/2z^3 + O(1/z^4) \quad (68)$$

We have azimuthal symmetry here, and since the potential satisfies Laplace's equation, we then know from this that the Legendre expansion for all angles is

$$4\pi\psi(\mathbf{x}) = q/r + qia/r^2 P_1(\cos(\theta)) - q(a^2 + R^2/2)/r^3 P_2(\cos(\theta)) + O(1/r^4) \quad (69)$$

So we can see from this that the monopole and dipole terms are the same as for the Kerr-Newman case and the line charge above, but the electric quadrupole moment, that is $(a^2 + R^2/2)/4\pi$, is different, and unfortunately it can never vanish, so this simple ring charge cannot describe an electron with zero electric quadrupole moment.

5.3 Another simple case of a barbell of two charges with complex shift

Consider two charges, each with half the charge, separated by a distance L and with the same complex shift a parallel to the direction of the separation. For this case, which also has azimuthal symmetry, we get

$$4\pi\psi(\mathbf{x}) = q/r + qia/r^2 P_1(\cos(\theta)) - q((a^2 - L^2/4))/r^3 P_2(\cos(\theta)) + O(1/r^4) \quad (70)$$

We see that the electric quadrupole moment here is proportional to $(a^2 - L^2/4)$, and this vanishes if $L = 2a$. We could replace the two charges on the end of the barbell with short line-charges, and this would allow us to have a large but finite mass and angular momentum.

5.4 A composite string model that can possibly describe an electron

We would like to find models that give correct values for four parameters of the electron: mass, spin, magnetic dipole, and electric quadrupole. The simple line charge had two parameters, L and a . The ring had two parameters R and a . If we get lucky, we might find a simple model which gives the correct value for all four parameters with just three degrees of freedom, and this would amount to a calculation of the fine structure constant. However, there is no obvious way to guess what such a structure might be, and so we can in the meantime consider composite systems with four free parameters to adjust. Here we will consider one such model of this type with four degrees of freedom. For simplicity, we will consider models with azimuthal symmetry.

Consider the model of a ring of charge with radius R with a complex shift utilizing the Frenet apparatus as in (60) with a constant charge density ρ as well

as constant real-valued shift parameters t , n , and b resulting in the potential integral in (66), but with a more complicated $\mathbf{x}_0(s)$ of the form

$$\mathbf{x}_0(s) = R\cos(\theta(s))\hat{x} + R\sin(\theta(s))\hat{y} + it\hat{\mathbf{t}}(\theta(s)) + in\hat{\mathbf{n}}(\theta(s)) + ib\hat{\mathbf{b}}(\theta(s)) \quad (71)$$

$$\hat{\mathbf{t}}(\theta) = -\sin(\theta)\hat{x} + \cos(\theta)\hat{y} \quad (72)$$

$$\hat{\mathbf{n}}(\theta) = -\cos(\theta)\hat{x} - \sin(\theta)\hat{y} \quad (73)$$

$$\hat{\mathbf{b}}(\theta) = \hat{z} \quad (74)$$

The far field expansion on the z-axis is found to be

$$4\pi\psi(z) = q/z + iqb/z^2 + q(t^2 + n^2 - 2b^2 - R^2 - i2Rn)/(2z^3) + O(1/z^4) \quad (75)$$

This expression has a monopole charge, a magnetic dipole given by qb , and an electric and magnetic quadrupole term. The electric quadrupole vanishes if the following condition is satisfied:

$$t^2 + n^2 - 2b^2 - R^2 = 0 \quad (76)$$

The magnetic quadrupole moment is $-qRn$, and this can vanish if $n = 0$, and therefore, it is possible to eliminate both quadrupole moments with this model. The binormal shift b is determined by the dipole moment alone. In the limit of $R \rightarrow 0$ we approach the Kerr-Newman solution once again, and therefore the electromagnetic mass will diverge if we don't include metric curvature. Obviously, for $R \rightarrow \infty$, the electromagnetic mass goes to zero. Consequently, somewhere in between these limits, it will equal the mass of the electron. If we require that both quadrupole moments be zero, then we can still get the correct mass, but will have no more parameters to adjust for the angular momentum, so either it just happens to be the correct one for the electron, i.e. $\hbar/2$, or we would have to allow one of the quadrupole moments to be nonzero. If the correct spin of the electron was derived this way, it would amount to a mathematical derivation of the fine structure constant, although we consider this lucky outcome to be very unlikely. Presumably, it would be more popular to set the electric quadrupole to zero and allow a nonzero magnetic quadrupole. Then we should be able to adjust the parameters to give the correct spin. So it seems likely that this model can describe an electron as a purely electromagnetic object. It does have a magnetic quadrupole moment though, which might be a way to test this picture further. Finding the parameters t , n , b , and R is beyond the scope of this paper. Different sets of parameters could be used for other charged particles, and so we have here a fairly general-purpose classical charged model.

