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Comment on “Harvesting information to control non-equilibrium states of active matter”

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In the article *Phys. Rev. E* **106**, 054617 “Harvesting information to control non-equilibrium states of active matter”, the authors study the transition from one non-equilibrium steady-state (NESS) to another NESS by changing the correlated noise that is driving a Brownian particle held in an optical trap. They find that the amount of heat that is released during the transition is directly proportional to the difference of spectral entropy between the two colored noises, in a fashion that is reminiscent of Landauer’s principle. In this comment I argue that the found relation between the released heat and the spectral entropy does not hold in general, and that one can provide examples of noises where it clearly fails. I also show that, even in the case considered by the authors, the relation cannot be rigorously true and is only approximately verified experimentally.

INTRODUCTION

In the article “Harvesting information to control non-equilibrium states of active matter” [1] the authors consider a Brownian particle, held in an optical trap, and submitted to an external correlated (*i.e.* “colored”) noise η . The displacement of the particle is accurately described by the overdamped Langevin equation:

$$\dot{x} = -\frac{\kappa}{\gamma}x + \sqrt{2D}\xi + \sqrt{2D_a}\eta \quad (1)$$

Where κ is the trap stiffness, γ is the Stokes viscous drag ($\gamma = 6\pi R\mu$ with R the particle’s radius, and μ the fluid’s viscosity), ξ is a Gaussian white noise with unit variance, accounting for the thermal fluctuations of the bath, D is the equilibrium diffusion coefficient ($D = k_B T/\gamma$ with k_B the Boltzmann constant and T the temperature), D_a is a control parameter that allows to change the amplitude of the colored noise η .

The authors then consider what they call a “STEP Protocol”: at time t_1 the particle is initially in a non-equilibrium steady-state (NESS) with a given colored noise η_1 , then at a given time $t_c > t_1$ the noise is abruptly changed to another colored noise η_2 , and the particle is let to reach a different NESS at time $t_2 > t_c$. Using the framework of stochastic thermodynamics [2, 3], they compute the average amount of *excess* heat that is released in the transition between the two NESS: ΔQ_{EX} . Their main result, is then that this amount of heat is directly proportional to the difference of spectral entropy (which is the Shannon entropy in the frequency domain [4]), ΔH_S between the two noises η_1 and η_2 :

$$\Delta Q_{EX} \propto \Delta H_S \quad (2)$$

They claim that this relation is “akin to Landauer’s principle”, as it relates a thermodynamics quantity (ΔQ_{EX}), to an informational quantity (ΔH_S). Their interpretation of the result is that “the protocol harvests information from the colored noise, turns it into heat necessary for the transition between the two NESS, and finally releases it to the surrounding environment”.

In this comment, I argue that the found relation between the heat released and spectral entropy is not true in general, and that, even in the exact case considered by the authors, it does not hold rigorously. In a first section I, starting from the definitions of stochastic heat and spectral entropy, I derive a general equation (8) to compute both quantities for any kind of colored noise. This result shows that the claimed relation 2 is invalid for the vast majority of noises. In a second section II I give several examples of noises for which the relation clearly fails. In the third section III, I discuss analytical results obtained when one considers the exact situation described by the authors, and show numerical simulations to verify that, even in this case, the relation 2 fails. Finally, I discuss a possible explanation to why the relation seems approximately verified in the experimental data of the article [1].

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I. THEORETICAL CALCULATIONS

We first consider the two quantities of interest ΔQ_{EX} and ΔH_S .

The spectral entropy H_S of a noise η is defined as the Shannon entropy applied to the Power Spectral Density (PSD) of that noise [4]:

$$H_S(\eta) = \sum_{j=1}^N S_\eta^*(\omega_j) \ln(S_\eta^*(\omega_j)) \quad (3)$$

with $S_\eta^*(\omega_j)$ the normalized PSD of the noise at the (discretized) angular frequency ω_j :

$$S_\eta^*(\omega_j) = S_\eta(\omega_j) / \sum_{j=1}^N S_\eta(\omega_j) \quad (4)$$

where S_η stands for the PSD of the noise η .

This quantity is a measurement of the “flatness” of the noise’s PSD. As stated by the authors: it vanishes for a monochromatic signal, it reaches its maximum $\ln(N)$ for a white noise, and any correlated noise has an intermediate value of H_S .

