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Global Elastodynamic Performance Criterion of Manipulators with Flexible Joints

Fabian Andres Lara-Molina [1] and Didier Dumur [2]

***Abstract*— This paper proposes a novel design criterion for manipulators with flexible joints based on elastodynamic performance. Consequently, the elastodynamic performance is assessed; the eigenvalue problem is solved to determine the first natural frequency based on the manipulator's inertia and stiffness matrices. Then, the elastodynamic criterion is evaluated within a region of the Cartesian workspace in order to obtain a global measure. The global elastodynamic performance of a planar serial manipulator and a planar parallel manipulator with flexible joints were obtained. The numerical results show a high dependency of the elastodynamic performance on the configuration and geometric parameters of the manipulator.**

I. INTRODUCTION

The design criteria are performance indices of the mechanical properties that measure the operation quality of robotic manipulators. Several design criteria have been stated to measure the manipulator's performance by examining the dynamic and kinematic proprieties. Mainly, the kinematic criteria use the Jacobian matrix, and they quantify the kinematic performance [1]. Moreover, the workspace and singularities are widely used as criteria of manipulators [2]. The dynamic criteria evaluate the dynamic characteristics such as the inertia and the behavior of the elastic elements (elastic joints or links) regarding the elastodynamics [3]. The design criteria can be classified as local and global; local criteria evaluate the performance at a single manipulator pose; contrastingly, global criteria examine the performance within a region of the manipulator's workspace. These design criteria are used for the optimization of robotic manipulators [4], [5], [6]. The design criteria definition is challenging because of the uncertainties in the tasks of the robot [7].

However, few research studies related to design criteria that consider the dynamic performance of robotic manipulators with flexible links and joints were reported in the literature. The information derived from these design criteria is crucial for the mechanical design and also for the further synthesis of control algorithms [8]. Several research studies focused on robotic manipulators considering flexible elements have been addressed to formulate the elastodynamic model [9], motion control [8], [10] and the optimization of the mechanical structure [11], [12]. However, these contributions aim at analyzing the behavior of design criteria based on the dynamic and elastodynamic performance of the robot as a function of the geometric parameters, i.e., the length of the links.

As shown by the research works, the design criteria are essential for the optimal design of robotic manipulators. Many authors have optimized criteria for specific performance resulting in an optimal design that considers a kinematic criterion [13], [11], [14]. Alternatively, several authors have carried out the optimal design of rigid manipulators and considering only dynamic performance indices [15], [11]. However, the present work proposes a design criterion for robotic manipulators with flexible joints that aim to assess the dimensions of the links in order to to optimize the elastodynamic proprieties.

This paper proposes a novel design criterion for manipulators with flexible joints based on elastodynamic performance. Consequently, the elastodynamic performance is assessed; the eigenvalue problem is solved to determine the first natural frequency based on the manipulator's inertia and stiffness matrices. Then, the elastodynamic criterion is evaluated within a region of the Cartesian workspace in order to obtain a global measure. The global elastodynamic performance of a planar serial manipulator and a planar parallel manipulator with flexible joints were obtained. The numerical results show a high dependency of the elastodynamic performance on the configuration and geometric parameters of the manipulator.

This paper is organized into five sections. Section II presents the modeling of a manipulator with flexible joints. Section III illustrated the novel global elostodynamic index. Then, section IV presents the numerical results of the application of the global elastodynamic index on a serial and a parallel manipulator with flexible joints. Finally, section V presents the conclusions and future work.

II. MODELING OF THE MANIPULATORS FLEXIBLE JOINTS

Figure 1 shows the model of a single flexible joint. Considering a manipulator with n joints, $\tau = \begin{Bmatrix} \tau_1 & \dots & \tau_n \end{Bmatrix}^T \in \mathbb{R}^{n\times 1}$ is the torque applied by the motor after the reduction, $\theta = \begin{Bmatrix} \theta_1 & \dots & \theta_n \end{Bmatrix}^T \in \mathbb{R}^{n\times 1}$ is the angular position of the motor after the reduction, and $\mathbf{q} = \begin{Bmatrix} q_1 & \dots & q_n \end{Bmatrix}^T \in \mathbb{R}^{n\times 1}$ is the angular position of the link. The flexible transmission of the motor to the link is modeled by a torsional spring with a stiffness coefficient k_i.