5.5 A spherical shell model which has only a charge monopole and a magnetic dipole

Consider a charged spherical shell of radius a , and no charge except on this. Assume azimuthal symmetry. The solution for Laplace's equation in spherical coordinates is:

$$\psi(\mathbf{x}) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos(\theta)) \quad (77)$$

Inside the shell, the B_l terms must vanish, and outside the A_l terms must vanish. Outside the shell, we write

$$\psi(\mathbf{x}) = \frac{q}{4\pi r} + \frac{iq}{4\pi} \frac{a_1}{r^2} P_1(\cos(\theta)), \quad r > a \quad (78)$$

Inside the shell we apply continuity requirements for Maxwell's equation to obtain

$$\psi(x) = \frac{iq a_1}{4\pi} r \left(-2/a^3 \right) P_1(\cos(\theta)), \quad r < a \quad (79)$$

The magnetic dipole moment is $M = q a_1$. This model does not have any internal chirality. It was analyzed and angular momentum and energy were calculated in [62], section 3.1. Inside the shell, the magnetic field is constant and directed along \hat{z} , the symmetry axis, and the electric field is zero. This ensures that the radial component of the magnetic field is continuous across the shell. With these fields, we can calculate the mass, the angular momentum, and the magnetic dipole moment. Since the electric field is zero inside the shell, the Poynting vector is zero there too, and so the hidden angular momentum comes only from the fields outside of the shell. The fields are all single-valued, and therefore the calculations of mass and angular momentum are straightforward. This model gives zero for all higher multipole moments which is consistent with a Dirac particle. There are two free parameters, the radius of the sphere a and the magnetic dipole moment controlling parameter a_1 . We would like to fit the mass, angular momentum, and magnetic dipole of the particle. If a_1 is chosen to give the measured dipole moment, then this leaves only the one parameter a to fit the mass and the angular momentum. The electrostatic energy calculated from [62] is

$$\mathcal{E} = \frac{q^2}{8\pi a} + \frac{M^2}{4\pi a^3} = \frac{q^2}{8\pi a} \left(1 + \frac{2a_1^2}{a^2} \right) \quad (80)$$

The electromagnetic angular momentum is

$$\mathcal{J} = \frac{q^2 a}{18\pi} \frac{3a_1}{a^2} = \frac{q^2}{6\pi} \frac{a_1}{a} \quad (81)$$

For an electron, this relation becomes

$$\frac{\hbar}{2} = \frac{e^2}{6\pi} \frac{a_1}{a} \quad (82)$$

But $e^2 = 4\pi\alpha$ and $\hbar = 1$ in our units. Therefore

$$\frac{a_1}{a} = \frac{3}{4\alpha} \approx 102.77699 \quad (83)$$

and then the mass of the electron, which is 1 in our units, is given by

$$m_{e=1} = \frac{\alpha}{2a} \left(1 + 2 \left(\frac{3}{4\alpha} \right)^2 \right) \quad (84)$$

and from this we can solve for a . In our units, the classical electron radius is simply α . The radius of the shell is

$$a = \frac{\alpha}{2} + \frac{9}{16\alpha} \approx 77.0863985 \approx 12.26868\lambda_e \quad (85)$$

where λ_e is the Compton wavelength of the electron. We can see from (80) and (83) that practically all the mass of the electron in this model is due to the magnetic field.

5.6 An extended fluid drop model with complex shift

We apply the complex shift method to the model in [13]. The particle there was a soliton in an inviscid charged fluid. The relativistic stress-energy tensor is

$$T^{\mu\nu} = \frac{1}{4\pi} F^\mu_\lambda F^{\lambda\nu} + (\rho + p) u^\mu u^\nu - g^{\mu\nu} \left(-\frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} + p \right) \quad (86)$$

The flow is assumed to be adiabatic, and the pressure as a function of the fluid density η is taken to be

$$p = -\kappa\eta^{6/5} \quad (87)$$

where this form was selected to allow a simple analytic solution, although other possibilities might also lead to interesting models. The pressure is negative and grows more negative the denser the fluid is. This leads to a cohesive force that balances the electrostatic repulsion. The resulting static electric field is found in [13] to be

$$\mathbf{E} = \frac{q\mathbf{r}}{4\pi} \left(b^2 + \mathbf{r}^2 \right)^{-3/2}, \quad b = (4\pi/3)^{1/2} (e^2/6\kappa)^{5/2} \quad (88)$$

with potential

$$\psi = \frac{q}{4\pi} \left(b^2 + \mathbf{r}^2 \right)^{-1/2} \quad (89)$$

Now we make the complex shift along the z -axis so that $z_{origin} \rightarrow ia$ to obtain

$$\psi = \frac{q}{4\pi} \left(b^2 + x^2 + y^2 + (z - ia)^2 \right)^{-1/2} \quad (90)$$