The average amount of *excess* heat that is released in the transition between two NESS is given by [2, 3]:

$$\Delta Q_{EX} = \frac{1}{2} \int_{t_1}^{t_2} \kappa \frac{d\langle x^2 \rangle}{dt} dt' \quad (5)$$

where κ is the trap stiffness, t_i (with $i \in \{1, 2\}$) is a time at which the particle is in a NESS with the colored noise η_i , and $\langle \cdot \rangle$ stands for the ensemble average. This relation can be written:

$$\Delta Q_{EX} = \frac{\kappa}{2} (\sigma_x^2(t_2) - \sigma_x^2(t_1)) \quad (6)$$

where σ_x^2 stands for the variance of the particle’s position ($\sigma_x^2 = \langle x^2 \rangle$).

Thus, we see that the amount of heat dissipated when the external noise is switch from η_1 to η_2 can simply be calculated by measuring the variance of the particle’s position in the first NESS $\sigma_{x_1}^2$ and in the second NESS $\sigma_{x_2}^2$. In the article [1], the authors have only considered those equations in the case of an exponentially-correlated noise, and state that “[the] extension to other classes of noises is less clear”. In the following, we show that it is actually possible to analytically compute both ΔH_S and ΔQ_{EX} for any kind of colored noise, as long as their PSD is known.

To do that, we recall that in a NESS the system is both stationary and ergodic [5]. Therefore, the Wiener–Khinchin theorem [6, 7] allows us to link the variance of the position to the PSD of the particle’s position S_x , through the relation:

$$\begin{aligned} \sigma_{x_i}^2 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{x_i}(\omega) d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega^2 + \omega_0^2} (2DS_\xi(\omega) + 2D_a S_{\eta_i}(\omega)) d\omega \end{aligned} \quad (7)$$

where ω_0 is the natural frequency of the system ($\omega_0 = \kappa/\gamma$), S_ξ is the PSD of the Gaussian white noise (which is a constant), and S_{η_i} is the PSD of the noise η_i .

In the end, for a STEP protocol where the noise is switch from η_1 to η_2 , we have the formulas:

$$\begin{aligned} \Delta H_S &= \sum_{j=1}^N [S_{\eta_1}^*(\omega_j) \ln(S_{\eta_1}^*(\omega_j)) - S_{\eta_2}^*(\omega_j) \ln(S_{\eta_2}^*(\omega_j))] \\ \Delta Q_{EX} &= \frac{\kappa}{2\pi} \int_{-\infty}^{\infty} \frac{2D_a}{\omega^2 + \omega_0^2} [S_{\eta_2}(\omega) - S_{\eta_1}(\omega)] d\omega \end{aligned} \quad (8)$$

As one can see, both quantities ΔQ_{EX} and ΔH_S are directly linked to the PSD of the two noises S_{η_1} and S_{η_2} . However, there are critical differences:

- $H_S(\eta)$ only depends on the “flatness” of S_η , but it does not depend on the noise amplitude, nor on the way that the power is distributed along frequencies. Since the terms of the sum 3 can be permuted without changing the result, a PSD that has “most of its power at high frequencies” can have the same spectral entropy as a PSD that has “most of its power at low frequencies”.
- On the contrary, the position variance σ_x^2 , depends explicitly on the noise amplitude, and the frequency content of the PSD. Indeed, as seen in equation 7, σ_x^2 is the integral of $S_\eta(\omega)$ weighted by the mechanical response of the system (the term $1/(\omega^2 + \omega_0^2)$ here). Since the overdamped Brownian particle trapped in optical tweezers acts as a first-order low-pass filter with cut-off frequency ω_0 , only the “low frequency content” of the noise’s PSD will have a significant impact on the variance of particle’s position.

We want to stress that equation 8 alone contradicts the main result of the article [1]. For the very vast majority of usual colored noises, the relation of direct proportionality between the spectral entropy difference ΔH_S and the released heat ΔQ_{EX} is not verified. In the following section II, as a pedagogical example, we provide several couples η_1 and η_2 that are designed such that they have the same spectral entropy ($\Delta H_S = 0$), but still produce a non-zero amount of heat released when the STEP protocol is applied ($\Delta Q_{EX} \neq 0$), which clearly shows the failure of the relation 2.