Considering that the angular velocity of the motor's rotors is produced only by their own spinning [16], the reduced

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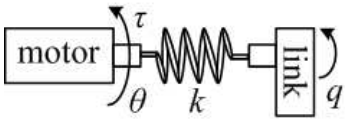


Fig. 1. Flexible joint.

dynamic model of a n-link robot manipulator with flexible joint can be written in the following form:

$$\mathbf{M}_L(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{V}_L(\mathbf{q},\dot{\mathbf{q}})+\mathbf{G}_L(\mathbf{q})+\mathbf{K}(\mathbf{q}-\theta)+\tau_{f\mathbf{q}}=0 \quad (1)$$

$$\mathbf{B}\ddot{\theta}+\mathbf{K}(\theta-\mathbf{q})+\tau_\theta=\tau \quad (2)$$

where the vectors $\dot{\mathbf{q}}=\begin{Bmatrix}\dot{q}_1 & \dots & \dot{q}_n\end{Bmatrix}^T\in\mathbb{R}^{n\times 1}$ and $\ddot{\mathbf{q}}=\begin{Bmatrix}\ddot{q}_1 & \dots & \ddot{q}_n\end{Bmatrix}^T\in\mathbb{R}^{n\times 1}$ are the angular velocity and acceleration of the links; $\mathbf{M}_L(\mathbf{q})\in\mathbb{R}^{n\times n}$ is the inertia matrix of the rigid links; $\mathbf{V}_L(\mathbf{q},\dot{\mathbf{q}})\in\mathbb{R}^{n\times 1}$ is the vector of the Coriolis and centrifugal torques; $\mathbf{G}_L(\mathbf{q})$ is the vector of the gravitational forces; $\tau_{f\mathbf{q}}$ and τ_θ are the joint frictions od the links and the motor, respectively; $\mathbf{B}\in\mathbb{R}^{n\times n}$ is the matrix of inertia of the motor's rotors; $\ddot{\theta}=\begin{Bmatrix}\ddot{\theta}_1 & \dots & \ddot{\theta}_n\end{Bmatrix}^T\in\mathbb{R}^{n\times 1}$ is the angular acceleration of the motor after the reduction; $\mathbf{K}=diag(k_1,\dots,k_n)$ is the diagonal matrix of the joint stiffness. The link and the motor equations (eqs. (1) and (2)) are dynamically coupled by the elastic torque $\mathbf{K}(\theta-\mathbf{q})$.

The rigid model of the manipulator considers an infinite joint stiffness ($\mathbf{K}\to\infty$). Thus, $\theta\to\mathbf{q}$, for this case the model can be stated as follows:

$$\mathbf{M}_L(\mathbf{q})+\mathbf{V}_L(\mathbf{q},\dot{\mathbf{q}})+\mathbf{G}_L(\mathbf{q})+\tau_{f\mathbf{q}}=\tau \quad (3)$$

This contribution is considered the dynamic model of n-link robotic manipulator with flexible joints; thus, n is the number of degrees of freedom.

A. Simplified model

The dynamic model, of eqs. (1) and (2), is reduced in order to obtain a simplified model to design the controller. In accordance with this, two assumptions are contemplated. First, the manipulator is considered at a constant configuration, therefore $\mathbf{V}_L(\mathbf{q},\dot{\mathbf{q}})=0$; and second, it is considered the worst case in which the manipulator is not damped, thus $\tau_\theta=\tau_{f_\mathbf{q}}=0$. Therefore can be express by using the matricial notation:

$$\mathbf{M}_T(\mathbf{q})\ddot{\mathbf{z}}+\mathbf{K}_T\mathbf{z}=\mathbf{u} \quad (4)$$

where, $\mathbf{z}=\begin{Bmatrix}\mathbf{q} & \theta\end{Bmatrix}^T$, $\mathbf{M}_T(\mathbf{q})=\begin{bmatrix}\mathbf{M}_L(\mathbf{q}) & 0_{n,n}\\ 0_{n,n} & \mathbf{B}\end{bmatrix}$ and $\mathbf{K}_T$ and $\mathbf{u}=\begin{Bmatrix}0_{1,n} & \tau\end{Bmatrix}^T$.