We can analyze the multipole expansion by setting $x = y = 0$ and calculating the Taylor-Laurent expansion at large z

$$\psi|_{\mathbf{r}=z\hat{z}} = \frac{q}{4\pi} \left(b^2 + (z - ia)^2 \right)^{-1/2} \quad (91)$$

$$\psi|_{\mathbf{r}=z\hat{z}} = \frac{q}{4\pi} \left[\frac{1}{z} + \frac{ia}{z^2} - \frac{b^2 + 2a^2}{2z^3} - \frac{i(3ab^2 + 2a^3)}{2z^4} \right] + O\left(\frac{1}{z^5}\right) \quad (92)$$

From this, we have the usual monopole and magnetic dipole terms. The electric quadrupole term is nonzero except in the limit $a = b = 0$. The magnetic octapole term can be zero if $3ab^2 + 2a^3 = 0$. This liquid drop model might be interesting for modeling nuclei.

6 Can wave-particle duality be understood in complex space-time models without formally quantizing them?

Arguments supporting this idea were presented in [47, 46]. First consider the electromagnetic field produced by a moving charge in conventional, i.e. real space-time. Assuming the particle is moving on a timelike curve, an observer at some instant sees the particle at exactly one point, which involves a retarded time calculation that makes the apparent position dependent on the past trajectory of the particle. There is no wave-particle duality here. The electromagnetic force is our window into reality. The Liénard-Wiechert potentials can be viewed as defining the properties of this window. Let's suppose that our classical charged particle is describable by a Hamilton-Jacobi (HJ) theory. It's well known that quantum mechanics can be transformed into HJ theory via Bohmian mechanics, and the converse is also true. Any HJ classical particle model can be transformed into a wave equation with a suitable potential function that may be nonlinear. This is true for multi-particle systems as well. Now suppose that each particle is moving in complex Minkowski space-time and is guided by some HJ equations. This problem was studied in [47] for a particular Bohmian system. The retarded time calculation is no longer unique. The apparent location of the source can now be multi-valued. If you allow a partition of unity of the particle trajectory and perform an independent analytic continuation of each partition, then you can obtain wave-particle duality. I called this procedure generalized analytic continuation in [47]. The Liénard-Wiechert potentials, when utilized in complex space-time, give us a blurry coke-bottle view of reality, but they produce more than just an apparent distortion because they determine how a particle interacts electromagnetically with all other particles. Given the multitude of apparent positions that a single charged particle can have due to

this effect, the natural question of causality and locality obviously needs to be addressed. Modern interpretations of quantum mechanics are moving towards the inclusion of time-symmetric formulations [63], and of course, quantum entanglement, in general, has suggested the possibility of nonlocal connections in physics. Since complex space-time is intimately connected with classical general relativity and with electromagnetism, one is led to wonder whether it could be possible that classical general relativity with electromagnetism could quantize itself. In other words, could quantum theory be derivable from this classical field theory? This would be a beautiful and unexpected solution to the quantum-gravity dilemma that physics currently faces, and it would be one that Einstein would probably be very pleased with. Given the results of this paper, it would also seem that string theory should play a critical role in this project.

7 Conclusion

The complex-shift method for generating charged electromagnetic particle-like solutions has been applied to extended charged objects in this paper, and it was found that several features of elementary particles can be phenomenologically described in this way. The string models are particularly interesting because they can have finite values for mass and angular momentum, they can have chirality, and they can be knotted and linked in complex and diverse ways. The linear charged model perfectly predicts the electron mass to be its measured mass given only the charge, spin, and magnetic moment of the electron. The sceptical reader can reproduce this result by executing the software listed in the appendix. This result is based entirely on electromagnetism, and it gives the same relation between mass, spin, and magnetic moment that the Dirac equation gives, but without quantum theory. The spin in this case is entirely due to electromagnetic angular momentum. Nothing is rotating. It can be compared with the expectation of the spin operator in a quantum Pauli equation which behaves much like a classical angular momentum [64].