II. EXAMPLES OF NOISES FOR WHICH THE SPECTRAL ENTROPY IS NOT DIRECTLY PROPORTIONAL TO THE HEAT RELEASED

We first note that, in the article [1], the authors have considered noises with the same amplitude to study only the influence of the noise’s “color” on ΔH_S and ΔQ_{EX} . Therefore, in the rest of this comment, we will only consider noises that are normalized so that their variance is equal to one.

The simplest way to design noises that have the same spectral entropy, but do not result in the same variance for the particle’s position, is to consider noises with a finite bandwidth, or noises with a finite band rejection in their PSD. Examples are shown in figure 1. In this case, the value of the spectral entropy H_S is the same for each noise with the same finite bandwidth (respectively band rejection), regardless of the central frequency of the band-pass (respectively band-stop) range. On the contrary, the value of the particle’s position variance σ_x^2 , highly depends on this central frequency. Indeed, we recall that the overdamped trapped particle acts as a low-pass filter: σ_x^2 will be higher for a noise with a high power at low frequency (such as the blue curve η_1 in figure 1a) than for a noise with a high power at high frequency (such as the green curve η_3 in figure 1a). Therefore, using such noises, it is possible to design a STEP protocol with no change of spectral entropy ($\Delta H_S = 0$) while having a non-zero amount of heat released ($\Delta Q_{EX} \neq 0$).

Of course, we are not limited to noises with a finite bandwidth or finite band rejection. It is also possible to design noises with a continuous PSD that have the same spectral entropy, but do not result in the same variance for the particle’s position. For example, one can consider a noise η_1 with a low-frequency content, and a noise η_2 with a high frequency content:

$$S_{\eta_1}(\omega) \propto \frac{1}{\omega_{c1}^2 + \omega^2} \quad \text{and} \quad S_{\eta_2}(\omega) \propto \frac{\omega^2}{\omega_{c2}^2 + \omega^2} \quad (9)$$

If the two cut-off frequencies ω_{c1} and ω_{c2} are chosen such that the two PSD of the noises crosses exactly in the middle of the considered frequency range, they will have the same spectral entropy ($\Delta H_S = 0$). However, it is clear that the two noises will not produce the same position variance when they are applied to the particle. For example, suppose that the two cut-off frequencies are bigger than the natural frequency of the particle $\omega_{c1} \gg \omega_0$ and $\omega_{c2} \gg \omega_0$. Then, η_1 will act nearly as a white noise on the particle, and on the contrary, η_2 will have nearly no influence on it’s position. This will result in $\sigma_{x1}^2 > \sigma_{x2}^2$, and therefore $\Delta Q_{EX} \neq 0$.

An example is shown in figure 2 for $\omega_{c1} = \omega_{c2} = 157\,079 \text{ rad s}^{-1}$ (corresponding to 25 000 Hz), and $\omega_0 = 884 \text{ rad s}^{-1}$ (corresponding to 140 Hz). As seen in figure 2a, the two noises η_1 and η_2 have the same variance and the same spectral entropy. However, as seen in figure 2b, the PSD of the particle’s position S_{x1} and S_{x2} , obtained by numerically