III. DEFINITION OF THE GLOBALL ELASTODYNAMIC CRITERION

The elastodynamic performance of a manipulator with flexible elements, such as flexible links or flexible joints, can be analyzed by evaluating the natural frequencies of dynamic model. The elastodynamic model depend on the intertia and stiffness of the manipulator [16].

The elastodynamics can be evaluated by assessing the natural frequencues of the manipulator. As presented in the previous section, the simplified dynamic model of the manipulator is represented by the expresion of Eq. (4). Consequently, the natural frequencies can be obtained by solving the eigenvalue problem of the simplified dynamic model which is written in the following way:

$$\left|-\boldsymbol{\omega}^2\mathbf{M}_T(\mathbf{q})+\mathbf{K}_T\right|=0 \quad (5)$$

where $\boldsymbol{\omega}=\begin{bmatrix}\omega_1 & \omega_2 & \dots & \omega_n\end{bmatrix}$ represents the vector of the n natural frequencies. The natural frequency ω_1 of the first elstic mode is used to evaluate the elastodynamics of the manipulator. The first natural frequency should be as high as possible to obtain a high elastodynamic performance.

In addition, one can obseve that the solution of the eigenvalue problem of Eq. 5 depends on the joint positions $\mathbf{q}$ because the mass matrix of the manipulator $\mathbf{M}_T(\mathbf{q})$ is joint position dependt. Thus, the elostadynamic performance can be evaluated only at single configuration indicating that it is a local property wich depends on the definition of joint positions. Nevertheless, in several cases is necessary to evaluate the elastodynamic performance over a desired region of the workspace.

The concept of the global criteria has been used to evaluate dynamic properties of the manipulator such as the global conditioning index (GCI) [4]. Consequently, this concept can be extended to evaluate the dynamics properties of the manipulators. The global elastodynamic index (GEI) aims at evaluating the behaviour of the elastodynamics over the workspace of the manipulator. Therefore, the following global elastodynamic index is proposed:

$$GEI=\frac{\int_w \omega_1 dw}{\int_w dw} \quad (6)$$

where ω_1 is the natural frequency of the first flexible model for the joint position $\mathbf{q}$, w represents the Cartesian workspace. The denominator represents the volume of the workspace. It is worth mentioning that the joint coordinates $\mathbf{q}$ should be inscrived within the Cartesian coordenates of workspace w; thus, $f(\mathbf{q})\in w$ where f represents the forward kinematic model.

The proposed concept of the global elastodynamic index will be applied to two different manipulators with flxible joints.

IV. NUMERICAL APPLICATIONS

A. Serial Planar Two-link Manipulator

Initially, a serial planar two-link manipulator with flexible joints is considered (see Fig. 3). According to the dynamic equation of Eq. (1) the rigid link dynamic model is completely described in [17]; thus, $\mathbf{M}_L$ is defined. Moreover, the stiffness matrix $\mathbf{K}_T=\begin{bmatrix}k_1 & 0 & -k_1 & 0\\ 0 & k_2 & 0 & -k_2\\ -k_1 & 0 & k_1 & 0\\ 0 & -k_2 & 0 & k_2\end{bmatrix}$ was defined in [18]. The geometric and dynamic proprieties of the maipulator are defined in tha appendix.

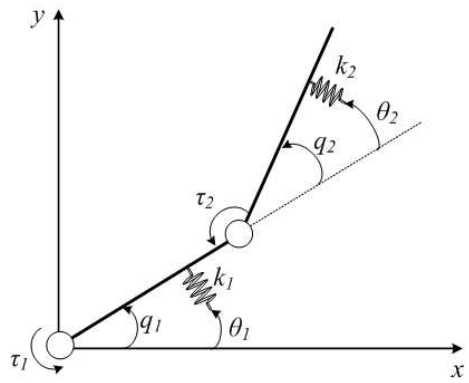


Fig. 2. Serial Planar Two-link Manipulator with flexible joints.

A dimensional analysis is considered in order to analyze the elastodynamics of the manipulator. In this way, the adimensional variable $\alpha \in [0,2]$ is introduces in order to define the links length; thus:

$$l1 = l_i(2-\alpha) \tag{7}$$
$$l2 = l_i\alpha \tag{8}$$

with l_i =0.25m.