A number of phenomena normally associated with quantum mechanics can be described qualitatively in this framework. These include charged particles with magnetic dipoles having Dirac g factors of 2 or anomalous g factors, particles with zero electric quadrupole moments, particles with internal chirality and topological symmetries related to knots and links, a possible explanation for wave-particle duality, and A qualitative understanding of the possible origins of quantum entanglement and nonlocality. These results seem to suggest a plausible path to emergent quantum mechanics from classical general relativity and electromagnetism in complex space-time. Charged string theory in complex space-time might turn out to be a theory of everything, including quantum mechanics itself. Although we can understand the possible origins of wave-particle duality, we don't have a detailed understanding of how nature might orchestrate a universe with a large multitude of Riemann sheets superimposed with one another to give us the laws of quantum mechanics. A statistical mechanics treatment is called for, perhaps one based on Adler's trace dynamics [65].

In place of Many Worlds, we have many Riemann sheets. The philosophy of emergent quantum mechanics is in sharp contrast to the prevailing “ER=EPR” movement which aims to derive everything, including the space-time manifold, from quantum theory. The two approaches might be dual in the sense that quantum mechanics might be emergent from classical theory while the inverse is also true.

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A Python script

```

# -*- coding : utf -8 -*-
"""
Written by Mark Davidson , March 31, 2023 This program is free software . You
can redistribute it and /or modify it under the terms of the GNU General
Public License as published by the Free Software Foundation , either
version 3 of the License , or (at your option ) any later version . See the
GNU General Public License for more details <https :// www. gnu. org/
licenses />.
Break the integration region into four regions . Multiply the integrals for J
,
Esq , and Bsqr by two to account for left + right symmetry , and a = J/m
should be 1/2 for the electron in our units ( hbar =1, mass of electron =
1)
"""
import scipy . constants as c; from scipy import integrate ; import time
import mpmath as mp; print ('StartProgramat',time . ctime ())
mp.mp.dps = 12; eabs = 1e-8; mp. pretty = True ; twofactor = 2
lda = mp.mpf (0.0229254974781)
q = mp. sqrt (4* mp. mpf (c.pi)*mp.mpf (c. alpha ))
factor = -(q /(4* mp.pi))
rm = mp. inf
def uf(s,z,a):
    return s-z+1j*a
def cf(u,r):
    return 1/ mp. sqrt (u **2+ r **2)
def leftsign (z,r,L,a):
    rt = 1
    if (z ==0) &(r <=a): rt = 0
    if (r <=a)&(z <0) &(z >=-L /2) : rt = -1
    return rt
def rightsign (z,r,L,a):
    rt = 1
    if (z ==0) &(r <=a): rt = 0
    if (r <=a)&(z >0) &(z <=L /2): rt = -1
    return rt
def F(L,a,z,r):

```

```

    return factor *(1/ L)* \
    ( rightsign (z,r,L,a)*cf(uf(L/2,z,a),r)
    *mp. matrix ([-1,- uf(L/2,z,a)/r])
    -leftsign (z,r,L,a)*cf(uf(-L/2,z,a),r)
    *mp. matrix ([-1,- uf(-L/2,z,a)/r]))
def E(L,a,z,r):
    return F(L,a,z,r). apply (mp.re)
def B(L,a,z,r):
    return F(L,a,z,r). apply (mp.im)
# Now define Poynting vector = E x B in the phi direction in cylindrical
# coordinates .
# z x r = phi unit vector equation
def Poynting (L,a,z,r):
    return E(L,a,z,r) [0]* B(L,a,z,r)[1] -E(L,a,z,r) [1]* B(L,a,z,r) [0]
# Now we define the angular momentum density in the z direction (r x P)_z
#It appears that Jz is continuous across the cut boundary
def Jz(L,a,z,r):
    return r* Poynting (L,a,z,r)
def Esq(L,a):
    def igE (z,r):
        Ez=E(L,a,z,r) [0]
        Er=E(L,a,z,r) [1]
        return 2* mp.pi*r*( Ez*Ez+Er*Er)
    sumE0 =mp. quad (igE ,[0 ,L/2] ,[a,rm], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumE1 =mp. quad (igE ,[L/2, rm ] ,[a,rm], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumE2 =mp. quad (igE ,[L/2, rm ] ,[0 ,a], error =True , method ='tanh-
        -sinh',
    ebsabs = eabs )
    sumE3 =mp. quad (igE ,[0 ,L/2] ,[0 ,a], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumEsq = sumE0 [0]+ sumE1 [0]+ sumE2 [0]+ sumE3 [0]
    return twofactor * sumEsq
def Bsqr(L,a):
    def igB (z,r):
        Bz=B(L,a,z,r) [0]
        Br=B(L,a,z,r) [1]
        return 2* mp.pi*r*( Bz*Bz+Br*Br)
    sumB0 =mp. quad (igB ,[0 ,L/2] ,[a,rm], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumB1 =mp. quad (igB ,[L/2, rm ] ,[a,rm], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumB2 =mp. quad (igB ,[L/2, rm ] ,[0 ,a], error =True , method ='tanh-
        -sinh',