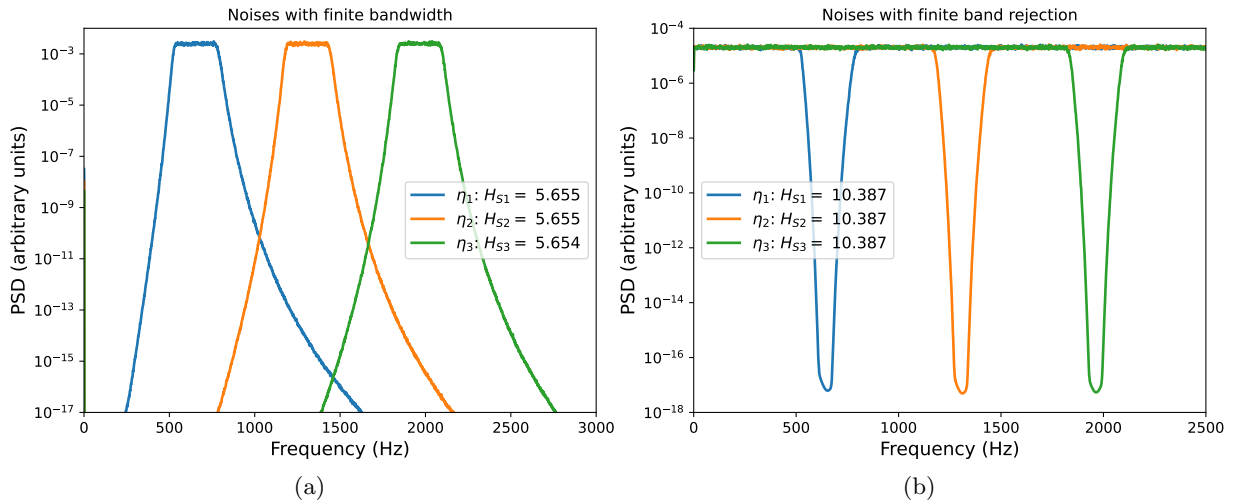


FIG. 1: Examples of noises with a finite bandwidth (a) or finite band rejection (b), centered around different frequencies. As shown in legend, all the noises in each category have the same spectral entropy H_S regardless of the value of the central frequency. Noises are generated by numerically filtering a Gaussian white noise, the spectral entropy is computed using equation 3.

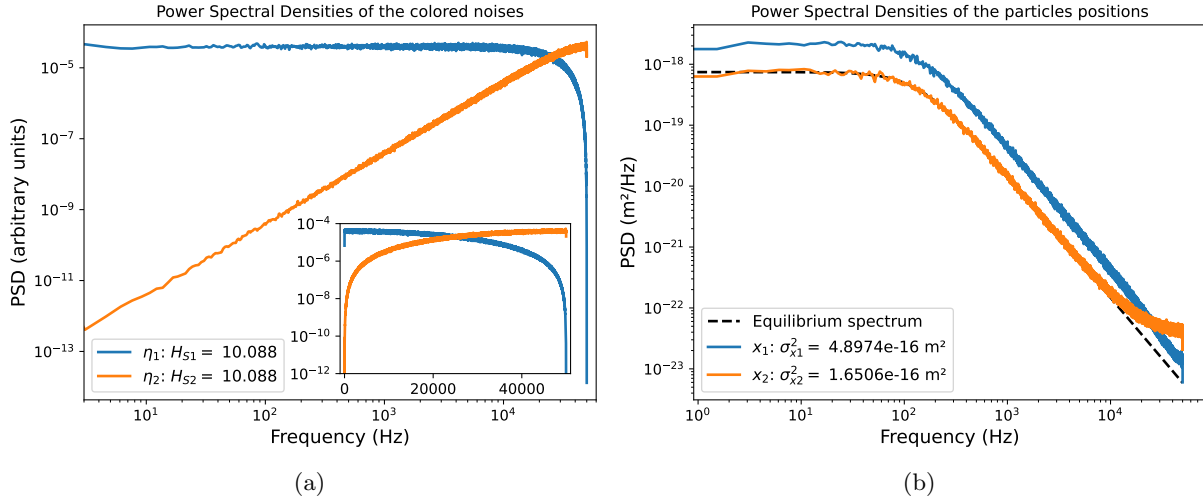


FIG. 2: Example of two continuous noises with the same variance and same spectral entropy that do not result in the same variance for the Brownian particle to which they are submitted. (a) PSD of two noises η_1 (blue) and η_2 (orange) designed to have the same spectral entropy H_S (values indicated in legend). Inset: same curves plotted in *semilogy* to show that the cut-off frequencies are indeed chosen such that the two PSD crosses in the middle of the considered frequency range. The noises are obtained by numerically filtering a Gaussian white noise. (b) PSD of the Brownian particle's position when it is submitted to no external noise (black-dashed), to η_1 (blue), and to η_2 (orange) respectively. PSD clearly show different behaviors (values of variances $\sigma_{x_1}^2$ and $\sigma_{x_2}^2$ indicated are in legend). The particle position is obtained by numerically integrating the Langevin equation 1 using Heun's method [8].

integrating the Langevin equation 1 with η_1 and η_2 respectively, are very different, and do not produce the same variance for the particle's position.