Initially, the first natural frequency is computed by solving the eigenproblem of Eq. (5). For this manipulator, the mass matrix $\mathbf{M}_T(q_2)$ depends on the second joint q_2. Thus, the Fig. 3 presents how the first natural frequency ω_1 varies as a function of q_2. One can observe that the first natural frequency increases according to q_2 because, for these cases, the total inertia of the manipulator gets higher. Moreover, the results indicate that the first natural frequency increases for α=1.0, i.e., the links have the same length. Thus the total inertia of the manipulator gets higher, increasing the natural frequency.

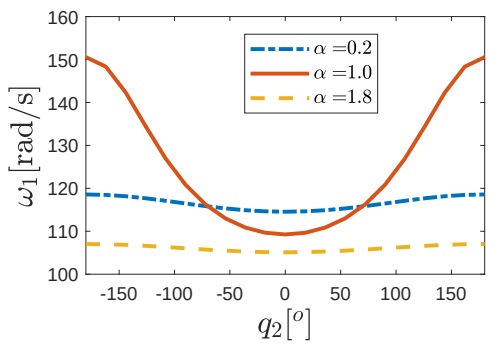


Fig. 3. Eigenvalue: Serial Planar Two-link Manipulator.

The global elastodynamic index (GIE) is computed for several links length definitions $\alpha \in [0,2]$ according to Eq. (8). For this case, the GEI corresponds to the mean of the curves presented in Fig. 3 because the elastodynamic performance only depends on q_2. The maximum global elastodynamic index (GIE) is obtained for $\alpha = 1$ because the inertia is maximized. For the cases in which one link is greater than the other, the GEI decreases. For the optimal design procedures, a value of $\alpha \geq 1.5$ in order to minimize the first natural frequency of the manipulator. Nevertheless, this selection should also attend to additional criteria based on kinematic and dynamic requirements.

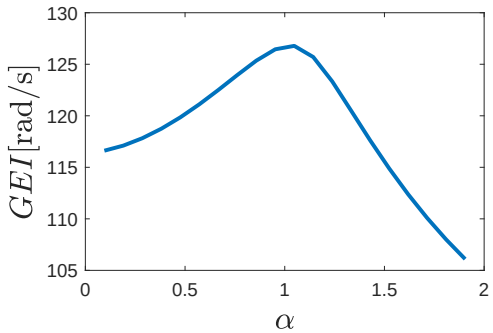


Fig. 4. Global Elastodynamic Index (GEI): Serial Planar Two-link Manipulator.

B. 5R Planar Parallel Manipulator

The 2-DOF planar parallel robot has two kinematic chains. Each kinematic chain includes an active or actuated joint, denoted as A_i, a passive or free joint, denoted as B_i for $i = 1,2$, and two rigid links. The geometry of planar parallel robot is defined according to Fig. 5(a). The active joints are rotational and they are located at the Cartesian coordinates $(x_{a_i}, 0)^T$, for $i = 1,2$. Additionally, the flexibility is considered in active joint, this flexibility is modeled as an elastic torsional spring k_i which couples the rotors of the motor with the links. In this contribution the robot is considered as symmetric, thus the length of the links are defined by $\bar{r}_1$, $\bar{r}_2$. The end effector of the mechanism is located at $\mathbf{p}$ where its position is defined by the Cartesian coordinates $(\bar{x}_p, \bar{y}_p)$. Additionally, the fixed reference frame O is defined in the middle of A_1A_2. The gravity acceleration acts perpendicular to the plane xy in which the mechanism works.

For the symmetrical parallel mechanism, the link lengths are stated by $\bar{r}_1$, $\bar{r}_2$ and $\bar{r}_3$ (see Fig. 5(a)). The link lengths are defined between zero and infinite. nevertheless, this dimensional length is eliminated in order to establish the design space as was presented in previous contributions [19]. In accordance with that, it is defined as $D = (\bar{r}_1 + \bar{r}_2 + \bar{r}_3)/3$, thus, the three non-dimensional parameters (r_i, for $i = 1,2,3$) are defined by:

$$r_1 = \bar{r}_1/D \qquad r_2 = \bar{r}_2/D \qquad r_3 = \bar{r}_3/D \tag{9}$$

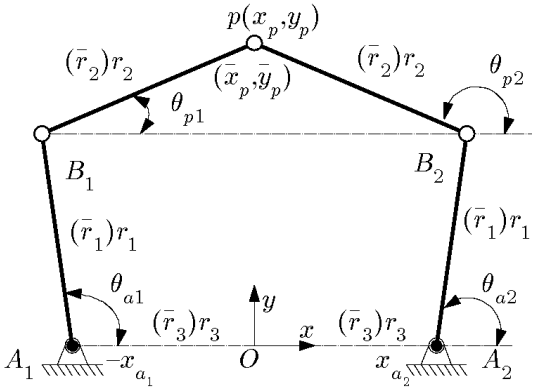
Therefore:

$$r_1 + r_2 + r_3 = 3 \tag{10}$$

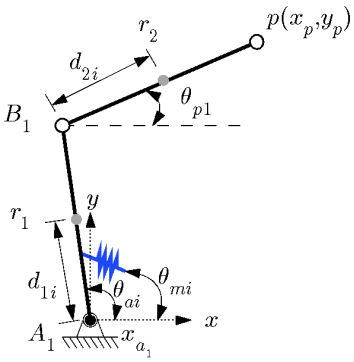
Moreover, the end effector coordinates are also non-dimensionalized as follows:

$$x_p = \bar{x}_p/D \qquad y_p = \bar{y}_p/D \tag{11}$$

The elastodynamic performance is evaluated as function of the link lengths in order to determine the link dimensions



(a)



(b)

Fig. 5. The 2-DOF Planar Parallel Robot

that maximizes the first natural frequency of the mechanism. Therefore, the design space assesses all the possible combinations of the links dimensions and their correspondent performance indices [20], [21].

The non-dimensional length of links were previously described by eqs. (9) and (10). Theoretically, $0 < r_i < 3$, nevertheless, the non-dimensional lengths of the links are constrained in order to avoid a failure of mechanism assembly [20], thus:

$$0 < r_1, r_2 < 3 \qquad \text{and} \qquad 0 \leq r_3 \leq 1.5 \tag{12}$$

Based on the Eq. (10) and the constraints of Eq. (12), the design space is defined as a trapezoid shown in Fig. 6.

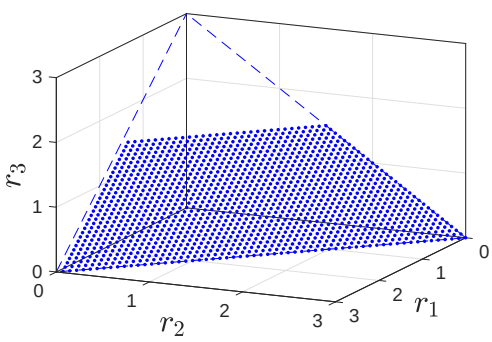


Fig. 6. Design space of the parallel symmetrical robot.

In the design process, the Maximum Inscribed Circle (MIC) is an index useful to evaluate the flatness of the usable workspace, the MIC is inscribed within the usable workspace and it is tangent with singular loci [5]. The Maximum Inscribed Workspace (MIW) is defined as the workspace bounded by the MIC. The MIC is characterized by the expression:

$$x^2 + (y - y_{MIC})^2 = r_{MIC}^2 \tag{13}$$

where r_{MIC} is the radius and $(0, y_{MIC})$ is the center. For the cases when $r_1 + r_3 < r_2$, the MIC is defined by

$$\begin{aligned} r_{MIC} &= (r_1 + r_2 - |r_1 - r_2|)/2 \\ y_{MIC} &= \sqrt{(r_1 + r_2 + |r_1 - r_2|)^2/4 - r_3^2} \end{aligned} \tag{14}$$

For the cases when $r_1 + r_3 > r_2$, the radius and center of the MIC are defined by:

$$\begin{aligned} r_{MIC} &= |y_{MIC}| - y_{col} \\ y_{MIC} &= \frac{(r_1 + r_2 + y_{col})^2 - r_3^2}{2(r_1 + r_2 + y_{col})} \end{aligned} \tag{15}$$

with $y_{col} = \sqrt{r_1^2 - (r_2 - r_3)^2}$. Figure 7 shows the workspace and the usable and MIW for the non-dimensional link lengths $r_1 = 1.2$, $r_2 = 1.0$ and $r_3 = 0.8$.