```

```

    ebsabs = eabs )
    sumB3 =mp. quad (igB ,[0 ,L/2] ,[0 ,a], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumBsq = sumB0 [0]+ sumB1 [0]+ sumB2 [0]+ sumB3 [0]
    return twofactor * sumBsq
def Jtotal (L,a):
    def igJz (z,r):
        return 2* mp.pi*r*Jz(L,a,z,r)
    sumJ0 =mp. quad (igJz ,[0 ,L/2] ,[a,rm], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )
    sumJ1 =mp. quad (igJz ,[L/2, rm ] ,[a,rm], error =True , method ='tanh
        -sinh',
    ebsabs = eabs )
    sumJ2 =mp. quad (igJz ,[L/2, rm ] ,[0 ,a], error =True , method ='tanh
        -sinh',
    ebsabs = eabs )
    sumJ3 =mp. quad (igJz ,[0 ,L/2] ,[0 ,a], error =True , method ='tanh-
        sinh',
    ebsabs = eabs )

    sumJ = sumJ0 [0]+ sumJ1 [0]+ sumJ2 [0]+ sumJ3 [0]
    return twofactor * sumJ
def Jst( ldiva ):
    result = Jtotal (ldiva ,1.0)
    return result
# Jrootf finds the precise value of L/a that gives a
# angular momentum of 1/2
def Jrootf ( ldiva ):
    return 1/2 - Jst(ldiva)
lda =0.0229252943718582 # Good starting point for fine tune with findroot
True_electron_mass = 0.51099895*1e6 # in eV
# Uncomment the next line to find the best value for L/a
Ldafound = mp. findroot ( Jrootf ,( lda -0.0001 , lda +0.0001) )
# Ldafound = mp. mpf (0.0229254974781) # For mp.mp.dps =12 and eabs =1e
    -8

""" Here is some code to do basic calculations """
a=mp. mpf (1/2)
L=mp. mpf ( Ldafound )*a
Jcalc = Jst (L/a)
E2calc =Esq (L,a)
B2calc =Bsq (L,a)
Mass =( E2calc + B2calc )/2
print ('Ju=',Jcalc )
print ('Esqu=',E2calc /2)
print ('Bsqu=',B2calc /2)
print ('Esqu-Bsqu=',( E2calc - B2calc ) *0.51099895*1000 , 'keV')
print ('Massu=',Mass )
print ('MassuErroru=',(Mass -1) * True_electron_mass ,'eV')

```

```

print ('Stop Program at',time . ctime ())
Lstart = Ldafound *a /10
increment = 1/2
Lfinish = 20* Ldafound
L= Lstart
""" Here is looping code to create spreadsheet data """
f= open ('JBEData.txt', 'a')
print ('L,a,rm,dps,prec,eabs=',L,a,rm ,mp.mp.dps ,mp.mp.prec , eabs )
print ('L,a,rm,dps,prec,eabs=',L,a,rm ,mp.mp.dps ,mp.mp.prec ,eabs ,
      file =f)
print ("a/L,L,a,Jcalc,(E2calc+B2calc)/2,(E2calc-B2calc)/2")
print ("a/L,L,a,Jcalc,(E2calc+B2calc)/2,(E2calc-B2calc)/2" , file =
      f)
f. close ()
while L< Lfinish :
    Lnext =(1+ increment )*L
    Jcalc = Jst (L/a)
    E2calc =Esq (L,a)
    B2calc =Bsq (L,a)
    f= open ('JBEData.txt', 'a')
    print (a/L,',',L,',',a,',',Jcalc ,',',( E2calc + B2calc )/2,',',
          E2calc - B2calc )/2)
    print (a/L,',',L,',',a,',',Jcalc ,',',( E2calc + B2calc )/2,',',
          E2calc - B2calc )/2,file =f)
    f. close ()
    L= Lnext

"""
Here is some previous output
Ldafound = 0.0229254974781
L,a,rm ,dps , prec = 0.0114627487391 0.5 + inf 12 43
J = 0.499999994429
Esq = 0.502865609542
Bsq = 0.49713438355
Esq - Bsq = 5.85730092837 kev
Mass = 0.99999993093
Mass Error = -0.00352955023643 ev
"""

```