Thus, we have seen that it is possible to design noises such that the relation of direct proportionality between ΔQ_{EX} and ΔH_S clearly fails in the STEP protocol [9]. This further emphasizes that the relation is not true in general, and cannot be used with any colored noise. In the next section we will show that, even in the exact case that is considered by the authors in the article [1], the relation is does not hold rigorously.

III. THE SPECTRAL ENTROPY IS NOT DIRECTLY PROPORTIONAL TO THE HEAT RELEASED EVEN IN THE CASE CONSIDERED BY THE AUTHORS

A. Analytical and simulated results of the case considered by the authors

In the article [1], the colored noise η , that is driving the Brownian particle, is generated by an Ornstein-Uhlenbeck process:

$$d\eta = -\omega_c \eta dt + \sqrt{2\alpha\omega_c} dW \quad (10)$$

where dW is a δ -correlated Wiener process. This noise η is characterized by its variance α , and by its cut-off angular frequency ω_c . It's Power Spectral Density (PSD) is given by:

$$S_\eta(\omega) = \frac{2\alpha\omega_c}{\omega_c^2 + \omega^2} \quad (11)$$

For the STEP protocol, the authors have chosen to keep the amplitude of the colored noise constant ($\alpha_1 = 1 = \alpha_2$), and to only change its cut-off frequency from ω_{c1} to ω_{c2} . Then, using the PSD of the noises 11, and the general formulas 8, we directly obtain:

$$\begin{aligned} \Delta H_S &= \ln\left(\frac{\omega_{c1}}{\omega_{c2}}\right) \\ \Delta Q_{EX} &= \frac{\kappa D_a}{\omega_0} \frac{(\omega_{c1} - \omega_{c2})}{(\omega_0 + \omega_{c2})(\omega_0 + \omega_{c1})} \end{aligned} \quad (12)$$

Therefore, we see that the relation $\Delta H_S \propto \Delta Q_{EX}$ fails analytically, even in the case considered by the authors. Note that those results are identical to the ones obtained by the authors, namely eqs. (H5) and (G9) in the Sup. Mat. of [1] (except for a missing κ prefactor in equation (G9) that is probably a typo).

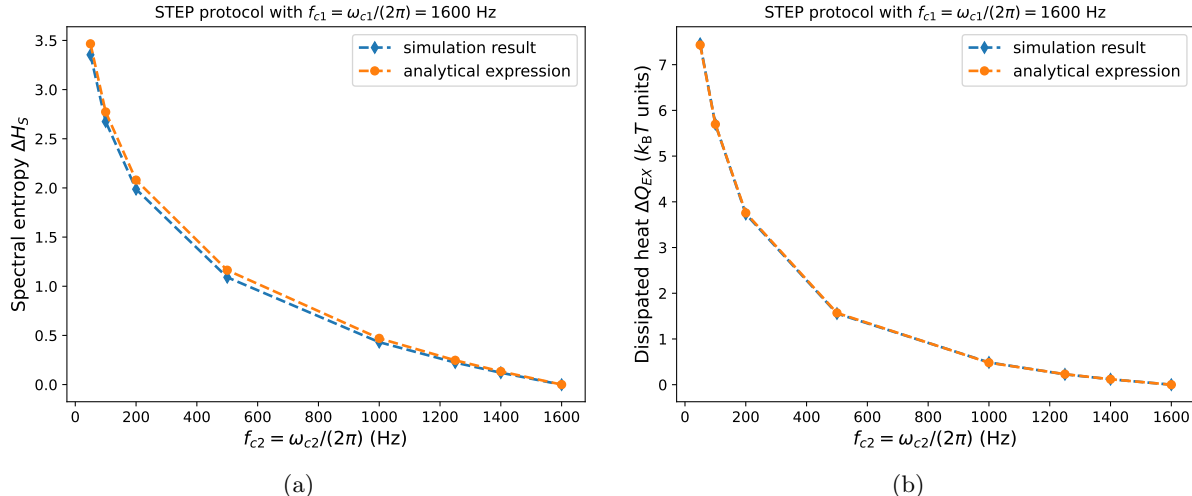


FIG. 3: Spectral entropy difference ΔH_S (a) and dissipated heat ΔQ_{EX} (b) obtained from numerical simulations of a STEP protocol where the colored noise is changed from η_1 ($\alpha_1 = 1$ and $f_{c1} = 1600$ Hz) to η_2 ($\alpha_2 = 1$ and $f_{c2} \in [50, 100, 200, 500, 1000, 1250, 1400, 1600]$ Hz). The simulated results are compared to the analytical expressions 12.