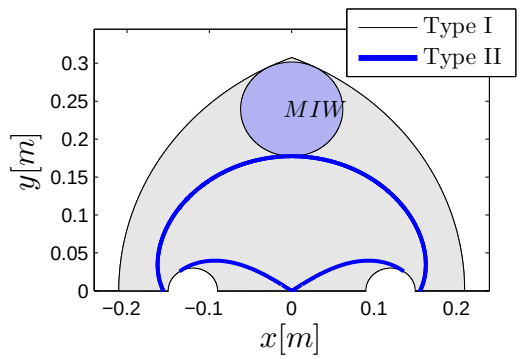


Fig. 7. Maximum Inscribed Workspace.

The parallel manipulator with flexible joints is considered; specifically, Fig. 5(b) shows the leg of the manipulator with flexibility in the active joints. According to the dynamic equation of Eq. (1), the rigid link dynamic model is completely described in [6], [22], [23]; thus, $\mathbf{M}_L$ is defined. Moreover, the stiffness matrix $\mathbf{K}_T = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & k_1 & 0 \\ 0 & -k_2 & 0 & k_2 \end{bmatrix}$ was defined in [18]. The geometric and dynamic proprieties of the maipulator are defined in tha appendix.

The non-dimensional parameters adopted in this contribution are m_{1i}=1.2, m_{2i}=0.8; this implies that the mass of the first link is 20 percent greater than the mass of the second link. In addition, k_{1i}=1 and k_{2i}=1, this means that the stiffness of active joints is equal. Additionally, the relationship between the non-dimensional first link mass and

the rotor mass is based on the parameters of the appendix: $j_i = 0.5m_1$.

Initially, the local elastodynamic performance is evaluated at each single posture by considering the specific set of non-dimensional link lengths r_1=1.2, r_2=1.0 and r_3=0.8. Figure 8 shows the first natural frequency ω_1 for each single posture within the usable workspace. One can observe that the maximum and minimum values of local first natural frequency $\max(\omega(x_p, y_p))$ and $\min(\omega(x_p, y_p))$ are located within the MIW, i.e., a wide range of variation of ω_1 that comprises the elastodynamic performance is inscribed within the maximum inscribed workspace (MIW). Consequently, the global elastodynamic index can be evaluated by considering the maximum inscribed workspace (MIW) as the workspace region w in the expression of Eq. (6).

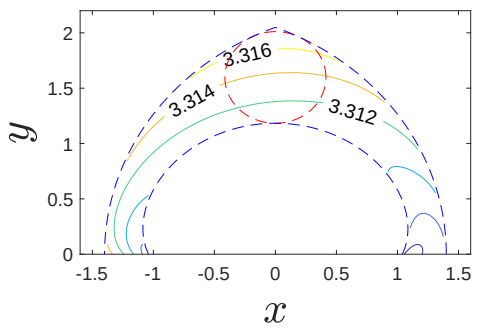
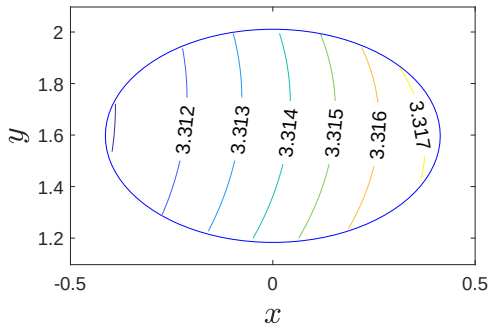


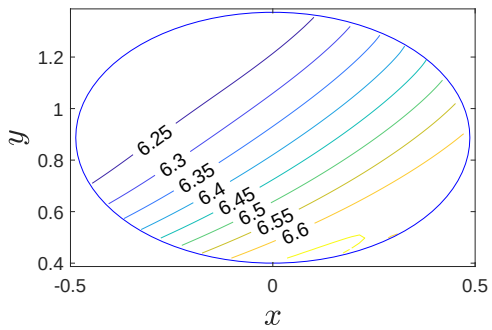
Fig. 8. First natural frequency over the workspace.