The failure of the linearity relation can be verified in numerical simulations. Using Heun's scheme [8], we have first integrated equation 10 to obtain colored noises η_i with the desired properties. Then, we have integrated the Langevin equation 1 during a STEP protocol, where we change the noise from η_1 to η_2 (characterized by $\alpha_1 = 1 = \alpha_2$ and $\omega_{c1} \neq \omega_{c2}$) in the middle of the trajectory. We have used parameters close to the experimental ones [10]: bead radius $R = 1.5 \mu\text{m}$, water viscosity $\mu = 1 \times 10^{-3}$ Pa.s, temperature $T = 298.15$ K, trap stiffness $\kappa = 25$ pN/ μm , Boltzmann constant $k_B = 1.381 \times 10^{-23}$ J/K, amplification of the noise $D_a = 10000D$ with

$D = k_B T / \kappa \approx 1.456 \times 10^{-13} \text{ m}^2/\text{s}$, cut-off frequency of first noise $f_{c1} = \omega_{c1}/(2\pi) = 1600 \text{ Hz}$, cut-off frequency of second noise $f_{c2} = \omega_{c2}/(2\pi) \in [50, 100, 200, 500, 1000, 1250, 1400, 1600] \text{ Hz}$, compared to $f_0 = \omega_0/(2\pi) \approx 140 \text{ Hz}$, and integration time-step $\delta t = 1 \times 10^{-5} \text{ s}$. Finally, we have computed both ΔH_S and ΔQ_{EX} for a thousand runs, and compared the mean results to the predictions 12. All the numerical simulation Python codes are available on a Zenodo depository [11], and can be consulted directly online on the associated GitHub page [12].

As seen in figure 3, both quantities correctly follow the analytical predictions 12, within the numerical accuracy. We stress that there is no reason to believe that these results would not hold experimentally, unless the Langevin equation 1 does not correctly describe the experimental system [13].

B. Why the relation seems verified experimentally?

So far, we have seen that the relation of direct proportionality between the spectral entropy difference and the heat released 2 does not hold in the general case. We have also seen that it is not verified analytically, nor numerically, in the case considered by the authors in [1]. Therefore, one can wonder how this relation can appear to be experimentally verified. In this subsection, we propose an explanation, based on the way that was used to verify the validity of the relation. We show that the graphical representation chosen by the authors can lead to wrong interpretations, and propose a more reliable way to plot the data.

Experimentally, the authors have found a good agreement for the relation:

$$\frac{\Delta Q_{EX}}{k_B T_{eq}} = \Delta H_S \quad (13)$$

where T_{eq} is an experimental quantity, equal to the effective temperature ($T_{eq} = \kappa \langle x^2 \rangle / k_B$) that is measured when the particle is submitted to a white noise with the same amplitude as the colored noise.

First, we stress that T_{eq} is a quantity only accessible experimentally, and that its value depends on the particular set-up that is used to measure it. In particular, here T_{eq} depends on the properties of the different apparatus used to generate the white noise acting on the trapped particle, such as the dynamic range of the digital-to-analogue card, and the response function of the acousto-optic modulator. Therefore, T_{eq} has no theoretical predicted value, and cannot be easily computed numerically. Moreover, equations using T_{eq} can hardly be seen as general results, since their value is system-dependent. Then, we note that the authors have plotted $\Delta Q_{EX}/(k_B T_{eq})$ as a function of ΔH_S to verify their relation (Figure 4 in [1]). Here, we have reproduced this figure with the results of our own numerical simulations in figure 4a. As one can see, *with this particular graphical representation*, it is possible to find that $\Delta Q_{EX} \approx k_B T_{eq} \Delta H_S$, with $T_{eq} = 622 \text{ K}$, which is compatible with the experimental values measured by the authors. Moreover, the validity of the linear relation is more likely to be found given that experimental values have inevitable error bars. Yet, it is only an approximation, and not a rigorous result.

A more reliable way to verify the relation of direct proportionality 2, would be to plot the ratio of the dissipated heat and the spectral entropy $T_{eff} = \Delta Q_{EX}/(k_B \Delta H_S)$, as a function of f_{c2} . In particular, this verification does not rely on theoretically unknown quantities, such as T_{eq} , or parameters that are difficult to measure experimentally, such as D_a . If the relation of direct proportionality is true, then T_{eff} must be a constant. The values of T_{eff} obtained in our numerical simulations are as shown in figure 4b. The quantity is clearly not constant, which is expected since we have already shown that the relation 2 is not verified in the numerical simulations.

IV. CONCLUSION

In conclusion, despite being approximately verified in a nice experimental configuration, the relation of direct proportionality between the spectral entropy and the dissipated heat $\Delta Q_{EX} \propto \Delta H_S$, is not true in the general case, and is not rigorously verified even in the framework of the original article that introduced it. We also recall that this relation is not predicted by any theoretical work [14]. All this cast serious doubts on the claim that this relation is “akin to Landauer’s principle”, and its interpretation in terms of “the protocol harvesting information from the

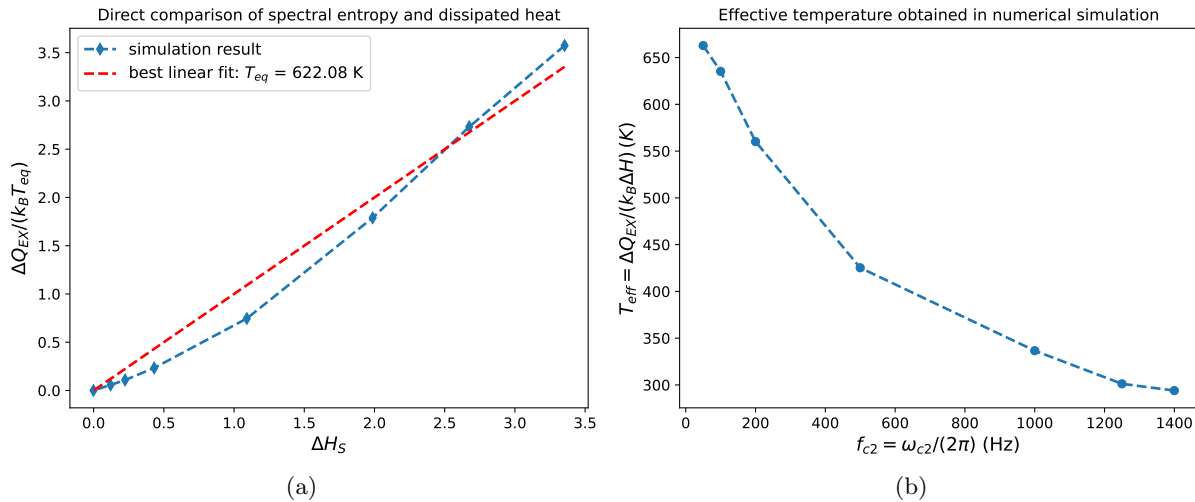


FIG. 4: (a) Normalized dissipated heat $\Delta Q_{EX}/(k_B T_{eq})$ plotted as a function of the spectral entropy difference ΔH_S . The value of T_{eq} is obtained by computing the best linear fit of $\Delta Q_{EX}/k_B$ as a function of ΔH_S . (b) Effective temperature $T_{eff} = \Delta Q_{EX}/(k_B \Delta H_S)$ computed for several STEP protocols. As expected, the value is not constant, as the dissipated heat and the spectral entropy are not directly proportional.

colored noise”. Therefore, we believe that it is very unlikely that this relation “could serve as a new tool for the study of non-equilibrium systems in non-trivial baths”.

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 - [10] Note that the exact values of the parameters are not important here, as the numerical results are only presented to “verify” the analytical calculations, which are expected to hold for any set of parameters.
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 - [12] <https://github.com/aberut/BrownianSimulation1D/blob/v1.5/arXiv-2212.06825.ipynb>.
 - [13] Note that if the Langevin equation does not accurately describe the system, the formulas that are derived from this equation are not valid anymore. In particular the definition of the stochastic heat 5, that is used by the authors in their article, does not hold in this case.
 - [14] The authors do not provide any reference to support the validity of this relation, besides their experimental measurements.