Then, the local elastodynamic performance is evaluated over the maximum inscribed workspace (MIW) for three cases that correspond to two definitions of the length of the links: *i*) r_1=1.2, r_2=1.0 and r_3=0.8. (see Fig. 9(a)), and *ii*) r_1=0.5, r_2=1.5 and r_3=1.0. (see Fig. 9(b)) and *iii*) r_1=1.7, r_2=0.5, r_3=0.8. One can observe that the first natural frequency is almost twice for the second case (Fig. 9(b)) compared to the first case (Fig. 9(a)). Moreover, the MIW is larger for the second case than the first one, indicating that the entire workspace is also larger. Consequently, the geometric parameters significantly influence the elastodynamic performance since they directly affect the inertia matrix of the manipulator. The MIW and the natural frequencies are the smallest for the third case; thus, the third case presents the worse case.

Figure 10 shows the global elastodynamic index as a function of the links lengths that was computed by using the expression of eq. (6). One can observe that the elastodynamic performance depends principally on r_1 and r_2. Moreover, it is observed in the GEI of Fig. 10 shows a rather small dependence on the on r_3. Consequently, the GEI depends on the r_1 and r_2 since the length of this link is directly proportional to the total inertia matrix of the robot $\mathbf{M}_T$ defined in eq. (4). Hence, based on the definition of Eqs. (10) and (9), the first natural frequency is maximized by selecting links lengths that follow the relation $r_1 \leq r_2$, this region in located in the left border of the design space showed in



(a)



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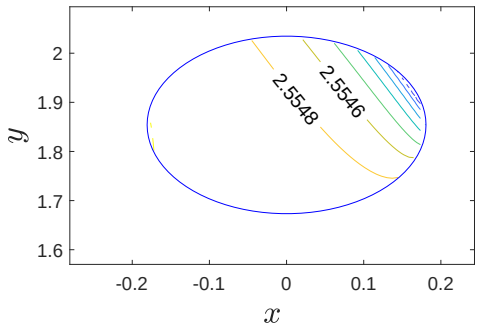

(c)

Fig. 9. ω_1 over the maximum inscrived workspace (*MIW*).

Fig. 10.

V. CONCLUSIONS

This paper proposed a novel design criterion based on the elastodynamic model of manipulators with flexible joints. The elastodynamic performance was evaluated considering a global performance measure that computes the first natural frequency solving the eigenvalue problem. The Global Elastodynamic Index (GEI) was computed for a serial two-link planar manipulator and a 5*R* parallel manipulator with flexible joints. This analysis considered how the elastodynamics varies according to the geometric parameters.

The numerical results present the behavior of the elastodynamic performance as a function of the geometric parameters and the manipulator pose. The elastodynamic criterion includes the manipulator pose by considering the mean first

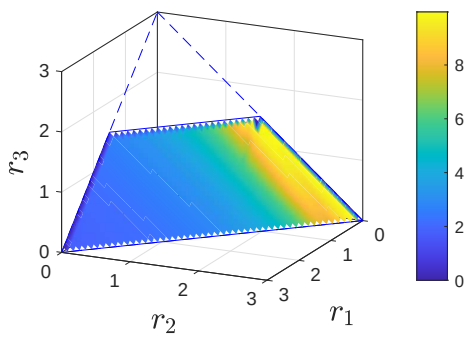

Fig. 10. Global elastodynamic index.

natural frequency over a considered region of the Cartesian space. Consequently, the global elastodynamic index permits quantifying the behavior of the first natural frequency not only at a single pose but over a region of the workspace.

This contribution is an initial surrogate toward the optimal design of the parallel robot with flexible joints. Future works will encompass the optimal design of parallel robots with flexible joints by considering the global elastodynamic index presented (GEI) in this contribution.

APPENDIX

The parameters of the planar manipulators with flexible joints are presented in Table I.

TABLE I

PARAMETERS OF FLEXIBLE MANIPULATOR.

Parameter of first motor/link	Symbol	Value
Mass of the link	m_1 (Kg)	0.5
Elasticity of the joint	k_1 (N/m)	10×10^3
Length of the link	l_1 (m)	0.25
Mass of the motor	$m_{r1}(kg)$	0.5
Reduction ratio	n_1	100
Rotor Inertia of the motor	$I_{r1_{zz}}$ (kg m^2)	14×10^{-6}
Mass of the link	m_2 (Kg)	0.5
Elasticity of the joint	k_2 (N/m)	10×10^3
Length of the link	l_2 (m)	0.25
Mass of the motor	$m_{r2}(kg)$	0.5
Reduction ratio	n_2	100
Rotor Inertia of the motor	$I_{r2_{zz}}$ (kg m^2)	14×10^{-6}

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